Modeling Learning via Progressive Alignment using Interim Generalizations

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Abstract

There is ample empirical evidence that children can sometimes learn during the course of even a few experimental trials. We propose that one mechanism for this is the use of analogical generalizations constructed in working memory, producing what we call *interim generalizations*. Prior research suggests that such generalizations can be constructed when there is high similarity between closely spaced items. This paper describes how structure-mapping simulations can be adapted to capture this phenomenon, using automatically encoded stimuli. It is an advance over prior models in that it automatically detects when rerepresentation should be tried and carries it out to improve its performance.

Keywords: Analogy; Computational modeling; Symbolic Modeling; Cognitive Development.

Introduction

People have relational comparison capacities that seem to outstrip any other primate (Gentner, 2003; Penn, Holyoak & Povinelli, 2008). Yet young children are prone to focus on object matches rather than relational matches. The Relational Shift hypothesis (Gentner & Rattermann, 1991) suggests that this difference is due to a lack of knowledge about relational structures in younger children, and that as they learn more, they gain the ability to make more relational matches. Indeed, there is evidence that, under the right conditions, preschool children can learn to carry out relational matches.

In one such study, modeled here, Kotovsky & Gentner (1996) explored children's performance on comparison tasks involving simple higher-order patterns, such as symmetry and monotonic increase (Figure 1). In each triad, the top figure is the standard, and the bottom two figures are the choices from which a participant must pick. One choice always has the same higher-order relationship between its entities as does the standard, while the other has the same entities as the relational choice, but permuted so that the relationship does not apply. The triads in Figure 1 illustrate the 2x2 manipulation, namely the polarity (same or opposite) of the higher-order relation and the dimension (size or brightness) over which the relationship holds. Children were asked to choose which one of bottom choices was most like the top one. No feedback was given at any time. However, some easy high-similar triads were provided as check trials.

The Relational Shift hypothesis predicts that older children will do better than younger children, and that all children will do better when there are lower-order commonalities supporting the higher-order commonalities. The results were consistent with these predictions: 4 year olds performed below chance on all but the same dimension/same polarity stimuli, where they were above chance. By contrast, 6 year old and 8 year old children were able to see the relational pattern to some degree without the support of first-order relational overlap, but better with it. The cross dimension/opposite polarity case was the hardest condition, even for eight year olds. Yet some children discovered this match over the course of the study. As Kotovsky & Gentner (1996) remark:

> *"The emerging appreciation of relational commonality can be seen in this comment by an eight year old, who after struggling with her first several cross-dimension matches, then excitedly articulated a startlingly apt description of relational similarity: "It's exactly the same, but different!" She proceeded to choose relationally for all the remaining triads"*

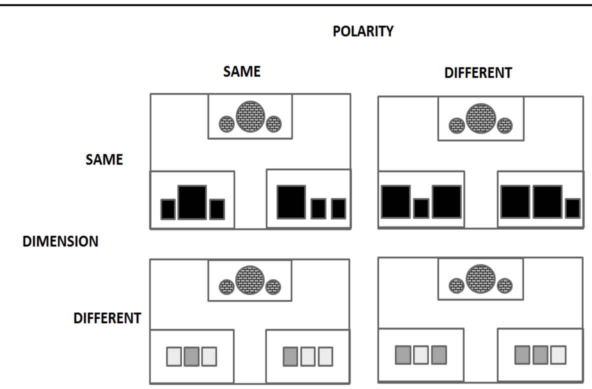

Figure1: An example of four types of triads for a size symmetry standard

How can we explain such learning within less than 20 trials, without feedback? It requires that a child be able to detect that they do not know a good answer. There is informal evidence for this in that children in the study often puzzled over the cross-dimensional triads, saying things like "A dark one and a big one make daddies. The other one has two twins and a daddy on the side." Children further need to figure out ways to *rerepresent* the stimuli so that the choice becomes clear. This rerepresentation process is aided by the experience of comparing and aligning relational structure across trials, as Kotovsky and Gentner showed in a second study. In that study, 4-year-olds were given a progressive alignment sequence: first 8 same-dimension (and same polarity) triads, which were relatively easy to align; and then 8 cross-dimension triads (also same-polarity). A control group received 8 initial size-change triads (so that they did not experience easy alignments over the saturation dimension. The progressive alignment group performed better on the subsequent cross-dimensional triads than did the control group). This suggests that successfully aligning the same-dimension triads led children to see the higher-order patterns that they had formerly missed—that is, to rerepresent the stimuli.

This paper describes a computational model of this phenomenon. We begin by summarizing the models of analogical processes that we are building on. Then we describe the idea of *interim generalizations,* that is, analogical generalizations constructed within working memory. Prior research (Gentner, Loewenstein & Hung, 2007) has suggested that such generalizations can be formed when very similar stimuli are shown in quick succession. Here, interim generalization serves as a rapid rerepresentation mechanism by focusing attention on relations that have proven beneficial in prior comparisons. Two simulation studies are described, showing how these ideas can account for Experiments 1 and 2 of Kotovsky & Gentner (1996). We close with related and future work.

Background

Our model is built on Gentner's (1983) structure-mapping theory. We build on existing models of matching and generalization, so we describe each in turn.

The Structure-Mapping Engine (SME; Falkenhainer et al. 1989) models the structural alignment process of comparison. It takes as input two structured descriptions, the base and target, which can contain entities, their attributes, relations, and higher-order relations. SME produces one or more mappings as its output, via a greedy merge process (Forbus et al., 1994). Each mapping consists of three things: (1) A set of correspondences, which describe what goes with what (i.e. what entities and statements match) (2) A score that provides a numerical estimate of similarity, and (3) a set of candidate inferences which describe how connected but non-overlapping structure can be projected from the base to the target (or in reverse, from the target to the base), according to the correspondences of that mapping.

The similarity score can be normalized to be in the range [0, 1]. There are two distinct normalization strategies, each measuring something different. The first is to normalize by the self-matching score of the base (or target), i.e. what SME would compute for matching that description with itself. This value measures how close the target (or base) is to the representation used for normalization. The second strategy is to normalize by the average self-matching scores of both the base and target. This value measures how close the two representations are to each other. As explained below, both types of scores are used in this model.

The Sequential Analogical Generalization Engine (SAGE; McLure et al 2010) provides a model of analogical generalization. For each concept being learned, SAGE maintains a *generalization context* which represents its current model of that concept. Each generalization context contains a set of generalizations and unassimilated exemplars. Generalizations are probabilistic structured descriptions, where the probability for each statement is the frequency with which assimilated exemplars contain a matching statement. We propose that there are two kinds of generalization contexts. *Persistent* generalization contexts are stored in long-term memory, and are used to accumulate models over substantial periods of time (e.g. learning words (Lockwood et al 2008), grammar (Taylor et al, 2011) and conceptual change (Friedman & Forbus, 2009)). *Interim* generalization contexts are part of working memory, and are used for short-term, within-task comparisons and learning (Day & Gentner, 2007). In interim generalization contexts, only a small number of descriptions are maintained and retrieval of them is based on similarity, biased via recency. Given a new example being processed, the most relevant abstraction from the generalization context is chosen on the basis of its similarity score normalized with respect to the abstraction, and modulated by recency.

Modeling the Forced Choice Task

We assume that children have default encoding strategies, and that given a forced-choice task, they compare the standard to each of the choices, with the standard serving as the base. The choice with the highest base-normalized similarity score is then selected as their choice, since they are seeking the closest to the standard.

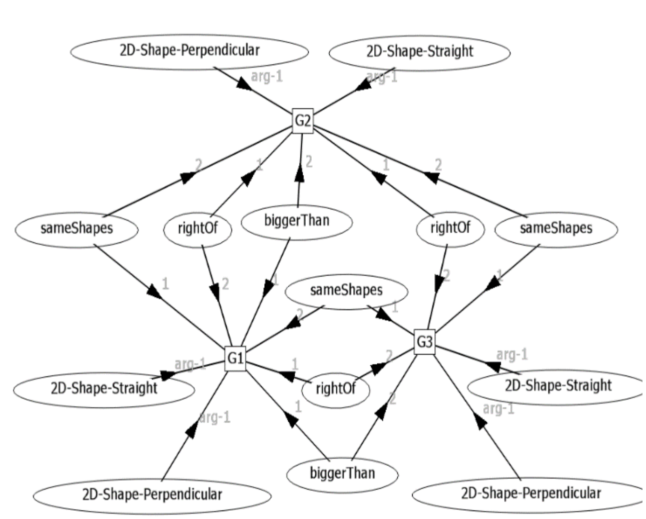

Figure 2: Graphical representation of the relational choice of the Different Dimension/Same Polarity saturation-change triad

But what if the two scores are very close? That is the criterion used to trigger rerepresentation efforts: An encoding that does not provide a clear choice is not adequate. Such encodings arise in part because the child's default encoding strategies may produce extra, irrelevant information that makes it harder for any relational comparison to emerge. We further assume that when a choice is clear, a generalization based on that comparison is added to the interim generalization context. Given a subsequent new task, if a generalization is retrieved for a portion of the stimulus, then only the overlap between the stimulus and the retrieved generalization is kept. This provides filtering, to make relevant structure more apparent based on experience. To illustrate, Figure 2 illustrates the (automatically produced) initial representation of one of the choices in a triad, while Figure 3 shows what remains after filtering via an interim generalization.

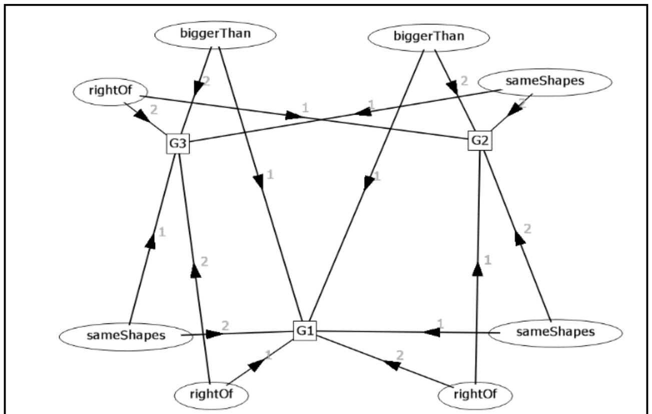

Figure 3: Graphical representation of the relational choice after rerepresentation based on reminding.

Filtering based on prior generalizations is one simple form of rerepresentation. Another, as proposed by Gentner et al. (1995), involves recognizing that dimension-specific comparative relations can be recast as a combination of functions that denote dimensions and a domain-independent comparative relation. For instance,

```
(darkerThan A B)
(biggerThan C D)
```

might be rerepresented as

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(greaterThan (Darkness A) (Darkness B))
(greaterThan (Area C) (Area D))
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where **`Area`** refers to the 2D area of the depicted entity. This greatly improves the match, because structure-mapping permits non-identical functions to align (here, **`Darkness`** and **`Area`**) when doing so would support a larger relational structure matching. Figure 4 illustrates how this changes the example of Figure 3.

What signal should be used to determine when—and which—rerepresentations are performed? That there is a problem making a decision is clear when the scores for the two choices are very close, as noted above. But there are many possible rerepresentations between any two descriptions. To decide which pair within the triad to focus on, our model uses the average self-matching score, since that measures which pair is more promising. Particular rerepresentation opportunities are spotted and carried out via techniques described in (Yan et al. 2003). The base-normalized score is then recomputed based on the new representations, and if there is now a clear difference, the most similar choice is selected. Otherwise, rerepresentation continues until the model runs out of techniques to try or a resource bound is hit.

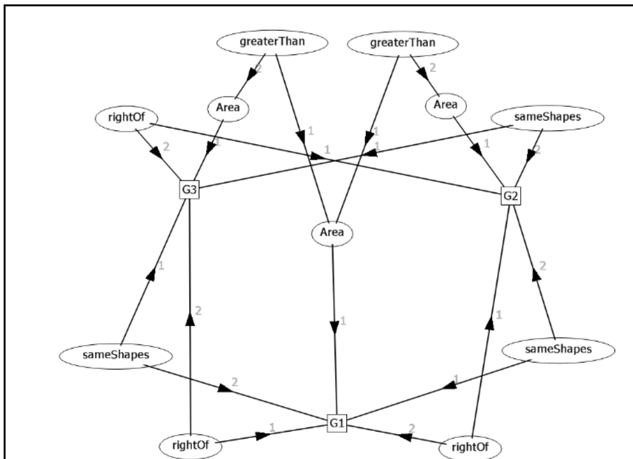

Figure 4: Graphical representation of the relational choice after rerepresentation based on candidate inferences.

Experiments

We used the Companion cognitive architecture (Forbus et al 2009) to conduct two simulation experiments, corresponding to the first two experiments in Kotovsky & Gentner (1996). Our goal was to model the behavior of four year olds in the studies. That is, in the first experiment, it should do well only on same dimension/same polarity triads, and in the second experiment, given progressive alignment, it should learn to do well on different dimension, different polarity triads.

In the original study, two shapes (circles and squares), two higher order relations (symmetry and monotonicity) and two dimensions for the standards (size and color) were used. This provided the basis for 16 relational patterns, 8 for each polarity. The opposite polarity cards were only used as relational choices, thus providing 8 standards. A stimulus that is a standard in one triad will appear as a relational choice in another. 16 non-relational choices were constructed by permuting the objects in the relational choices. We constructed 32 cards as PowerPoint slides and used CogSketch (Forbus et al 2011), an open-domain sketch understanding system, to generate qualitative visual and spatial representations. CogSketch is useful for this purpose because it can provide automatic encoding for experiments (e.g. Kandaswamy & Forbus 2012), with a representation vocabulary that has proven useful in modeling human visual problem solving (Lovett & Forbus, 2011). Figure 2 illustrates a portion of the representation created for two symmetry standards of different dimensions.

Simulation Experiment 1

Recall that the four types of triads (ordered in terms of predicted difficulty) are:

1. Same dimension/same polarity (SDSP)
2. Same dimension/different polarity (SDDP)
3. Different dimension/same polarity (DDSP)
4. Different dimension/different polarity (DDDP)

We created two ordered sets of 16 triads grouped by polarity, shuffled so that there would be no more than two of the same triad types consecutively, as in Experiment 1 of Kotovsky & Gentner (1996). In particular, as in that study, same-dimension triads (like the top left triad in Fig. 1) and cross-dimension triads (bottom left, Fig. 1) were mixed semi-randomly across the study. We evaluated our model on the two sets. The model performs the triads task sequentially following the determined order. The model uses three parameters. The assimilation threshold (0.95) is used by SAGE to determine when to assimilate examples into generalization. It is also applied during the reminding phase to choose the most similar generalization. The *rerepresentation threshold* (0.55) controls when a mapping between a base and a target looks promising enough to attempt rerepresentation. The *size limit* (5) determines the maximum number of items in the interim generalization context. These values were set based on pilot experiments. When the model has no clear choice, it does not make a decision, unlike the children, who always had to make a choice. (Importantly, the children were not given feedback as to whether their choices were correct or not.) The Kotovsky & Gentner experiments measured the proportion of relational responses. Table 1 shows the results for four year olds along with the model's responses. As noted above, the correct choice is always the relational choice, so the children were above chance only for the SDSP case.

Table1: Proportions of choice types for Experiment 1

	Children Reln %	Model Reln %	Model NonReln%	Model No-choice
SDSP	68%	100%	0%	0%
SDDP	49%	0%	87.5%	12.5%
DDSP	49%	37.5%	0%	62.5%
DDDP	48%	12.5%	12.5%	75%

The results of the model are qualitatively consistent with the children's behavior. First, the SDSP cases are easiest. The model gets 100% of these correct because the automatic encoding process, using CogSketch, is deterministic and uniform, whereas children (68% correct) are likely to vary more in their encodings. Second, when the no-choice model answers are randomly distributed between the two possible choices, the model is at chance for DDDP, somewhat better than chance for DDSP, and far worse than chance for SDDP. In the SDDP stimuli, there is sufficient relational overlap between even a non-relational standard to make the base-normalized comparison scores different enough to satisfy the system that it has a reasonable answer. We suspect that increasing the required difference in similarity between the two alternatives would eliminate this behavior. In the DDSP case, while the same dimension triads were not consecutive, they were sometimes close enough that occasionally interim generalizations were getting created. This suggests that our model can form interim generalizations a bit more readily than children do.

Table 2: Order of triad pairs in progressive alignment condition as in Kotovsky & Gentner 1996

Dimension	Dimension of Standard	High Order Relation
same	size	monotonic-increase
same	size	symmetry
same	color	symmetry
same	color	monotonic-increase
cross	size	symmetry
cross	color	symmetry
cross	size	monotonic-increase
cross	color	monotonic-increase

Simulation Experiment 2

Experiment 2 was designed to test the Progressive Alignment hypothesis, i.e. that children who first received highly similar (i.e., highly alignable) closely spaced trials could then do tasks that were beyond them previously. The stimuli consisted of only same polarity triads. There were two conditions.

1. Experimental condition: Eight same dimension triads followed by eight cross dimension triads. The same dimension triads consisted of both saturation-change and size-change triads. To encourage progressive alignment the triads were ordered as shown in Table 2. The children received two of each type.
2. Control condition: Same as in the progressive alignment condition, but (as in the Kotovsky & Gentner 1996 study) the eight same dimension triads are all size-change triads, with no saturation-change triads.

The procedure is the same as in Simulation Experiment 1. The proportions of relational choices are shown in Table 3 and Table 4 respectively. Consistent with the human pattern, the model was extremely accurate on the same-dimension triads in both conditions. Also consistent with the human data, the model was far more accurate on the subsequent cross-dimensional triads in the experimental (progressive alignment) condition than in the control condition. In the progressive alignment condition, the model formed interim generalizations for both size-change and saturation-change. This drove rerepresentation, leading to relational choices being preferred. By contrast, in the control condition, the model did not form any interim generalizations involving saturation-change.

These results are qualitatively consistent with the results of Kotovsky & Gentner (1996). Like the children, the model performed better on cross-dimensional triads after progressive alignment on both dimensions than after progressive alignment only on the size/area dimension. However, there are some discrepancies. The simulation performs too well, especially on the same-dimension triads. The model's high degree of uniform encoding, and aggressive use of rerepresentation, appears to be going beyond what the children are doing.

Table 3: Proportions of choice types for Experiment 2, Experimental Condition

Dimension	Relational choice	Non-relational choice	No choice
same	100%	0%	0%
different	100%	0%	0%

Table 4: Proportions of choice types for Experiment 2, Control Condition

Dimension	Relational choice	Non-relational choice	No choice
same	100%	0%	0%
different	50%	0%	50%

Related Work

The DORA model (Doumas & Hummel, 2010) has been used to model learning of 3D geon representations, starting with synthetic 3D stimuli. Like our model, they extract patterns of overlapping relations and apply them to subsequent stimuli, although these are more like SAGE's persistent generalizations, since subsequent trials are intended to model the course of development over multiple years, versus within a single experiment, as in our model.

The interaction of perception and cognition has been heavily emphasized in the work of Hofstadter's group, e.g. (Mitchell, 1993; French, 1995). Their models have focused on highly interleaving these processes, but building domain-specific models of them, whereas our model does less interleaving, but its components are domain-general and our representations are calibrated against human visual problem solving in multiple tasks (Lovett & Forbus, 2011; Lovett et al 2007).

There has been increasing interest in rerepresentation in analogy research. Kokinov et al. (2009) examined it in the context of rapid visual perception, arguing that it could cause shifts to an alternate model retrieved from long-term memory. Davies & Goel (2008) used rerepresentation in visual problem-solving, focusing on the classic Duncker radiation problem. Both share our concern with understanding how analogy interacts with perception, but neither of their efforts have been concerned with modeling rapid learning.

Discussion

This paper has shown that a model based on structure mapping and interim generalizations can simulate the progressive alignment effects on 4 year olds found in (Kotovsky & Gentner 1996). We conjecture that the use of non-discriminating similarity comparisons to drive rerepresentation, and the use of interim generalizations to focus on relevant relational structure, are commonly used in learning and reasoning.

There are several avenues of future work to explore. First, even though the model's behavior is qualitatively consistent with that of four year olds in the experiment, there are differences. Our model currently does not do several things that children probably do during the course of development. For example, it does not change its encoding strategy to shift to a more abstract comparative relation, nor does it introduce a new higher-order relationship (symmetry or monotonicity) to encode the newly-discovered pattern. Since the model's behavior is qualitatively consistent with 4 year olds without these operations, it may be that the children are not doing this, but there is insufficient evidence to tell one way or the other. Second, we note that the simulation's responses are uniform and performance improves rapidly, whereas children exhibit a wider range of behavior. For example, even the 8 year olds in the original experiment were not at ceiling in this task. Adding the operations suggested above, and expanding the range of rerepresentation operations available, as well as looking for rerepresentation opportunities in both pairs, would widen the search space of the model and perhaps capture the more gradual improvement trajectory of children. Finally, to explore these questions we plan to test the model with a wider range of forced-choice tasks, to better triangulate what combinations of processes and representations can better explain these aspects of cognitive development.

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