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UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

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QUALITATIVE KINEMATICS OF LINKAGES

by

Hyun-Kyung Kim

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Hyun-Kyung Kim

Qualitative Reasoning Group
Department of Computer Science
Beckman Institute, University of Illinois
405 North Mathews Avenue
Urbana, Illinois, 61801, USA
email: hkim@cs.uiuc.edu

Abstract

An important aspect of qualitative spatial reasoning is understanding mechanisms. This paper presents a qualitative analysis of motion for mechanical linkages. In particular, we describe how to analyze the behavior of a mechanism which has a movable axis. The basic idea of our approach is to represent *relative motions* of the link in terms of quadrants (qualitatively representing the direction relative to a global reference frame) and *relative inclinations* relative to the x-axis. Using this representation, we can derive all the possible motions of a system of linkages. This idea has been implemented and tested on several examples.

1 Introduction

In order to capture commonsense understanding of the physical world, it is crucial to understand how dynamics and geometries interacts. Without spatial reasoning, dynamics cannot explain the physical world. For example, an object's response to a force depends on exactly where that force is applied. Thus, to have a complete theory for reasoning about the everyday physical world requires a theory of spatial reasoning as well as dynamics.

Since the general spatial reasoning problem is too unconstrained to be tractable, recently qualitative spatial reasoning research has focused on mechanical systems [6,7]. Describing and predicting the behaviors of mechanical systems is important for several reasons. Motions are constrained by the shapes of parts and the way in which each part is connected. Furthermore, compared to other spatial problems: (1) people are good at reasoning about mechanisms (unlike, say, laying out a building without external aids), (2) the behaviors of mechanisms are in general highly constrained, and (3) it has practical application in mechanical design and analysis.

We are extending the framework of [5,7]. Previous techniques focused on fixed axis mechanisms [6,7], and thus could not analyze many important systems. For example, they cannot analyze the slider-crank mechanism which transmits the vertical motion of the piston to the rotation of the crankshaft in an automobile.

This paper focuses on *qualitative kinematics* which can be considered as a subarea of qualitative mechanics. Mechanism kinematics focuses on motion without considering force. The particular class of mechanisms we consider are 2D linkages. A linkage is defined as "an assemblage of rigid bodies or links in which each link is connected with at least two other links by either pin connections or sliding blocks" [1]. If the linkage is not sufficiently constrained, it is not considered to be a mechanism. A four-bar mechanism is four-bar linkage with one link fixed, so that its motions become relatively constrained. The analysis of four-bar mechanisms is important since it is the most commonly used linkage and it provides a basis for more complicated linkages. Its motion is constrained to have a single degree of freedom given a driver. In general, if we have an n-bar mechanism with m degrees of freedom and m links are driven, every remaining link has a corresponding and predictable motion because of constraints. Our analysis can be generalized to more complicated linkages by propagating constraints across pairs of adjacent links. If the n-bar linkage is not constrained enough to have ambiguous behaviors, our analysis captures the possible motions for each link in a particular positions of the links.

We assume as input the connection between the links and their relative lengths. The result of the analysis is the qualitative states and state transitions which explain the possible behaviors of the linkage. The heart of our analysis is the representation of angles in terms of quadrants and inclination. We demonstrate by implemented examples how this representation allows us to predict the behavior of systems of linkages.

Section 2 presents the theory for representing the behaviors of linkages in qualitative and geometric terms. Instead of using exact angles, the inequalities between the inclinations are used to reason about angles. Section 3 explains an algorithm which qualitatively simulates the behaviors of linkages, illustrated using several four-bar linkages. In section 4 we summarize our results and discuss possible extensions.

Figure 1: Two example configurations with different inclinations

2 A Qualitative Theory of Linkages

We want our qualitative states to explain the behaviors of a mechanism given minimal geometric information. We generalize the qualitative vector used in [7] with inequalities between the inclinations to reason about angular changes during the rotation of the links. We represent the position, motion, and constraints in relative terms. Each state consists of a static component representing the orientations of each link, and a dynamic component representing the motions.

2.1 Symbolic Direction

In spatial reasoning, the concept of direction is essential to describe position, force, and motion. In our theory, direction (or orientation) is represented by *sense* and *inclination*. Sense indicates where the direction is pointing relative to some frame of reference. For example, we can say one end-point of a link has a sense of "upward to the right" relative to the other end-point. We assume a single global reference frame. This reference frame can be translated but not rotated. Sense is represented by a qualitative vector [7]. In our two-dimensional space representation, the first component of a qualitative vector represents the qualitative direction along the x-axis and the second one is for the y-axis. To represent the x-axis direction, we use "+" for "right" and "−" for "left" and "0" for center. For the y-axis, "+" is used for "up" and "−" for "down" and "0" for center.

Sense itself is not enough to represent the direction. For example, Figure 1a and 1b shows two different kinematic states even though their links have the same senses. Inclination is used to disambiguate these states.

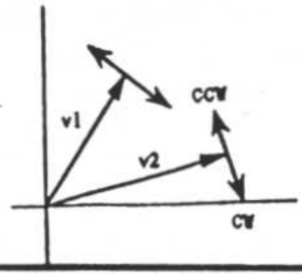
Definition 1 (Relative-Angle) *Relative-Angle*($v1$,$v2$) is the angle measured counter-clockwise from qualitative vector $v1$ to $v2$.

Definition 2 (Inclination) If `Relative-Angle`($(+0)$,v) is less than π, then *Inclination*(v) is equal to `Relative-Angle`($(+0)$,v). Otherwise, if `Relative-Angle`($(+0)$,v) is greater than or equal to π, *Inclination*(v) is equal to `Relative-Angle`($(+0)$,v) $- \pi$.

Inclination represents how much the link is inclined relative to x-axis. For our analysis, the exact angle is not needed; instead inequalities between link angles is sufficient. Thus we can qualitatively distinguish the states in Figure 1 since in (a) `Inclination(L3)` > `Inclination(L4)`, and in (b) `Inclination(L4)` > `Inclination(L3)`.

Definition 3 (Inversion) *Inversion*(v) is the qualitative vector v rotated by 180 degrees. `Inclination(Inversion(`v`))` is same as `Inclination(`v`)`.

Figure 2: Directions of `Rotate-90` of two links with similar inclinations



Definition 4 (Acute Inclination) A vector v has *acute inclination* if `Inclination`(v) is acute.

Thus, a link has a sense of (++) or (−−), exactly when it has acute inclination.

Definition 5 (Obtuse Inclination) A vector v has *obtuse inclination* if the `Inclination` (v) is obtuse.

Thus, a link has a sense of (−+) or (+−), exactly when it has obtuse inclination.

Definition 6 (Similar Inclinations) Two vectors have *similar inclinations* if both have acute inclinations or both have obtuse inclinations.

If two links have similar inclinations, they can be distinguished by three inequalities of inclinations between them (>, <, =).

We now generalize Open-Half-Plane and vector rotations used in [7] with inclinations.

Definition 7 (Rotate-90) *Rotate-90*(v,d) is the vector which is perpendicular to v by rotation in direction d. Rotational direction is represented as "+" for counter-clockwise and "−" for clockwise rotation.

Law 1 (Rotation-90) If the vectors $v1$ and $v2$ have similar inclinations and `Inclination` ($v1$) > `Inclination`($v2$), then inclination of `Rotate-90`($v1$,d) is greater than that of `Rotate-90`($v2$,d).

Figure 2

Definition 8 (Open-Half-Plane) *Open-Half-Plane*(v) are the vectors whose vector dot product with v is "+".

Vector having the same sense as `Rotate-90`(v,−) and whose inclination is greater than that of `Rotate-90`(v,−) are also included in `Open-Half-Plane`(v) even though its dot product is qualitatively ambiguous. Similarly, vectors having the same sense as `Rotate-90`(v,+) and whose inclination is less than that of `Rotate-90`(v,+) are included in `Open-Half-Plane`(v).

Since a global reference frame is used for representing directions, the angular relationship between the directions of two vectors can be defined as follows:

Definition 9 (CW) *CW*($v1$,$v2$) is true if `Relative-Angle`($v1$,$v2$) is greater than zero and less than π.

Definition 10 (CCW) *CCW*($v1$,$v2$) is true if `Relative-Angle`($v1$,$v2$) is greater than π.

2.2 Relative Position for Links

For each link, we represent the orientation of that link as the position of one end relative to the other. If the other end-point is fixed to the ground, that relative position represents the absolute position. In general, the relative position can be defined for any two points:

Definition 11 (Relative-Position) *Relative-Position*(P1,P2) is the qualitative vector which represents the direction from point P2 to point P1.

Consider a link with end-points P1 and P2, which are pin-jointed to other adjacent links. To get the possible orientations of the link, imagine P1 is fixed and P2 rotates about P1. The direction of the link is represented by `Relative-Position(P2,P1)`.

Since each link is connected to the other by either a pin joint or a sliding block carrying a pin joint, connected links can only rotate relative to each other. Quadrants are used to represent the changes of angular positions. Since a qualitative vector in each quadrant and at the quadrant boundaries all have unique senses, there are 8 possible qualitative angular positions for a link.

Information about the relative position can be propagated using transitivity:

Law 2 (Transitivity of Relative-Position) For any points P1,P2, and P3,`Relative-Position(P1,P3)` is computed by adding given values `Relative-Position(P1,P2)` and `Relative-Position(P2,P3)`.

2.3 Relative Motions

To represent the direction of motion, we also use qualitative direction. Just as the orientation of a link is represented by the relative position of two end points of that link, the motion of a link is represented by the relative motion of its two end points.

Definition 12 (Rotation) *Rotation*(P1,P2) represents the rotational direction of the line connecting P1 and P2. Possible values are + (CCW), − (CW), and 0 (no motion).

The rotation of a link with end-points P1 and P2 can be represented by `Rotation(P1,P2)`.

Definition 13 (Relative-Motion) *Relative-Motion* (P1,P2) is the qualitative vector which represents the direction of the motion of point P1 relative to point P2.

Given vector v `Relative-Position(P2,P1)` for link-i, its the motion is represented by `Relative-Motion(P2,P1)`. The possible values of `Relative-Motion(P2,P1)` are Rotate-90(v,+), `Rotate-90`(v,−) and (00) for no motion. We call these non-zero relative motions *CCW rotation vector* and *CW rotation vector*, respectively. Figure 3 shows the 8 angular positions and corresponding unique CCW rotation vectors.

Figure 4 shows the orientations of the clockwise and counter-clockwise rotation of L2. Since `Relative-Position(B, A)` is (++), `Relative-Motion(B,A)` can be (+−) for clockwise rotation or (−+) for counter-clockwise or (00) for no motion. In this case the directions of these motions are absolute since pin joint A is fixed to the ground.

When two links have similar inclinations, their rotation vectors will also have similar inclinations. Let $v1$ and $v2$ represent the directions of `link1` and `link2` respectively and have similar inclinations. From the Law of `Rotation-90`, it follows that if `Inclination`($v1$) > `Inclination`($v2$), then inclination of the motion vector of $v1$ is greater than that of $v2$. Relative-Motion, like Relative-Position is transitive:

Law 3 (Transitivity of Relative-Motion) `Relative-Motion(P1,P3)` is computed by adding given values `Relative-Motion(P1,P2)` and `Relative-Motion(P2,P3)`.

Figure 3: Angular positions and motion vectors in quadrants

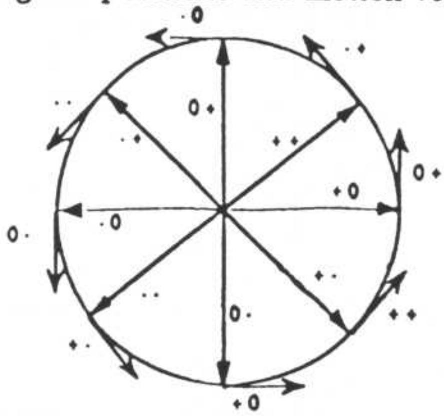
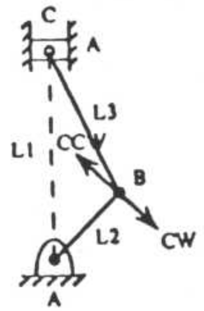


Figure 4: Slider-crank mechanism



2.4 Coupled Vectors

To compute the states from all the possible relative positions and motions of every pair of links, an adaptation of the *method of coupled vectors* is used. The idea of this traditional method is to express a vector as the sum of two vectors. We now apply it to our qualitative representations.

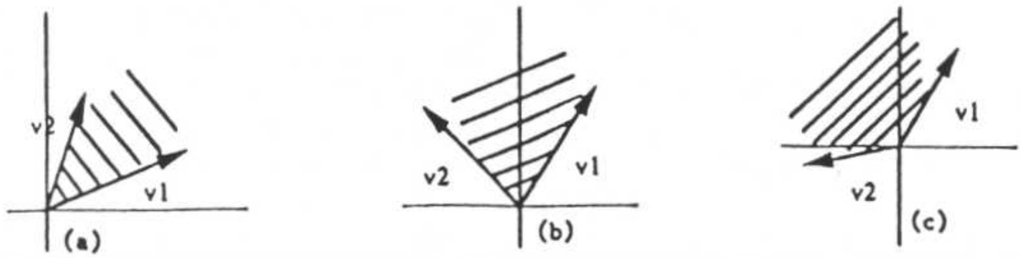
Consider the slider-crank mechanism of Figure 4. By the law of `Transitivity of Relative-Position`, we can compute `Relative-Position(C,A)` by summing the `Relative -Position(B,A)` and `Relative-Position(C,B)` since `L2` and `L3` are connected by the pin joint `B`. Since `Relative-Position(C,A)` is constrained to move only in vertical direction, the illegal configurations of `L2` and `L3` are filtered out. Similarly, `Relative-motion(C,B)` can be computed by this method according to the the law of `Transitivity of Relative-Motion`.

We extend the qualitative vector arithmetic with relative angles and relative lengths to enable us to filter more ambiguities. The sense of the sum of two vectors is the qualitative sum of their senses. When we add two qualitative vectors, the sense is computed by adding their senses. The inclination of the summed vector is constrained by the inclinations and lengths of the two vectors.

Law 4: If CW($v1$,$v2$) and $v = v1 + v2$, then CCW(v,$v1$) and CW(v,$v2$) are true. If $v1$ and $v2$ have the same inclinations, then v has the same inclinations.

Figure 5 shows some examples. The possible summation of $v1$ and $v2$ is hatched. In 5a,

Figure 5: Vector Arithmetic



v has the same sense as $v1$ and $v2$ but `Inclination(`$v1$`)` is between the inclinations of $v1$ and $v2$. In 5b, v has the sense whose y-direction is + and its inclination is constrained by $v1$ and $v2$. In 5c, when we add (++) and (−−), our answer can be any of the qualitative vectors. Such cases of ambiguous addition provide no useful information. Our vector arithmetic avoids such ambiguities using relative angles and lengths. The summation of $v1$ and $v2$ of 5c can not be (+0), (+−), (0−), (++) with less inclination than that of $v1$, or (−−) with greater inclination than that of $v2$. If we have information about the inequalities between the magnitude of vectors, more constraint can be imposed. For example, if we know magnitude of $v1$ is greater than that of $v2$ in 5c, then the y value of $v1 + v2$ should be "+".

If a vector is represented as the sum of two vectors, the change in the angle and magnitude (i.e., the change in direction of the motion) can also be expressed as sums. Since the `Relative-Position` for the link in a linkage cannot change magnitude, the computation of the motion of the summed vector of two links can be simplified. In the following section we give our method for using this simplification to maximally exploit the qualitative geometry to find angular constraint between two connected links.

Computing absolute motions or positions in a movable mechanism is complicated unless the links are directly connected to the ground. Our method find the absolute motions and positions by summing the relative ones of adjacent links.

2.5 Triangle Constraints

To produce consistent global kinematic states from the local information about the position of each link, we need geometric constraints in addition to the vector arithmetic. The following laws have rational interpretations in qualitative terms and are used to filter out illegal kinematic states of linkages given the relative lengths of the links.

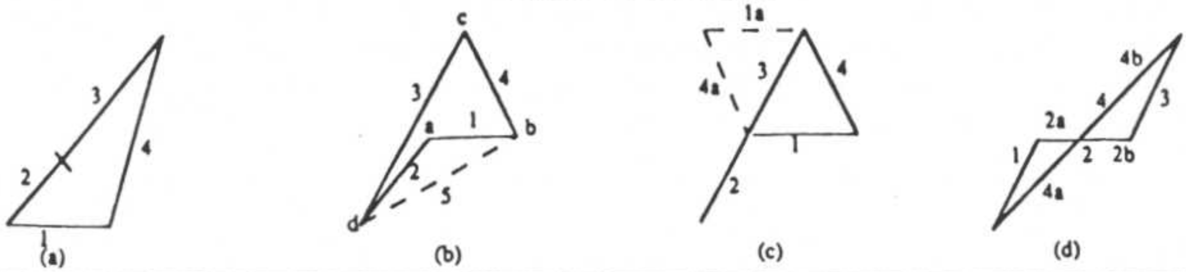
Law 5: In a triangle, the lengths of any two sides is longer than the remaining side.

Law 6: In the case of a right triangle, the hypotenuse is always longer than either leg. Similarly for an obtuse triangle, the side opposite the obtuse angle is longer than either side adjacent to the obtuse angle(Figure 6a).

Law 7: Given four-sided polygon 1-2-3-4 with a reflex angle **a**, we can draw a segment **5** from `d` to `b` (Figure 6b). By observation, we can see that for the smaller triangle 1-2-5 to be "inside" the larger triangle 3-4-5, $3 + 4 > 1 + 2$.

Law 8: Now consider a special case of the Law 7 where the side 2 is folded onto side 3 (Figure 6c). In agreement with the Law 7, $3 + 4 > 1 + 2$. Furthermore we can show $1 + 3 > 2 + 4$. First we consider a parallelogram where $1a = 1$ and $4a = 4$. By examining the

Figure 6: Triangles



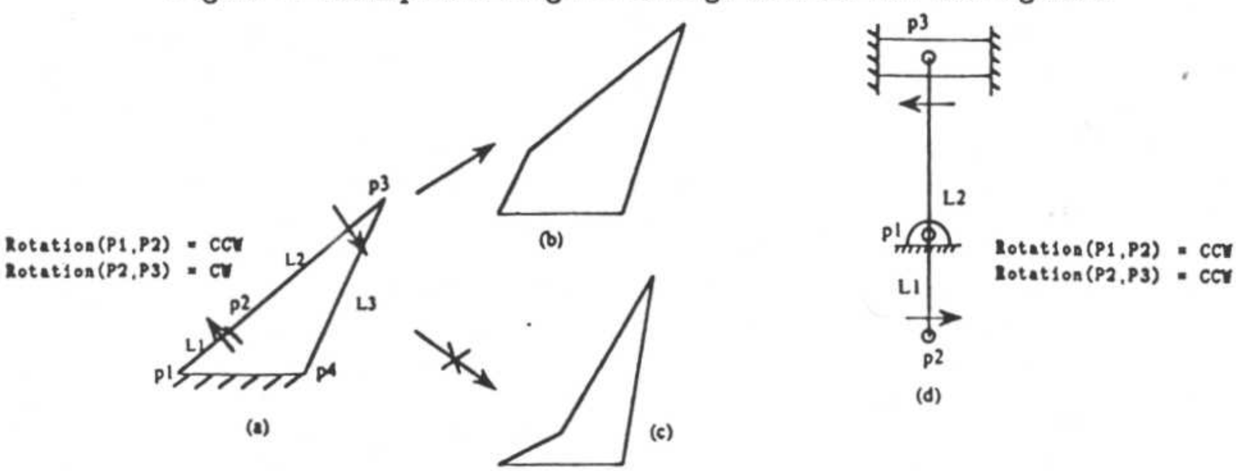
new 4-sided polygon, we can see **1a + 3 > 2 + 4a** which implies **1 + 3 > 2 + 4**.
Law 9: In the four-sided polygon 1-2-3-4 with 2 and 4 crossed, we can show 2 + 4 > 1 + 3 since 2a + 4a > 1 and 2b + 4b > 3 by Law 5 (Figure 6d).

These constraints help filter illegal kinematic states which vector arithmetic cannot. By these laws and the method of coupled vectors, we could find the every possible kinematic states for different kinds of four-bar mechanisms. To handle linkages which have more than 4 links, we may need more geometric constraints about polygons. But these constraints about triangles and four-sided polygons will be still applicable since adjacent links can be summed into one link.

2.6 Angle between two Links

To predict transitions between states, changes in relative positions and motions are not sufficient. In Figure 7, for example, reasoning about angular change between two links is needed.

Figure 7: Examples of Angular Change between two moving links



Angular relationships with other links can change if it is moving, as shown in Table 1. This table filters the illegal transition from the state of Figure 7a to 7c. But for the ambiguous cases (i.e., N1 - N6 in the table), we need to know the inequalities of the angular velocities between the links. In Figure 7d, for example, linear velocities of P1 and P3 relative to P2 are the same. Since the linear velocity of the end point of a rotating link is its length times its angular velocity and L2 is longer than L1, the angular velocity of L1 is greater than that of L2. Let $v1$ and $v2$ represent Relative-Position(P2,P1) and Relative-Position(P3,P2) respectively. We can eliminate Inclination($v1$) < Inclination($v2$) and Inclination($v1$) = Inclination($v2$) from the next possible state since it will be (CW $v1$ $v2$). Therefore, the only next possible state for Figure 7d has $v1 = (+-)$, $v2 = (-+)$, with Inclination($v1$) > Inclination($v2$).

Table 1: Changes in Angular Relationship for given Motions

For the sake of convenience, CW(A,B), CCW(A,B) are represented as A < B and A > B, respectively. When A and B have the same inclinations, A =o B indicates A and B have the opposite sense and A =s B indicates A and B have the same sense. Ds[A] and Dm[A] represent the sign and magnitude of the angular velocity of A.

For A > B:

Ds[A] \ Ds[B]	-1(CW)	0(No Motion)	1(CCW)
-1(CW)	N1	=s	=s
0	=o	>	=s
1(CCW)	=o	=o	N2

N1: Dm[A] > Dm[B] then =s, otherwise =o
N2: Dm[A] < Dm[B] then =o, otherwise =s

For A =s B:

Ds[A] \ Ds[B]	-1(CW)	0(No Motion)	1(CCW)
-1(CW)	N3	<	<
0	>	=s	<
1(CCW)	>	>	N4

N3: Dm[A] > Dm[B] then <
Dm[A] < Dm[B] then >
Dm[A] = Dm[B] then =s

N4: Dm[A] > Dm[B] then >
Dm[A] < Dm[B] then <
Dm[A] = Dm[B] then =s

For A =o B:

Ds[A] \ Ds[B]	-1(CW)	0(No Motion)	1(CCW)
-1(CW)	N5	>	>
0	<	=o	>
1(CCW)	<	<	N6

N5: Dm[A] > Dm[B] then >
Dm[A] < Dm[B] then <
Dm[A] = Dm[B] then =o

N6: Dm[A] > Dm[B] then <
Dm[A] < Dm[B] then >
Dm[A] = Dm[B] then =o

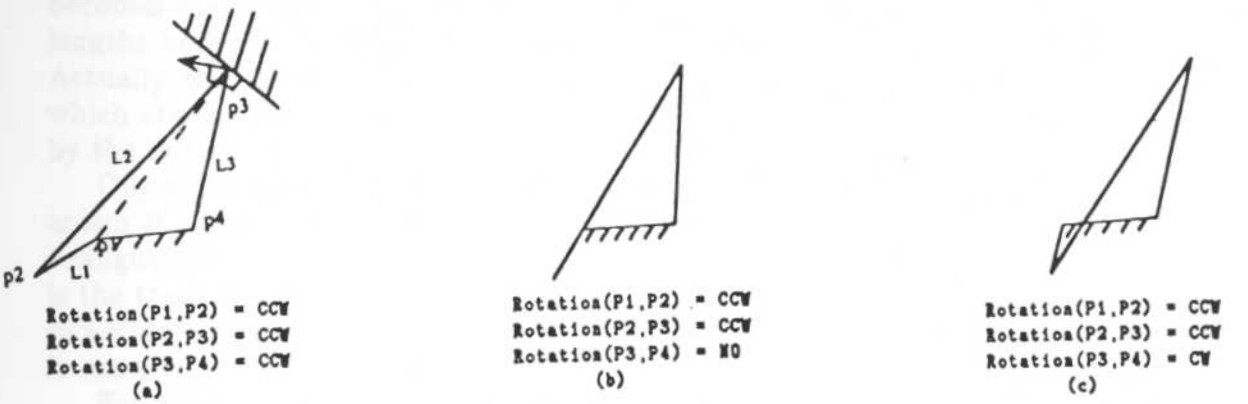
We may not always be able to determine inequalities between angular velocities. For those cases, another method finds transitions that exploit the available constraints, That method focuses on the angular change between two adjacent links. The analysis of this change is important since it can intuitively show how the shape of a linkage changes with motion (e.g., two adjacent links are approaching – may be folded, or departing – may be lined-up).

Law 10 (Angular Change) Suppose L1 and L2 are two adjacent links with different inclinations. If their end points are (P1 & P2) and (P2 & P3) respectively, then by the Law of Transitivity of Relative-Motion we can compute Relative-Motion(P3,P1) by adding Relative-Motion(P3,P2) and Relative-Motion(P2,P1). If Relative-Motion(P3, P1) is contained in Open-Half-Plane(Relative-Position(P3,P1)), then L1 and L2 are *departing*. If Relative-Motion(P3, P1) is contained in Open-Half-Plane(Inversion(Relative-Position(P3,P1))), then L1 and L2 are *approaching*. Otherwise, there is no angular change between L1 and L2.

Using this law for two adjacent links with different inclination yields the same result as Table 1. In the table, the next state having =s means two links are approaching while =o means they are departing.

We use approaching or departing instead for the decreasing or increasing of the relative angle since it is not clear whether the angle is the reflexive angle or obtuse one. Whether the angle is decreasing or increasing depends on the shape of the linkage. Our

Figure 8: State Transition in Crank-Rocker Mechanism



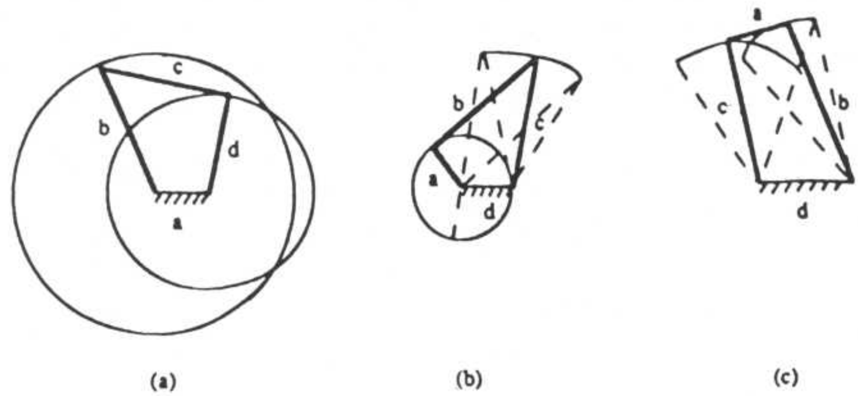
strategy is especially useful for finding additional constraints when two adjacent links move in same the direction and the angular velocities are not known. If `Relative-Motion(P3,P1)` is computed by adding the motions of two adjacent links, it may have several qualitative vectors rather than one. But this vector is constrained to have an unique value since every link in a mechanism is constrained to have unique motions in a given kinematic state.

Figure 8 shows three qualitative states of the crank-rocker mechanism that we explain later. Table 1 and the law predict the transition from Figure 8a to b and to c by reasoning about the angular relationship between L1 and L2. In Figure 8a, L1 and L2 are rotating CCW and L3 is rotating CCW. If we add the relative motions of L1 and L2 to compute `Relative-Motion(P3,P1)`, there are several possible vectors. But `Relative-Motion(P3,P1)` is constrained to be (−+) perpendicular to `Relative-Position(P3,P4)` since `Relative-Motion(P3,P4)` is constrained to move in that direction and both P1 and P4 are fixed to ground. If we consider triangle P1-P2-P3, the length of P1-P2 and P2-P3 are fixed but P1-P3 is decreasing, which implies angle P1-P2-P3 is decreasing. Since the angle P1-P2-P3 is decreasing, L1 and L2 are approaching each other. This implies that L1 and L2 may be folded at the next state (Figure 8b). In Figure 8b, both L1 and L2 rotate CCW. Since the angular velocity of L1 is greater than that of L2 in Figure 8b, the table for A =s B shows that the inclination of L2 will be less than that of L1 in the next state (Figure 8c). Since L1 and L2 are predicted to be departing from Figure 8c, they are not folded in the next state. If L1 is driven to rotate in the CW direction, the motions of the remaining links will be opposite. Then using the same analysis, we can predict that the direction of the transition will be reversed.

3 Envisioning Linkages

A qualitative state consists of the union of kinematic state and dynamic state. A kinematic state represents a particular configuration of links. The dynamic state describes the motions of the links. Motions can change the kinematic state of the link. If the linkage is a mechanism, each link has only one motion in a given kinematic state. Otherwise, each link may have more than one motion. Given a description of four-bar linkages in terms of relative lengths of links and the connections between them, the following algorithm computes an envisionment for that system.

Figure 9: Four-Bar Mechanisms



1. Compute the kinematic states for the given mechanism.
    (a) Generate all possible positions of the links in terms of relative positions of their two points.
    (b) Filter out all illegal combinations using the coupled vector method based on the extended vector arithmetic and triangle constraints.
2. Compute the dynamic state for each kinematic state.
    (a) Generate all motions for the given position of each link.
    (b) Filter out all illegal motions in given kinematic state using the coupled vector method based on the extended vector arithmetic and triangle constraints..
3. Compute all possible transition between the states.
    (a) For each state, find all kinematic transitions by using directions of motion and information associated with quadrant and angular change.
    (b) For each state, find all dynamic transitions.
    (c) Find all consistent combinations of dynamic and kinematic transitions.

Our LINKAGE program uses this algorithm to produce a qualitative analysis of linkages given minimal geometric information. It has analyzed slider-crank mechanisms, four-bar mechanisms, a mechanism where two slider-crank mechanisms are connected by sliding blocks, and linkages which were not not fully constrained.

Taking the four-bar mechanism as an example, there are three possible mechanisms to consider. In a four-bar mechanism, the link that is attached to ground may rotate completely or may oscillate. We call the former a crank and the latter a rocker. The three possible mechanisms are (1)two links attached to the ground rotate completely (*drag-link* mechanism) (2)one grounded link rotates and the other oscillates (*crank-rocker* mechanism) (3)both grounded links oscillate (*double-rocker* mechanism). In kinematics textbooks, these different mechanisms are categorized based on the relative lengths between the links. Suppose a,b,c, and d represents the links. Link a is the shortest link and b the longest and $a + b < c + d$. If a is fixed, this becomes the drag-link Figure 9a. If a is used as the driver, this becomes crank-rocker Figure 9b. If a connects the two grounded links, it becomes the double-rocker Figure 9c. When LINKAGE is given the connections and relative lengths between the links, it can produce the behaviors of the corresponding mechanism. Actually, the number of qualitative states for representing the behaviors of the mechanisms which are described above is less than the total states since the behaviors are determined by the initial setting of the linkage.

Our approach can exactly determine unique qualitative behaviors of an n-bar mechanism if the frame path of each link's end points can be qualitatively known in every configuration and if the relative motion between pairs of links are known. The frame path is the trace of the possible motion of the point relative to the fixed ground. For example, in a slider-crank mechanism, the point which is attached to the sliding block has a straight frame path.

For n-bar mechanism whose frame path of every end point cannot be determined, we may get more than one dynamic state for a given kinematic state since exact angles between links are sometimes needed to constrain the frame path of each link. For every kinematic state, there is a unique and corresponding dynamic state. However, since only relative inclinations are used to represent the kinematic state instead of the exact value of the angle between the links, every qualitatively consistent dynamic state is computed. For n-bar linkage which is not constrained enough, every possible dynamic state is also computed for each kinematic state.

4 Discussion

This paper presents a theory of linkages in two-dimensional space. Given any relative lengths and connections between the links, we can determine the consistent behaviors of a linkage.

Angle is represented by quadrants and relative inclination. When a link rotates, relative positions of one end-point to the other have 8 qualitatively different senses. If two links have different senses, it implies they have different angular positions. Since the sense is not enough to represent the angles of the two links which have the same or opposite sense, the relative inclination for the two links is also used. The orientations and motions of links are also expressed in our representation.

Our theory has been implemented in a program called LINKAGE. LINKAGE has been successfully tested on slider-crank mechanisms, four-bar mechanisms and a mechanism where two slider-crank mechanisms are connected by sliding blocks.

Our analysis extends to 2D motions even when not all parts of the linkages lie in the same plane. For example, in an actual engine the piston, block, crank-shaft, and the connecting rod do not lie in the same plane. However, since all parts move in parallel planes, 2D analysis suffices. Linkages which require three-dimensional space are very rare. For example, in [1] there is no such example. It is a reasonable first step to develop a 2D theory and then use this to attack the 3-dimensional problem. Consider a 3-dimensional linkage. Its possible configurations and motions can be analyzed in xy-plane, yz-plane, and zx-plane and then the analyses in each plane could be combined to give the positions or motions in three-dimensional space. Compared to the two-dimensional domain, the more ambiguities are expected, due to more ambiguity about the relative lengthes in each plane. In some cases, the ambiguities in relative lengthes in each plane cannot be avoidable without the exact numerical values for positions on that plane.

Since we are only concerned here with kinematics, we ignore changes due to forces. Integrating kinematics with the dynamics of physical processes is left as future work. What we hope to analyze eventually is a real system, such as internal combustion engine, which can only be explained by tightly integrating dynamics and kinematics. Our theory for analyzing linkages is the first step towards that goal.

5 Acknowledgements

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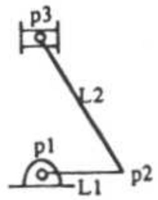
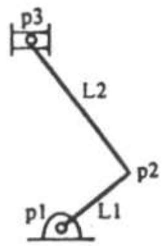
Appendix A : Slider-Crank Envisionment

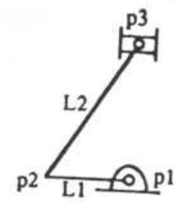
Each qualitative state consists of a kinematic state representing the orientation of each link, and a dynamic state representing the motions. The orientation of links are represented by qualitative vectors and inequalities of inclination. The qualitative vector corresponds to the signs of the numerical values in a right-handed Cartesian coordinate system. The motion is also represented by the signs of numerical values in a right-handed Cartesian coordinate system. A counter-clockwise rotation will be "+" and clockwise rotation will be "−". The following table shows the qualitative states and their transitions. Diagrams of the kinematic states are included to the right.

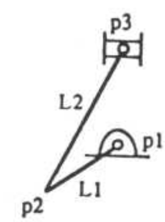
Kinematic State K.S. No	Senses P2/P1	Senses P3/P2	Incl L1/L2	Dynamic State L1/L2/L3	Next K.S. No	Next Dynamic state
1	+0	−+		+0+	2	000
				+0+	2	+ − +
				−0−	8	− − −
				−0−	8	000
				000	1	−0−
				000	1	+0+
2	++	−+		+ − +	3	000
				+ − +	3	+ − 0
				+ − +	2	000
				− + −	1	−0−
				− + −	1	000
				− + −	2	000
				000	2	+ − +
				000	2	− + −
3	0+	0+		+ − 0	4	000
				+ − 0	4	+ − −
				− + 0	2	− + −
				− + 0	2	000
				000	3	− + 0
				000	3	+ − 0

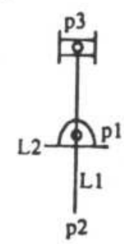


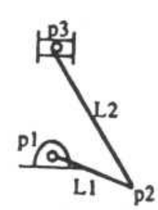


	Kinematic State			Dynamic	Next	
K.S.	Senses		Incl	State	K.S.	Dynamic
No	P2/P1	P3/P2	L1/L2	L1/L2/L3	No	state
4	−+	++		+ − −	5	000
				+ − −	5	+0−
				+ − −	4	000
				− + +	4	000
				− + +	3	− + 0
				− + +	3	000
				000	4	− + +
				000	4	+ − −
5	−0	++		+0−	6	000
				+0−	6	+ + −
				−0+	4	− + +
				−0+	4	000
				000	5	−0+
				000	5	+0−
6	−−	++	<	+ + −	7	000
				+ + −	7	+ + 0
				+ + −	6	000
				− − +	6	000
				− − +	5	−0+
				− − +	5	000
				000	6	− − +
				000	6	+ + −
7	0−	0+		+ + 0	8	000
				+ + 0	8	+ + +
				− − 0	6	− − +
				− − 0	6	000
				000	7	− − 0
				000	7	+ + 0
8	+−	−+	>	+ + +	1	000
				+ + +	1	+0+
				+ + +	8	000
				− − −	8	000
				− − −	7	− − 0
				− − −	7	000
				− − −	8	− − −
				000	8	+ + +









Appendix B : Drag-Link Envisionment

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
1	+0	−+	−−				+ + +	7 + + +
							− − −	80 − − −
2	+0	−+	0−				+ + +	9 + + +
							− − −	80 − − −
3	+0	−+	+−		>		+ + +	9 + + +
							− − −	82 − − −
4	+0	−−	++		<		+ + +	13 + + +
							− − −	86 − − −
5	+0	−−	0+				+ + +	15 + + +
							− − −	86 − − −
6	+0	−−	−+				+ + +	15 + + +
							− − −	88 − − −
7	++	−+	−−			>	+ + +	8 + + +
							+ + +	10 + + +
							− − −	1 − − −
							− − −	9 − − −
8	++	−+	0−				+ + +	9 + + +
							− − −	7 − − −
9	++	−+	+−		>		+ + +	12 + + +
							− − −	3 − − −
							− − −	2 − − −
							− − −	8 − − −
10	++	−0	−−				+ + +	17 + + +
							− − −	7 − − −
11	++	−0	0−				+ + +	20 + + +
							− − −	7 − − −

	Kinematic State						Dynamic	
		Senses			Incl		State	Next
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
12	++	−0	+−				+ + +	20 + + +
							− − −	9 − − −
13	++	−−	++	<	<	>	+ + +	14 + + +
							− − −	4 − − −
14	++	−−	0+	<			+ + +	15 + + +
							− − −	13 − − −
15	++	−−	−+	<			+ + +	16 + + +
							+ + +	22 + + +
							+ + +	21 + + +
							− − −	6 − − −
							− − −	5 − − −
							− − −	14 − − −
16	++	−−	−0	<			+ + +	18 + + +
							− − −	15 − − −
17	++	−−	−−	>	<	>	+ + +	19 + + +
							− − −	10 − − −
18	++	−−	−−	<	>	<	+ + +	23 + + +
							− − −	16 − − −
19	++	−−	0−	>			+ + +	20 + + +
							− − −	17 − − −
20	++	−−	+−	>			+ + +	27 + + +
							− − −	12 − − −
							− − −	11 − − −
							− − −	19 − − −
21	++	0−	−+				+ + +	24 + + +
							− − −	15 − − −
22	++	0−	−0				+ + +	26 + + +
							− − −	15 − − −
23	++	0−	−−			<	+ + +	26 + + +
							− − −	18 − − −
24	++	+−	−+		<		+ + +	25 + + +
							− − −	21 − − −
25	++	+−	−0				+ + +	26 + + +
							− − −	24 − − −

	Kinematic State						Dynamic State	Next
	Senses			Incl				
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
26	++	+−	−−			<	+ + +	28 + + +
							− − −	23 − − −
							− − −	22 − − −
							− − −	25 − − −
27	0+	−−	+−				+ + +	37 + + +
							− − −	20 − − −
28	0+	+−	−−				+ + +	43 + + +
							− − −	26 − − −
29	−+	+0	−−				+ + +	32 + + +
							− − −	43 − − −
30	−+	+0	0−				+ + +	34 + + +
							− − −	43 − − −
31	−+	+0	+−			<	+ + +	34 + + +
							− − −	46 − − −
32	−+	++	−−		<		+ + +	33 + + +
							− − −	29 − − −
33	−+	++	0−				+ + +	47 + + +
							− − −	32 − − −
34	−+	++	+−			<	+ + +	47 + + +
							− − −	33 − − −
							− − −	31 − − −
							− − −	30 − − −
35	−+	−−	+0				+ + +	36 + + +
							− − −	37 − − −
36	−+	−−	++		>		+ + +	39 + + +
							− − −	35 − − −
37	−+	−−	+−			<	+ + +	35 + + +
							+ + +	38 + + +
							+ + +	40 + + +
							− − −	27 − − −
38	−+	0−	+0				+ + +	42 + + +
							− − −	37 − − −
39	−+	0−	++				+ + +	42 + + +
							− − −	36 − − −

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
40	−+	0−	+−				+ + +	45 + + +
							− − −	37 − − −
41	−+	+−	+0	>			+ + +	42 + + +
							− − −	45 − − −
42	−+	+−	++	<			+ + +	48 + + +
							− − −	38 − − −
							− − −	39 − − −
							− − −	41 − − −
43	−+	+−	−−	<			+ + +	44 + + +
							+ + +	30 + + +
							+ + +	29 + + +
							− − −	28 − − −
44	−+	+−	0−	<			+ + +	46 + + +
							− − −	43 − − −
45	−+	+−	+−	>	<	>	+ + +	41 + + +
							− − −	40 − − −
46	−+	+−	+−	<	>	<	+ + +	31 + + +
							− − −	44 − − −
47	−0	++	+−				+ + +	57 + + +
							− − −	34 − − −
48	−0	++	++				+ + +	64 + + +
							− − −	42 − − −
49	−−	+0	++			>	+ + +	55 + + +
							+ + +	53 + + +
							− − −	64 − − −
							− − −	48 − − −
50	−−	+0	0+				+ + +	56 + + +
							− − −	64 − − −
51	−−	+0	−+				+ + +	56 + + +
							− − −	66 − − −
52	−−	++	+0	<			+ + +	54 + + +
							− − −	57 − − −
53	−−	++	++	>	<	<	+ + +	55 + + +
							− − −	49 − − −

	Kinematic State						Dynamic State	Next
	P2/P1	Senses P3/P2	P4/P3	L1/L2	Incl L2/L3	L3/L1	L1/L2/L3	
54	−−	++	++	<	>	<	+ + +	59 + + +
							− − −	52 − − −
55	−−	++	0+	>			+ + +	56 + + +
							− − −	53 − − −
56	−−	++	−+	>			+ + +	67 + + +
							− − −	55 − − −
							− − −	51 − − −
							− − −	50 − − −
57	−−	++	+−	<			+ + +	52 + + +
							+ + +	58 + + +
							+ + +	60 + + +
							− − −	47 − − −
58	−−	0+	+0				+ + +	62 + + +
							− − −	57 − − −
59	−−	0+	++			<	+ + +	62 + + +
							− − −	54 − − −
60	−−	0+	+−				+ + +	63 + + +
							− − −	57 − − −
61	−−	−+	+0				+ + +	62 + + +
							− − −	63 − − −
62	−−	−+	++			<	+ + +	68 + + +
							− − −	58 − − −
							− − −	59 − − −
							− − −	61 − − −
63	−−	−+	+−		<		+ + +	61 + + +
							− − −	60 − − −
64	−−	+−	++		>		+ + +	65 + + +
							+ + +	50 + + +
							+ + +	49 + + +
							− − −	48 − − −
65	−−	+−	0+				+ + +	66 + + +
							− − −	64 − − −
66	−−	+−	−+		>		+ + +	51 + + +
							− − −	65 − − −

	Kinematic State						Dynamic	
		Senses			Incl		State	Next
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
67	0−	++	−+				+ + +	69 + + +
							− − −	56 − − −
68	0−	−+	++				+ + +	75 + + +
							− − −	62 − − −
69	+−	++	−+			>	+ + +	70 + + +
							+ + +	73 + + +
							+ + +	72 + + +
							− − −	67 − − −
70	+−	++	−0				+ + +	71 + + +
							− − −	69 − − −
71	+−	++	−−		>		+ + +	74 + + +
							− − −	70 − − −
72	+−	0+	−+			>	+ + +	77 + + +
							− − −	69 − − −
73	+−	0+	−0				+ + +	80 + + +
							− − −	69 − − −
74	+−	0+	−−				+ + +	80 + + +
							− − −	71 − − −
75	+−	−+	++	<			+ + +	76 + + +
							+ + +	84 + + +
							+ + +	83 + + +
							− − −	68 − − −
76	+−	−+	0+	<			+ + +	78 + + +
							− − −	75 − − −
77	+−	−+	−+	>	<	>	+ + +	79 + + +
							− − −	72 − − −
78	+−	−+	−+	>	>	<	+ + +	85 + + +
							− − −	76 − − −
79	+−	−+	−0	>			+ + +	80 + + +
							− − −	77 − − −

Appendix C : Crank-Rocker Envisionment

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
1	+0	++	−−		<		+ − −	11 + − −
							− + +	85 − + +
2	+0	++	0−				+ − −	11 + − −
							− + +	87 − + +
							− + +	88 −0+
3	+0	++	+−				+ − −	17 + − −
							− + +	87 − + +
4	+0	+−	++				+ − −	28 + − −
							− + +	98 − + +
5	+0	+−	0+				+ − −	28 + − −
							− + +	104 − + +
6	+0	+−	−+		>		+ − −	30 + − −
							− + +	104 − + +
7	++	++	−−	>	<	<	+ + +	32 +0+
							+ + +	31 + + +
							+ + +	8 +0+
							− − −	8 −0−
8	++	++	−−	>	<	=	+0+	7 + + +
							−0−	9 − + −
9	++	++	−−	>	<	>	+ − +	32 +0+
							+ − +	12 + − +
							+ − +	8 +0+
							− + −	10 − + 0
							− + −	8 −0−
10	++	++	−−	=	<	>	+ − 0	9 + − +
							− + 0	11 − + +

	Kinematic State						Dynamic	
		Senses			Incl		State	Next
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
11	++	++	−−	<	<	>	+ − −	10 + − 0
							− + +	2 − + +
							− + +	1 − + +
							− + +	14 − + +
12	++	++	0−	>			+ − +	15 + − +
							− + −	9 − + −
13	++	++	0−	=			+ − 0	32 +0+
							+ − 0	12 + − +
							− + 0	14 − + +
14	++	++	0−	<			+ − −	11 + − −
							− + +	17 − + +
15	++	++	+−	>			+ − +	33 + − +
							− + −	16 − + 0
							− + −	12 − + −
							− + −	13 − + 0
16	++	++	+−	=			+ − 0	15 + − +
							− + 0	17 − + +
17	++	++	+−	<			+ − −	16 + − 0
							+ − −	14 + − −
							+ − −	13 + − 0
							− + +	3 − + +
18	++	−−	++	>	>	<	+ + +	34 + + +
							− − −	19 − − 0
19	++	−−	++	=	>	<	+ + 0	18 + + +
							− − 0	20 − − +
20	++	−−	++	<	>	<	+ + −	35 + + 0
							+ + −	19 + + 0
							+ + −	21 +0−
							+ + −	23 + + −
							+ + −	24 +0−
							− − +	21 −0+
21	++	−−	++	<	>	=	+0−	20 + + −
							−0+	22 − + +
22	++	−−	++	<	>	>	+ − −	21 +0−
							− + +	25 − + +

	Kinematic State						Dynamic	Next
	Senses			Incl			State	
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
23	++	0−	++			<	+ + −	26 + + −
							− − +	20 − − +
24	++	0−	++			=	+0−	23 + + −
							+0−	25 + − −
25	++	0−	++			>	+ − −	22 + − −
							− + +	28 − + +
26	++	+−	++			<	+ + −	36 + + −
							− − +	24 −0+
							− − +	23 − − +
							− − +	27 −0+
27	++	+−	++			=	+0−	26 + + −
							−0+	28 − + +
28	++	+−	++			>	+ − −	25 + − −
							− + +	5 − + +
							− + +	4 − + +
							− + +	29 − + +
							− + +	27 −0+
29	++	+−	0+				+ − −	28 +0−
							− + +	30 − + +
30	++	+−	−+		>		+ − −	37 +0−
							+ − −	29 + − −
							− + +	6 − + +
31	0+	++	−−		<		+ + +	38 + + +
							− − −	7 − − −
32	0+	++	0−				+0+	40 + + +
							−0−	9 − + −
33	0+	++	+−				+ − +	42 + − +
							− + −	15 − + −
34	0+	−−	++		>		+ + +	43 + + +
							− − −	18 − − −
35	0+	0−	++				+ + 0	45 + + +
							− − 0	20 − − +

	Kinematic State						Dynamic	Next
		Senses			Incl		State	
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
36	0+	+−	++				+ + −	47 + + −
							− − +	26 − − +
37	0+	+−	0+				+0−	47 + + −
							−0+	30 − − +
38	−+	++	−−		<		+ + +	56 + + +
							+ + +	55 + + +
							+ + +	39 + + +
							− − −	31 − − −
39	−+	++	0−				+ + +	40 + + +
							− − −	38 − − −
40	−+	++	+−			<	+ + +	57 + + +
							+ + +	41 + + +
							− − −	41 −0−
							− − −	39 − − −
							− − −	32 −0−
41	−+	++	+−			=	+0+	40 + + +
							+0+	42 + − +
							−0−	42 − + −
42	−+	++	+−			>	+ − +	41 +0+
							− + −	41 −0−
							− + −	33 − + −
							− + −	32 −0−
43	−+	−−	++		>		+ + +	44 + + +
							− − −	34 − − −
44	−+	0−	++				+ + +	45 + + +
							− − −	43 − − −
45	−+	+−	++	>			+ + +	59 + + +
							+ + +	58 + + +
							+ + +	48 + + +
							+ + +	45 + + +
							− − −	44 − − −
							− − −	46 − − −
							− − −	35 − − −
46	−+	+−	++	=			+ + 0	45 + + +
							− − 0	47 − − +

	Kinematic State						Dynamic	
		Senses			Incl		State	Next
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
47	−+	+−	++	<			++− −−+	46 ++0 37 −0+
48	−+	+−	0+	>			+++ −−−	50 +++ 45 −−−
49	−+	+−	0+	=			++0 −−0	48 +++ 37 −0+
50	−+	+−	−+	>	>	<	+++ −−− −−−	60 +++ 51 −−0 48 −−−
51	−+	+−	−+	=	>	<	++0 −−0	50 ++− 52 −−+
52	−+	+−	−+	<	>	<	++− ++− −−+	51 ++0 49 ++0 53 −0+
53	−+	+−	−+	<	>	=	+0− −0+	52 ++− 54 −++
54	−+	+−	−+	<	>	>	+−−	53 +0−
55	−0	++	−−		<		+++ −−−	65 +++ 38 −−−
56	−0	++	0−				+++ −−−	70 +++ 38 −−−
57	−0	++	+−				+++ −−−	70 +++ 40 −−−
58	−0	+−	++				+++ −−−	75 +++ 45 −−−
59	−0	+−	0+				+++ −−−	77 +++ 45 −−−
60	−0	+−	−+		>		+++ −−−	77 +++ 50 −−−
61	−−	++	−−	>	<	<	−++	62 −0+
62	−−	++	−−	>	<	=	+0− −0+	61 +−− 63 −−+

	Kinematic State						Dynamic	Next
		Senses			Incl		State	
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
63	−−	++	−−	>	<	>	+ + −	62 +0−
							− − +	66 − − 0
							− − +	64 − − 0
64	−−	++	−−	=	<	>	+ + 0	63 + + −
							− − 0	65 − − −
65	−−	++	−−	<	<	>	+ + +	66 + + 0
							+ + +	67 + + +
							+ + +	64 + + 0
							− − −	55 − − −
66	−−	++	0−	=			+ + 0	78 +0−
							− − 0	67 − − −
67	−−	++	0−	<			+ + +	70 + + +
							− − −	65 − − −
68	−−	++	+−	>			+ + −	79 + + −
							+ + −	78 +0−
							+ + −	80 + + 0
							+ + −	69 + + 0
							+ + −	66 + + 0
							− − +	69 − − 0
69	−−	++	+−	=			+ + 0	68 + + −
							− − 0	70 − − −
70	−−	++	+−	<			+ + +	80 + + 0
							+ + +	69 + + 0
							+ + +	71 + + +
							− − −	67 − − −
							− − −	57 − − −
							− − −	56 − − −
71	−−	0+	+−				+ + +	72 + + +
							− − −	70 − − −
72	−−	−+	+−		<		+ + +	81 + + +
							− − −	71 − − −
73	−−	−+	++			<	+ − +	83 +0+
							+ − +	82 + − +
							+ − +	74 +0+
							− + −	74 −0−

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
74	−−	−+	++			=	+0+	73 + − +
							−0−	75 − − −
75	−−	−+	++			>	+ + +	83 +0+
							− − −	74 −0−
							− − −	58 − − −
76	−−	−+	0+				+ + +	77 + + +
							− − −	75 − − −
77	−−	−+	−+		>		+ + +	84 + + +
							− − −	76 − − −
							− − −	60 − − −
							− − −	59 − − −
78	0−	++	0−				+0−	85 + − −
							−0+	68 − − +
79	0−	++	+−				+ + −	89 + + −
							− − +	68 − − +
80	0−	0+	+−				+ + 0	95 + + −
							− − 0	70 − − −
81	0−	−+	+−		<		+ + +	95 + + 0
							+ + +	97 + + +
							− − −	72 − − −
82	0−	−+	++				+ − +	100 + − +
							− + −	73 − + −
83	0−	−+	0+				+0+	106 + − +
							−0−	75 − − −
84	0−	−+	−+		>		+ + +	108 + + +
							− − −	77 − − −
85	+−	++	−−		<		+ − −	1 + − −
							− + +	86 − + +
							− + +	78 −0+
86	+−	++	0−				+ − −	1 + − −
							+ − −	85 + − −
							− + +	90 − + +

	Kinematic State						Dynamic	
		Senses			Incl		State	Next
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
87	+−	++	+−			<	+−−	3 +−−
							+−−	2 +−−
							+−−	88 +0−
							+−−	86 +−−
							−++	88 −0+
							−++	90 −++
							−++	91 −0+
88	+−	++	+−			=	+0−	87 +−−
							−0+	89 −−+
89	+−	++	+−			>	++−	88 +0−
							++−	92 ++−
							++−	91 +0−
							−−+	88 −0+
							−−+	79 −−+
90	+−	0+	+−			<	+−−	87 +−−
							−++	93 −++
91	+−	0+	+−			=	+0−	90 +−−
							−0+	92 −−+
							−0+	80 −−0
92	+−	0+	+−			>	++−	95 ++−
							−−+	89 −−+
93	+−	−+	+−	>	<	<	+−−	90 +−−
							−++	94 −0+
94	+−	−+	+−	>	<	=	+0−	93 +−−
							−0+	95 −−+
95	+−	−+	+−	>	<	>	++−	94 +0−
							−−+	91 −0+
							−−+	92 −−+
							−−+	94 −0+
							−−+	96 −−0
							−−+	80 −−0
96	+−	−+	+−	=	<	>	++0	95 ++−
							−−0	97 −−−
97	+−	−+	+−	<	<	>	+++	96 ++0
							−−−	80 −−0
							−−−	81 −−−

	Kinematic State						Dynamic State	Next
	Senses				Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
98	+−	+−	++	>			+ − −	4 + − −
							− + +	102 − + 0
							− + +	101 − + +
							− + +	99 − + 0
99	+−	+−	++	=			+ − 0	98 + − −
							− + 0	100 − + −
100	+−	+−	++	<			+ − +	102 + − 0
							+ − +	103 + − +
							+ − +	99 + − 0
							− + −	82 − + −
101	+−	+−	0+	>			+ − −	98 + − −
							− + +	104 − + +
102	+−	+−	0+	=			+ − 0	101 + − −
							− + 0	103 − + −
103	+−	+−	0+	<			+ − +	106 + − +
							− + −	100 − + −
104	+−	+−	−+	>	>	<	+ − −	6 + − −
							+ − −	5 + − −
							+ − −	101 + − −
							− + +	105 − + 0
105	+−	+−	−+	=	>	<	+ − 0	104 + − −
							− + 0	106 − + −
106	+−	+−	−+	<	>	<	+ − +	107 +0+
							− + −	105 − + 0
							− + −	107 −0−
							− + −	103 − + −
							− + −	83 −0−
107	+−	+−	−+	<	>	=	+0+	106 + − +
							− + 0	108 − − −
108	+−	+−	−+	<	>	>	+ + +	107 +0+
							− − −	84 − − −
							− − −	83 −0−

Appendix D : Double-Rocker Envisionment

	Kinematic State						Dynamic	
		Senses			Incl		State	Next
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
1	+0	+0	+−				+ − +	25 + − +
							− + −	9 − + −
2	+0	++	−−	>	<	>	+ − +	6 + − +
							− + −	3 − + 0
3	+0	++	−−	=	<	>	+ − 0	2 + − +
							− + 0	4 − + +
4	+0	++	−−	<	<	>	+ − −	3 + − 0
							− + +	8 − + +
5	+0	++	−−	<	>	>	+ + +	12 + + +
6	+0	++	0−	>			+ − +	9 + − +
							− + −	2 − + −
7	+0	++	0−	=			+ − 0	6 + − +
							− + 0	8 − + +
8	+0	++	0−	<			+ − −	4 + − −
							− + +	11 − + +
9	+0	++	+−	>			+ − +	27 + − +
							+ − +	26 + − +
							+ − +	1 + − +
							− + −	10 − + 0
							− + −	6 − + −
							− + −	7 − + 0
10	+0	++	+−	=			+ − 0	9 + − +
							− + 0	11 − + +
11	+0	++	+−	<			+ − −	10 + − 0
							+ − −	8 + − −
							+ − −	7 + − 0
							− + +	13 − + +

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
12	+0	0+	−−			>	+ + +	14 + + +
							− − −	5 − − −
13	+0	0+	+−				+ − −	11 + − −
							− + +	15 − + +
14	++	−+	−−			>	− − −	12 − − −
15	++	−+	+−		<		+ − −	28 + − 0
							+ − −	13 + − −
16	++	−+	+−		=		0 − −	15 + − −
							0 + +	17 + + +
17	++	−+	+−		>		+ + +	30 + + +
							+ + +	31 + + +
							+ + +	18 + + +
							− − −	16 0 − −
18	++	−0	+−				+ + +	19 + + +
							− − −	17 − − −
19	++	−−	+−	>			+ + +	32 + + +
							+ + +	33 + + 0
							+ + +	20 + + 0
							− − −	18 − − −
20	++	−−	+−	=			+ + 0	21 + + −
							− − 0	19 − − −
21	++	−−	+−	<			+ + −	22 + + −
							− − +	20 − − 0
22	++	0−	+−				+ + −	23 + + −
							− − +	21 − − +
23	++	+−	+−		<		+ + −	35 0 + −
							+ + −	34 + + −
							+ + −	24 0 + −
							− − +	22 − − +
24	++	+−	+−		=		0 − +	23 − − +
							0 + −	25 − + −

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
25	++	+−	+−		>		+ − +	35 0 − +
							+ − +	36 + − +
							+ − +	24 0 − +
							− + −	1 − + −
26	0+	+0	+−				+ − +	51 + − +
							− + −	9 − + −
27	0+	++	+−				+ − +	38 + − +
							− + −	9 − + −
28	0+	0+	+−				+ − 0	38 + − +
							− + 0	15 − + +
29	0+	−+	+−		=		0 + +	30 + + +
30	0+	−+	+−		>		+ + +	43 + + +
							− − −	17 − − −
31	0+	−0	+−				+ + +	45 + + +
							− − −	17 − − −
32	0+	−−	+−				+ + +	45 + + +
							− − −	19 − − −
33	0+	0−	+−				+ + 0	49 + + −
							− − 0	19 − − −
34	0+	+−	+−		<		+ + −	49 + + −
							− − +	23 − − +
35	0+	+−	+−		=		0 − +	34 − − +
							0 + −	36 − + −
36	0+	+−	+−		>		+ − +	51 + − +
							− + −	25 − + −
37	−+	+0	+−			>	+ − +	51 + − +
							− + −	38 − + −
38	−+	++	+−			>	+ − +	37 + − +
							− + −	27 − + −
							− + −	28 − + 0
39	−+	−+	+−	>	<	>	− + −	40 − + 0

	Kinematic State						Dynamic	Next
		Senses			Incl		State	
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
40	−+	−+	+−	=	<	>	+ − 0	39 + − +
							− + 0	41 − + +
41	−+	−+	+−	<	<	>	+ − −	40 + − 0
							− + +	42 0 + +
							− + +	29 0 + +
42	−+	−+	+−	<	=	>	0 − −	43 − − −
							0 − −	41 + − −
							0 + +	43 + + +
43	−+	−+	+−	<	>	>	+ + +	44 + + +
							− − −	42 0 − −
							− − −	30 − − −
							− − −	29 0 − −
44	−+	−0	+−			>	+ + +	45 + + +
							− − −	43 − − −
45	−+	−−	+−			>	+ + +	46 + + +
							− − −	44 − − −
							− − −	31 − − −
							− − −	32 − − −
46	−+	0−	+−			>	+ + +	47 + + +
							− − −	45 − − −
47	−+	+−	+−	>	<	>	+ + +	48 + + 0
							− − −	46 − − −
48	−+	+−	+−	=	<	>	+ + 0	49 + + −
							− − 0	46 − − −
							− − 0	47 − − −
49	−+	+−	+−	<	<	>	+ + −	50 0 + −
							− − +	48 − − 0
							− − +	33 − − 0
							− − +	34 − − +

	Kinematic State						Dynamic	Next
		Senses			Incl		State	
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
50	−+	+−	+−	<	=	>	0 − +	49 − − +
							0 + −	51 − + −
51	−+	+−	+−	<	>	>	+ − +	50 0 − +
							− + −	37 − + −
							− + −	36 − + −
							− + −	26 − + −
52	−−	+0	++			<	+ − +	66 + − +
							− + −	53 − + −
53	−−	++	++	>	<	<	+ − +	68 + − +
							+ − +	67 + − +
							+ − +	52 + − +
							− + −	54 − + −
54	−−	++	++	>	=	<	0 − +	53 + − +
							0 + −	55 + + −
55	−−	++	++	>	>	<	+ + −	70 + + −
							+ + −	71 + + 0
							+ + −	56 + + 0
							− − +	54 0 − +
56	−−	++	++	=	>	<	+ + 0	70 + + −
							+ + 0	71 + + 0
							+ + 0	57 + + +
							− − 0	55 − − +
57	−−	++	++	<	>	<	+ + +	58 + + +
							− − −	56 − − 0
58	−−	0+	++			<	+ + +	59 + + +
							− − −	57 − − −
59	−−	−+	++			<	+ + +	72 + + +
							+ + +	73 + + +
							+ + +	60 + + +
							− − −	58 − − −
60	−−	−0	++			<	+ + +	61 + + +
							− − −	59 − − −

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
61	−−	−−	++	>	<	<	+ + +	75 0 + +
							+ + +	74 + + +
							+ + +	62 0 + +
							− − −	60 − − −
62	−−	−−	++	>	=	<	0 − −	61 − − −
							0 + +	63 − + +
63	−−	−−	++	>	>	<	+ − −	75 0 − −
							+ − −	62 0 − −
							− + +	64 − + 0
64	−−	−−	++	=	>	<	+ − 0	63 + − −
							− + 0	65 − + −
65	−−	−−	++	<	>	<	+ − +	64 + − 0
66	−−	+−	++			<	+ − +	76 + − 0
							+ − +	77 + − +
							− + −	52 − + −
67	0−	+0	++				+ − +	95 + − +
							− + −	53 − + −
68	0−	++	++		<		+ − +	79 + − +
							− + −	53 − + −
69	0−	++	++		=		0 − +	68 + − +
							0 + −	70 + + −
70	0−	++	++		>		+ + −	81 + + −
							− − +	55 − − +
71	0−	0+	++				+ + 0	83 + + −
							+ + 0	85 + + +
							− − 0	55 − − +
72	0−	−+	++				+ + +	85 + + +
							− − −	59 − − −
73	0−	−0	++				+ + +	87 + + +
							− − −	59 − − −

			Kinematic State				Dynamic	Next
		Senses			Incl		State	
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
74	0−	−−	++		<		+ + +	87 + + +
							− − −	61 − − −
75	0−	−−	++		=		0 − −	74 − − −
							0 + +	74 + + +
76	0−	0−	++				+ − 0	89 + − −
							− + 0	66 − + −
77	0−	+−	++				+ − +	95 + − +
							− + −	66 − + −
78	+−	+0	++				+ − +	95 + − +
							− + −	79 − + −
79	+−	++	++		<		+ − +	78 + − +
							− + −	80 0 + −
							− + −	68 − + −
							− + −	69 0 + −
80	+−	++	++		=		0 − +	79 + − +
							0 + −	81 + + −
81	+−	++	++		>		+ + −	82 + + −
							− − +	80 0 − +
							− − +	70 − − +
							− − +	69 0 − +
82	+−	0+	++				+ + −	83 + + −
							− − +	81 − − +
83	+−	−+	++	>			+ + −	84 + + 0
							− − +	82 − − +
84	+−	−+	++	=			+ + 0	85 + + +
							− − 0	83 − − +

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
85	+−	−+	++	<			+ + +	86 + + +
							− − −	84 − − 0
							− − −	71 − − 0
							− − −	72 − − −
86	+−	−0	++				+ + +	87 + + +
							− − −	85 − − −
87	+−	−−	++		<		+ + +	88 0 + +
							− − −	86 − − −
							− − −	73 − − −
							− − −	74 − − −
88	+−	−−	++		=		0 − −	87 − − −
							0 + +	89 − + +
89	+−	−−	++		>		+ − −	88 0 − −
							− + +	91 − + +
							− + +	76 − + 0
90	+−	−−	−+			<	+ + +	92 + + +
91	+−	0−	++				+ − −	89 + − −
							− + +	93 − + +
92	+−	0−	−+			<	+ + +	99 + + +
							− − −	90 − − −
93	+−	+−	++	>			+ − −	91 + − −
							− + +	97 − + 0
							− + +	96 − + +
							− + +	94 − + 0
94	+−	+−	++	=			+ − 0	93 + − −
							− + 0	95 − + −
95	+−	+−	++	<			+ − +	97 + − 0
							+ − +	98 + − +
							+ − +	94 + − 0
							− + −	78 − + −
							− + −	77 − + −
							− + −	67 − + −

	Kinematic State						Dynamic State	Next
		Senses			Incl			
	P2/P1	P3/P2	P4/P3	L1/L2	L2/L3	L3/L1	L1/L2/L3	
96	+−	+−	0+	>			+ − −	91 + − −
							+ − −	93 + − −
							− + +	100 − + +
97	+−	+−	0+	=			+ − 0	96 + − −
							− + 0	98 − + −
98	+−	+−	0+	<			+ − +	102 + − +
							− + −	95 − + −
99	+−	+−	−+	>	<	<	− − −	92 − − −
100	+−	+−	−+	>	>	<	+ − −	96 + − 0
							− + +	101 − + 0
101	+−	+−	−+	=	>	<	+ − 0	100 + − −
							− + 0	102 − + −
102	+−	+−	−+	<	>	<	+ − +	100 + − 0
							− + −	98 − + −

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16. Abstracts

An important aspect of qualitative spatial reasoning is understanding mechanisms. This paper presents a qualitative analysis of motion for mechanical linkages. In particular, we describe how to analyze the behavior of a mechanism which has a movable axis. The basic idea of our approach is to represent *relative motions* of the link in terms of quadrants (qualitatively representing the direction relative to a global reference frame) and *relative inclinations* relative to the x-axis. Using this representation, we can derive all the possible motions of a system of linkages. This idea has been implemented and tested on several examples.

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