Mathematical Aspects of Qualitative Modeling

Peter Struss
Technical University of Munich
and
OCC'M Software GmbH

Qualitative Modeling

The Goal:

- Models that make the essential and possible distinctions only

Yes!
"Small" is beautiful ...

Reflects

- Goals
  - What has to be distinguished?
- Input
  - Observations, hypotheses
  - What can be distinguished?
  - Partially specified information
  - Noise
  - Limited knowledge

For Example

- Position of the accelarator pedal is above the idle threshold
- Increase in pressure increases negative acceleration of the wheel
- A leakage in the control pipe is not relevant to the turbo pressure

Reflects

- Goals
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Everything is Real ...

- Position of the accelarator pedal is above the idle threshold
- Increase in pressure increases negative acceleration of the wheel
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One Possible View:

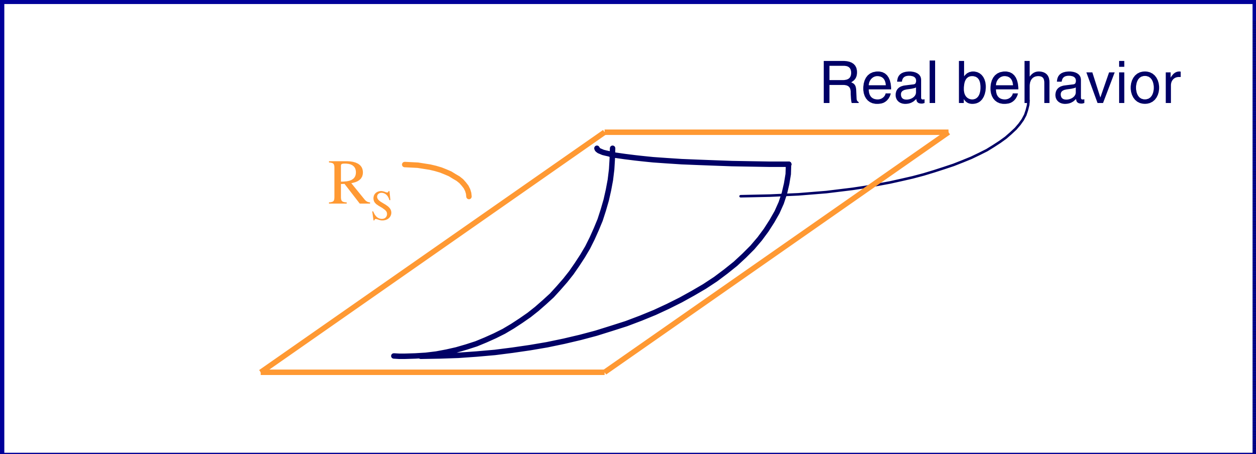
- There is a fine-grained (real-valued) model that describes the system accurately
- Base Representation:
  - $DOM_0(\underline{v}_S)$
  - $= \mathbb{R}^n$
- Model abstraction:
  - $\text{Model}_0 \rightarrow \text{Model}_1$

Relational Models

- Independently of the syntactical form:
- What set of states is allowed by the model?
- $R_S \subset DOM(\underline{v}_S)$

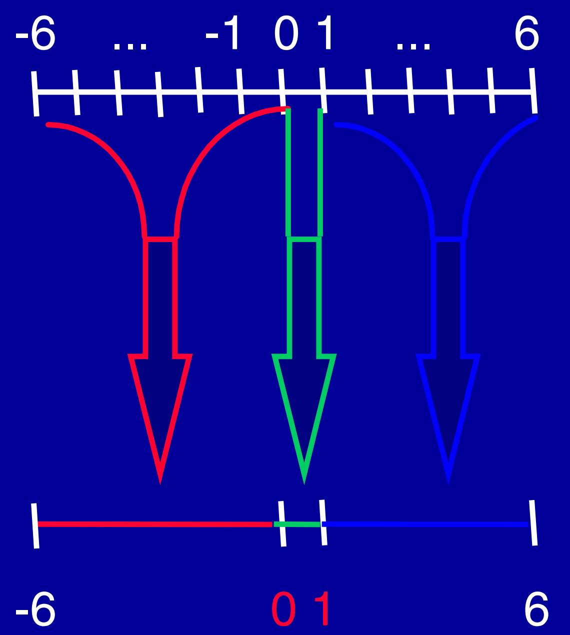
A valid model of a behavior:

- R_S covers all states of the behavior
- $\forall s \in SIT \;\; Val(\underline{v}_S \,, \underline{v}_{S,0}, s) \;\Rightarrow\; \underline{v}_{S,0} \in R_S$



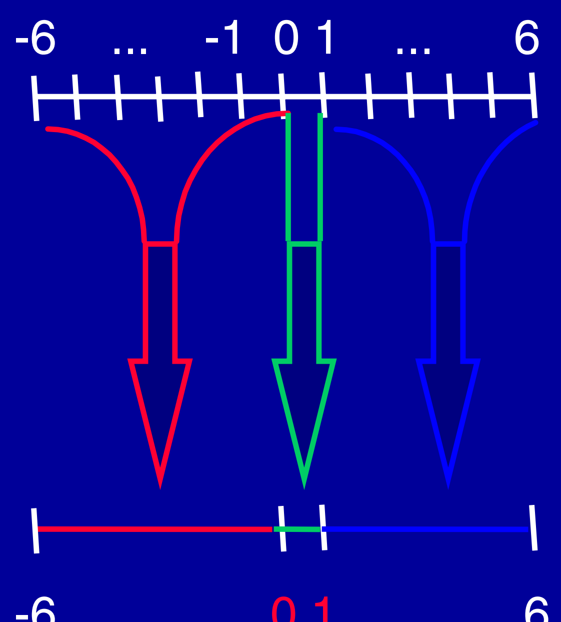
Domain Abstraction

- Position of the accelarator pedal is above the idle threshold
- Increase in pressure increases negative acceleration of the wheel
- A leakage in the control pipe is not relevant to the turbo pressure



- Aggregate values leading to the same class of behaviors
- e.g. between “landmarks”: intervals

Domain Abstraction - Formally



General:

- $\tau_i: DOM_0(v_i) \rightarrow DOM_1(v_i)$

Aggregation of values:

- $\tau_i: DOM_0(v_i) \rightarrow DOM_1(v_i) \subset P(DOM_0(v_i))$

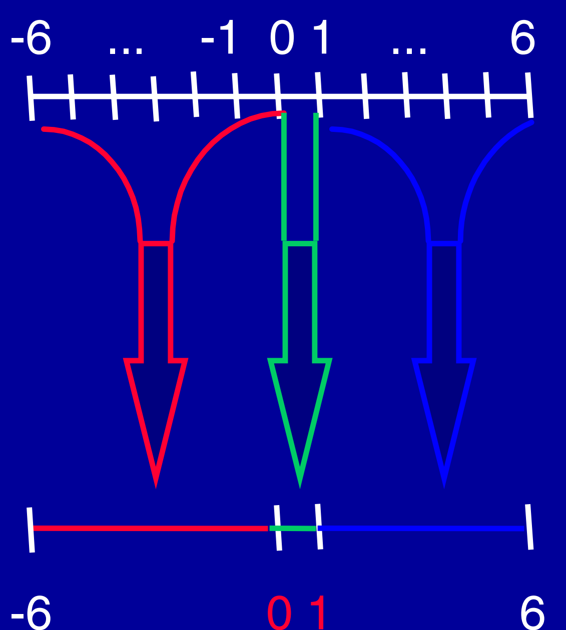
(Generalized) Intervals:

- $\tau_i: IR_\infty \rightarrow DOM_1(v_i) \subset I(IR_\infty)$

Real landmarks and intervals between them:

- $L \subset IR_\infty$
- $\tau_i: IR_\infty \rightarrow DOM_1(v_i) \subset I_L(IR_\infty)$

Symbolic Landmarks



Totally or partially ordered symbols:

- $(L, \leq_L) \subset IR_\infty$
- $I_L: \{(l_i, l_j), (l_i, l_i) \mid l_i, l_j \in L\}$

Interpretation over IR_∞ :

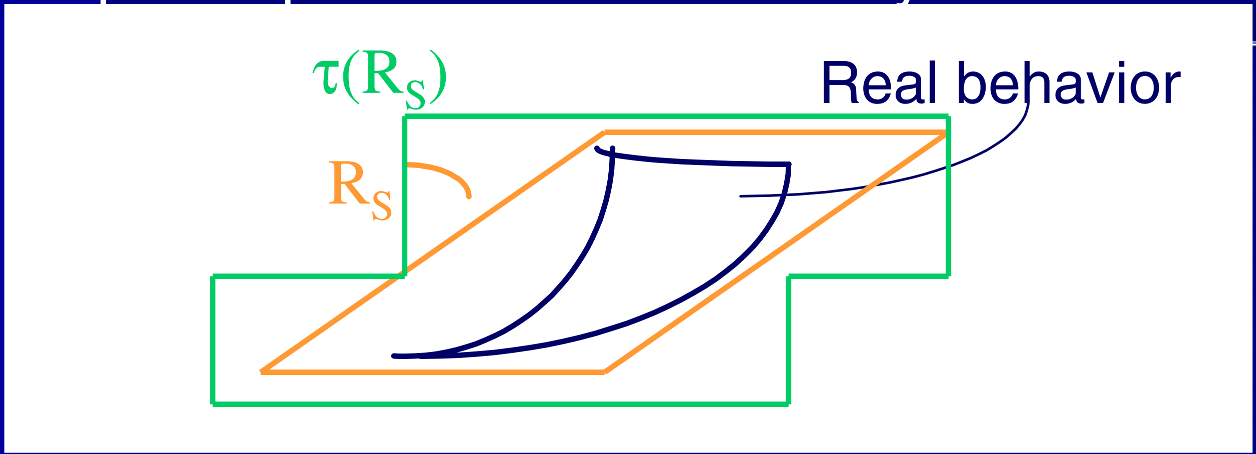
- $\iota: L \to IR_\infty$
- $\iota: L^2 \to I(IR_\infty)$
- $l_i \leq_L l_j \;\to\; \iota(l_i) \leq_{IR} \iota(l_j)$

Model Abstraction

- Domain abstraction
- $\tau: DOM_0(v_S) \rightarrow DOM_1(v_S)$
- induces model abstraction
- $R_S \subset DOM(\underline{v}_S) \rightarrow \tau(R_S) \subset DOM_1(v_S)$

Theorem:

- If the base relation is a valid model of a behavior
- then so is its abstraction
- Important for consistency check



States and Behaviors

- R_S and $\tau(R_S)$ describe the set of states
- What about behaviors?
- = sequences of states

- Usually
- the constraints on state transitions are general
- i.e. apply to all models
- Reflecting continuity, integration, derivative relations

Theorem

- For two models $model_1$, $model_2$
- $STATES(model_1) = STATES(model_2)$
  iff $BEHAVIORS(model_1) = BEHAVIORS(model_2)$

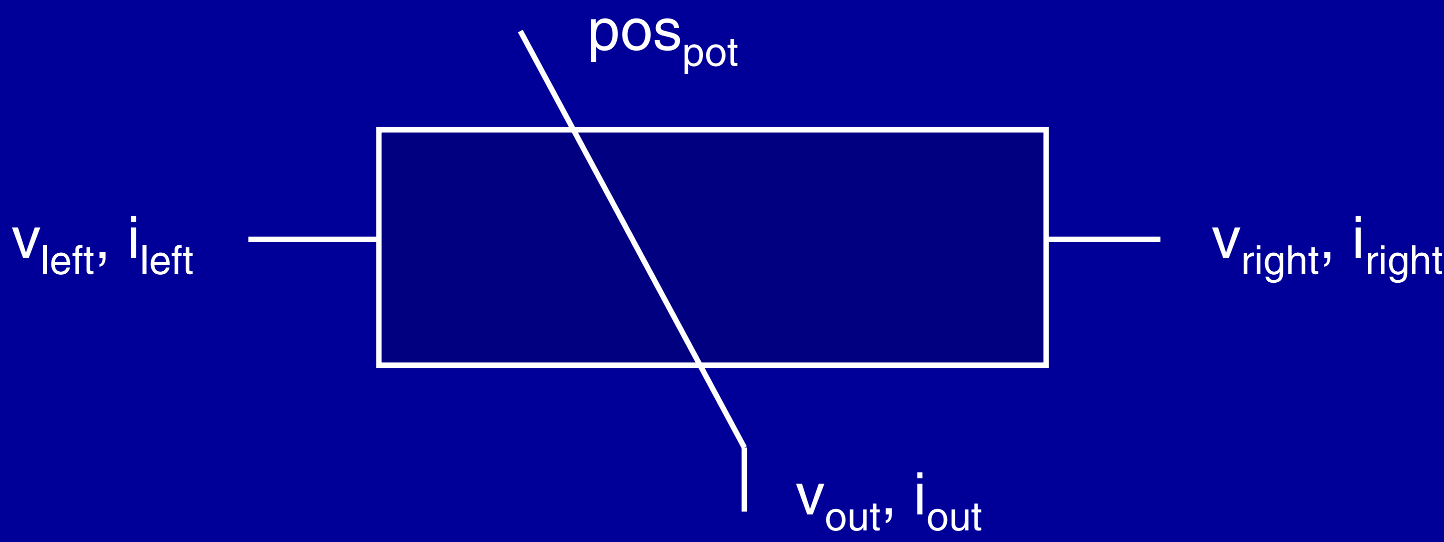
Transformation of Model Descriptions

- But:
- we do not want to compute R_S and $\tau(R_S)$

Instead

- Transform e.g.
- (Ordinary differential) equations into model descriptions over $\tau(DOM(\underline{v}_S))$ and solve them
- How are these solutions related to $\tau(R_S)$?
- Soundness and completeness?

A Simple Exercise: Qualitative Model of a Potentiometer

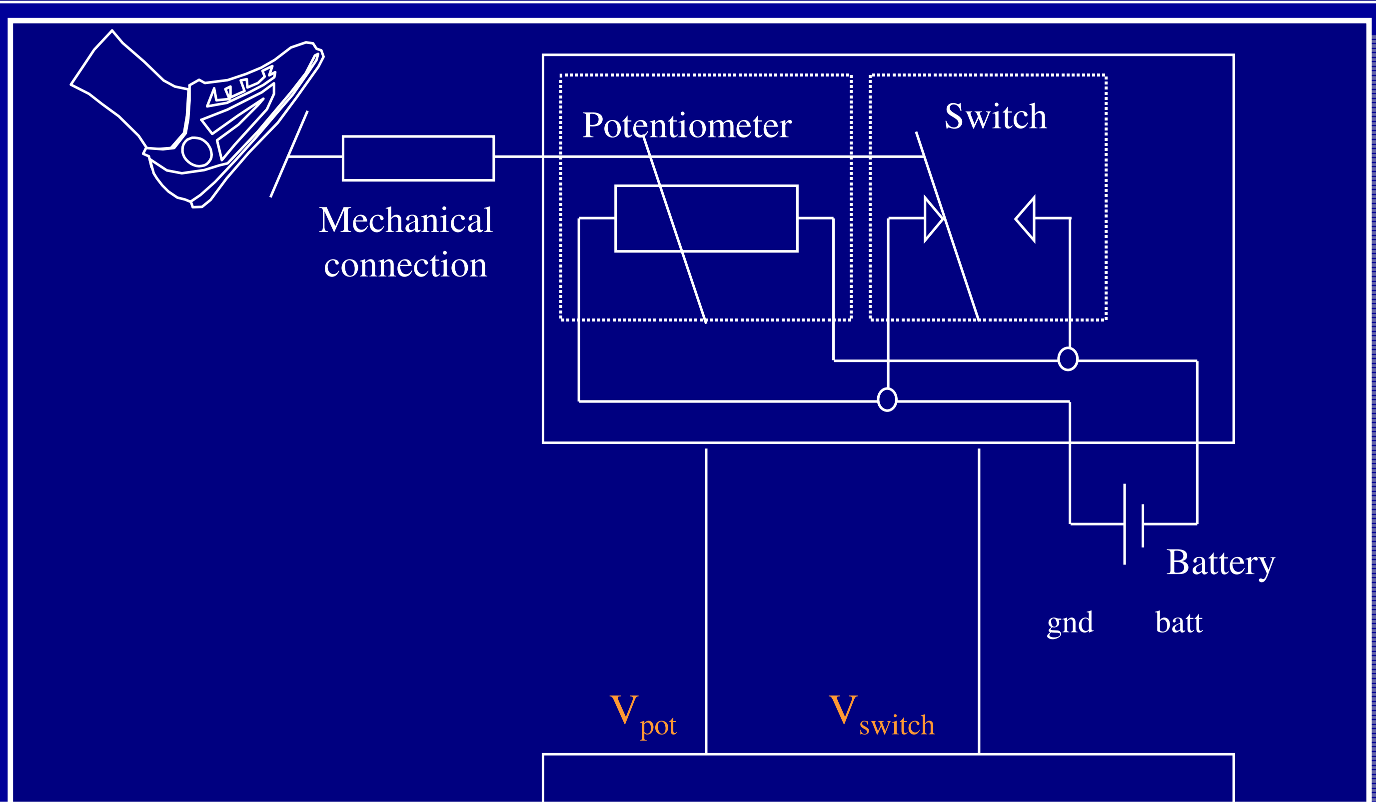


Library

- **Potentiometer**

pos_{pot}	i_{out}	v_{left}	v_{right}	v_{out}	…
0	0	{gnd, betw, batt}	{gnd, betw, batt}	$= v_{left}$	
$pos_{p,max}$	0	{gnd, betw, batt}	{gnd, betw, batt}	$= v_{right}$	
$(0, pos_{p,max})$	0	gnd	gnd	gnd	
$(0, pos_{p,max})$	0	batt	{betw, batt}	betw	
$(0, pos_{p,max})$	0	batt	batt	batt	
…					

The Pedal Position Sensor

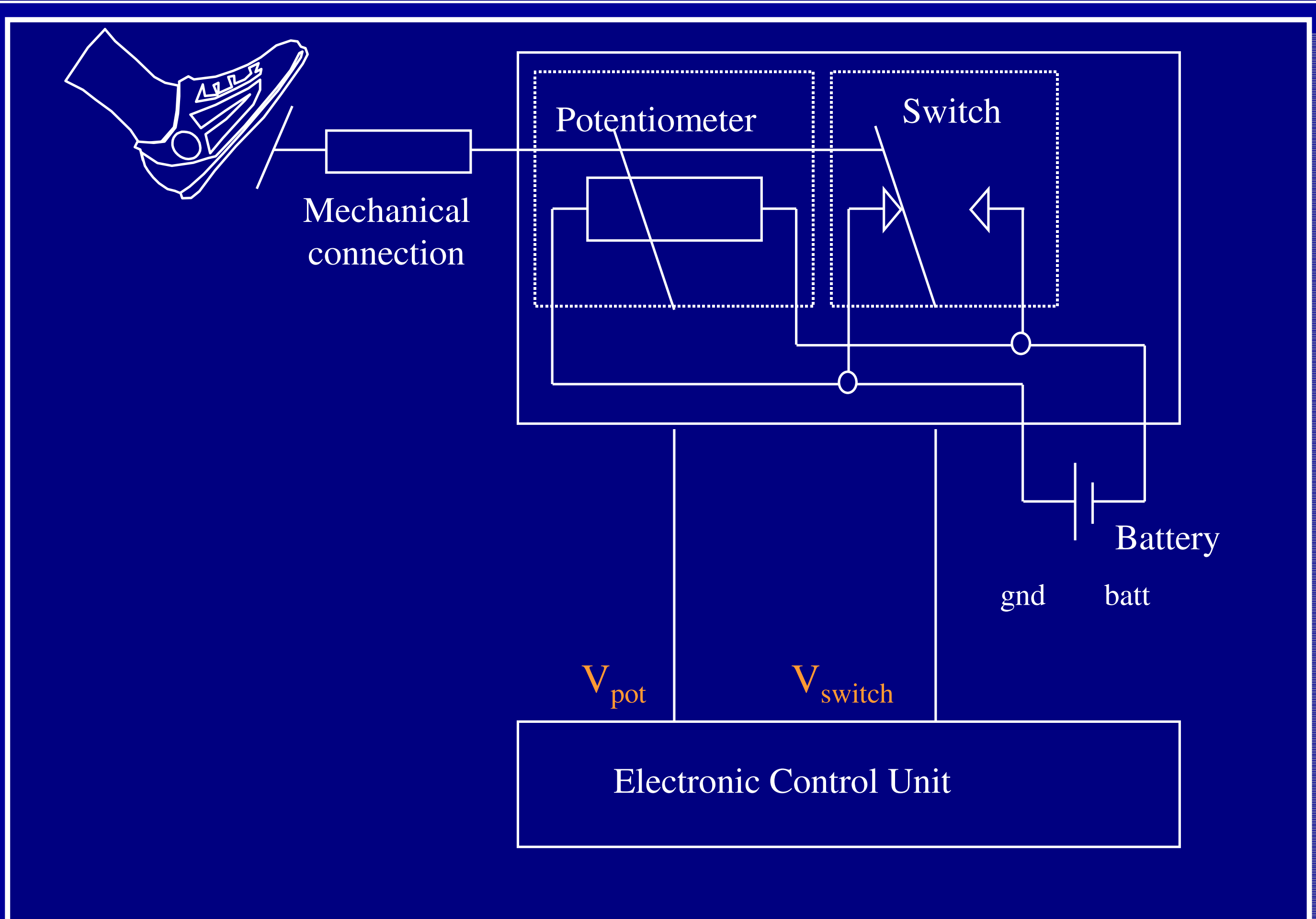


Library

- **Switch**

pos_{switch}	state	V_{left}	V_{right}	V_{switch}
$[0, pos_{s,med}]$	left	{gnd, betw, batt}	{gnd, betw, batt}	$= v_{left}$
$(pos_{s,med}, pos_{s,max}]$	right	{gnd, betw, batt}	{gnd, betw, batt}	$= vl_{eft}$

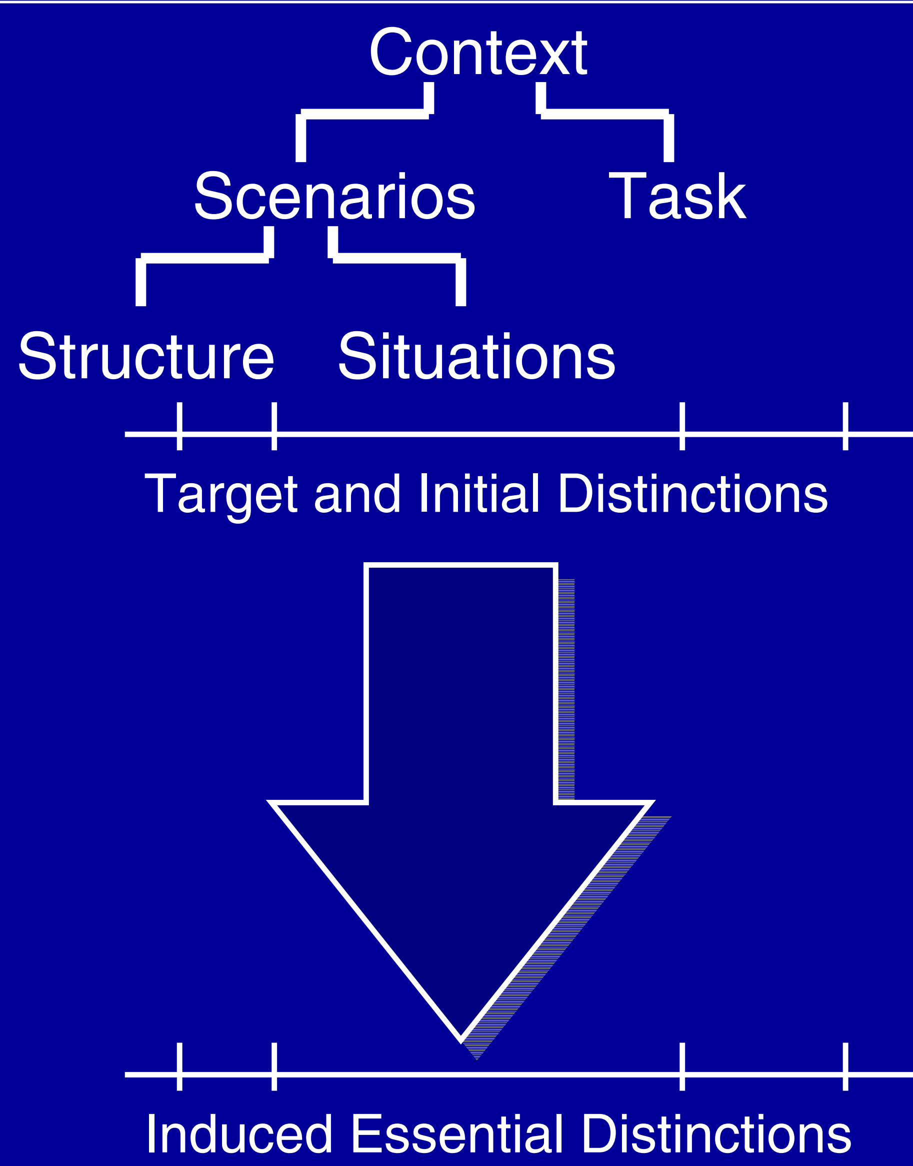
The Problem



- Generate a (qualitative) model that is appropriate for a particular device and task

Elements of the Context

- **Task and situation**:
  - requirements for distinctions on certain variables
  - possible distinctions
- **Structure and behavior models**: induce distinctions on other variables
- Required: a “base model”



Model-based Qualitative Model Abstraction

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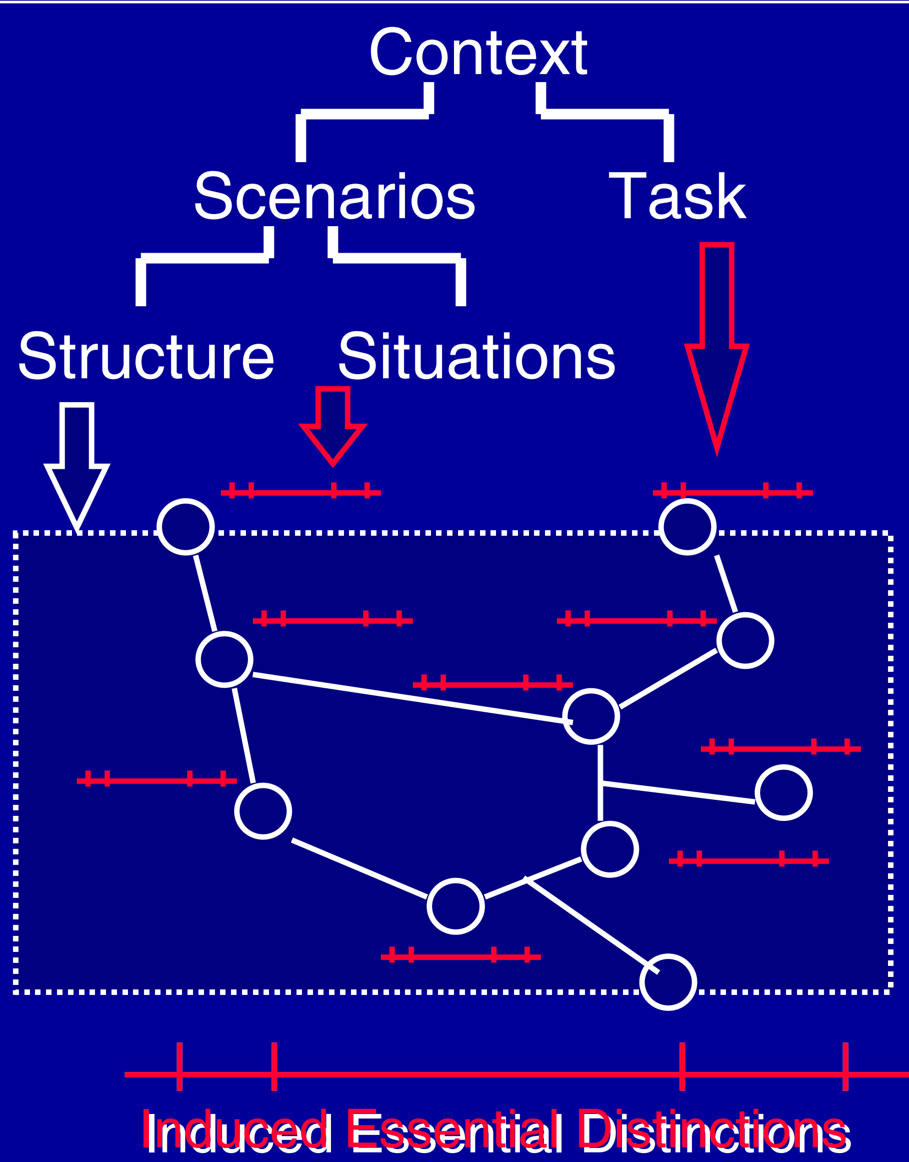
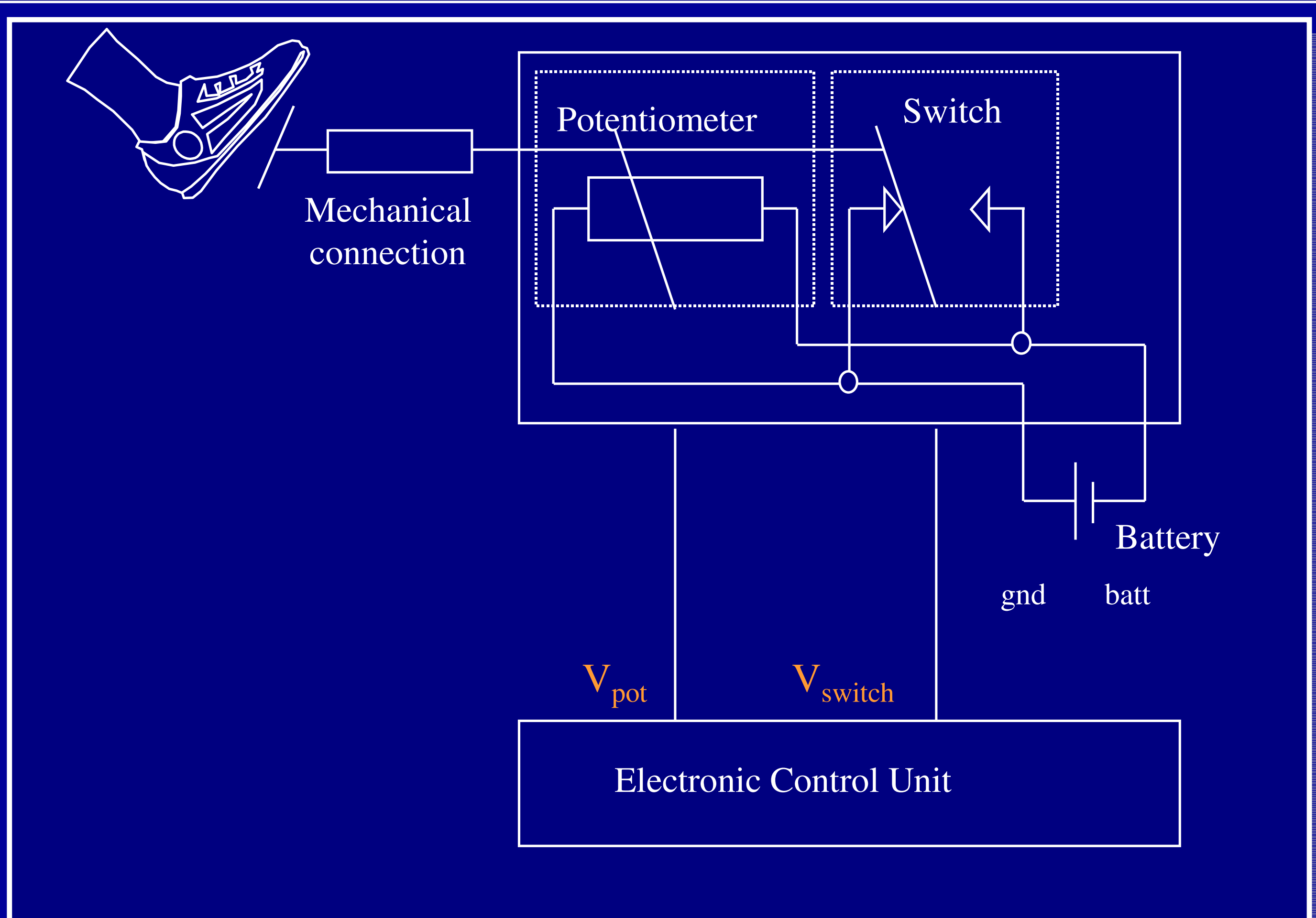


"Small" is Beautiful



- **But difficult!**