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Analogical Reasoning and Conceptual Change: A Case Study of Johannes Kepler

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The work of Johannes Kepler offers clear examples of conceptual change. In this article, using Kepler's work as a case study, we argue that analogical reasoning

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facilitates change of knowledge in four ways: (a) highlighting, (b) projection, (c) rerepresentation, and (d) restructuring. We present these four mechanisms within the context of structure-mapping theory and its computational implementation, the structure-mapping engine. We exemplify these mechanisms using the extended analogies Kepler used in developing a causal theory of planetary motion.

> The roads by which men arrive at their insights into celestial matters seem to me almost as worthy of wonder as those matters in themselves.
>
> —Johannes Kepler (as cited in Koestler, 1963, p. 261)

Analogy is an important mechanism of change of knowledge. Researchers studying transfer of learning have shown that analogies to prior knowledge can foster insight into new material (Bassok, 1990; Bassok & Holyoak, 1989; Catrambone & Holyoak, 1989; Dunbar, 1994; Forbus, Gentner, & Law, 1995; Gentner & Gentner, 1983; Gentner, Rattermann, & Forbus, 1993; Gick & Holyoak, 1980, 1983; Holyoak, Junn, & Billman, 1984; Holyoak & Thagard, 1989; Keane, 1988; Novick & Holyoak, 1991; Novick & Tversky, 1987; Ross, 1987; Spellman & Holyoak, 1993). These laboratory results are supported by direct and indirect observations of the scientific process. The journals of Boyle, Carnot, Darwin, Faraday, and Maxwell (and Kepler) contain many examples of generative uses of analogy (Darden, 1992; Gentner, 1982; Gentner & Jeziorski, 1993; Nersessian, 1985, 1986, 1992; Nersessian & Resnick, 1989; Ranney & Thagard, 1988; Thagard, 1989; Tweney, 1991; Wiser, 1986; Wiser & Carey, 1983). Modern scientists like Oppenheimer (1956) and Glashow (1980) have commented explicitly on the usefulness of analogy in their work. Nersessian's (1992) detailed analyses of the analogies used by Faraday and Maxwell provide evidence that analogy was useful in the development of electromagnetic field theory. Finally, direct field observations of molecular biologists at work demonstrate that analogy is frequently used in the everyday practice of science (Dunbar, 1994).

Our goal in this article is to show how analogy promotes conceptual change. We first lay out four theoretically driven specific mechanisms by which analogy can act to create changes in knowledge and consider the sorts of changes these processes can bring about. In particular, we ask whether analogical mechanisms can bring about changes in concepts as well as changes in the theoretical structure relating the concepts.

We draw on the works of Kepler (1571–1630) to illustrate our points. The goal of modeling the thought processes of a mind like Kepler's is daunting, to say the least. We make no claim to have captured anything close to Kepler's full cognitive processes. Yet, we consider Kepler a particularly apt subject for the study of analogy and conceptual change. First, his work spanned and contributed to a period of immense change in theory. He inherited from Copernicus a conception of the solar system in which the planets moved in perfect circles at uniform speed. By the end of his career, he had abandoned this simple and beautiful view for a model in which the planets travel in elliptical paths at nonuniform speed, with the Sun as the cause of their motion. Second, Kepler was a prolific analogizer. In his books, journals, and letters he constantly used analogies, some only fleetingly and others with tenacious persistence. In some cases, he returned to an analogy repeatedly across different works, extending and analyzing it further on successive bouts. Third, Kepler's writings are unusually rich in descriptions of his thought processes, including fulsome descriptions of his blind alleys and mistakes. The candor and detail of Kepler's writings helps to mitigate the problems inherent in inferring thought processes after the fact from written records. At least part of Kepler's inclusiveness seems to have stemmed from a fascination with the mental paths that led to his conceptual shifts, as evidenced by the quote at the beginning of this article.

In this article, we trace Kepler's extended analogy between light and the *vis motrix* (a precursor of gravity) and also his further analogy between magnetism and the *vis motrix*. Our goal is to characterize the processes by which these analogies led to changes of knowledge, using structure-mapping theory as a framework. We first describe the basic theory. Then we discuss four mechanisms by which analogy brings about change of beliefs. Finally, we apply this framework to Kepler's analogies.

STRUCTURE-MAPPING THEORY

Structure-mapping theory (SMT; Gentner, 1983, 1989) is based on the assumption that analogy involves a process of alignment and projection. Assertions in a base (or source) domain are placed into correspondence with assertions in a target domain, and further assertions true of the base domain are then inferred to be potentially true of the target. For example, when (as we later discuss) Kepler compared the target domain of the Sun and planet to the base domain of two lodestones, he inferred that if the Sun and planet also have polarity, they may alternately attract and repel one another, depending on whether their "friendly" or "unfriendly" poles are proximate. This illustrates the power of an analogy to provide a whole system of inferences about a novel domain. But a mechanism for inferring new knowledge must be constrained. To be cognitively plausible, a theory of analogical mapping must provide some natural limit to what will be inferred based on the mapping. It must also explain the fact that some analogies and some interpretations of a given analogy are preferred over others, even when no differences in factual accuracy are at stake.

SMT (Gentner, 1983, 1989) and its computational counterpart, the structure-mapping engine (SME; Falkenhainer, Forbus, & Gentner, 1989) meet this need by making strong assumptions about the nature of cognitive representation and how it is used in the mapping process. Structure mapping assumes that domain knowledge is in the form of symbolic structural descriptions that include objects, relations between objects, and higher order relations among whole propositions. On this view, the analogical process is one of structural alignment between two mental representations to find the maximal structurally consistent match between them. A *structurally consistent* match is one that satisfies the constraints of *parallel connectivity* and *one-to-one mapping* (Falkenhainer et al., 1989; Gentner, 1983, 1989; Gentner & Markman, 1993, in press; Halford, 1993; Holyoak & Thagard, 1989; Keane, 1988; Markman & Gentner, 1993a, 1993b; Medin, Goldstone, & Gentner, 1993). *Parallel connectivity* says that if two predicates are matched then their arguments must also match. For example, if the predicate HEAVIER(a,b) matches the predicate HEAVIER(x,y) then a must match x and b must match y. *One-to-one mapping* requires that each element in one representation corresponds to at most one element in the other representation.

To explain why some analogies are better than others, structure mapping uses the principle of systematicity: a preference for mappings that are highly interconnected and contain deep chains of higher order relations (Forbus & Gentner, 1989; Forbus et al., 1995; Gentner, 1983, 1989; Gentner et al., 1993). Thus, the probability that an individual match will be included in the final interpretation of a comparison is greater if it is connected by higher order relations to a common system of predicates (Bowdle & Gentner, 1996; Clement & Gentner, 1991; Gentner & Bowdle, 1994). We focus on two predictions that derive from this framework. First, the correspondences mandated by a comparison are governed not only by local similarity but also by the degree to which the elements play the same roles in the common higher order structure (e.g., Clement & Gentner, 1991; Gentner, 1988; Gentner & Clement, 1988; Spellman & Holyoak, 1993). Relational commonalities thus tend to outweigh object commonalities in determining the interpretation of a comparison. Second, because comparison promotes a structural alignment, differences relevant to the common structure are also highlighted by a comparison (Gentner & Markman, 1994; Markman & Gentner, 1993a, 1993b, 1996, in press). Thus, paradoxically, comparisons can illuminate differences as well as commonalities.

SME simulates the comparison process (Falkenhainer et al., 1989; Falkenhainer, Forbus, & Gentner, 1986). To capture the necessary structural distinctions we use an nth-order typed predicate calculus. Entities stand for the objects or reified concepts in the domain (e.g., planet, orbit). Attributes are unary predicates used to describe independent descriptive properties of objects (e.g., HEAVY(planet)). Functions[1] are used primarily to state dimensional properties (e.g., BRIGHT-

[1]Functions, unlike attributes and relations, do not take truth values but rather map objects onto other objects or values. For brevity, we sometimes use the term *predicate* to refer to all three categories: relations, attributes, and functions.

NESS(planet)). Relations are multiplace predicates that represent links between two or more entities, attributes, functions, or relations (e.g., REPELS(lodestone-1, lodestone-2); using magnetism as the domain).

To represent beliefs about physical domains, we use the qualitative process (QP) theory as a representation language (Forbus, 1984, 1990; Forbus & Gentner, 1986; Forbus, Nielsen, & Faltings, 1991; see Forbus, 1984, for a full description of the QP language and its model building capabilities). QP theory allows the representation of qualitative proportionalities between quantities and relations. For example, the statement QPROP + (a, b) expresses a positive qualitative relation between the quantities a and b: That is, that a is a monotonic positive function of (at least) b. QPROP – (a, b) expresses a negative qualitative relation.

Relations can hold between expressions as well as entities. Such higher order relations allow the construction of large representational structures that can describe, for example, the relation between magnetism and lodestone attraction:

IMPLIES(AND(MAGNETIC(lodestone-1), COMPOSED-OF(filing-1, iron)),
ATTRACTS(lodestone-1, filing-1))

It is the presence of structurally interconnected representations that is the key to implementing structure mapping. Given two representations in working memory, SME operates in a local-to-global manner to find one or a few structurally consistent matches. In the first stage, SME proposes matches between all identical predicates at any level (attribute, relation, higher order relation, etc.) in the two representations. At this stage, there may be many mutually inconsistent matches. In the next stage, these local correspondences are coalesced into large mappings, called *kernels,* by enforcing structural consistency (one-to-one mapping and parallel connectivity). SME allows correspondences between nonidentical entities and dimensions (represented as functions), in accordance with the principle that lower order information need not match identically. However, relations must match identically, reflecting the principle that comparison is implicitly directed toward finding structural commonality. For example, ATTRACTS(Sun, planet) may map to ATTRACTS(magnet, nail), but can never map to COMPOSED-OF(nail, iron).

In the next step, SME gathers these structurally consistent clusters into one or a few global interpretations. At this point, it projects candidate inferences into the target. It does this by adding to the target representation any predicates that currently belong to the common structure in the base but are not yet present in the target. These predicates function as possible new inferences imported from the base to the target. The mappings are given a structural evaluation, reflecting the size and depth of the system of matches.

SME has many useful properties for modeling conceptual change. First, the final interpretation preserves large-scale connected structure. Second, this global interpretation does not need to be explicit at the outset. The assertions that will constitute the final point of the analogy need not be present initially in the target and need not have been extracted as a separable "goal structure" or "problem-solution structure" in the base before the comparison processes begins. SME begins blindly, using only local matches, and the final global interpretation emerges via the pull toward connectivity and systematicity in the later stages of the process. Third, SME makes spontaneous, structurally consistent inferences from its comparison process, unlike many other models of analogy (cf. Holyoak & Thagard, 1989; Markman, 1996). Finally, this model of the analogy process allows us to delineate four specific subprocesses that can change conceptual structure: highlighting, projection, rerepresentation, and restructuring (Gentner & Wolff, in press).

The Four Analogical Processes of Conceptual Change

Highlighting. SME's first result is a matching system of predicates between the base and target. This models the psychological assumption that the process of alignment causes the matching aspects of the domains to become more salient (Elio & Anderson, 1981, 1984; Gentner & Wolff, in press; Gick & Holyoak, 1980, 1983; Markman & Gentner, 1993a, 1993b; Medin et al., 1993; Miller, 1979; Ortony, 1979). This process of highlighting is important because human representations, we suggest, are typically large, rich, and thickly interwoven nets of concepts. In particular, early representations tend to be conservative, in the sense that they retain many specific details of the context of learning: They are particularistic and contextually embedded (e.g., Brown, Collins, & Duguid, 1989; Forbus & Gentner, 1986; Medin & Ross, 1989). Highlighting can create a focus on a manageable subset of relevant information. Moreover, the relational identity constraint, combined with rerepresentation processes, means that the output of an analogy may reveal hitherto unnoticed relational commonalities. There is considerable psychological evidence that comparison can reveal nonobvious features (Gentner & Clement, 1988; Gentner & Imai, 1995; Markman & Gentner, 1993a; Medin et al., 1993; Ortony, Vondruska, Foss, & Jones, 1985; Tourangeau & Rips, 1991) and that highlighting of common information can influence category formation (Elio & Anderson, 1981, 1984; Medin & Ross, 1989; Ross, 1984, 1989; Skorstad, Gentner, & Medin, 1988).

Projection of candidate inferences. As previously described, SME projects candidate inferences from the base to the target domain. These projected inferences, if accepted, add to the knowledge in the target domain. However, not all inferences made by SME will be correct. Postmapping processes, such as the application of semantic and pragmatic constraints, are necessary to ensure the correctness of the inferences (Falkenhainer, 1990; Kass, 1994; Kolodner, 1992, 1993; Novick & Holyoak, 1991).

Rerepresentation. In rerepresentation, the representation of either or both domains is changed to improve the match. Typically, this involves a kind of tinkering in order that two initially mismatching predicates can be adjusted to match. For example, suppose an analogy matches well but for a mismatch between BRIGHTER-THAN(x,y) and FASTER-THAN(a,b) (as in Kepler's analogy that is discussed later). These relations can be rerepresented as GREATER-THAN(BRIGHTNESS(x), BRIGHTNESS(y)), and GREATER-THAN(SPEED(a), SPEED(b)) to allow comparison. This involves a kind of decomposition that separates the GREATER-THAN magnitude relation (which is common to both) from the specific dimension of increase (which is distinctive). Studies of the development of children's comparison abilities support the psychological validity of such rerepresentation in learning: Children are better able to match cross-dimensional analogies when they have been induced to rerepresent the two situations to permit noticing the common magnitude increase (Gentner & Rattermann, 1991; Gentner, Rattermann, Markman, & Kotovsky, 1995; Kotovsky & Gentner, in press). We discuss SME's implementation of rerepresentation later in this article.

Restructuring. Restructuring is the process of large-scale rearrangement of elements of the target domain to form a new coherent explanation. This rearrangement can take the form of adding or deleting causal links in the target domain as well as altering specific concepts. It should perhaps be considered separately from the other three processes or possibly as arising from a combination of the other three. For example, when little is known about a target domain, a mapping from the base can provide casual linkages that significantly alter the connectivity in the target. However, on this account, there must be some minimal alignment as a basis for inference; even if no initial relational match exists, there must be at least a partial object mapping (which could be suggested by local similarities or pragmatically stipulated; Forbus & Oblinger, 1990; Holyoak & Thagard, 1989; Winston, 1980). We conjecture that substantial restructuring during a single mapping is comparatively rare because normally the candidate inferences projected from the base domain will be at least compatible with the existing target structure. Furthermore, as Nersessian (1992) pointed out, massive restructuring from a single base can be dangerous: She noted that Faraday's modeling of magnetic fields by analogy with the concrete lines of iron filings created by magnets led to an overly concrete, partly erroneous model of the fields. In general, we suspect that most restructuring occurs as a result of multiple analogies iteratively applied as well as other processes. With these tools in hand, we now return to Kepler. We begin with some historical background.

KEPLER AND THE SOLAR SYSTEM

Kepler[2] (1571–1630) is best known today for his three laws of planetary motion.[3] His far more important contributions in changing our conception of the solar system are difficult to appreciate. Ironically, this is in part because of his very success. The conceptual structure that existed prior to Kepler's work is now almost impossible for us to call forth. Medieval cosmology differed from our own not only in the specific conceptual structure but also in the character of its explanations: They sought to find mathematical regularities, not causal mechanisms. It is here that Kepler's contribution lies. As Caspar (1993) put it: "It is Kepler's greatest service that he substituted a dynamic system for the formal schemes of the earlier astronomers, the law of nature for mathematical rule, and causal explanation for the mathematical description of motion" (p. 136). Holton (1973) went further: "Kepler's genius lies in his early search for a physics of the solar system. He is the first to look for a universal physical law based on terrestrial mechanics to comprehend the whole universe in its quantitative details" (p. 71).

To understand the magnitude of the conceptual change involved, an account of the prior state of belief is necessary.[4] Western cosmology in the 16th century, continuing the tradition laid down by the Greeks, stated the laws of planetary motion in purely mathematical terms. It postulated a universe with the Earth at the center,

[2]Opinions on Kepler's standing have varied. School children are taught that he was a mathematician who made his discoveries by trying all possible mathematical combinations, much in the manner of Langley, Bradshaw, and Simon's (1983) Bacon program. Koestler (1963) portrayed him as a neo-Platonic mystic, a "sleepwalker" who stumbled on his discoveries by accident. Many of his commentators consider that he ranks among the great scientists (e.g., Caspar, 1993; Gingerich, 1993; Holton, 1973; Koyre, 1973; Toulmin & Goodfield, 1961). Furthermore, as we make clear, he proceeded not by mechanical application of formulae but by the bold application of analogies and causal principles. This discussion of Kepler's work was compiled from a variety of sources: Barker (1991, 1993), Barker and Goldstein (1994), Baumgardt (1952), Butterfield (1957), Caspar (1993), Gingerich (1993), Hanson (1958), Holton (1973), Koestler (1963), Koyre (1973), Kuhn (1957), Layzer (1984), Mason (1962), Stephenson (1994a, 1994b), Toulmin and Goodfield (1961), and Vickers (1984)

[3]The first law states that the orbits of the planets are elliptical with the Sun at one focus. The second law (chronologically the first) states that the equal areas are swept in equal times by a line connecting a planet and the Sun. The third law states that the product of the square of the period of a planet's revolution and the cube of its mean distance from the Sun is constant.

[4]This account is taken chiefly from Butterfield (1957), Hanson (1958), Koyre (1973), Kuhn (1957), Layzer (1984), Mason (1962), Sambursky (1975), and Toulmin and Goodfield (1961). It is necessarily much abbreviated and oversimplified. There were dissenters, both among the Greeks—notably Aristarchus of Samos (310–230 B.C.), called "the Copernicus of Antiquity" for his heliocentric theory (Kuhn, 1957)—and in the Western scholastic tradition—including William of Ockham (1295–1349), who argued that postulating a spinning earth would simplify the explanations (an instance of Ockham's razor), Buridan (c. 1297–1358), Albert of Saxony (c. 1360), Oresme (c. 1323?–1382), and Nicolas of Cusa (1401–1464). However, even scholars willing to postulate a rotating earth did not generally countenance an earth that revolved around the Sun.

around which revolved crystalline spheres containing the heavenly bodies. The set of beliefs laid down by Plato and Aristotle and culminating in Ptolemy's system of the 2nd century A.D. was roughly as follows:

1. The Earth is at the center of the universe and is itself unmoving.
2. The Earth is surrounded by physically real crystalline spheres,[5] containing the heavenly bodies, which revolve around the Earth.
3. The heavenly bodies move in perfect circles at uniform velocity, as befits incorruptible bodies. (Epicycles and eccentrically positioned circles were admitted into the system to account for the observed motions.)
4. All motion requires a mover. The outermost sphere, containing the fixed stars, is moved by an "unmoved mover," the Primum Mobile. Each sphere imparts motion to the next one in; in the Aristotelian universe, there is no action-at-a-distance. In addition, each sphere is controlled by its own spirit that mediates its motion.[6] (The heavenly bodies were known not to move in synchrony.)
5. Celestial phenomena must be explained in entirely different terms from earthly phenomena. Indeed, heavenly bodies and their spheres are made of different matter altogether. They are composed not of the four terrestrial elements—earth, air, fire, and water—but instead of a fifth element (the quintessence), crystalline aether (pure, unalterable, transparent, and weightless). The further from Earth, the purer the sphere.

This Aristotelian–Ptolemaic system was integrated with Catholic theology, largely by Magnus (1206–1280) and Aquinas (1225–1274). Angelic spirits were assigned to the celestial spheres in order of rank: The outermost sphere, that of the Primum Mobile, belonged to the Seraphim; next inward, the Cherubim controlled the sphere of the fixed stars; next came Thrones, Dominations, Virtues, Powers, Principalities, Archangels, and finally Angels, who controlled the sphere of the moon. The resulting conceptual scheme, dominant until the 16th century, was one of extreme clarity, intricacy, and cohesion.

Thirteen centuries after Ptolemy's model, Copernicus (1473–1543) published (in 1543, the year of his death) *De Revolutionibus Orbium Celestium,* proposing the revolutionary idea that the Earth and other planets moved rather than the Sun.[7]

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[5]There were variations on this basic scheme with different numbers of spheres. Aristotle's (384–322) system contained 55 spheres. However, when the Greek system was merged with Christian theology, the resulting system had 9 (or 10, depending on what is counted) spiritually significant spheres.

[6]In Aristotle's theory of motion, a homogeneous body required an external mover. There was a kind of analogy of the form *spirit | planet || soul | body || mover | moved.*

[7]Copernicus's theory was only partly heliocentric. For mathematical reasons, he placed the center of the solar system at the center of the Earth's orbit, rather than at the Sun itself.

Copernicus argued for his system on the grounds of mathematical elegance and sufficiency. He complained of the number of eccentric circles and epicycles required. He argued further that the Ptolemaic system had in effect departed from the ancient principle of perfect circularity and regularity of movement (by using "equants"—hypothetical points around which the centers of the planetary epicycles revolved—as a way of saving the fit to data).[8] The Copernican system was not widely accepted. Even among those learned enough to appreciate the problems with the Ptolemaic system, a more popular proposal was Brahe's system in which the five planets revolved around the Sun but the Sun itself revolved around a stationary Earth.

Kepler was a confirmed Copernican from the beginning, having studied the theory at Tubingen with Maestlin. In 1591, at the age of 20, he began as lecturer in mathematics at Graz. In his first book, the *Mysterium Cosmographicum,* in 1596, he defended the Copernican view and presented his own heliocentric proposal. The *Mysterium Cosmographicum* attracted the interest of Brahe (1546–1601), and in 1600 Kepler spent time as an assistant in Tycho's observatory. When Tycho died in 1601, Kepler was appointed his successor as Imperial Mathematician of the court in Prague.

Kepler acquired from Tycho the largest and most accurate store of astronomical observations available. He also acquired the task of determining the orbit of Mars, a task that proved far more difficult and ultimately more revealing than Kepler had foreseen. Kepler spent the next years trying to construct a consistent heliocentric model of the solar system based on his principle (discovered in the *Mysterium Cosmographicum*) that the planets move faster when closer to the Sun (a precursor of his second law, "equal area in equal times"). Unfortunately, he also retained the virtually universal, self-evident ancient principle that the orbits of the planets were perfect circles or were at least composed of perfect (although possibly eccentric) circles. Ultimately, the fact that his calculations for Mars's orbit differed from Tycho's observations (by the famous mere 8° of arc) forced him to a dismaying rejection of the ancient assumption of circularity. It is hard today to grasp how tenaciously these beliefs were held. Kepler, in the preface to the *Astronomia Nova,* commented on the incredible labor required to establish the existence of the solar force, largely due to the power of the assumption of circular motion "because I had bound them to the millstones (as it were) of circularity, under the spell of common opinion. Restrained by such fetters, the movers could not do their work" (p. 67).[9]

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[8]In fact, although Copernicus was able to divest himself of the "major epicycles" that accounted for the planets' apparent retrograde motions and of the notion of the *equant* (an imaginary point from which the calculated orbit would appear more uniform), he was forced to maintain a complex set of eccentrics and minor epicycles (34 circles in all) as compared with Kepler's six ellipses (Mason, 1962).

[9]Galileo (1564–1642), Kepler's brilliant contemporary and a fellow Copernican, never abandoned the belief that the planets moved in perfect circles at uniform velocity, despite receiving Kepler's evidence for elliptical orbits.

He next tried fruitlessly to model the planetary path with an ovoid, before at last accepting the ellipse as the shape of the orbit.[10] This led to a more precise statement of the Second Law of planetary motion, that a line between the Sun and any planet sweeps out equal areas in equal intervals of time and to the First Law, that the planetary orbits are ellipses with the Sun at one focus.[11] With this new model Kepler could replace Copernicus's 34 circles with just six ellipses.[12]

Kepler published this new view in 1609 as the *Astronomia Nova: A New Astronomy Based on Causation, or a Physics of the Sky*. It records the discoveries and the saga of his quest to derive the orbit of the planets—in particular, Mars, the most resistant to calculation—from causal principles.[13] He understood well that his causal explanation moved him out of the kind of astronomy practiced at the time and announced in the introductory summary: "Ye physicists, prick your ears, for now we are going to invade your territory" (as cited in Koestler, 1963, p. 325). Kepler's causal explanation of planetary motion and his three laws were a major step toward our modern conception of the solar system. As Gingerich (1993) put it:

> Kepler's most consequential achievement was the mechanizing and perfecting of the world system. By the *mechanization* of the solar system, I mean his insistence on "a new astronomy based on causes, or the celestial physics," as he tells us in the title of his great book. By the *perfection* of the planetary system, I mean the fantastic improvement of nearly two orders of magnitude in the prediction of planetary positions. (p. 333)[14]

We now return to the beginning, to the *Mysterium Cosmographicum* (1596), to ask how Kepler arrived at this revolutionary position. One last bit of context setting is necessary. Besides Copernicus's treatise, there were two astronomical events, both solidly documented by Brahe, that helped to prepare the ground for new conceptions of the heavens. The first was a nova (or supernova) in 1572. The

[10]After abandoning the circle, Kepler at first used the ellipse merely as a mathematical approximation to the ovoid, or egg, which had the advantage of possessing only one focus. He resisted the ellipse as a solution for physical reasons: If the Sun was the unique cause of planetary motion, then there should be one unique place for it, not an arbitrary selection from between two foci as with an ellipse (Hanson, 1958).

[11]The Second Law appears in rough form in the *Mysterium Cosmographicum* (1596) and appears explicitly in Book III of the *Astronomia Nova,* before the First Law in Book IV. It was in fact crucial to his derivation of the First and Third Laws. The Third Law appears in the *Harmonice Mundi* in 1619.

[12]However, Kepler's system was not accepted by his contemporaries. Even those few willing to consider Kepler's and Copernicus's heliocentric views (including Kepler's old mentor, Maestlin) rejected his notion of a celestial physics governed by the same causal law as earthly phenomena.

[13]Hanson (1958), echoing Charles Sanders Peirce, called Kepler's discovery of the orbit of Mars "the greatest piece of retroductive reasoning ever performed" (p. 85).

[14]Gingerich (1993) noted that it was the success of these predictions (the Rudophine Tables) that kept Kepler's theory alive during the 2 centuries after its publication.

addition of a new fixed star was evidence against the Aristotelian doctrine of the unchanging and incorruptible firmament. The second was a comet in 1577 (and others not long after), whose path ran through the planetary spheres and which challenged the physical reality of the crystalline spheres. Fueled in part by these challenges to Aristotelian cosmology, there was a revival of the Stoic cosmology in the late 16th century (see Barker, 1991, for a more detailed account). Like the Aristotelian view, the Stoic view was geocentric and had a sphere of fixed stars; it differed in that it postulated that the heavens were filled not with pure aether but with a kind of intelligent pneuma (a combination of fire and air), which became more pure with distance from the earth. The heavenly bodies, made of pneuma, were intelligent and capable of self-direction. Although Kepler firmly dismissed the view that the planets were each attached to their own crystalline spheres, he continued to wrestle with the idea that the planets move themselves intelligently.

The Sun as Prime Mover: The Light–*Anima Motrix* Analogy

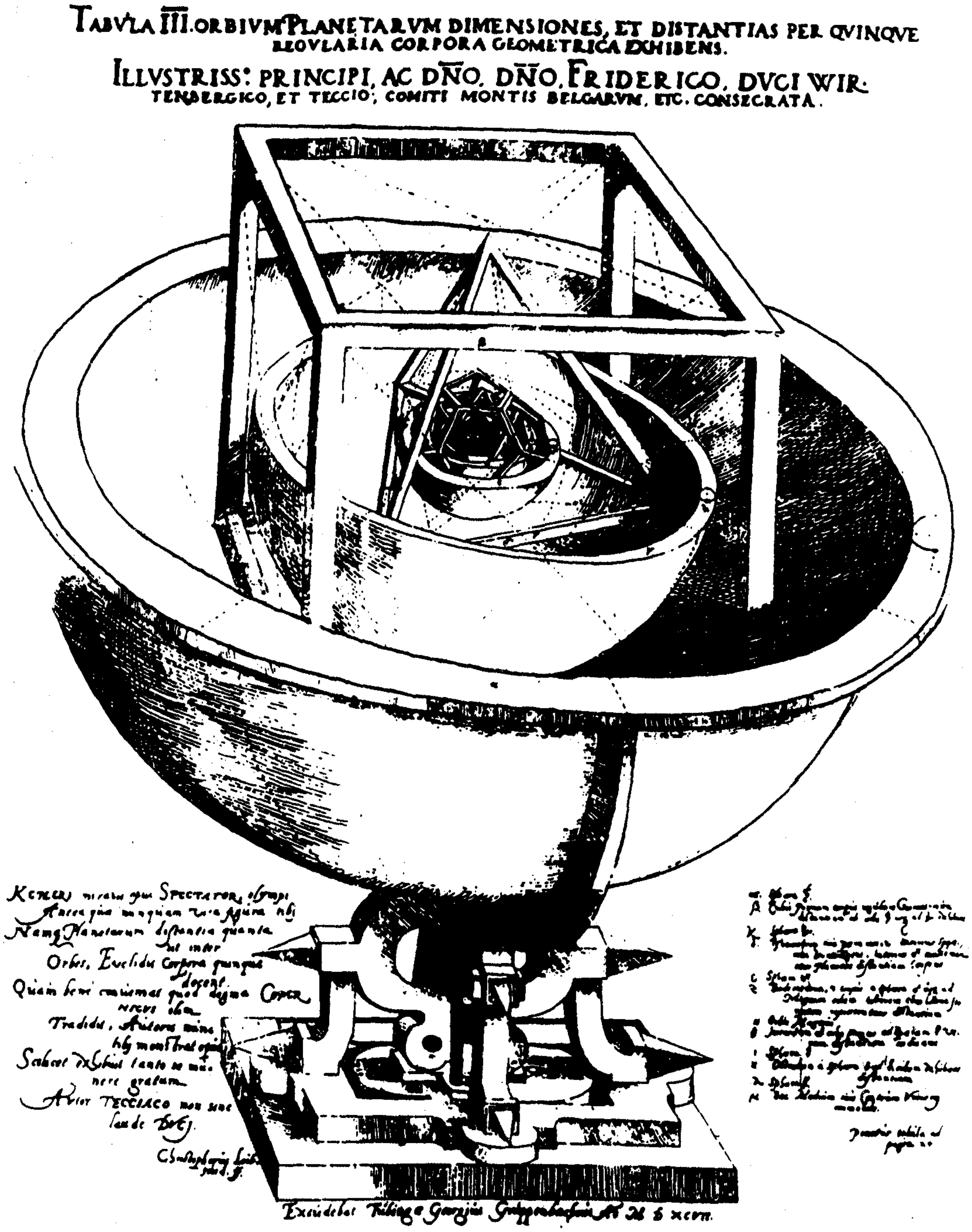
As Toulmin and Goodfield (1961) put it:

> The lifelong, self-appointed mission of Johann Kepler ... was to reveal the new, inner coherence of the Sun-centered planetary system. His central aim was to produce a "celestial physics," a system of astronomy of a new kind, in which the forces responsible for the phenomena were brought to light. (p. 198)

Kepler combined a neo-Platonist's love of mathematical regularity, a commitment to explanation in terms of physical causation, and an equally strong belief in empirical tests. In the preface to the *Mysterium Cosmographicum*, the 25-year-old Kepler stated his purpose: "There were three things of which I persistently sought the reasons why they were such and not otherwise: the number, size and motion of the circles" (Kepler, 1596/1981, p. 63).

Kepler's solution to the first two questions was a system of inscribed solids that predicted the distances of the planets from the Sun (see Figure 1). This is a rather quixotic model that clearly shows Kepler's passion for mathematically regularities. The extreme particularity of this initial model is striking: The distance of a given planet from the Sun could only be calculated by knowing the orbit of the next inward planet.

The work is interesting in at least two more respects. The first is Kepler's reworking of the Copernican theory to be more consistently heliocentric. Rejecting the Copernican placement of the center of the solar system as at the center of the Earth's orbit, Kepler proposed a mathematically small but physically significant change: The center of the solar system was the Sun itself. As Aiton (1976) pointed out, Kepler's causal interpretation of Copernicus's theory led to a reaxiomitization



Johannes Kepler, *Mysterium cosmographicum*. Copper engraving from the first edition (Tübingen, 1596).

FIGURE 1 Kepler's model of the solar system from the *Mysterium Cosmographicum*, showing the inscribed solids. From *Mysterium Cosmographicum I, II* (p. 1), by J. Kepler, 1981, New York: Abaris Books. Copyright by Abaris Books, Norwalk, CT. Reprinted with permission.

of astronomy. Kepler also posed an important question. He noticed that the periods of the outer planets were longer, relative to those of the inner planets, than could be predicted simply from the greater distances they had to travel. That is, the planets further away from the Sun moved slower than those closer to the Sun. Were the moving souls simply weaker in the faraway planets? Kepler reasoned:

> One of two conclusions must be reached: either the moving souls [*motricis animae*] are weaker the further they are from the Sun; or, there is a single moving soul [*motricem animam*][15] in the center of all the spheres, that is, in the Sun, and it impels each body more strongly in proportion to how near it is. (Kepler, 1596/1981, p. 199)

Kepler went on to apply this hypothesis to the paths of the individual planets. If motion is caused by a single *anima motrix* in the Sun that weakens with distance, this would explain why each individual planet should move slower when further from the Sun. (This insight requires noting the nonuniform speed of the planets, a fact that emerges only when the observations are recast from Ptolemaian epicycles into a heliocentric system.) To reason further, he used an analogy with light:

> Let us suppose, then, as is highly probable, that motion is dispensed by the Sun in the same proportion as light. Now the ratio in which light spreading out from a center is weakened is stated by the opticians. For the amount of light in a small circle is the same as the amount of light or of the solar rays in the great one. Hence, as it is more concentrated in the small circle, and more thinly spread in the great one, the measure of this thinning out must be sought in the actual ratio of the circles, both for light and for the moving power [motrice virtute]. (Kepler, 1596/1981, p. 201)

Pushing the Analogy

Kepler returned repeatedly to the analogy between light and the motive power. In the *Mysterium cosmographicum* (1596/1981), the analogy functioned as a kind of existence proof that the Sun's influence could be assumed to weaken in an orderly way with distance. Kepler's many subsequent uses of this analogy served to extend and refine this notion of the *vis motrix*. He devoted multiple chapters of his greatest work, the *Astronomia Nova* (1609/1992) to its explanation and returned to it again in the *Epitome of Copernican Astronomy* (1621/1969). Kepler also delved into the domain of light and optics. He published a treatise on astronomical optics *Astronomiae Pars Optica* in 1604 and another piece on optics, the *Dioptrice* in 1611.

[15]Kepler's annotation in 1621/1981 stated: "If for the word 'soul' [*Anima*] you substitute the word 'force' [*Vim*], you have the very same principle on which the Celestial Physics is established" (p. 201). (We return to this shift from *soul* to *forc* in the Discussion section.)

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> Let us suppose, then, as is highly probable, that motion is dispensed by the Sun in the same proportion as light. Now the ratio in which light spreading out from a center is weakened is stated by the opticians. For the amount of light in a small circle is the same as the amount of light or of the solar rays in the great one. Hence, as it is more concentrated in the small circle, and more thinly spread in the great one, the measure of this thinning out must be sought in the actual ratio of the circles, both for light and for the moving power [motrice virtute]. (Kepler, 1596/1981, p. 201)

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With this knowledge of the behavior of light, Kepler had a base domain systematic enough to provide considerable inferential resources for the target (Bassok & Holyoak, 1989; Bowdle & Gentner, 1996; Clement & Gentner, 1991; Gentner & Bowdle, 1994; Gentner & Gentner, 1983).

In the *Astronomia Nova,* Kepler developed this analogy of motive power with light much further. Early on, he raised the challenge of action at a distance:

> For it was said above that this motive power is extended throughout the space of the world, in some places more concentrated and in others more spread out. ... This implies that it is poured out throughout the whole world, and yet does not exist anywhere but where there is something movable. (Kepler, 1609/1992, p. 382)

He answered this challenge by invoking the light analogy.

> But lest I appear to philosophize with excessive insolence, I shall propose to the reader the clearly authentic example of light, since it also makes its nest in the Sun, thence to break forth into the whole world as a companion to this motive power. Who, I ask, will say that light is something material? Nevertheless, it carries out its operations with respect to place, suffers alteration, is reflected and refracted, and assumes quantities so as to be dense or rare, and to be capable of being taken as a surface wherever it falls upon something illuminable. Now just as it is said in optics, that light does not exist in the intermediate space between the source and the illuminable, this is equally true of the motive power. (Kepler, 1609/1992, p. 383)

Kepler also used the light analogy to buttress a prior claim, namely, that the *vis motrix* is diminished with distance not through being lost but through being spread out (a kind of conservation principle). He used two further potential analogs here: odors and heat. These are near misses (Winston, 1980), which differ with respect to the key behavior and thus serve to sharpen the parallel between light and the *vis motrix.*

> Since there is just as much power in a larger and more distant circle as there is in a smaller and closer one, nothing of this power is lost in traveling from its source, nothing is scattered between the source and the movable body. The emission, then, in the same manner as light, is immaterial, unlike odours, which are accompanied by a diminution of substance, and unlike heat from a hot furnace, or anything similar which fills the intervening space. (Kepler, 1609/1992, p. 381)

To trace the analogical process, we represented parts of Kepler's expressed knowledge about light and the motive power. We applied SME to these representations to simulate the process of analogical reasoning that Kepler may have used in rethinking his conceptual model of the solar system.

Our representation of Kepler's knowledge of the nature of light[16] is shown in Figure 2. Specifically, we ascribe to Kepler five beliefs: (a) A source produces light that travels instantaneously and undetectibly through space until it reaches an object, at which point the light is detectable; (b) the brightness of an object decreases with distance from a source; (c) the concentration of light affects the brightness of an object, with a greater concentration resulting in greater brightness; (d) as light spreads from a source there is an increase in volume and a decrease in concentration so that multiplying the volume by the concentration will produce a constant; and hence, (e) the concentration of light decreases as an object's distance from the source increases.

Kepler's initial knowledge of the motive power was of course considerably less rich than his knowledge about light. In our representation of this knowledge (see Figure 3), we use the term *vis motrix*, reflecting Kepler's shift to calling the Sun's influence *virtus motrix* or *vis motrix* (motive power or motive force) rather than *anima motrix* (motive spirit). His terminology over time had become less animate and more mechanical.

The *Vis Motrix* Analogy and the Process of Conceptual Change

Highlighting. When given these representations of Kepler's knowledge of light and of the Sun's motive force, SME produces the interpretation shown in Figure 4. This interpretation highlights commonalities, for example, the similarity that in both cases the emanation makes itself known when it strikes a planet and, respectively, illuminates or moves the planet.

Projection. As we have noted, highlighting influences conceptual change in part by identifying relevant aspects of the two domains and thereby permitting abstraction. It also provides the alignable structure over which two other processes of conceptual change operate: projection and rerepresentation. This is crucial, for by constraining the candidate inferences to be those connected to the aligned structure we can model an inferential process that is generative without overshooting into "wanton inferencing" (Eric Diettrich, personal communication, February 1994). The *vis motrix*–light analogy leads to several candidate inferences. Figure 5 shows SME's inferences, which seem reasonably like those Kepler appears to have made. First, SME infers that the *vis motrix* travels from the Sun to the planet through space. Second, it infers that the product of volume and concentration of

[16]These representations are of course not intended to be exhaustive representations of Kepler's knowledge but of the subset necessary to make our points about analogy and conceptual change. We do not attempt a full explanation of how Kepler selected the relevant information from his larger knowledge of light. Although this is clea ly important, it is beyond the scope of this article.

2a:

```
(PRODUCE Sun light)

(CAUSE (TRAVEL light Sun object space)
       (REACH light object))

(INSTANTANEOUS (TRAVEL light Sun object space))

(WHILE (AND (TRAVEL light Sun object space)
            (NOT (REACH light object)))
       (NOT (DETECTABLE light)))

(WHILE (AND (TRAVEL light Sun object space)
            (REACH light object))
       (DETECTABLE light))
```

2b:

```
(QPROP- (CONCENTRATION light object)
        (DISTANCE object Sun))
```

2c:

```
(CAUSE (REACH light object)
       (PROMOTE (BRIGHTNESS object)))

(QPROP+ (BRIGHTNESS object)
        (CONCENTRATION light object))
```

2d:

```
(CAUSE (AND (QPROP+ (VOLUME light)
                    (DISTANCE Sun object))
            (QPROP- (CONCENTRATION light object)
                    (DISTANCE object Sun)))
       (CONSTANT (* (VOLUME light)
                    (CONCENTRATION light object))))
```

(continued)

2e:

```
(IMPLIES (AND (QPROP- (CONCENTRATION light object)
                      (DISTANCE object Sun))
              (QPROP+ (BRIGHTNESS object)
                      (CONCENTRATION light object)))
         (QPROP- (BRIGHTNESS object)
                 (DISTANCE object Sun)))
```

FIGURE 2 Schematic representation of the belief structure for light in the light–*vis motrix* analogy.

3a:

```
(CAUSE (REACH vis-motrix planet)
       (PROMOTE (SPEED planet)))
```

3b:

```
(QPROP- (SPEED planet)
        (DISTANCE planet Sun))
```

FIGURE 3 Schematic representation of the *vis motrix* beliefs.

4:

```
(CAUSE (REACH light object)                (CAUSE (REACH vis-motrix planet)
       (PROMOTE (BRIGHTNESS object)))             (PROMOTE (SPEED planet)))

(QPROP- (BRIGHTNESS object)                (QPROP- (SPEED planet)
        (DISTANCE object Sun))                     (DISTANCE planet Sun))
```

FIGURE 4 Structure-mapping engine interpretation for the light–*vis motrix* analogy.

the *vis motrix* is a constant. Third, SME explains that because the concentration of the *vis motrix* decreases with distance, and the concentration of the *vis motrix* governs the speed of the planet, the speed of the planet will decrease with distance from the Sun. Finally, SME infers that the *vis motrix* will be detectable once it contacts the planet but not while it travels to the planet (the last two inferences shown in Figure 5). Taken together, these inferences explain the apparent phenomenon of action at a distance.

Rerepresentation. We suggested that the process of alignment can lead to rerepresentating parts of one or both representations in such a way as to improve the alignment. Figure 6 shows this process as well as highlighting and projecting inferences. Such a process may have operated on a large scale to contribute to Kepler's gradual shift toward thi king of the motive power as a physical phenomenon rather than an animistic one. However, a more locally contained example can be found shortly after the passage quoted previously in the *Astronomia Nova.* Kepler noted a discrepancy—an important alignable difference—and tried to resolve it:

> Moreover, although light itself does indeed flow forth in no time, while this power creates motion in time, nonetheless the way in which both do so is the same, if you consider them correctly. Light manifests those things which are proper to it instantaneously, but requires time to effect those which are associated with matter. It illuminates a surface in a moment, because here matter need not undergo any alteration, for all illumination takes place according to surfaces, or at least as if a property of surfaces and not as a property of corporeality as such. On the other hand, light bleaches colours in time, since here it acts upon matter qua matter, making it hot and expelling the contrary cold which is embedded in the body's matter and is not on

```
5:
(CAUSE (TRAVEL vis-motrix Sun planet (:SKOLEM space))
       (REACH vis-motrix planet))

(CAUSE (AND (QPROP+ (VOLUME (:SKOLEM space))
                    (DISTANCE Sun planet))
            (QPROP- (CONCENTRATION vis-motrix planet)
                    (DISTANCE planet Sun)))
       (CONSTANT (* (VOLUME (:SKOLEM space))
                    (CONCENTRATION vis-motrix planet))))

(IMPLIES (AND (QPROP- (CONCENTRATION vis-motrix planet)
                      (DISTANCE planet Sun))
              (QPROP+ (SPEED planet)
                      (CONCENTRATION vis-motrix planet)))
         (QPROP- (SPEED planet) (DISTANCE planet Sun)))

(WHILE (AND (TRAVEL vis-motrix Sun planet (:SKOLEM space))
            (NOT (REACH vis-motrix planet)))
       (NOT (DETECTABLE vis-motrix)))

(WHILE (AND (TRAVEL vis-motrix Sun planet (:SKOLEM space))
            (REACH vis-motrix planet))
       (DETECTABLE vis-motrix))
```

FIGURE 5 Structure-mapping engine's candidate inferences for the light–*vis motrix* analogy.

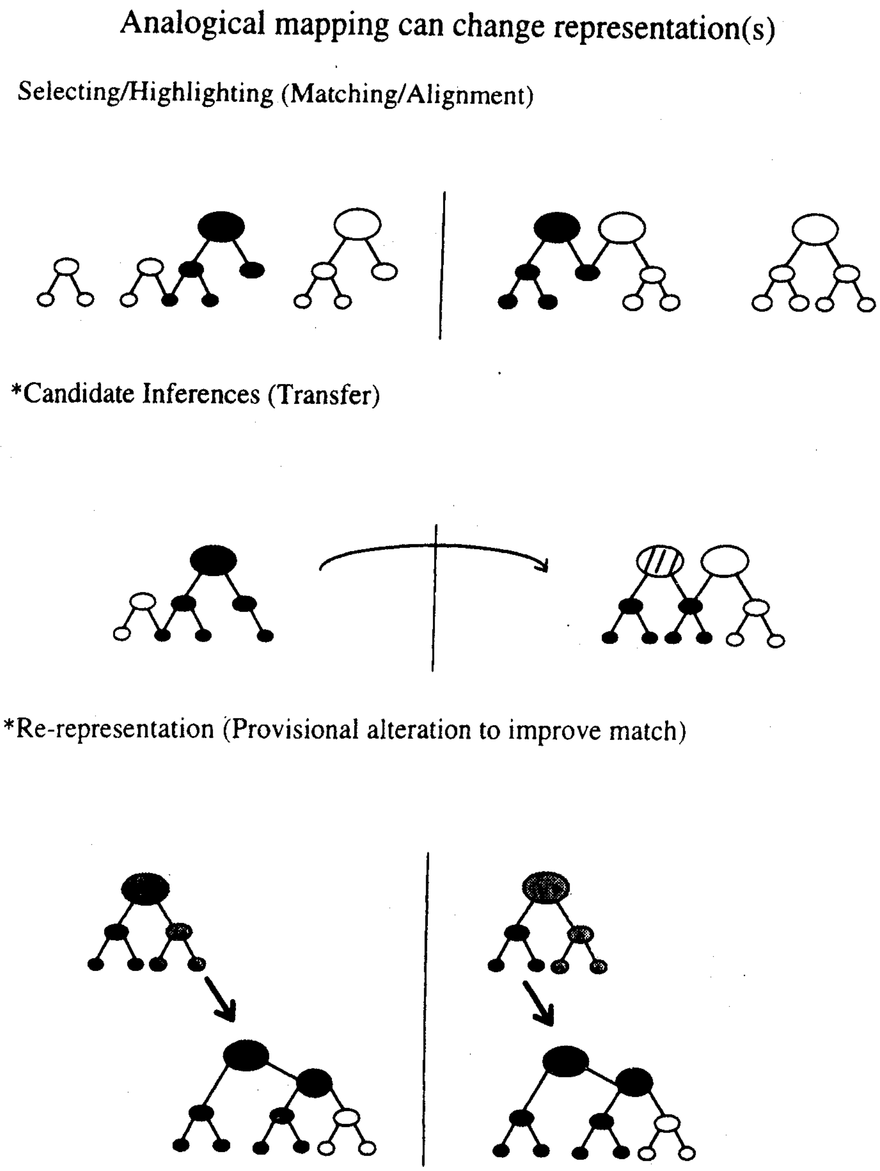

FIGURE 6 Ways analogy can create change.

its surface. In precisely the same manner, this moving power perpetually and without any interval of time is present from the Sun wherever there is a suitable movable body, for it receives nothing from the movable body to cause it to be there. On the other hand, it causes motion in time, since the movable body is material. (Kepler, 1609/1992, p. 383)

Kepler believed (according to the conventional wisdom of the time) that light moved instantaneously from the Sun to light up the planets:

INSTANTANEOUS (AFFECT (light, Sun, planet, space))

However, he believed that the *vis motrix* required time to affect the motion of the planets. At a rough level, then, Kepler faced a mismatch between the candidate inference from light and his existing knowledge about the planetary motion:

INSTANTANEOUS (AFFECT (*vis-motrix,* Sun, planet, space))
TIME-OCCURRING (AFFECT (*vis-motrix,* Sun, planet, space))

Kepler (1609/1992) admitted the problem but suggested a rerepresentation "although light itself does indeed flow forth in no time, while this power creates motion in time, nonetheless the way in which both do so is the same, if you consider them correctly" (p. 383). His solution was to be more precise about the notion of AFFECT (influence, planet). For such an effect to occur, he reasoned, the influence must travel to the planet and interact with the planet somehow. Kepler suggested that travel is instantaneous for both kinds of influences (the *vis motrix* and light). However, whereas light need only interact with the surfaces of bodies to illuminate them (which, Kepler believed, can be done instantaneously), the *vis motrix* must interact with the body of the planet itself to cause motion, and this requires time. Thus, Kepler gained a partial identity by decomposing and rerepresenting the previously nonmatching statements. Instead of the nonmatching pair:

INSTANTANEOUS (AFFECT (*vis-motrix,* Sun, planet, space)
TIME-OCCURING (AFFECT (*vis-motrix,* Sun, planet, space)

he now had the partial match:

INSTANTANEOUS (TRAVEL (light, Sun, planet, space))
INSTANTANEOUS (PROMOTE (BRIGHTNESS (planet)))

INSTANTANEOUS (TRAVEL (*vis-motrix,* Sun, planet, space))
TIME-OCCURRING (PROMOTE (SPEED (planet)))

Alignable differences. Given a structural alignment, connected differences become salient. Kepler used these differences to deal with the question of whether the Sun's light and the motive power may in fact be the same thing (a reasonable question given the force of the analogy). He answered that they cannot be the same, because light can be impeded by an opaque blocker (e.g., during an eclipse), yet the motive power is not thereby impeded (otherwise motion would stop during an eclipse; see Figure 7):

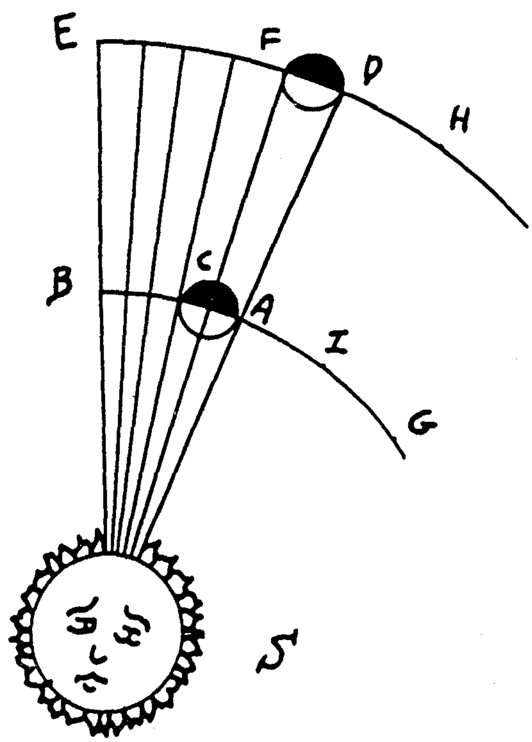

FIGURE 7 Kepler's depiction of the Sun's light radiating outward. From *Epitome of Copernican Astronomy* (p. 103), by J. Kepler, 1618/1969, New York: Kraus Reprint. Copyright by St. John's College Press, Annapolis, MD. Reprinted with permission.

> The analogy between light and motive power is not to be disturbed by rashly confusing their properties. Light is impeded by the opaque, but is not impeded by a body. ... Power acts upon the body without respect to its opacity. Therefore, since it is not correlated with the opaque, it is likewise not impeded by the opaque. (Kepler, 1609/1992, p. 392)

A more important alignable difference concerns the degree of decrease with distance. By the time of the *Astronomia Nova*, Kepler was clear about the fact that the concentration of light diminishes as the inverse square of distance from its source. He therefore held himself responsible for either mapping this fact into the target, or explaining why it should not be mapped. As it happens, he still required a simple inverse law for the *vis motrix*, because in his model the *vis motrix* directly caused the planetary motion.[17] As usual, he tackled this discrepancy head on and

[17]Kepler's dynamics was Aristotelian: He believed that velocity was caused by (and proportional to) the motive force (as opposed to the Newtonian view that forces cause *changes* in velocity). He held the belief of his time that the planets would cease to move if not pushed around that the Sun. Thus, he conceived of the motive force as acting directly to impart counterclockwise speed to the planets (rather than imparting inward acceleration, as in Newton's system). As Koestler (1963) noted, Kepler had made the insightful move of decomposing planetary motion into two separate components, but had reversed the roles of gravity and planetary inertia. Kepler thought that the planets' forward motion was caused by the Sun and their inward motion by magnetism specific to each planet. In the Newtonian system, the planets' inward motion is caused by the Su., and their forward motion by inertia specific to each planet.

produced, in the *Astronomia Nova,* a long mathematical argument that, because the *vis motrix* can cause motion only in planes perpendicular to the Sun's axis of rotation, the proper analog to the *vis motrix* is light spreading out not in a sphere around the Sun, but only in a plane. Thus, he justified the alignable difference that the concentration of *vis motrix* should decrease as a simple inverse of distance, even though the concentration of light decreases with inverse-square distance.

Restructuring. From what we have said so far, it appears that the *vis motrix* analogy may have contributed to Kepler's restructuring of his model of the solar system. It provided him with a structure from which to argue for a single causal "soul" in the Sun, rather than moving souls in each of the planets, and to the gradual mechanization of this soul to a power or force. The analogy may also have promoted the shift from crystalline spheres containing the planets to paths continually negotiated between the Sun and the planets. We return to this issue in the Discussion section.

RICHER ASPECTS OF THE ANALOGICAL PROCESS

The analogy between the *vis motrix* and light provides insight into some aspects of Kepler's conceptual change. However, this analogy is part of a much larger process. Kepler used several other analogies—including a sailor steering a ship, a balance scale, and a magnet. Some of these were used only once or twice, but at least one other was intensely developed and extended. This was an analogy between the *vis motrix* and magnetism, which Kepler used to reason out aspects of the phenomenon that the analogy with light could not explain. He modeled the planets and the Sun as magnets and tried to explain the inward and outward movements of the planets in terms of attractions and repulsions resulting from which poles were proximate.[18]

Although it is beyond the scope of this article to provide a full model of the development of Kepler's thought, at least three additional mechanisms are needed to capture his analogy process. First, a mechanism is needed to mediate between multiple analogies. For example, how did Kepler intersect the magnetism mapping (which explained why a planet varied in distance from the Sun in terms of alternating attraction and repulsion between two magnets that revolve around one another) with the light analogy? One computational approach may be found in Burstein's (1986) CARL, which combined different analogies to build a repre-

[18]By the time of the *Astronomia Nova* in 1609, Kepler had become familiar with the work of William Gilbert (*De Magnete,* 1600) and drew extensively on Gilbert's proposal that the Earth may function as a giant magnet. Kepler's analogy went further in applying this model to the Sun and planets. Interestingly, although Gilbert believed that the Earth rotated on its axis, he retained a Tychonic model in which the Sun and its satellite planets revolved around the Earth.

sentation of how a variable works. Spiro, Feltovich, Coulson, and Anderson (1989) have also traced the way in which multiple analogies are combined to produce a domain model.

A second mechanism needed is incremental analogizing. As new information about a domain is learned or brought in, the system must be able to extend the original mapping. It has been shown that participants are sensitive to a recent mapping, and will more quickly extend that mapping than create a new one (Boronat & Gentner, 1996; Gentner & Boronat, 1992). A few incremental analogical mapping models exist (Burstein, 1986; Keane, 1990), including an incremental version of SME (ISME), which can extend an analogy after the initial mapping has been made (Forbus, Ferguson, & Gentner, 1994). ISME draws further information from its long term knowledge to add to the working memory descriptions. It then remaps the analogy, building on the results of the initial mapping, thus enriching the overall analogical mapping. ISME can model the process of extended analogizing in problem solving, and we think it has promise for capturing creative extension processes.

Finally, it should be possible to embed these mapping processes in a process that can test the projected inferences of the mapping and make rerepresentations when needed. The system that comes closest to this is PHINEAS (Falkenhainer, 1990), which constructs physical theories by analogy with previously understood examples, by iterating through what Falkenhainer called a map–analyze cycle. In this cycle, PHINEAS starts with a qualitative description of a physical systems behavior and a set of domain theories. If it does not have an applicable theory to explain the new behavior, then it uses analogy to find an explanation. PHINEAS has an index of previously explained examples, arranged using an abstraction hierarchy of observed behaviors. PHINEAS selects and evaluates potentially analogous examples from this hierarchy and then uses SME to generate a set of correspondences between the novel behavior and the understood example. The explanation for the new behavior is then projected from the explanation of the old behavior. PHINEAS then tests this new explanation to make sure that it is coherent with its rules about physical domains. When there is conflict, Phineas can rerepresent some predicates. It then simulates the operation of the new theory to replicate the observed behavior.

DISCUSSION

Kepler used analogies both widely and deeply in his quest for an understanding of planetary motion. We have traced some of these analogies and modeled the processes using SME. We suggest that these analogies were instrumental to Kepler's conceptual change. To argue this point, we must justify some key assumptions.

Did Kepler Use Analogy in Thinking?

The frequent use of analogies in Kepler's texts is no guarantee that these analogies drove his conceptual change. He could have used analogy simply as a rhetorical device. Although there is no way to decide this issue definitively, there are reasons to believe that at least some of Kepler's analogies were instrumental in his thought processes. First, as discussed earlier, his major analogies were pursued with almost fanatical intensity across and within his major works. There are numerous detailed diagrams of base and target, long passages that spell out the commonalities, the inferences, and the incremental extensions, as well as alignable differences between base and target and Kepler's assessment of their import.

The open and inclusive character of Kepler's general writing practice offers a second line of encouragement for the belief that the extended analogies used in his text were actually used in his thought processes. Many of Kepler's commentators note the exceptional—at times even excessive—candor and detail of his scientific writing. Holton (1973), in noting that Kepler was relatively neglected among the great early scientists, stated:

> [Modern scientists are] taught to hide behind a rigorous structure the actual steps of discovery—those guesses, errors, and occasional strokes of good luck without which creative scientific work does not usually occur. But Kepler's embarrassing candor and intense emotional involvement force him to give us a detailed account of his tortuous process. ... He gives us lengthy accounts of his failures, though sometimes they are tinged with ill-concealed pride in the difficulty of his task. With rich imagination he frequently finds analogies from every phase of life, exalted or commonplace. He is apt to interrupt his scientific thoughts, either with exhortations to the reader to follow a little longer through the almost unreadable account, or with trivial side issues and textual quibbling, or with personal anecdotes or delighted exclamations about some new geometrical relation, a numerological or musical analogy. (pp. 69–70)

Kepler's writings are studded with personal comments that would be inadmissible in modern papers: "In what follows, the reader should overlook my credulity, since I am judging everything by my own wits." (Kepler, 1609/1992, p. 95); "Consider, thoughtful reader, and you will be transfixed by the force of the argument" (Kepler, 1609/1992, p. 290); and "And we, good reader, will not indulge in this splendid triumph for more than one small day ... restrained as we are by the rumours of a new rebellion, lest the fabric of our achievement perish with excessive rejoicing" (Kepler, 1609/1992, p. 290). On this last occasion, Kepler's foreboding proved correct, for he was then working on an egg-shaped orbit which proved a failure. When he at last rejected the egg in favor of the ellipse (hitherto used only as a mathematical approximation) he again reacted feelingly: "O me ridiculum!

[How ridiculous of me!]: As though the libration in diameter could not lead to the elliptical path" (as cited in Hanson, 1958, p. 83).

Kepler's (1609/1992) inclusiveness stemmed in part from his interest in "the roads by which men arrive at their insights into celestial matters" (as cited in Koestler, 1963, p. 261). In the introduction to the *Astronomia Nova,* he stated this agenda explicitly:

> Here it is a question not only of leading the reader to an understanding of the subject matter in the easiest way, but also, chiefly, of the arguments, meanderings, or even chance occurrences by which I the author first came upon that understanding. (Kepler, 1609/1992, p. 78)[19]

His writings include long sections detailing calculations made in pursuit of false assumptions; often the hapless reader is only informed afterwards that this effort has been misguided. A similar example occurred with the publication in 1621 of the second edition of his first book, the *Mysterium Cosmographicum* (1596/1981). Kepler's ideas had changed radically in the 25 intervening years, yet he chose not to rewrite but to leave the original text intact, adding notes that specified how and why his ideas had changed. The annotations in 1621 again reveal a zest for tracing the cognitive paths of discovery:

> The remaining hints at the truth that are offered by erroneous values, and which I quote everywhere, are fortuitous, but do not deserve to be deleted; yet I enjoy recognizing them, because they tell me by what meanders, and by feeling along what walls through the darkness of ignorance, I have reached the shining gateway of truth. (Kepler, 1621/1981, p. 215)

Even if some of Kepler's analogies are later additions, it seems likely that many of them formed a serious part of his journey.

A third indication that Kepler may have used analogies in thinking is that the sheer fecundity of his analogizing, suggests that analogy was a natural mode of thought for him. In the *Epitome of Copernican Astronomy* (1621/1969), he likened the Earth to a spinning top to answer why it revolves only in one direction. Later (Kepler, 1621/1969), he compared his celestial physics—in which planetary paths arise out of interacting forces—with the fixed firmament of the ancients:

> Here we entrust the planet to the river, with an oblique rudder, by the help of which the planet, while floating down, may cross from one bank to the opposite. But the ancient astronomy built a solid bridge—the solid spheres—above this river,—the

[19]However, Stephenson (1994a), although noting the "almost confessional style" of the *Astronomia Nova,* argued that Kepler shaped the book in this way to persuade astronomers of his new views.

> latitude of the zodiac—and transports the lifeless planet along the bridge as if in a chariot. But if the whole contrivance is examined carefully, it appears that this bridge has no props by which it is supported, not does it rest upon the earth, which they believed to be the foundation of the heavens. (pp. 182–183)

In pursuit of a causal model of the planetary system, Kepler analogized Sun and planet to sailors in a river, magnets, and orators gazing at a crowd, among other domains. Analogies are used for matters personal as well as public, playful as well as serious. For example, he wrote Fabricius in 1608, criticizing his (Fabricius's) model: "You say that geometry bore you a daughter. I looked at her, she is beautiful, but she will become a very bad wench who will seduce all the men of the many daughters which mother physics has borne me." During one of his frequent bouts of financial travails (as Imperial Mathematician, he held a post of high honor and intermittant renumeration) he wrote a friend: "My hungry stomach looks up like a little dog to the master who once fed it" (as cited in Caspar, 1993, p. 157).

A fourth reason to take Kepler's analogies seriously is that he himself did so. This is apparent from his explicit comments. For example, Vickers (1984) discussed how in the *Optics* (1904), Kepler treated the conic sections by analogy with light through a lens. Kepler justified this unorthodox treatment as follows:

> But for us the terms in Geometry should serve the analogy (for I especially love analogies, my most faithful masters, acquainted with all the secrets of nature) and one should make great use of them in geometry, where—despite the incongruous terminology—they bring the solution of an infinity of cases lying between the extreme and the man, and where they clearly present to our eyes the whole essence of the question. (pp. 149–150)

Kepler does not, however, consider analogy a substitute for logical proof: "Analogy has shown, and Geometry confirms" (p. 150).

What Did Kepler Mean by "Analogy"?

Alchemy, the dominant approach to natural phenomena in medieval Europe, was still a major presence during Kepler's life. The alchemists used analogies and metaphors in great quantity and relied on them as a guide to truth. Yet, from the viewpoint of current scientific practice, their use of analogy was wildly unconstrained. Many-to-one mappings and other structural inconsistencies were normal practice. Richness and ambiguity, rather than clarity and systematicity, were valued (see Gentner & Jeziorski, 1993, for a comparison of alchemical analogies with current scientific analogies). A final indication of how seriously Kepler took analogy was his sharp criticisms of this sort of analogizing, which stand in striking contrast to his normal collegial charity. In the *Harmonice Mundi* (1619) he strove to distinguish the proper use of analogy from the methods of alchemists, hermeticists, and others of that ilk: "I have shown that Ptolemy luxuriates in using comparisons in a poetical or rhetorical way, since the things that he compares are not real things in the heavens" (as cited in Vickers, 1984, p. 153). He is equally critical of the Theosophist Fludd:

> One sees that Fludd takes his chief pleasure in incomprehensible picture puzzles of the reality, whereas I go forth from there, precisely to move into the bright light of knowledge the facts of nature which are veiled in darkness. The former is the subject of the chemist, followers of Hermes and Paracelsus, the latter, on the contrary, the task of the mathematician. (as cited in Caspar, 1993, pp. 292–293)

A letter to a colleague in 1608 makes it clear that Kepler believed both that analogy is heuristic, not deductive, and that to be worthwhile analogies must preserve interrelationships and causal structure:

> I too play with symbols, and have planned a little work, Geometric Cabala, which is about the Ideas of natural things in geometry; but I play in such a way that I do not forget that I am playing. For nothing is proved by symbols ... unless by sure reasons it can be demonstrated that they are not merely symbolic but are *descriptions of the ways in which the two things are connected and of the causes of this connexion* [italics added]. (as cited in Vickers, 1984, p. 155)

Analogy and Conceptual Change

We have established that Kepler used analogy intensively and that he meant by *analogy* roughly what we do. We now come to the crucial questions: To what extent did Kepler undergo conceptual change? Was analogy instrumental in this change? Some instances of the change of knowledge accomplished over Kepler's lifework are as follows:

1. The planetary system changed from one governed by mathematical law to one governed by physical causality. As noted by Gingerich (1993): "Copernicus gave the world a revolutionary helio*static* system, but Kepler made it into a heliocentric system. In Kepler's universe, the Sun has a fundamental physically motivated centrality that is essentially lacking in *De revolutionibus.* We have grown so accustomed to calling this the Copernican system that we usually forget than many of its attributes could better be called the Keplerian system" (p. 333).

2. Formerly, the planets' orbits were conceived of either as crystalline spheres containing the planets or as eternal circles traveled by planetary intelligences. Kepler came to see them as paths continually negotiated between the Sun and the planets. As Toulmin and Goodfield (1961) noted: "One cannot find before Kepler

any clear recognition that the heavenly motions called for an explanation in terms of a *continuously* acting physical force" (p. 201).

3. Formerly, celestial phenomena were considered completely separate from earthly physics. From the start, Kepler extended terrestrial knowledge to astronomical phenomena. Over the course of his work, he projected analogies from the domains of light, magnetism, balance scales, sailing, and the optics of lenses, among others.

4. Formerly, the paths of the planets were composed of perfect circles of uniform speed. As early as the *Mysterium*, Kepler gave up uniform speed. Over the next several years, Kepler also gave up on circularity, shifting to the belief that the planets move in ellipses with the Sun at one focus, faster when closer and slower when further. This was a far more radical change than most of us can today appreciate: "Before Kepler, circular motion was to the concept of a planet as 'tangibility' is to our concept of 'physical object'" (Hanson, 1958, p. 4).

5. Early in Kepler's work, he proposed the *anima motrix* as the "spirit" in the Sun that could move the planets. Later, he called it the *vis motrix* or *virtus motrix*. This change could be considered an ontological change, an instance of what Thagard (1992) called "branch jumping" from animate to inanimate. It may alternatively be better analyzed as akin to Wiser and Carey's (1983) "degree of heat"—a case of an anima–mechanistic notion that differentiated or specialized into a mechanical notion. In either case, it marked a shift toward a mechanistic notion of the influence from the Sun.

6. Early in Kepler's work, the planets (on the Stoic account) were, or possessed, intelligences (Barker, 1991). Kepler struggled with the notion of a planetary intelligence throughout his career. It was not merely a question of persuading others that an animate spirit was superfluous. The more fundamental issue was that Kepler himself had to find a way of thinking about the planets that constrained and motivated their lawful interactions, although assigning to them the minimal possible degree of sentience. Lacking any established notion of force, Kepler developed these ideas by gradually stripping away from the notion of "planetary intelligence" more and more of its specific properties. For example, he asked himself whether he could explain the fact that planets go faster when nearer the sun by granting them only the ability to "perceive" the Sun's diameter. Thus, the notion of "mind" underwent a kind of progressive abstraction. Indeed, Stephenson (1987) suggested that in many cases Kepler's speculations about celestial minds were really hypothetical analyses of abstract physical constraints.

How should we characterize the magnitude of these changes? The term *conceptual change* is sometimes used to refer to any significant change in conceptual structure. However, it is often useful to distinguish three grades of change (see Thagard, 1992, for a more detailed discussion of degrees of conceptual change). *Belief revision* is a change in facts believed. *Theory change* is a change in the global knowledge structure. Conceptual change, in some sense the most drastic, is a change in the fundamental concepts that compose the belief structure. Conceptual change thus requires at least locally nonalignable or incommensurable beliefs (Carey, 1985). Of the six changes mentioned previously regarding Kepler's lifework, we suggest that most if not all of them would qualify as theory change, and that Points 2, 5, and 6 have a good claim to be full-fledged changes of concepts.

Was analogy instrumental in these changes or merely a rhetorical device for conveying them? As mentioned earlier, the first evidence for this point is the intense, closely reasoned extended analogy passages in Kepler's writing (of which we had space to show only a small portion). In his 1621 annotations to the *Mysterium Cosmographicum* is more direct evidence that Kepler commented explicitly on the role of analogy in his knowledge revision process. In the original version, in 1596, he had argued that there was "a single moving soul [*motricem anima*] in the center of all the spheres, that is, in the Sun, and it impels each body more strongly in proportion to how near it is" (Kepler, 1596/1981, p. 199). In 1621, he wrote:

> If for the word "soul" [*Anima*] you substitute the word "force" [*Vim*], you have the very same principle on which the Celestial Physics is established. ... For once I believed that the cause which moves the planets was precisely a soul, as I was of course imbued with the doctrines of J. D. Scaliger on moving intelligences. But, when I pondered that this moving cause grows weaker with distance, and that the Sun's light also grows thinner with distance from the Sun, from that I concluded, that this force is something corporeal, that is, an emanation which a body emits, but an immaterial one. (Kepler, 1621/1969, p. 201)

Kepler Compared With Current Practice

One final, indirect argument for the position that analogy was instrumental in Kepler's changes of belief comes from Dunbar's (1994) observations of microbiology laboratories. His observations of the research process suggest that analogy plays a role in working scientists' online creative thinking. Dunbar's question is, of course, quite different from ours. He asked what makes for change of knowledge in a laboratory, whereas we are asking what makes for change of knowledge in an individual. However, his observations are valuable because they serve as a partial check on whether the historical retrospective account we have devised has any online plausibility. There are some striking commonalities. The microbiology laboratories that showed the most progress were those that used a large variety of analogies. Dunbar's detailed analyses of transcripts show that analogies are taken very seriously by the successful lab groups; they are extended and "pushed" in group discussions. Another interesting commonality is that Dunbar found that creativity is best fostered by multiple analogies, each treated quite analytically, and this accords with our conclusions from Kepler's works. Dunbar also found that a variety of base domains is useful and this too is characteristic of Kepler. Indeed, Kepler seems to have profited considerably from a comparison of the magnet and light analogs with each other as well as with the intended target domain of the motive power of the Sun.

There are also commonalities not directly related to analogy. Attention to inconsistencies is another factor Dunbar singled out in his analysis of creative laboratories. Kepler worried about inconsistencies and was driven by them to keep pushing old analogies and in some cases to reject them. However, we would amplify Dunbar's analysis slightly, in that we consider attention to inconsistencies a motivator for conceptual change, rather than (like analogy) a process leading to conceptual change.

There are also some interesting differences between Dunbar's (1994) observations and Kepler's behavior. First, by far the vast majority of the analogies Dunbar observed are close literal similarities (what he called local analogies), typically involving the same organism type or species, similar diseases, similar genetic materials, and so forth. Kepler did in fact use close analogs on many occasions. He tested his reasoning about the Sun and the planets by applying that same reasoning to the Earth and the Moon, which he regarded as a basically analogous system. He used analogies between the planets on many occasions profitably, notably the analogy between Mars and the Earth, which was instrumental in his computing of Mars's orbit. However, it should be noted that these analogies appear closer now than they did in 1625. Moreover, in contrast to the microbiologists, Kepler used many distant analogies. This stems in part from the different historical stages of the domains. Kepler was forming the new science of astrophysics, more or less in the absence of a usable physics. Distant analogies were in many cases his only option. There was no literal similarity to be had. In contrast, in the microbiology laboratories that Dunbar (1994) studied, the historical moment is one of a well-developed (but not yet fully explored) framework in which many close analogies exist that are likely to be extremely fruitful. Thus, we suspect that whether close analogies or far analogies are used depends in part on the historical context. Local analogies are useful for filling in an established framework, whereas distant analogies are used for creating a new framework.

Analogy and Business as Usual

Analogical reasoning does not always promote conceptual change. In fact, we believe analogy and similarity are most frequently used to retrieve and use prior cases from memory without significantly altering conceptual structure. Similarity-based access to long-term memory most often produces mundane literal similarity matches (Gentner et al., 1993; Reeves & Weisberg, 1994; Ross, 1989). Previous research has shown that people use prior cases or problems to conserve reasoning time when attempting to solve a novel problem (Bassok & Holyoak, 1989; Gick & Holyoak, 1980; Holyoak, Koh, & Nisbett, 1989; Novick & Holyoak, 1991; Novick & Tversky, 1987; Ross, 1987; Ross, Ryan, & Tenpenny, 1989). Case-based reasoning researchers have modeled this behavior with a variety of computer simulations (e.g., Birnbaum & Collins, 1989; Collins, 1989; Hammond, 1986, 1989; Kolodner, 1992, 1993; Kolodner & Simpson, 1989; Schank, 1982).

There is even evidence that analogy can sometimes inhibit conceptual change (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi & VanLehn, 1991). Chi et al. found that participants who were poor physics problem solvers were more likely than good physics problem solvers to refer back to worked out examples. Using analogy in this manner is characterized by making concrete matches, often local attribute matches, without matching higher order structure: for example, searching for the same term in base and target algebra problems. We have characterized this kind of analogizing as *analogy as recipe:* The analogist uses analogy to avoid hard thought, as when we fill out our tax form by cribbing from last years.' However, such uses contrast sharply with Kepler's kind of analogizing, which we can characterize as *analogy as X-ray.* This use of analogy is characterized by pursuing an alignment, noting differences, articulating common systems, and in general allowing the comparison to illuminate the topics. As Kepler's writings show, such analogies can promote deep conceptual change.

Creativity, Structure, and Conceptual Change

There is a common intuition that creative thinking is characterized by fuzzy concepts and shifting conceptual boundaries. This intuition has manifested itself in dissatisfaction with symbolic systems, which have been criticized as rigid, brittle, and unable to show transfer beyond the tasks for which they were designed. Indeed, on the face of it, ideas like "fluid representations" and "flexible processes" seem highly congenial to creative processing and conceptual change. Yet, we suggest that the true case, at least for scientific creativity, is closer to the opposite: Creativity is best realized with deeply structured representations that are relatively firm, but that admit limited, structurally guided alterations. Fluid, dynamic models may be appropriate for capturing the kind of gradual generalizations that occur across close similarity matches, as in learning to recognize handwriting; however, these kinds of changes are often recombinant shifts of small, anonymous subclusters. The shifting subclusters may fail to result in noticeable differences. In contrast, in structured representations the presence of higher order relational structure can permit rapid conceptual change between significantly different belief structures.

Of all the proposed mechanisms of learning—including accretion, tuning and compilation (Anderson, 1982), differentiation (Wiser & Carey, 1983), and generalization—analogy is the only one that offers the possibility of a self-generated large-scale transformation of knowledge in a concerted period. For example, SME, a system that thrives on structured representations, behaves in what may be considered to be a creative manner when it notices cross-dimensional structural matches, projects candidate inferences, infers skolomized entities and incrementally extends its mapping. Falkenhainer's PHINEAS, which extended SME with rerepresentation capabilities, went even further in this direction. Although these models are still a long way from the goal, and the right combination of fluidity and rigidity may still be in the offing, we suspect that the route to modeling creative conceptual change lies through, not around, structure.

Analogy's power to reveal common structure and to import structure from a well-articulated domain into a less coherent domain makes it the foremost instrument of major theory change. Our analysis of Kepler's writings reveals that analogy was indeed his "most Faithful Servant."

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