Qualitative Spatial Reasoning

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Overview (1)

- Motivation
- Introduction to QSR + ontology
- Representation aspects of pure space
  - Topology
  - Orientation
  - Distance & Size
  - Shape

Overview (2)

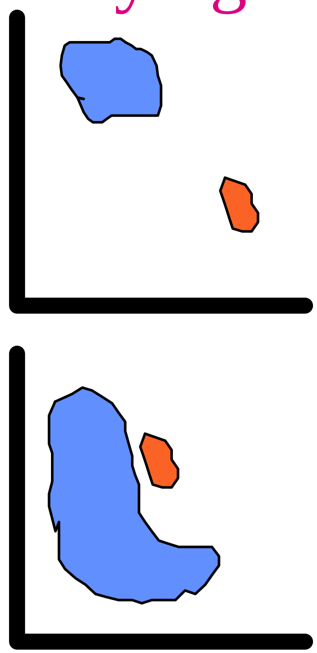
- Reasoning (techniques)
  - ☛ Composition tables
  - ☛ Adequacy criteria
  - ☛ Decidability
  - ☛ Zero order techniques
  - ☛ completeness
  - ☛ tractability

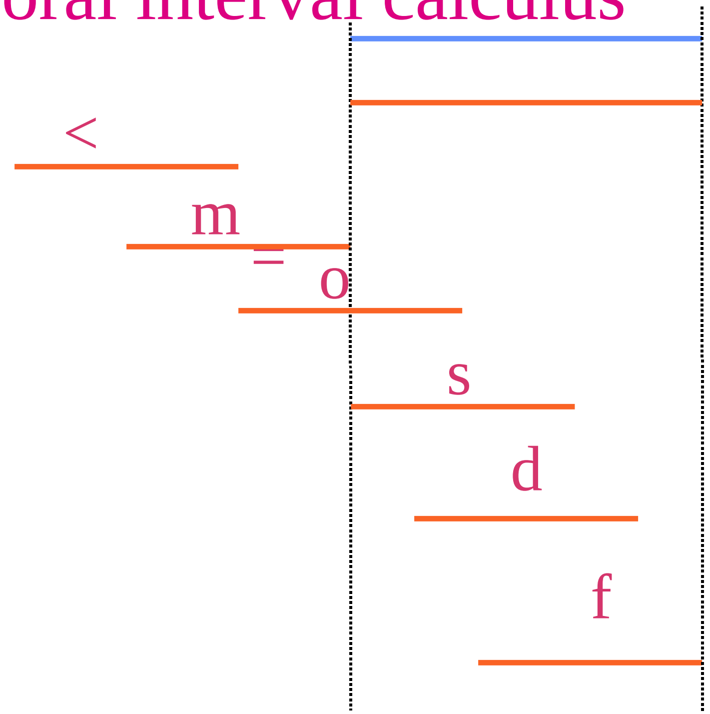
Overview (3)

- Spatial representations in context
  - Spatial change
  - Uncertainty
  - Cognitive evaluation
- Some applications
- Future work
- *Caveat: not a comprehensive survey*

What is QSR? (1)

- Develop QR representations specifically for space
- Richness of QSR derives from multi-dimensionality
  - Consider trying to apply temporal interval calculus in 2D:





- Can work well for particular domains -- e.g. envelope/address recognition (Walischemwski 97)

What is QSR? (2)

- Many aspects:
  - ontology, topology, orientation, distance, shape...
  - spatial change
  - uncertainty
  - reasoning mechanisms
  - pure space v. domain dependent

What QSR is not (at least in this lecture!)

- Analogical
- metric representation and reasoning
  - ☛we thus largely ignore the important spatial models to be found in the vision and robotics literatures.

“Poverty Conjecture” (Forbus et al, 86)

- “There is no purely qualitative, general purpose kinematics”
- Of course QSR is more than just kinematics, but...
- 3rd (and strongest) argument for the conjecture:
  - “No total order: Quantity spaces don’t work in more than one dimension, leaving little hope for concluding much about combining weak information about spatial properties"

“Poverty Conjecture” (2)

- transitivity: key feature of qualitative quantity space
  - ☛can this be exploited much in higher dimensions ??
  - ☛ “we suspect the space of representations in higher dimensions is sparse; that for spatial reasoning almost nothing weaker than numbers will do”.
- The challenge of QSR then is to provide calculi which allow a machine to represent and reason with spatial entities of higher dimension, without resorting to the traditional quantitative techniques.

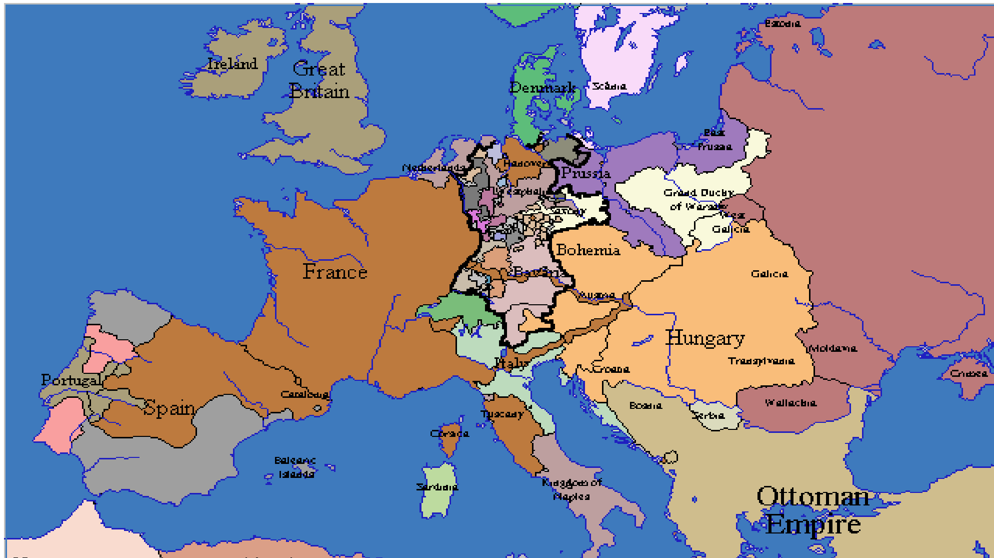
Why QSR?

- Traditional QR spatially very inexpressive
- Applications in:
  - Natural Language Understanding
  - GIS
  - Visual Languages
  - Biological systems
  - Robotics
  - Multi Modal interfaces
  - Event recognition from video input
  - Spatial analogies
  - ...

Reasoning about Geographic change

- Consider the change in the topology of Europe's political boundaries and the topological relationships between countries
    - disconnected countries
    - countries surrounding others
        - Did France ever enclose Switzerland? (Yes, in 1809.5)
    - continuous and discontinuous change
    - ...

☛http:/www.clockwk.com CENTENIA



Ontology of Space

- extended entities (regions)?
- points, lines, boundaries?
- mixed dimension entities?
- What is the embedding space?
  - ☛connected? discrete? dense? dimension? Euclidean?...
- What entities and relations do we take as primitive, and what are defined from these primitives?

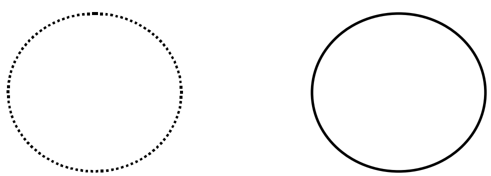
Why regions?

- encodes indefiniteness naturally
- space occupied by physical bodies
  - ☛ a sharp pencil **point** still draws a line of finite thickness!
- points can be reconstructed from regions if desired as infinite nests of regions
- unintuitive that extended regions can be composed entirely of dimensionless points occupying no space!

- However: lines/points may still be useful abstractions

Topology

- Fundamental aspect of space
  - ☛ "rubber sheet geometry"
  - connectivity, holes, dimension …
- interior: $i(X)$ union of all open sets contained in X



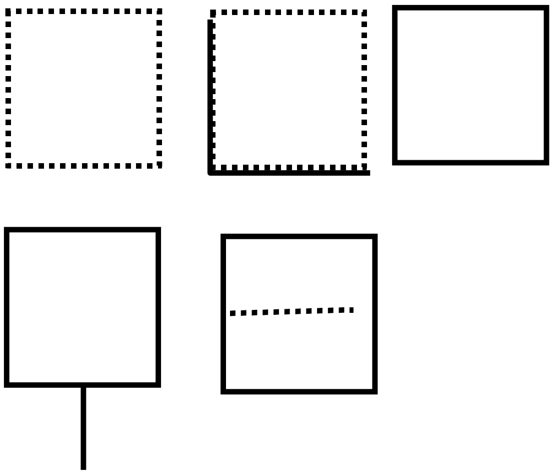
- $i(X) \subseteq X$
- $i(i(X)) = i(X)$
- $i(U) = U$
- $i(X \cap Y) = i(X) \cap i(Y)$
- Universe, U is an open set

Boundary, closure, exterior

- Closure of *X*: intersection of all closed sets containing *X*
- Complement of *X*: all points not in *X*
- Exterior of *X*: interior of complement of *X*
- Boundary of *X*: closure of X $\cap$ closure of exterior of *X*

What counts as a region? (1)

- Consider R^n:
  - any set of points?
  - empty set of points?
  - mixed dimension regions?
  - regular regions?
    - regular open: interior(closure(x)) = x
    - regular closed: closure(interior(x)) = x
    - regular: closure(interior(x)) = closure(x)
  - scattered regions?
  - not interior connected?



What counts as a region? (2)

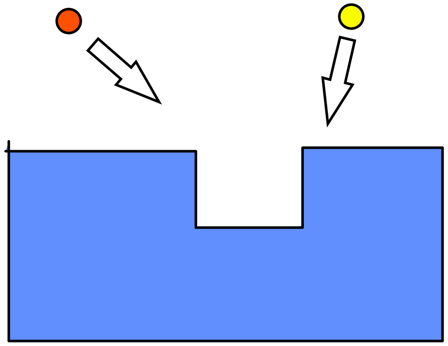
- Co-dimension = n-m, where m is dimension of region
  - 10 possibilities in R^3
- Dimension :
  - differing dimension entities
    - cube, face, edge, vertex
    - what dimensionality is a road?
  - mixed dimension regions?

Is traditional mathematical point set topology useful for QSR?

- more concerned with properties of different kinds of topological spaces rather than defining concepts useful for modelling real world situations
- many topological spaces very abstract and far removed from physical reality
- not particularly concerned with computational properties

History of QSR (1)

- Little on QSR in AI until late 80s
  - some work in QR
  - E.g. FROB (Forbus)
    - bouncing balls (point masses) – can they collide?
    - place vocabulary: direction + topology



History of QSR (2)

- Work in philosophical logic
  - Whitehead(20): “Concept of Nature”
    - defining points from regions (*extensive abstraction*)
  - Nicod(24): intrinsic/extrinsic complexity
    - Analysis of temporal relations (cf. Allen(83)!)
  - de Laguna(22): ‘*x* can connect *y* and *z*’
  - Whitehead(29): revised theory
    - binary “connection relation” between regions

History of QSR (3)

- Mereology: formal theory of part-whole relation
  - Lesniewski(27-31)
  - Tarski (35)
  - Leonard & Goodman(40)
  - Simons(87)

History of QSR (4)

- Tarski's Geometry of Solids (29)
  - mereology + sphere(*x*)
  - made "categorical" indirectly:
    - points defined as nested spheres
    - defined equidistance and betweeness obeying axioms of Euclidean geometry
  - reasoning ultimately depends on reasoning in elementary geometry
    - decidable but not tractable

History of QSR (5)

- Clarke(81,85): attempt to construct system
  - more expressive than mereology
  - simpler than Tarski's
- based on binary connection relation (Whitehead 29)
  - C(x,y)
    - $\forall x,y\ [\mathrm{C}(x,y) \rightarrow \mathrm{C}(y,x)]$
    - $\forall z\ \mathrm{C}(z,z)$
  - spatial or spatio-temporal interpretation
  - intended interpretation of C(x,y) : x & y share a point

History of QSR (6)

- topological functions: interior(x), closure(x)
- quasi-Boolean functions:
  - ☛ sum(x,y), diff(x,y), prod(x,y), compl(x,y)
  - ☛ "quasi" because no null region
- Defines many relations and proves properties of theory

Problems with Clarke(81,85)

- second order formulation
- unintuitive results?
  - is it useful to distinguish open/closed regions?
  - remainder theorem does not hold!
    - *x* is a proper part of *y* does not imply *y* has any other proper parts
- Clarke's definition of points in terms of nested regions causes connection to collapse to overlap (Biacino & Gerla 91)

RCC Theory

- Randell & Cohn (89) based closely on Clarke
- Randell et al (92) reinterprets C(x,y):
  - don't distinguish open/closed regions
    - same area
    - physical objects naturally interpreted as closed regions
    - break stick in half: where does dividing surface end up?
  - closures of x and y share a point
  - distance between x and y is 0

Defining relations using C(x,y) (1)

- DC(x,y) $\equiv_{df}$ ¬C(x,y)

  x and y are disconnected

- P(x,y) $\equiv_{df}$ $\forall z$ [C(x,z) → C(y,z)]

  x is a part of y

- PP(x,y) $\equiv_{df}$ P(x,y) ∧ ¬P(y,x)

  x is a proper part of y

- EQ(x,y) $\equiv_{df}$ P(x,y) ∧ P(y,x)

  x and y are equal

  ☛ alternatively, an axiom if equality built in

Defining relations using C(x,y) (2)

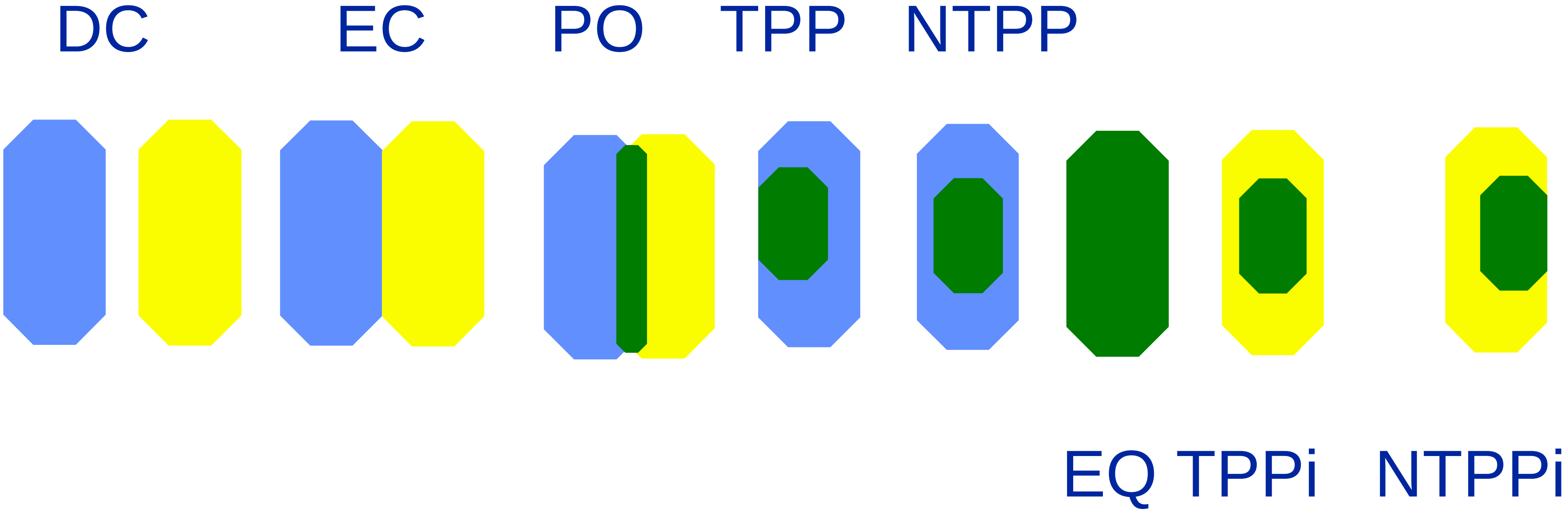
- $\mathrm{O}(x,y) \equiv_{df} \exists z[\mathrm{P}(z,x) \wedge \mathrm{P}(z,y)]$
  - ☛ *x* and *y* overlap
- $\mathrm{DR}(x,y) \equiv_{df} \neg \mathrm{O}(x,y)$
  - ☛ *x* and *y* are discrete
- $\mathrm{PO}(x,y) \equiv_{df} \mathrm{O}(x,y) \wedge \neg \mathrm{P}(x,y) \wedge \neg \mathrm{P}(y,x)$
  - ☛ *x* and *y* partially overlap

Defining relations using C(x,y) (3)

- $EC(x,y) \equiv_{df} C(x,y) \wedge \neg O(x,y)$
  - ☛ *x* and *y* externally connect
- $TPP(x,y) \equiv_{df} PP(x,y) \wedge \exists z[EC(z,y) \wedge EC(z,x)]$
  - ☛ *x* is a tangential proper part of *y*
- $NTPP(x,y) \equiv_{df} PP(x,y) \wedge \neg TPP(x,y)$
  - ☛ *x* is a non tangential proper part of *y*

RCC-8

- 8 provably *jointly exhaustive pairwise disjoint* relations (JEPD)



An additional axiom

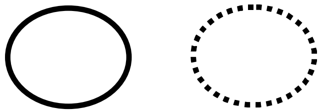
- $\forall x \exists y$ NTPP(y,x)
- "replacement" for interior(x)
- forces no atoms
  - Randell et al (92) considers how to create atomistic version

Quasi-Boolean functions

- sum(x,y), diff(x,y), prod(x,y), compl(x)
- u: universal region
- axioms to relate these functions to C(x,y)
- "quasi" because no null region
  - ☛note: sorted logic handles partial functions
  - ☛e.g. compl(x) not defined on u
- (note: no topological functions)

Properties of RCC (1)

- Remainder theorem holds:
  - ☛A region has at least two distinct proper parts
  - ☛$\forall x,y\ [\mathrm{PP}(y,x) \rightarrow \exists z\ [\mathrm{PP}(z,x) \wedge \neg \mathrm{O}(z,y)]]$



- Also other similar theorems
  - e.g. *x* is connected to its complement

A canonical model of RCC8

- Above models just delineate a possible space of models
- Renz (98) specifies a canonical model of an arbitrary ground Boolean wff over RCC8 atoms
  - ☛ uses modal encoding (see later)
  - ☛ also shows how *n*-D realisations can be generated (with connected regions for n > 2)

Asher & Vieu (95)'s Mereotopology (1)

- development of Clarke's work
  - corrects several mistakes
  - no general fusion operator (now first order)
- motivated by Natural Language semantics
- primitive: C(x,y)
- topological and Boolean operators
- formal semantics
  - quasi ortho-complemented lattices of regular open subsets of a topological space

Asher & Vieu (95)'s Mereotopology (2)

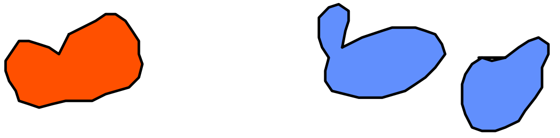
- Weak connection:
  - $\mathrm{Wcont}(x,y) \equiv_{df} \neg C(x,y) \wedge C(x, n(c(y)))$
  - $n(x) =_{df} \iota y\, [P(x,y) \wedge \mathrm{Open}(y) \wedge \forall z\, [[P(x,z) \wedge \mathrm{Open}(z) \rightarrow P(y,z)]$
- True if x is in the *neighbourhood* of y, $n(y)$
- Justified by desire to distinguish between:
  - stem and 'cup' of a glass
  - wine in a glass
- should this be part of a theory of pure space?

Expressivenesss of C(x,y)

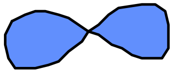
- Can construct formulae to distinguish many different situations
  - connectedness
  - holes
  - dimension

Notions of connectedness

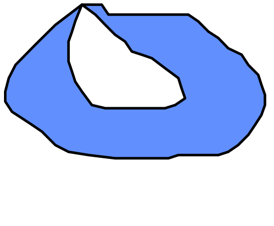
- One piece



- Interior connected

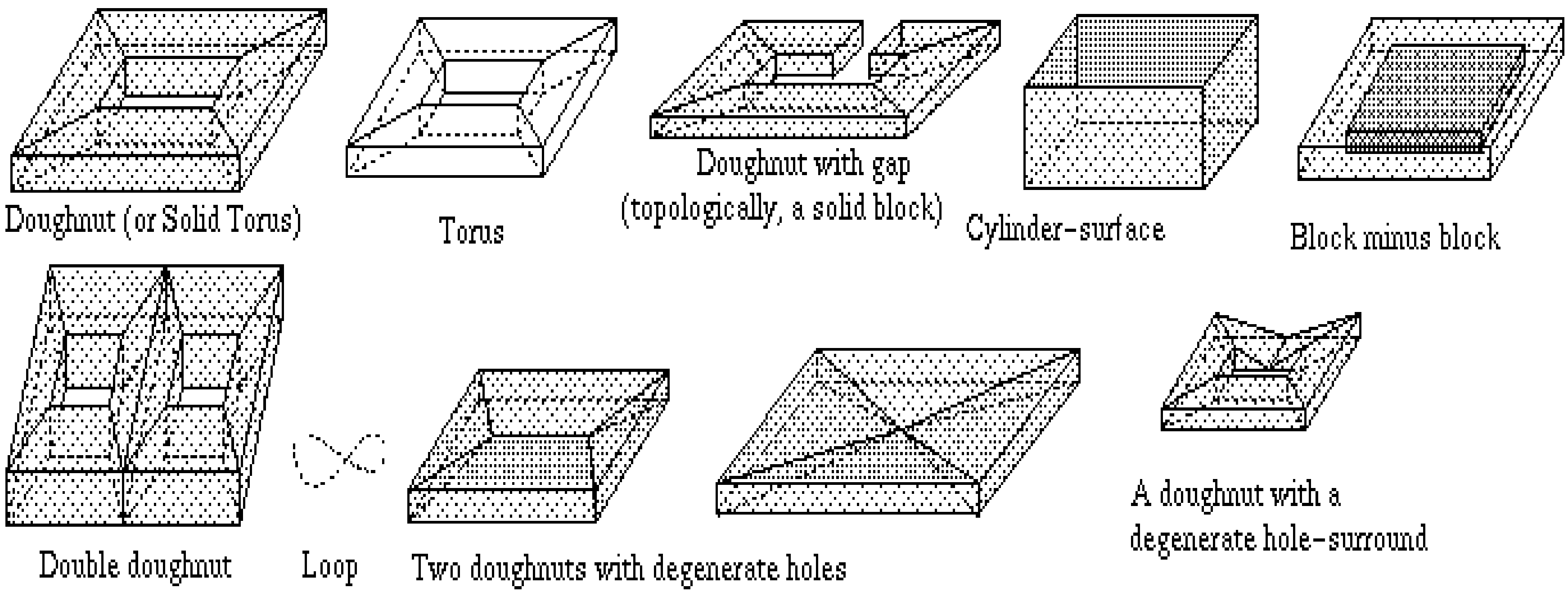


- Well connected



Gotts(94,96): “How far can we C?”

- defining a doughnut



Other relationships definable from C(*x*,*y*)

- E.g. FTPP(*x*,*y*)
  - *x* is a firm tangential part of *y*

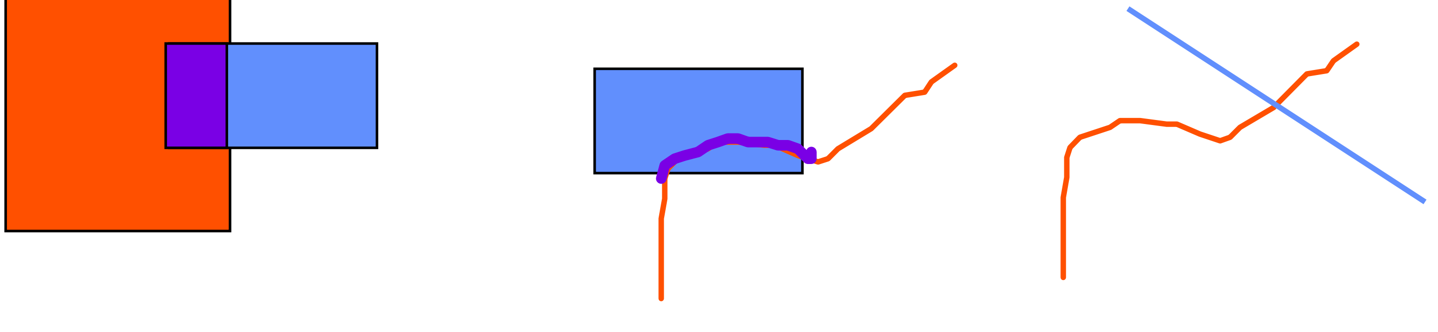
- Intrinsic TPP: ITPP(*x*)
  - TPP(*x*,*y*) definition requires externally connecting *z*
  - universe can have an ITPP but not a TPP

Characterising Dimension

- In all the C(x,y) theories, regions have to be same dimension
- Possible to write formulae to fix dimension of theory (Gotts 94,96)
  - very complicated
- Arguably may want to refer to lower dimensional entities?

The INCH calculus (Gotts 96)

- INCH(*x*,*y*): *x* includes a chunk of *y* (of the same dimension as *x*)
- symmetric iff *x* and *y* are equi-dimensional



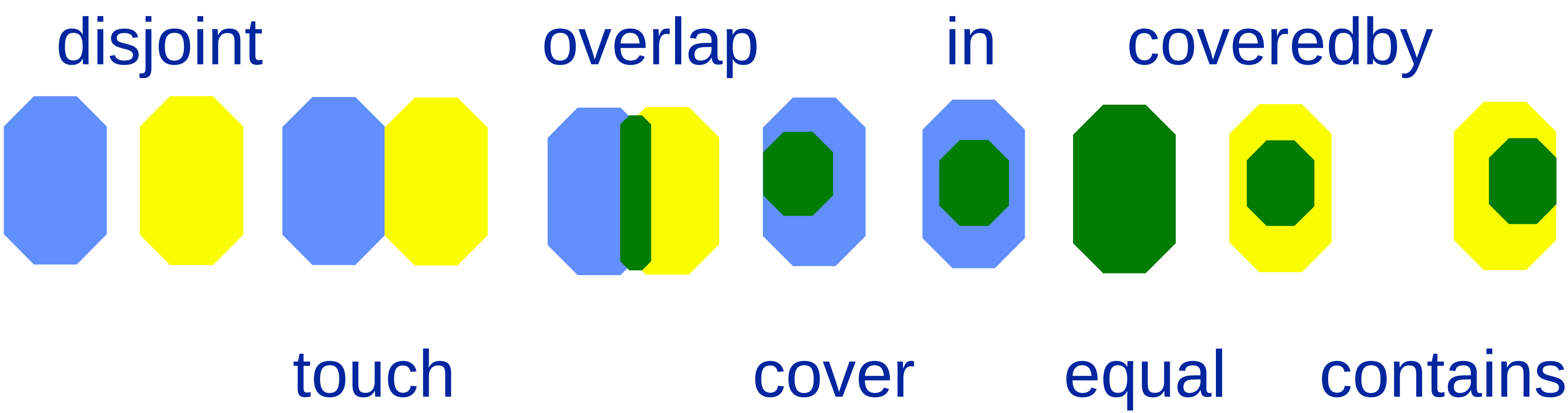
Galton's (96) dimensional calculus

- 2 primitives
  - mereological: P(*x*,*y*)
  - topological: B(*x*,*y*)
- Motivated by similar reasons to Gotts
- Related to other theories which introduce a boundary theory (Smith 95, Varzi 94), but these do not consider dimensionality
- Neither Gotts nor Galton allow mixed dimension entities
  - ontological and technical reasons

4-intersection (4IM) Egenhofer & Franzosa (91)

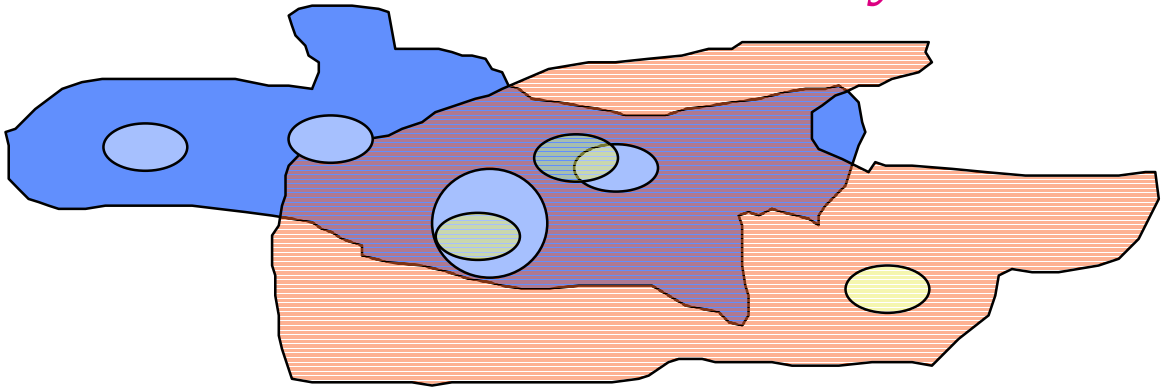
$\cap$	boundary(y)	interior(y)
boundary(x)	$\neg$	$\emptyset$
interior(x)	$\emptyset$	$\emptyset$

- $2^4 = 16$ combinations
- 8 relations assuming planar regular point sets



Extension to cover regions with holes

- Egenhofer(94)
- Describe relationship using 4-intersection between:
  - *x* and *y*
  - *x* and each hole of *y*
  - *y* and each hole of *x*
  - each hole of *x* and each hole of *y*



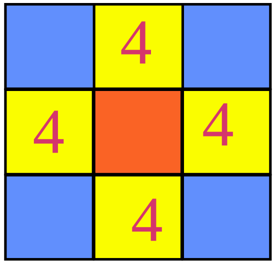
9-intersection model (9IM)

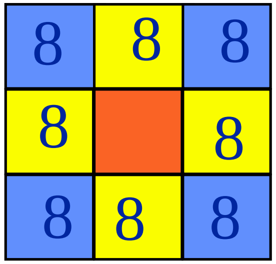
$\cap$	boundary(y)	interior(y)	exterior(x)
boundary(x)	¬	∅	¬
interior(x)	∅	∅	∅
exterior(x)	¬	∅	¬

- $2^9 = 512$ combinations
  - 8 relations assuming planar regular point sets
- potentially more expressive
- considers relationship between region and embedding space

Modelling discrete space using 9-intersection (Egenhofer & Sharma, 93)

- How many relationships in Z^2 ?
- 16 (superset of R^2 case), assuming:
  - boundary, interior non empty
  - boundary pixels have exactly two 4-connected neighbours
    - interior and exterior not 8-connected
  - exterior 4-connected
  - interior 4-connected and has ≥ 3 8-neighbours





“Dimension extended” method (DEM)

- In the case where array entry is ‘¬’, replace with dimension of intersection: 0,1,2
- 256 combinations for 4-intersection
- Consider 0,1,2 dimensional spatial entities
  - 52 realisable possibilities (ignoring converses)
  - (Clementini et al 93, Clementini & di Felice 95)

"Calculus based method" (Clementini et al 93)

- Too many relationships for users
- notion of interior not intuitive?

"Calculus based method" (2)

- Use 5 polymorphic binary relations between *x*,*y*:
  - disjoint: $x \cap y = \emptyset$
  - touch (a/a, l/l, l/a, p/a, p/l): $x \cap y \subseteq b(x) \cup b(y)$
  - in: $x \cap y \subseteq y$
  - overlap (a/a, l/l): $\dim(x)=\dim(y)=\dim(x \cap y) \wedge x \cap y \neq \emptyset \wedge y \neq x \cap y \neq x$
  - cross (l/l, l/a): $\dim(\mathrm{int}(x)) \cap \mathrm{int}(y))=\max(\mathrm{int}(x)),\mathrm{int}(y)) \wedge x \cap y \neq \emptyset \wedge y \neq x \cap y \neq x$

“Calculus based method” (3)

- Operators to denote:
  - boundary of a 2D area, x: b(x)
  - boundary points of non-circular (non-directed) line:
    - t(x), f(x)
  - *(Note: change of notation from Clementini et al)*

“Calculus based method” (4)

- Terms are:
  - spatial entities (area, line, point)
  - t(x), f(x), b(x)
- Represent relation as:
  - conjunction of $R(\alpha,\beta)$ atoms
    - R is one of the 5 relations
    - α,β are terms

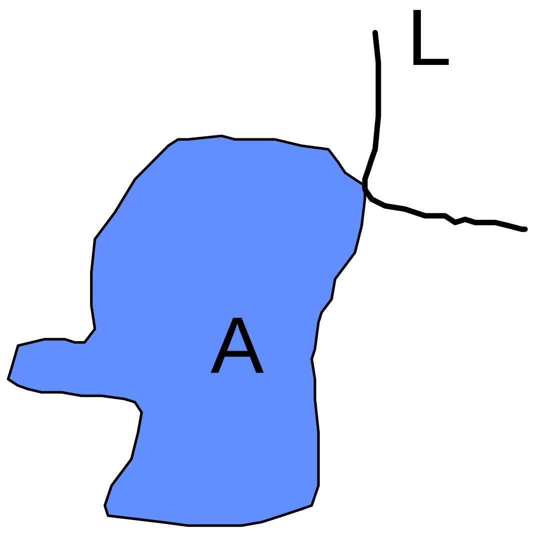
Example of "Calculus based method"

touch(L,A) $\wedge$

cross(L,b(A)) $\wedge$

disjoint(f(L),A) $\wedge$

disjoint(t(L),A)



“Calculus based method” v. “intersection” methods

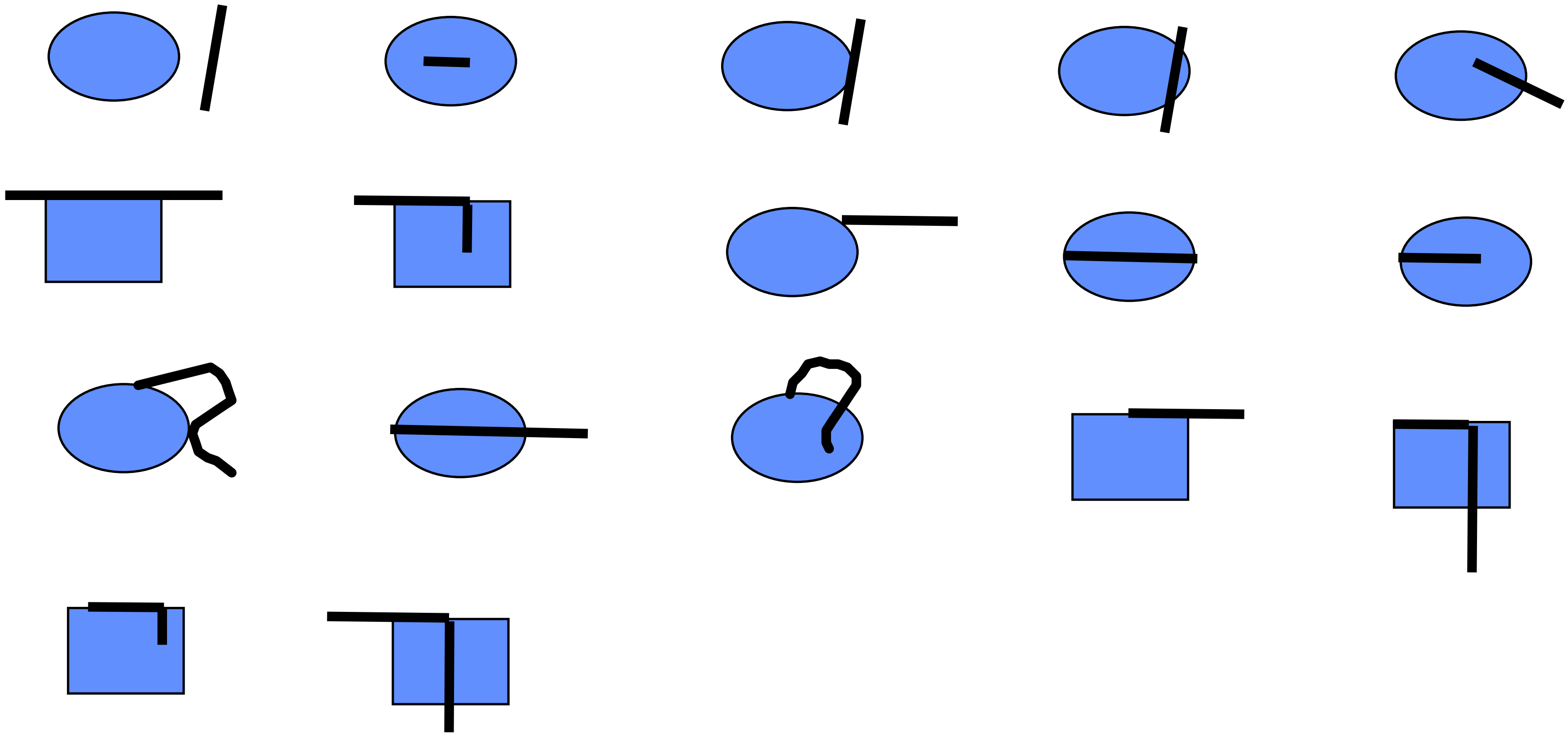
- more expressive than DEM or 9IM alone
- minimal set to represent all 9IM and DEM relations

	A/A	L/A	P/A	L/L	P/L	P/P	Total
4IM	6	11	3	12	3	2	37
9IM	6	19	3	23	3	2	56
DEM	9	17	3	18	3	2	52
DEM+9IM or CBM	9	31	3	33	3	2	81

(Figures are without inverse relations)

- Extension to handle complex features (multi-piece regions, holes, self intersecting lines or with > 2 endpoints)

The 17 different L/A relations of the DEM



Mereology and Topology

- Which is primal? (Varzi 96)
- Mereology is insufficient by itself
  - ☛can't define connection or 1-pieceness from parthood

1. generalise mereology by adding topological primitive
2. topology is primal and mereology is sub theory
3. topology is specialised domain specific sub theory

Topology by generalising Mereology

1) add $\mathsf{C}(x,y)$ and axioms to theory of $\mathsf{P}(x,y)$

2) add $\mathsf{SC}(x)$ to theory of $\mathsf{P}(x,y)$

- ☛ $\mathsf{C}(x,y) \equiv_{df} \exists z\, [\mathsf{SC}(z) \wedge \mathsf{O}(z,x) \wedge \mathsf{O}(z,y) \wedge \forall w[\mathsf{P}(w,z) \rightarrow [\mathsf{O}(w,x) \vee \mathsf{O}(w,y)]]$

3) Single primitive: *x* and *y* are connected parts of *z* (Varzi 94)

- ◆ Forces existence of boundary elements.
- ◆ Allows colocation without sharing parts
  - ☛ e.g holes don't share parts with things in them

Mereology as a sub theory of Topology

- define P(x,y) from C(x,y)
  - e.g. Clarke, RCC, Asher/Vieu,...
- single unified theory
- colocation implies sharing of parts
- normally boundaryless
  - EC not necessarily explained by sharing a boundary
  - lower dimension entities constructed by ‘nested sets’

Topology as a mereology of regions

- Eschenbach(95)
- Use restricted quantification
  - $C(x,y) \equiv_{df} O(x,y) \wedge R(x) \wedge R(y)$
  - $EC(x,y) \equiv_{df} C(x,y) \wedge \forall z[[C(z,x) \wedge C(z,y)] \rightarrow \neg R(z)]$
- In a sense this is like (1) - we are adding a new primitive to mereology: $R(x)$

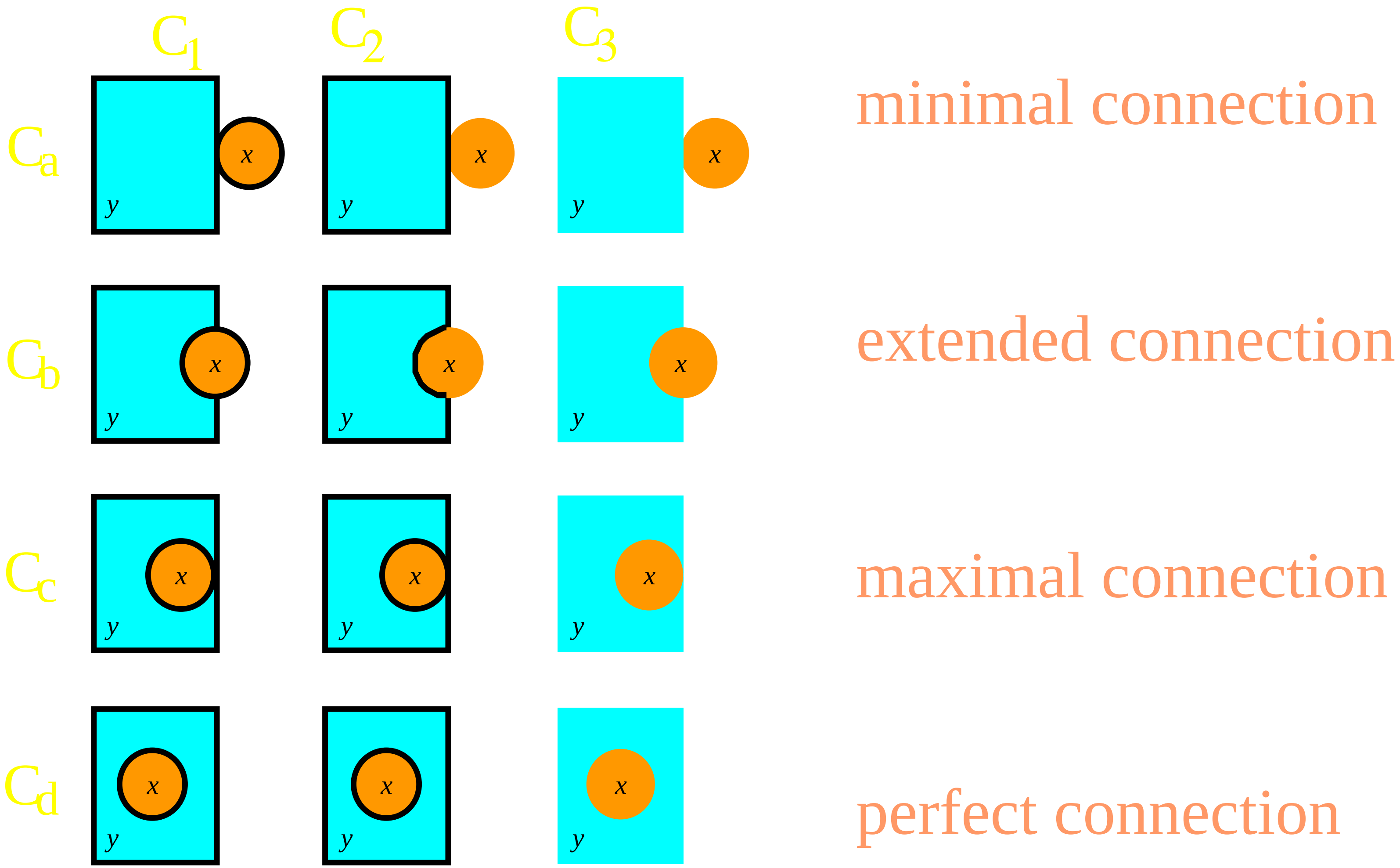
A framework for evaluating connection relations (Cohn & Varzi 98)

- many different interpretations of connection and different ontologies (regions with/without boundaries)
- framework with primitive connection, part relations and fusion operator (normal topological notions)
- define hierarchy of higher level relations
- evaluate consequences of these definitions
- place existing mereotopologies into framework

$C(x,y)$: 3 dimensions of variation

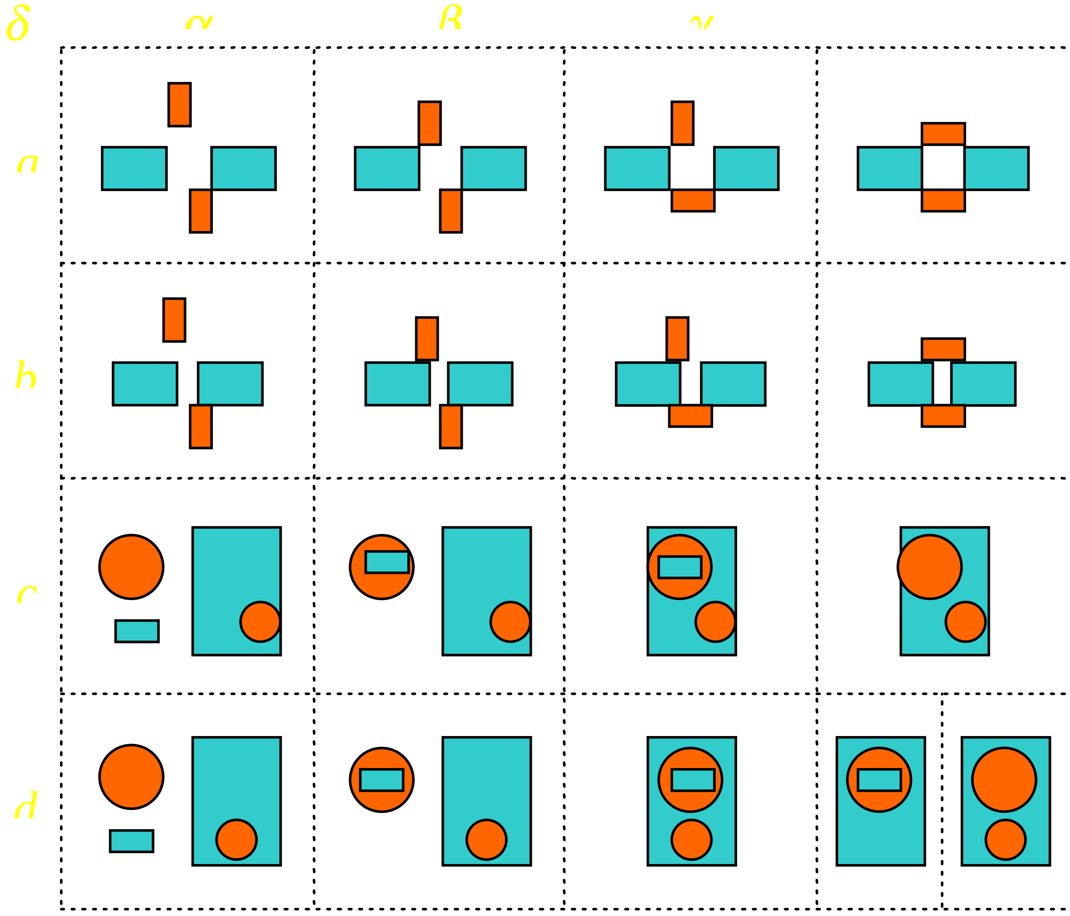
- Closed or open
  - $C_1(x, y) \Leftrightarrow x \cap y \neq \emptyset$
  - $C_2(x, y) \Leftrightarrow x \cap c(y) \neq \emptyset$ or $c(x) \cap y \neq \emptyset$
  - $C_3(x, y) \Leftrightarrow c(x) \cap c(y) \neq \emptyset$
- Firmness of connection
  - point, surface, complete boundary
- Degree of connection between multipiece regions
  - All/some components of *x* are connected to all/some components of *y*

First two dimensions of variation



- Cf RCC8 and conceptual neighbourhoods

Second two dimensions of variation

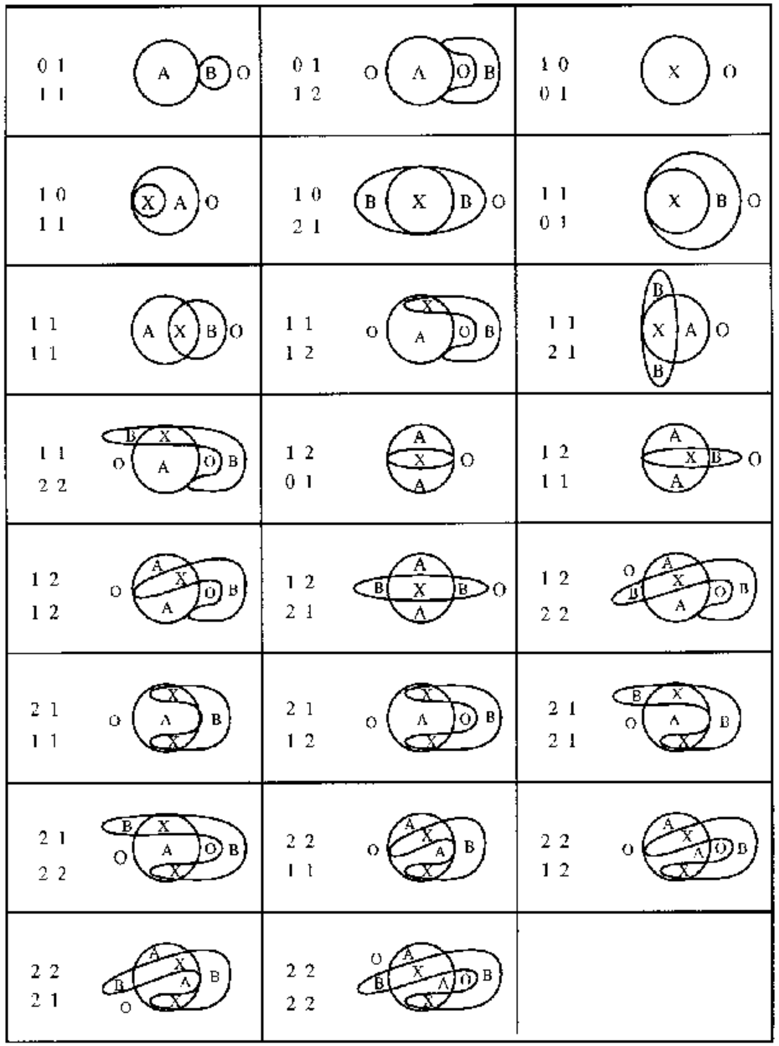


Algebraic Topology

- Alternative approach to topology based on "cell complexes" rather than point sets - Lienhardt(91), Brisson (93)
- Applications in
  - GIS, e.g. Frank & Kuhn (86), Pigot (92,94)
  - CAD, e.g. Ferrucci (91)
  - Vision, e.g. Faugeras , Bras-Mehlman & Boissonnat (90)
  - …

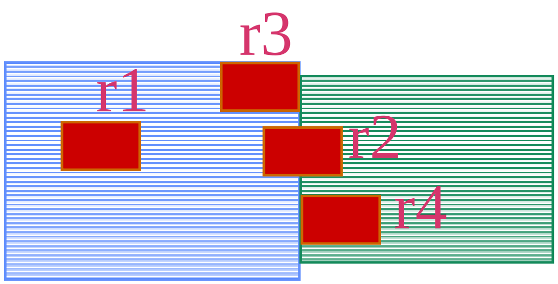
Expressiveness of topology

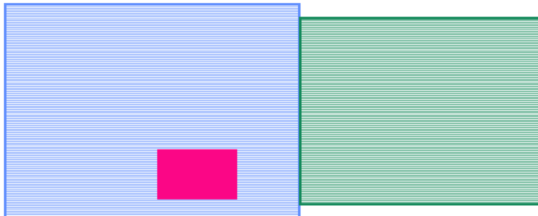
- can define many further relations characterising properties of and between regions
  - e.g. "modes of overlap" of 2D regions (Galton 98)
  - 2x2 matrix which counts number of connected components of AB, A\B, B\A, compl(AB)
  - could also count number of intersections/touchings
    - but is this qualitative?



Position via topology (Bittner 97)

- fixed background partition of space
  - e.g. states of the USA
- describe position of object by topological relations w.r.t. background partition
- ternary relation between
  - 2 internally connected background regions
    - well-connected along single boundary segment
  - and an arbitrary figure region
  - consider whether there could exist r1,r2,r3,r4 P or DC to figure region
    - 15 possible relations
    - e.g. <r1:+P,r2:+DC,r3:-P,r4:-P>



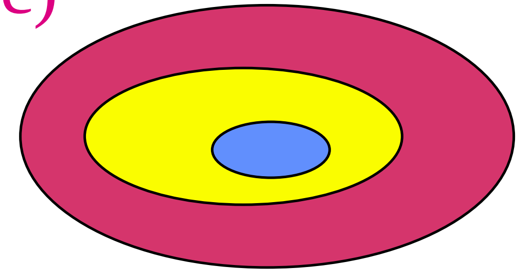


Reasoning Techniques

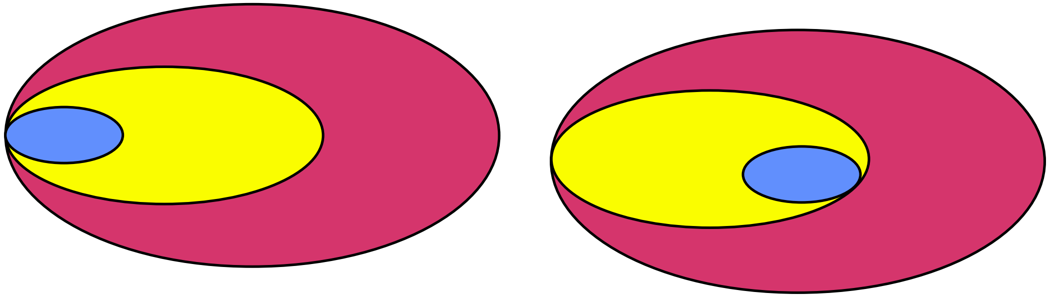
- First order theorem proving?
- Composition tables
- Other constraint based techniques
- Exploiting transitive/cyclic ordering relations
- 0-order logics
  - reinterpret proposition letters as denoting regions
  - logical symbols denote spatial operations
  - need intuitionistic or modal logic for topological distinctions (rather than just mereological)

Reasoning by Relation Composition

- R1(a,b), R2(b,c)
  - R3(a,c)



- In general R3 is a disjunction
  - Ambiguity



Composition tables are quite sparse

	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ
DC	?	DR,PO, PP	DR,PO, PP	DR,PO, PP	DR, PO, PP	DC	DC	DC
EC	DR,PO, PPi	DR,PO, TPP,TPi	DR,PO, PP	EC,PO, PP	PO, PP	DR	DC	DC
PO	DR,PO, PPi	DR, PO, PPi	?	PO,PP	PO, PP	DR, PO, PPi	DR, PO, PPi	PO
TPP	DC	DR	DR,PO, PP	PP	NTPP	DR,PO, TPP,TPi	DR, PO, PPi	TPP
NTPP	DC	DC	DR,PO, PP	NTPP	NTPP	DR,PO, PP	?	NTPP
TPPi	DR,PO, PPi	EC,PO, PPi	PO,PPi	PO,TPP ,TPi	PO, PP	PPi	NTPPi	TPPi
NTPPi	DR,PO, PPi	PO,PPi	PO,PPi	PO,PPi	O	NTPPi	NTPPi	NTPPi
EQ	DC	EC	PO	TPP	NTPP	TPPi	NTPPi	EQ

- cf poverty conjecture

Other issues for reasoning about composition

- Reasoning by Relation Composition
  - topology, orientation, distance,...
  - problem: automatic generation of composition tables
  - generalise to more than 3 objects
    - Question: when are 3 objects sufficient to determine consistency?

Reasoning via Helle's theorem (Faltings 96)

- A set R of n convex regions in d-dimensional space has a common intersection iff all subsets of d+1 regions in R have an intersection
  - In 2D need relationships between triples not pairs of regions
  - need convex regions
    - conditions can be weakened: don't need convex regions just that intersections are single simply connected regions
- Given data: intersects(r_1,r_2,r_3) for each r_1,r_2,r_3
  - can compute connected paths between regions
    - decision procedure
    - use to solve, e.g., piano movers problem

Other reasoning techniques

- theorem proving
  - ☛general theorem proving with 1st order theories too hard, but some specialised theories, e.g. Bennett (94)
- constraints
  - ☛e.g. Hernandez (94), Escrig & Toledo (96,98)
- using ordering (Roehrig 94)
- Description Logics (Haarslev et al 98)
- Diagrammatic Reasoning, e.g. (Schlieder 98)
- random sampling (Gross & du Rougemont 98)

Between Topology and Metric representations

- What QSR calculi are there "in the middle"?
- Orientation, convexity, shape abstractions…
- Some early calculi integrated these
  - ☛we will separate out components as far as possible

Orientation

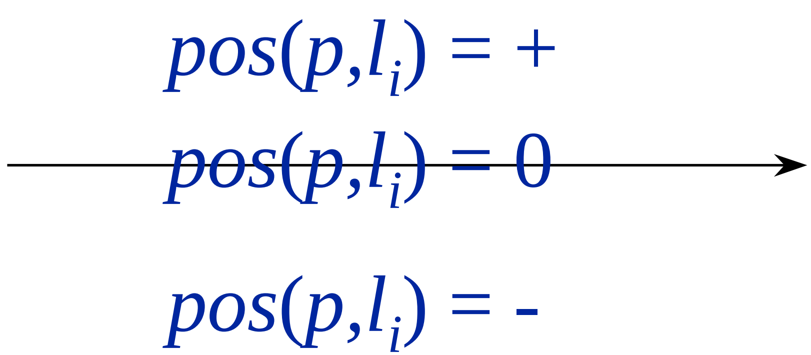
- Naturally qualitative: clockwise/anticlockwise orientation
- Need reference frame
  - deictic: *x* is to the left of *y* (viewed from observer)
  - intrinsic: *x* is in front of *y*
    - *(depends on objects having fronts)*
  - absolute: *x* is to the north of *y*
- Most work 2D
- Most work considers orientation between points

Orientation Systems (Schlieder 95,96)

- Euclidean plane
  - ☛set of points Π
  - ☛set of directed lines Λ
- $C=(p_1,\ldots,p_n) \in \Pi^n$: *ordered configuration of points*
- $A=(l_1,\ldots,l_m) \in \Lambda^m$: *ordered arrangement of d-lines*
  - ☛such *reference axes* define an *Orientation System*

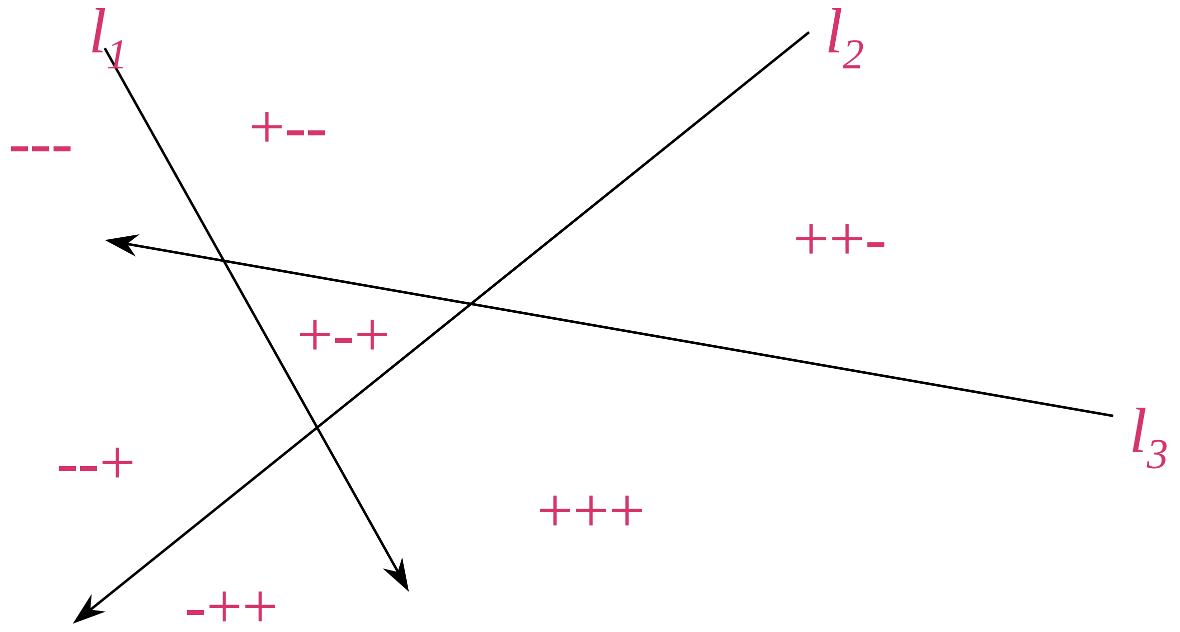
Assigning Qualitative Positions (1)

- $pos: \Pi \times \Lambda \rightarrow \{+,0,-\}$
- $pos(p,l_i) = +$ iff p lies to left of l_i
- $pos(p,l_i) = 0$ iff p lies on l_i
- $pos(p,l_i) = -$ iff p lies to right of l_i



Assigning Qualitative Positions (2)

- $Pos: \Pi \times \Lambda \rightarrow \{+,0,-\}^m$
- $Pos(p,A) = (pos(p,l_1),\ldots, pos(p,l_m))$
- Eg:

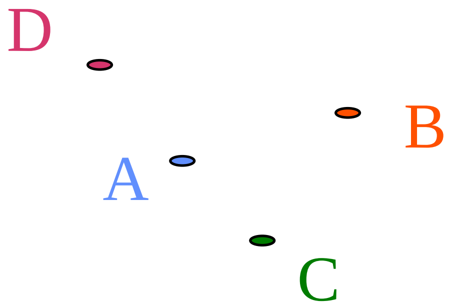


Note: 19 positions (7 named) -- 8 not possible

Inducing reference axes from reference points

- Usually have point data and reference axes are determined from these
  - $o: \Pi^n \rightarrow \Lambda^m$
  - E.g. join all points representing landmarks
  - *o* may be constrained:
    - incidence constraints
    - ordering constraints
    - congruence constraints

Triangular Orientation (Goodman & Pollack 93)



ABC = -

ACB = +

CAB = -

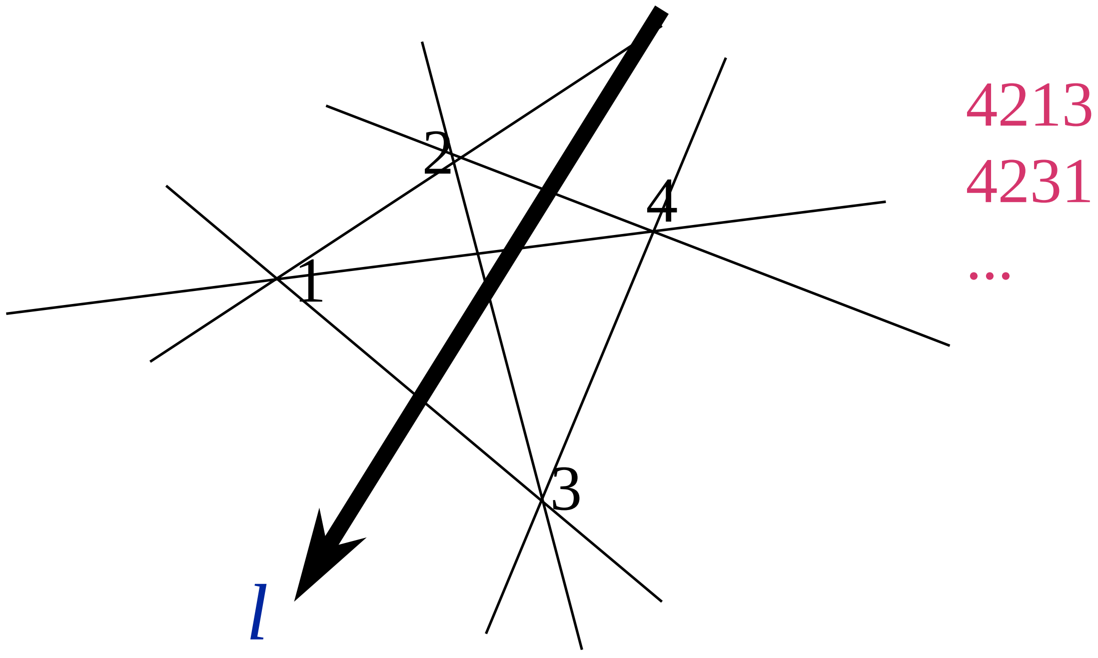
CBA = +

DAC = 0   DA B = +

- 3 possible orientations between 3 points
- Note: single permutation flips polarity
- E.g.: A is viewer; B,C are landmarks

Permutation Sequence (1)

- Choose a new directed line, *l*, not orthogonal to any existing line
- Note order of all points projected
- Rotate *l* counterclockwise until order changes

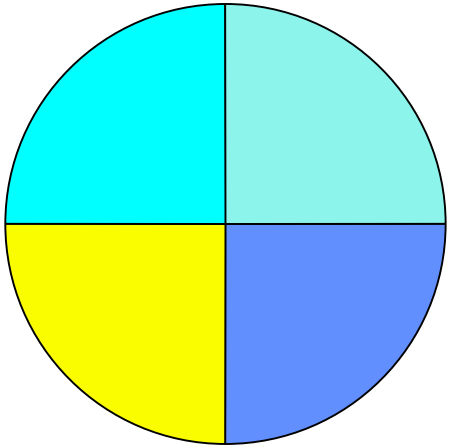


Permutation Sequence (2)

- Complete sequence of such projections is *permutation sequence*
- more expressive than triangle orientation information

Exact orientations v. segments

- E.g absolute axes: N,S,E,W
- intervals between axes
- Frank (91), Ligozat (98)

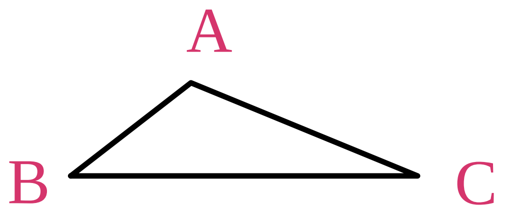


Qualitative Trigonometry (Liu 98) -- 1

- Qualitative distance (wrt to a reference constant, *d*)
  - less, slightlyless, equal, slightlygreater, greater
  - *x/d*: 0…2/3… 1 … 3/2… infinity
- Qualitative Angles
  - acute, slightlyacute, rightangle, slightlyobtuse, obtuse
  - 0 … π/3 … π/2 … 2π/3 … 2π

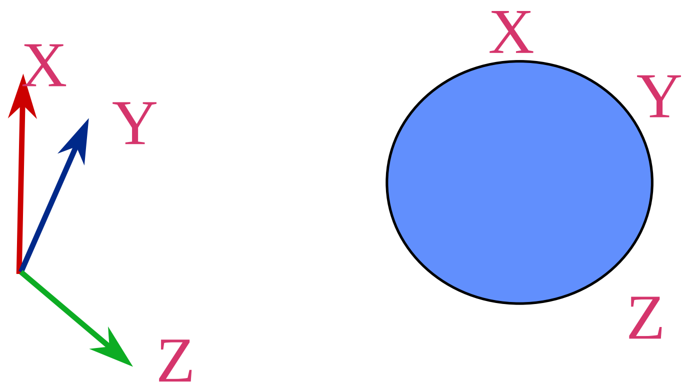
Qualitative Trigonometry (Liu 98) -- 2

- Composition table
  - given any 3 q values in a triangle can compute others
  - e.g. given AC is slightlyless than BC and C is acute then A is slightlyacute or obtuse, B is acute and AB is less or slightlyless than BC

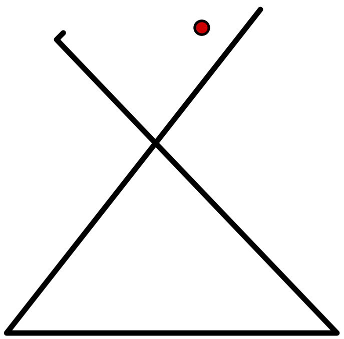


- compute quantitative visualisation by simulated annealing
- application to mechanism velocity analysis
  - deriving instantaneous velcocity relationships among constrained bodies of a mechanical assembly with kinematic joints

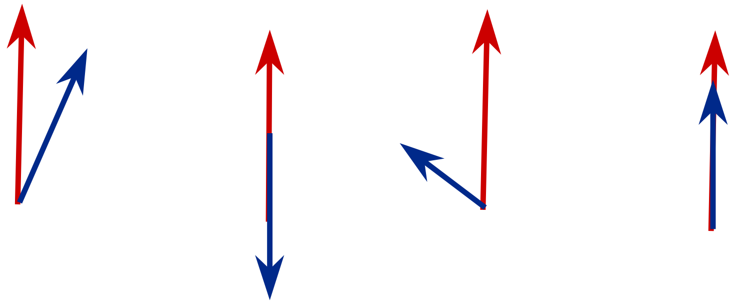
2D Cyclic Orientation



- CYCORD(X,Y,Z) (Roehrig, 97)
  - (XYZ = +)
  - axiomatised (irreflexivity, asymmetry,transitivity, closure, rotation)
  - Fairly expressive, e.g. “indian tent”
  - NP-complete

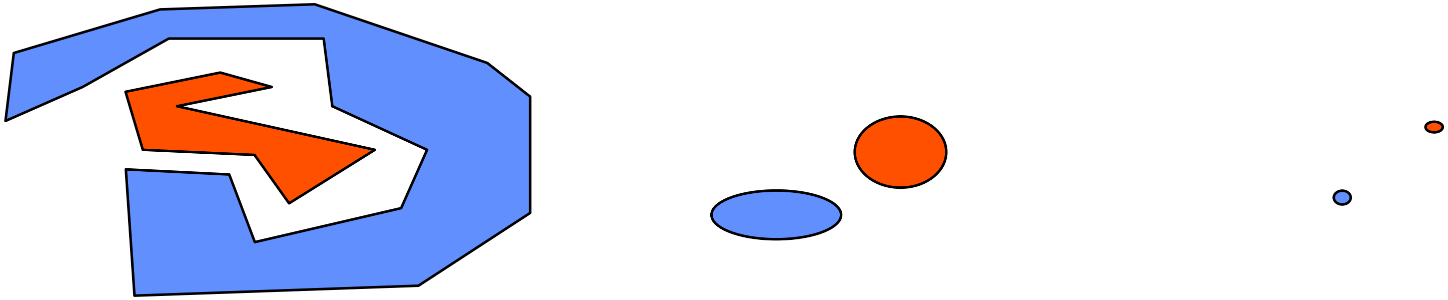


Algebra of orientation relations (Isli & Cohn 98)



- binary relations
  - BIN = {l,o,r,e}
  - composition table
    - 24 possible configurations of 3 orientations
- ternary relations
  - 24 JEPD relations
    - eee, ell, eoo, err, lel, lll, llo, llr, lor, lre, lrl, lrr, oeo, olr, ooe, orl, rer, rle, rll, rlr, rol, rrl, rro, rrr
    - CYCORD = {lrl,orl,rll,rol,rrl,rro,rrr}

Orientation: regions?

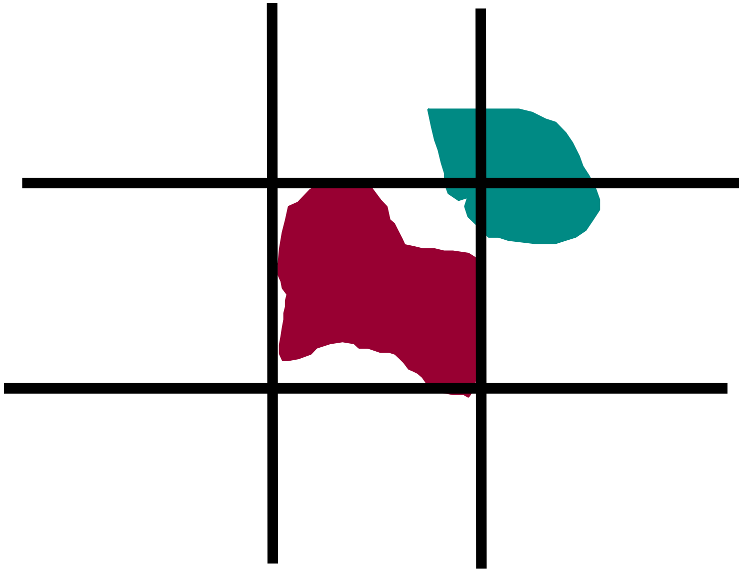


- more indeterminacy for orientation between regions vs. points

Direction-Relation Matrix (Goyal & Sharma 97)

- cardinal directions for extended spatial objects

0	1	1
0	1	1
0	0	0



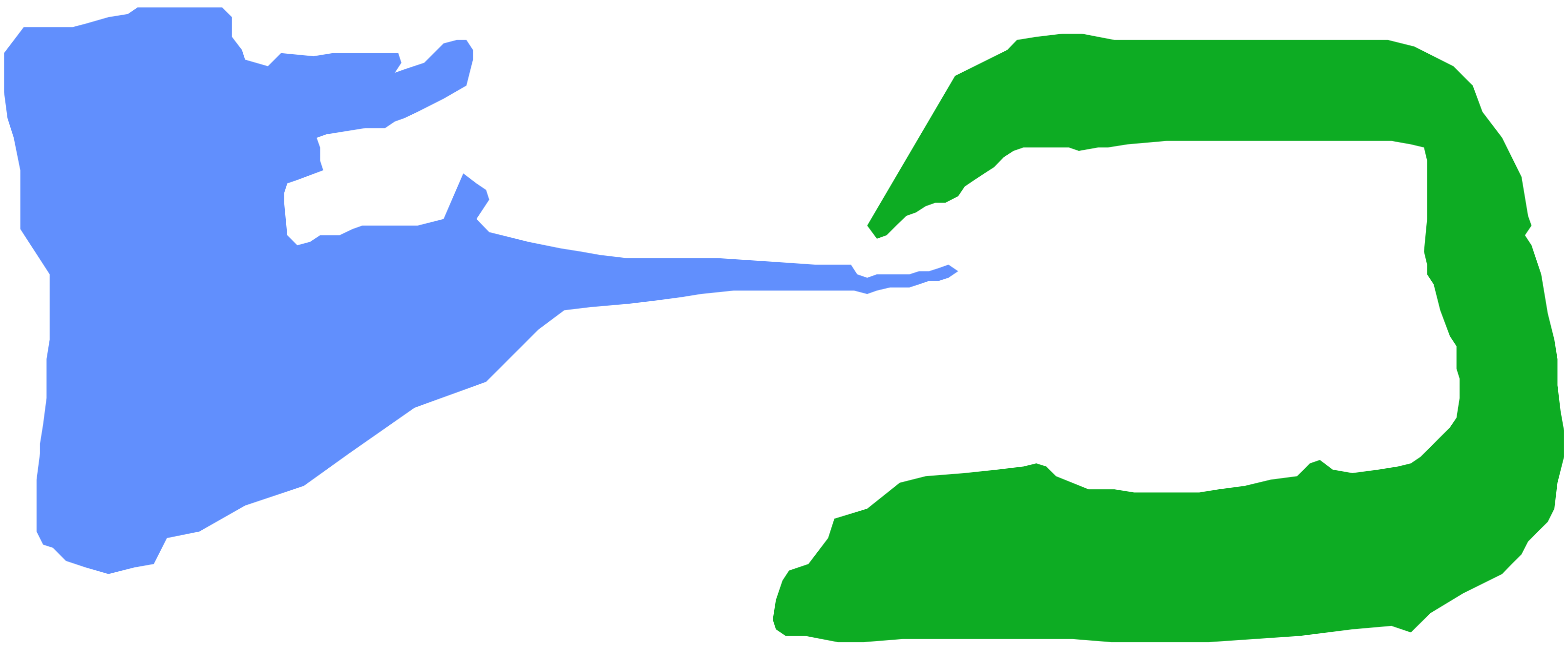
- also fine granularity version with decimal fractions giving percentage of target object in partition

Distance/Size

- Scalar qualitative spatial measurements
  - area, volume, distance,...
  - coordinates often not available
  - Standard QR may be used
    - named landmark values
    - relative values
- comparing v. naming distances
  - linear; logarithmic
  - order of magnitude calculi from QR
    - (Raiman, Mavrovouniotis)

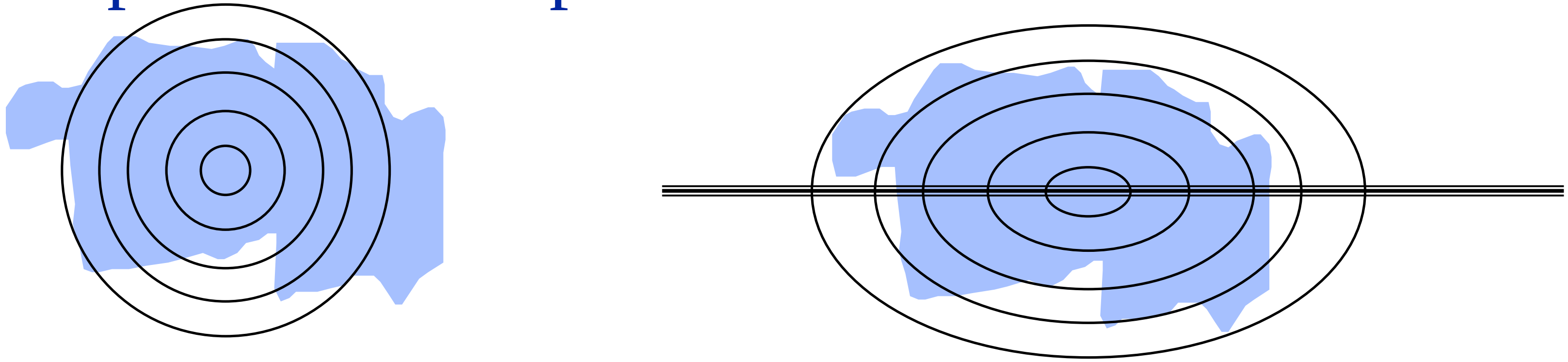
How to measure distance between regions?

- nearest points, centroid,…?
- Problem of maintaining triangle inequality law for region based theories.



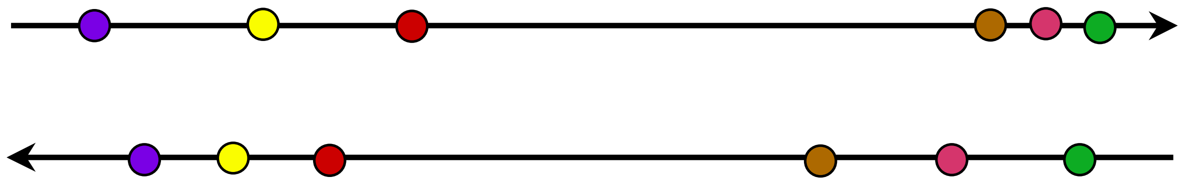
Distance distortions due to domain (1)

- isotropic v. anisotropic



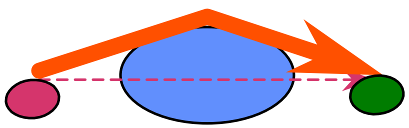
Distance distortions due to domain (2)

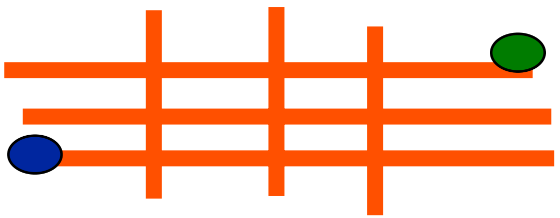
- Human perception of distance varies with distance
  - Psychological experiment:
    - Students in centre of USA ask to imagine they were on either East or West coast and then to locate a various cities wrt their longitude
    - cities closer to imagined viewpoint further apart than when viewed from opposite coast
    - and vice versa



Distance distortions due to domain (3)

- Shortest distance not always straight line in many domains





Distance distortions due to domain (4)

- kind of scale
  - ☛figural
  - ☛vista
  - ☛environmental
  - ☛geographic
- Montello (93)

Shape

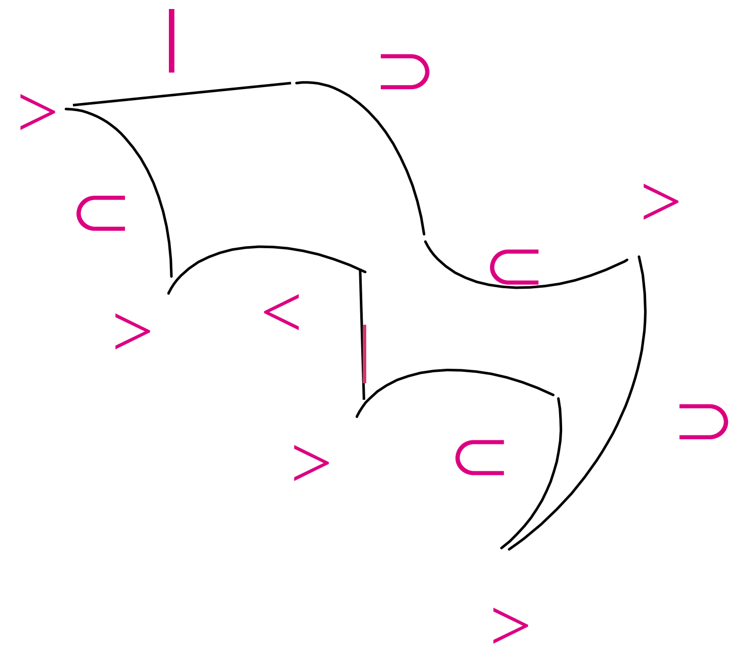
- topology ....fully metric
  - what are useful intermediate descriptions?
- metric same shape:
  - transformable by rotation, translation, scaling, reflection(?)
- What do we mean by qualitative shape?
  - in general very hard
  - small shape changes may give dramatic functional changes
  - still relatively little researched

Qualitative Shape Descriptions

- boundary representations
- axial representations
- shape abstractions
- synthetic: set of primitive shapes
  - Boolean algebra to generate complex shapes

boundary representations (1)

- Hoffman & Richards (82): label boundary segments:
  - curving out ⊃
  - curving in ⊂
  - straight |
  - angle outward >
  - angle inward <
  - cusp outward ➢
  - cusp inward ⮘



boundary representations (2)

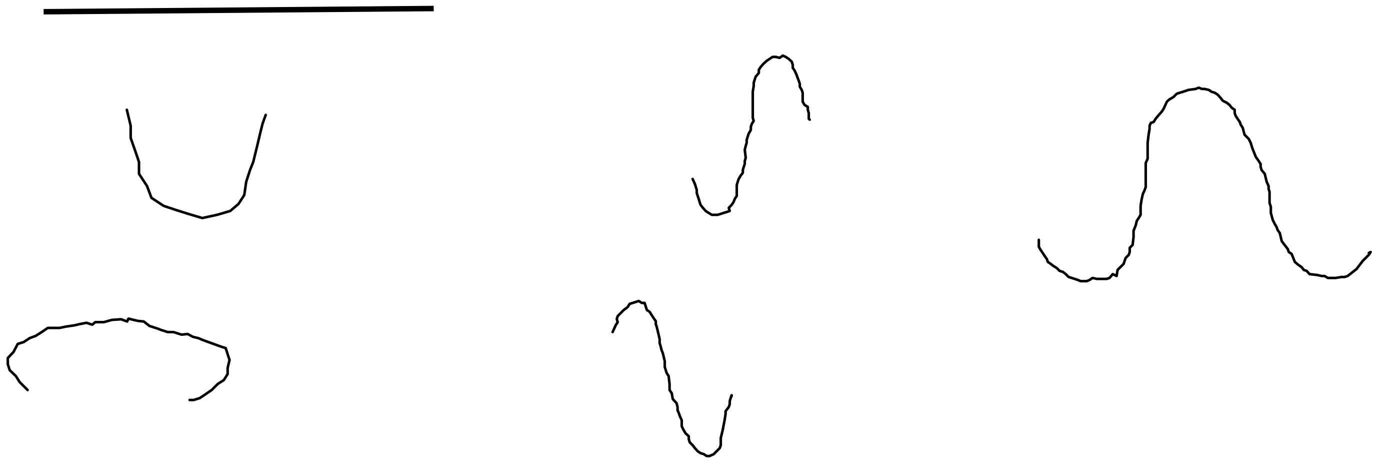
- constraints:
  - ☛consecutive terms different
  - ☛no 2 consecutive labels from {<,>, ➢, ⮘}
  - ☛< or > must be next to ➢ or⮘
- 14 shapes with 3 or fewer labels
- {⊃,|,>}: convex figures
- {<,|,>}: polygons

boundary representations (3)

- maximal/minimal points of curvature (Leyton 88)
  - Builds on work of Hoffman & Richards (82)
  - M^+: Maximal positive curvature
  - M^-: Maximal negative curvature
  - m^+: Minimal positive curvature
  - m^-: Minimal negative curvature
  - 0: Zero curvature

boundary representations (4)

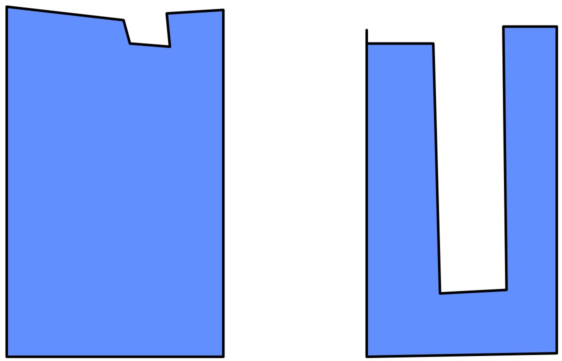
- six primitive *codons* composed of 0, 1, 2 or 3 curvature extrema:



- extension to 3D
- shape process grammar

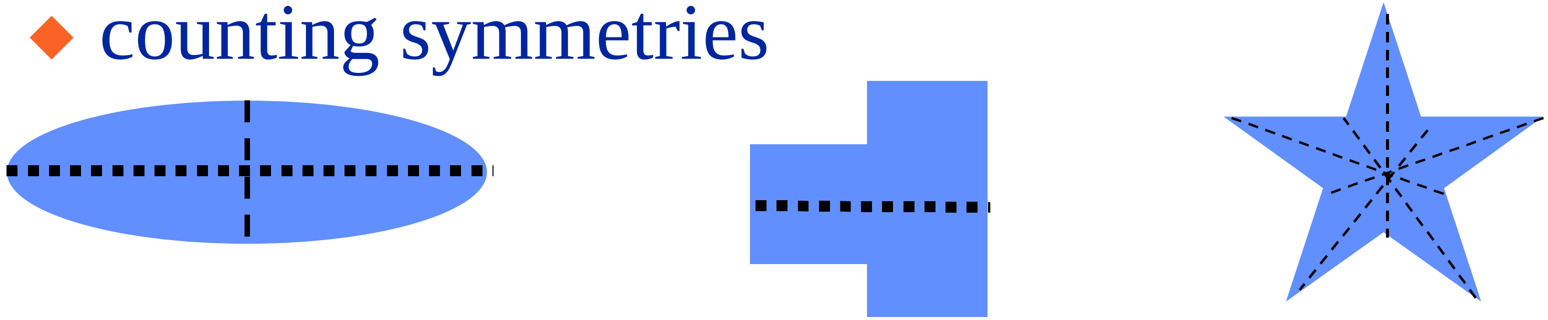
boundary representations (5)

- Could combine maximal curvature descriptions with qualitative relative length information



axial representations (1)

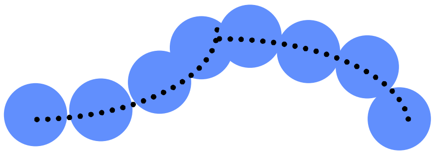
- counting symmetries



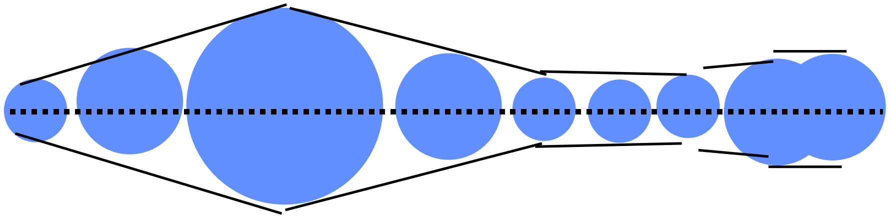
- generate shape by sweeping geometric figure along axis
  - axis is determined by points equidistant, orthogonal to axis
    - consider shape of axis
    - straight/curved
    - relative size of generating shape along axis

axial representations (2)

- generate shape by sweeping geometric figure along axis
- axis is determined by points equidistant, orthogonal to axis
- consider shape of axis



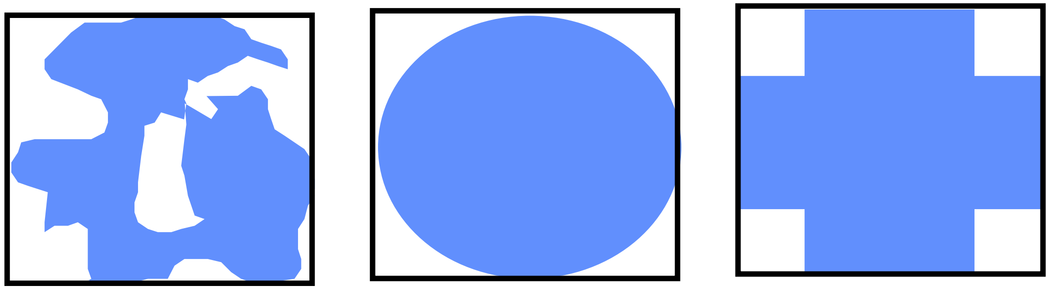
  - ☛straight/curved
  - ☛relative size of generating shape along axis



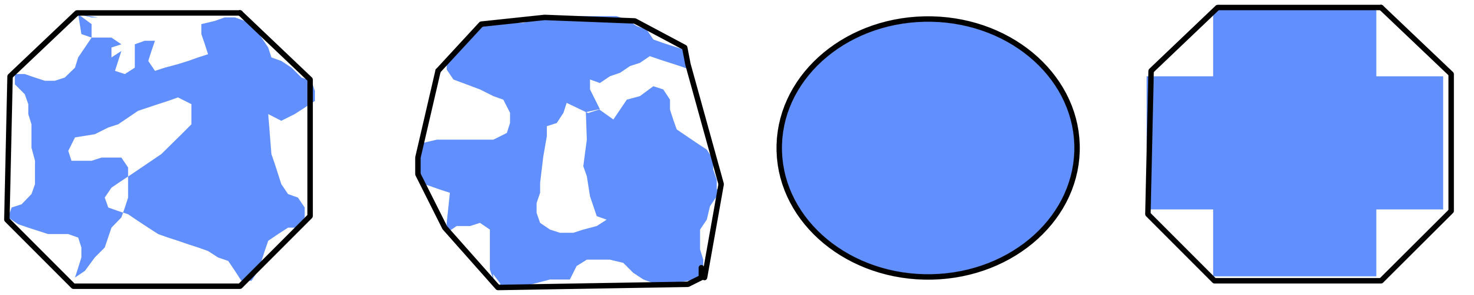
❑ increasing,decreasing,steady,increasing,steady

Shape abstraction primitives

- classify by whether two shapes have same abstraction
  - ☛bounding box

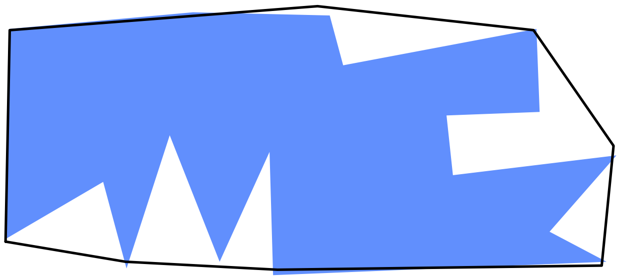


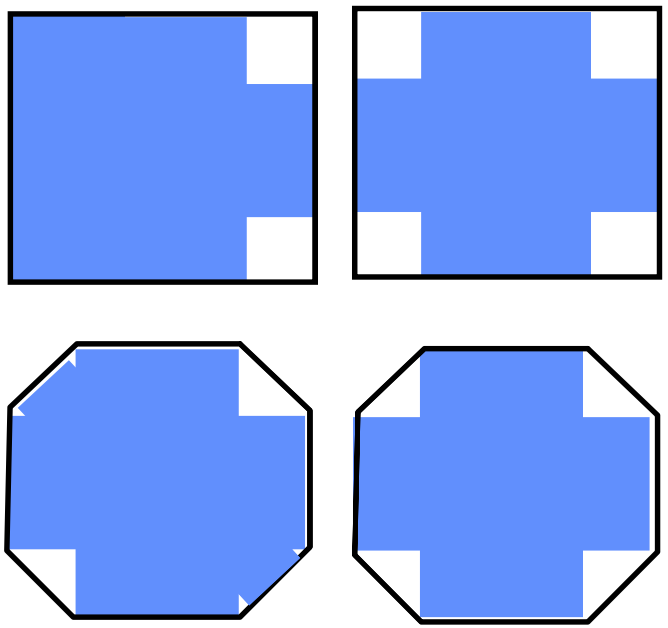
  - ☛convex hull



Combine shape abstraction with topological descriptions

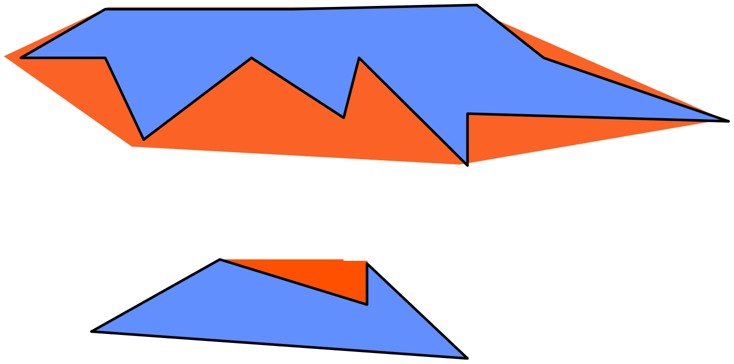
- compute difference, *d*, between shape, *s* and abstraction of shape, *a*.
- describe topological relation between:
  - components of *d*
  - components of *d* and *s*
  - components of *d* and *a*
- shape abstraction will affect similarity classes





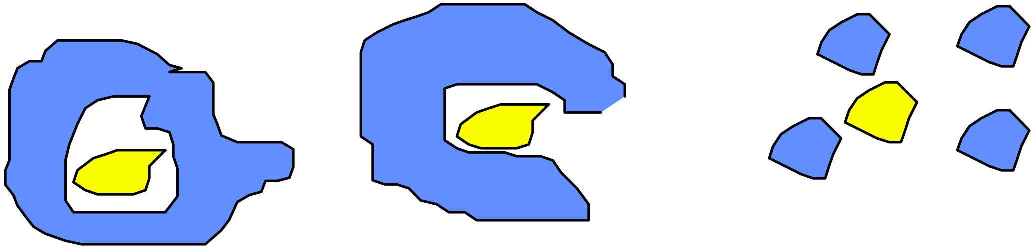
Hierarchical shape description

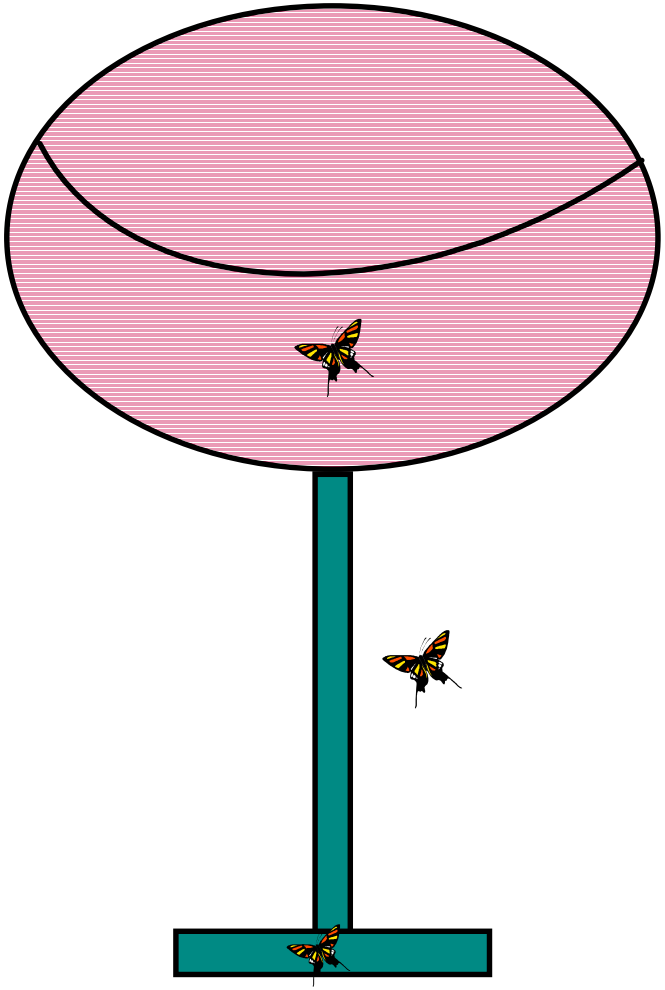
- Apply above technique recursively to each component which is not idempotent w.r.t. shape abstraction
  - ☛Cohn (95), Sklansky (72)



Describing shape by comparing 2 entities

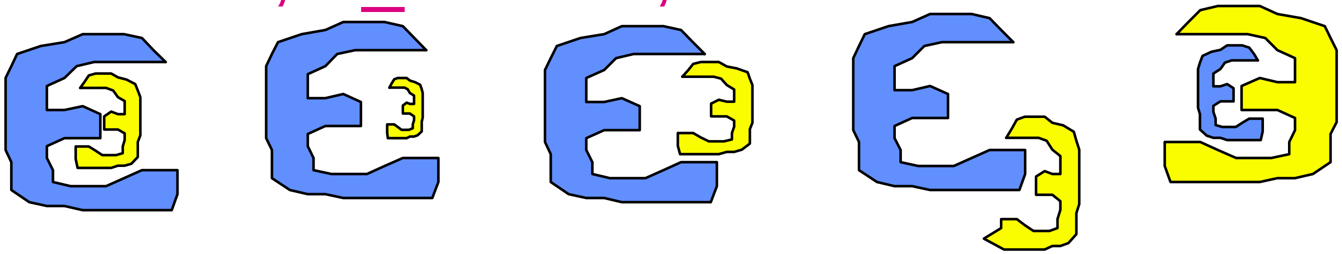
- conv(x) + C(x,y)
  - ☛ topological inside
  - ☛ geometrical inside
  - ☛ “scattered inside”
  - ☛ “containable inside”
  - ☛ ...

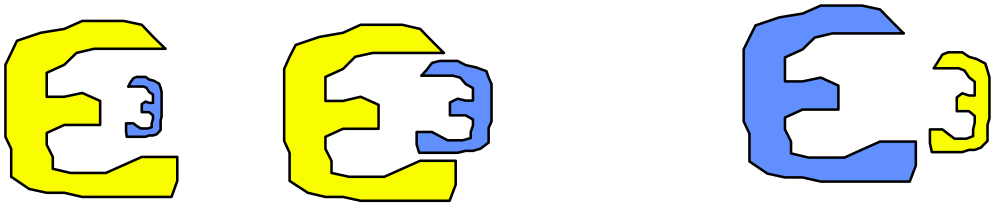




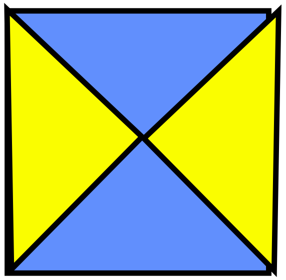
Making JEPD sets of relations

- Refine DC and EC:
  - INSIDE, P_INSIDE, OUTSIDE:





- INSIDE_INSIDEi_DC does not exist (except for weird regions).

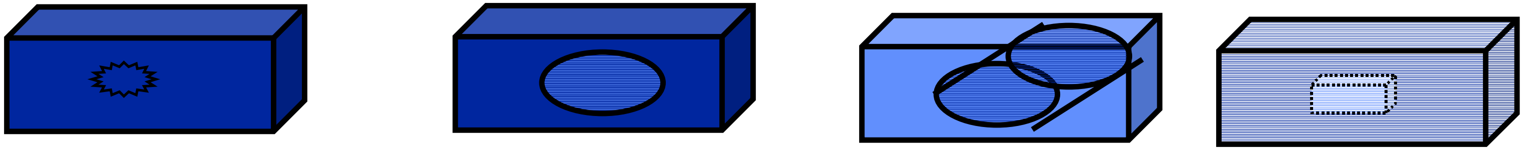


Expressiveness of conv(x)

- Constraint language of EC(x) + PP(x) + Conv(x)
  - can distinguish any two bounded regular regions not related by an affine transformation
  - Davis et al (97)

Holes and other superficialities
Casati & Varzi (1994), Varzi (96)

- Taxonomy of holes:
  - depression, hollow, tunnel, cavity



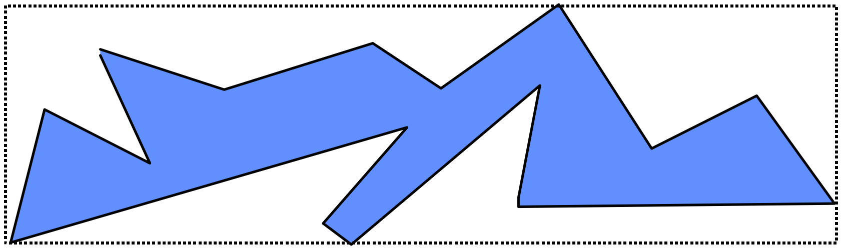
- "Hole realism"
  - hosts are first class objects
- "Hole irrealism"
  - "x is holed"
  - "x is α-holed"

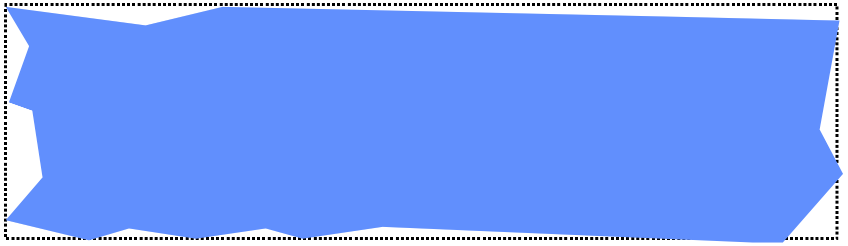
Holes and other superficialities
Casati & Varzi (1994), Varzi (96)

- Outline of theory
  - H(*x*): *x* is a hole in/though *y* (its host)
  - mereotopology
  - axioms, e.g.:
    - the host of a hole is not a hole
    - holes are one-piece
    - holes are connected to their hosts
    - every hole has some one piece host
    - no hole has a proper hole-part that is EC with same things as hole itself

Compactness (Clementini & di Felici 97)

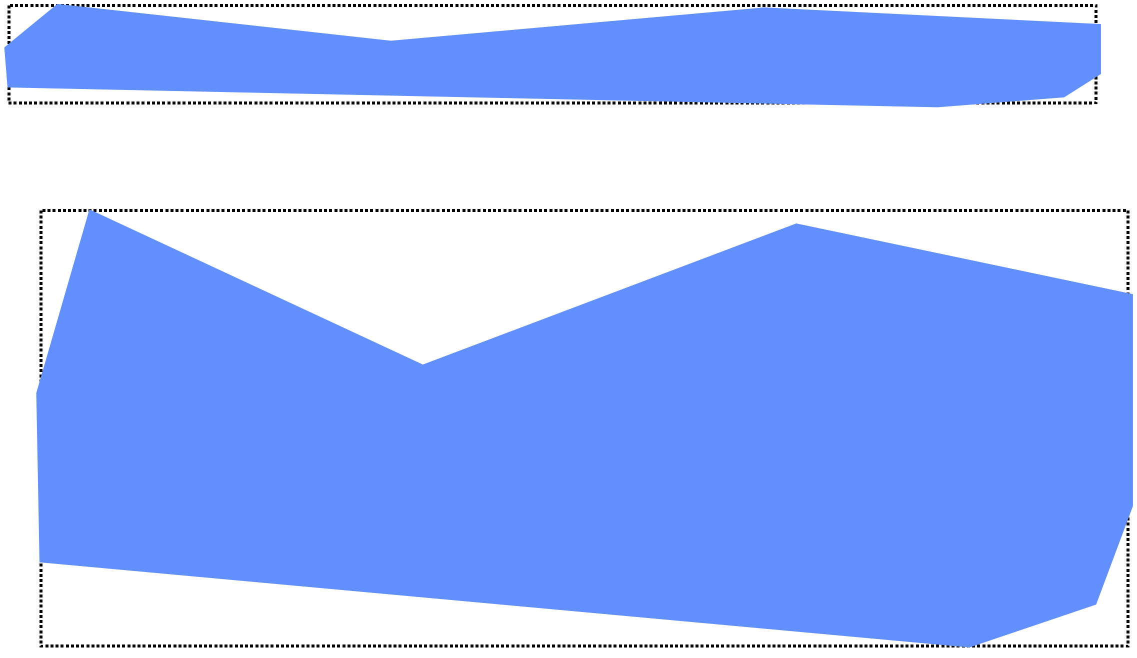
- Compute minimum bounding rectangle (MBR)
  - consider ratio between shape and MBR –shape
  - use order of magnitude calculus to compare
    - e.g. Mavrovouniotis & Stephanopolis (88)
    - a<<b, a<b, a~<b, a=b, a~>b, a>b, a>>b





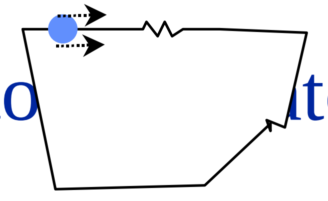
Elongation (Clementini & di Felici 97)

- Compare ratio of sides of MBR using order of magnitude calculus



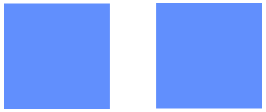
Shape via congruence (Borgo et al 96)

- Two primitives:
  - CG(*x*,*y*): *x* and *y* are congruent
  - topological primitive
- more expressive than conv(x)
  - build on Tarski’s geometry
  - define sphere
  - define Inbetween(*x*,*y*,*z*)
  - define conv(*x*)
- Notion of a “grain” to … te small surface irregularities

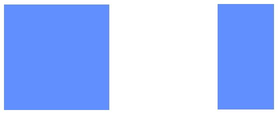


Shape via congruence and topology

- can (weakly) constrain shape of rigid objects by topological constraints (Galton 93, Cristani 99):
  - ☛congruent -- DC,EC,PO,EQ -- CG

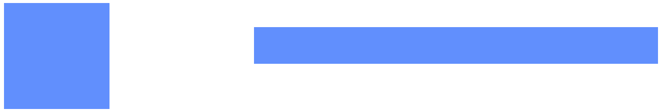


  - ☛just fit inside - DC,EC,PO,TPP -- CGTPP
    - ❑ *(& inverse)*



  - ☛fit inside - DC,EC,PO ,TPP,NTPP -- CGNTPP
    - ❑ *(& inverse)*

  - ☛incomensurate: DC,EC,PO -- CNO



"Shape" via Voronoi hulls (Edwards 93)

- Draw lines equidistant from closest spatial entities
- Describe topology of resulting set of *"Voronoi regions"*
  - ☛proximity, betweeness, inside/outside, amidst,...
- Notice how topology changes on adding new object

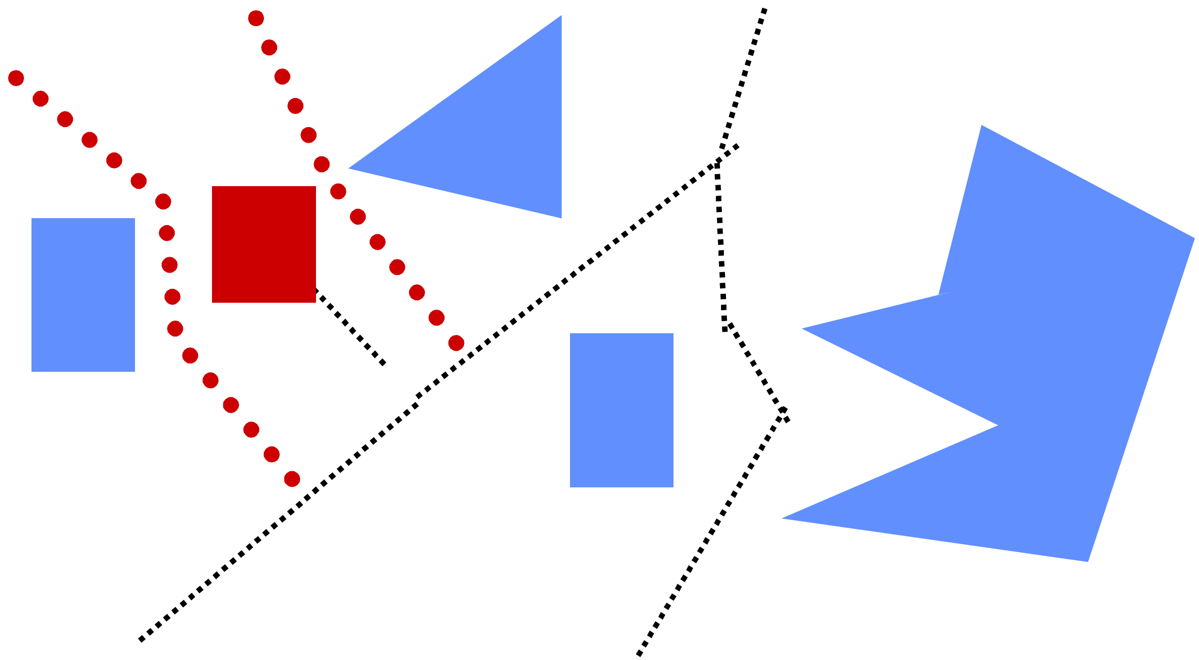
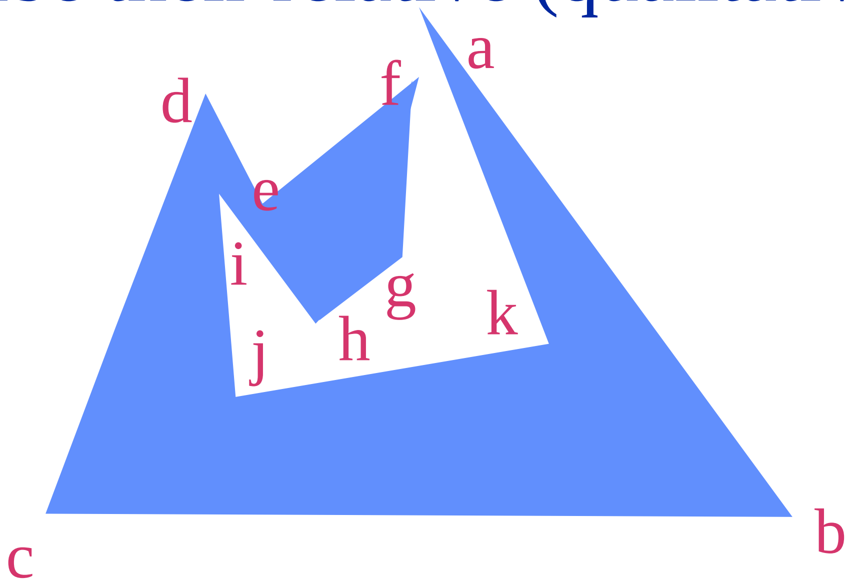


Figure drawn by hand - very approximate!!

Shape via orientation

- pick out selected parts (points) of entity
  - ☛(e.g. max/min curvatures)
- describe their relative (qualitative) orientation
- E.g.:



abc = –

acd = –

…

cgh = 0

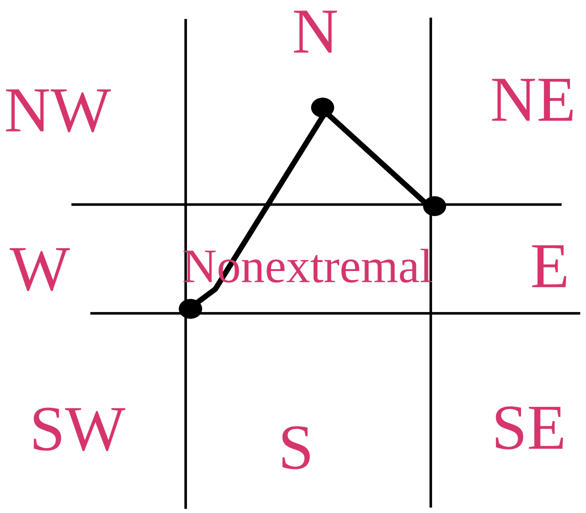
…

ijk = +

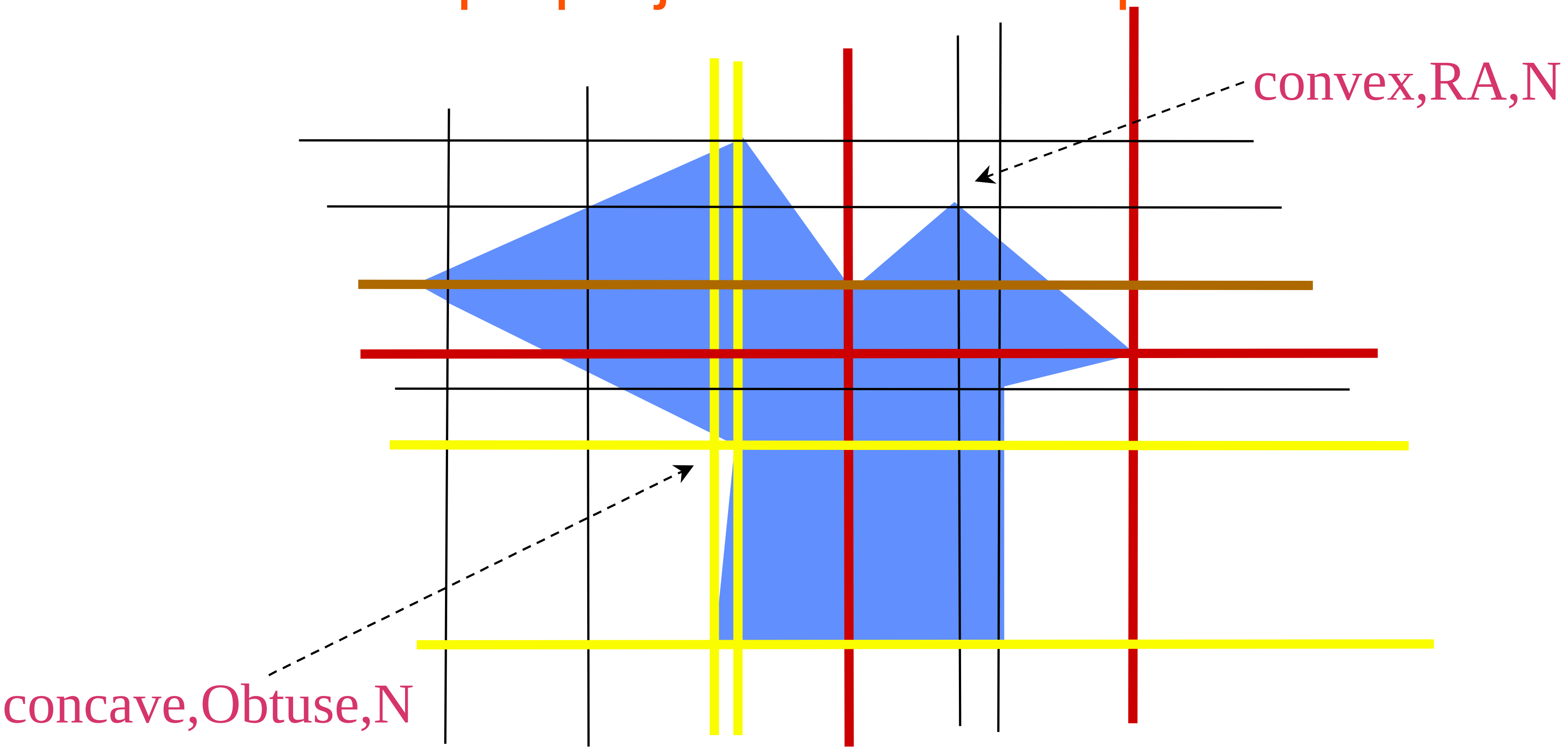
...

Slope projection approach

- Technique to describe polygonal shape
  - equivalent to Jungert (93)
- For each corner, describe:
  - convex/concave
  - obtuse, right-angle, acute
  - extremal point type:
    - non extremal
    - N/NW/W/SW/S/SE/E/NE
  - Note: extremality is local not global property



Slope projection -- example



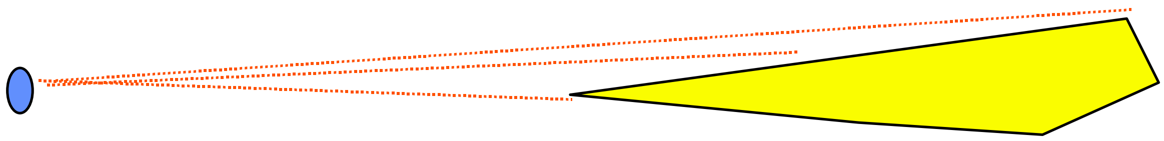
- Give sequence of corner descriptions:
  - convex,RA,N … concave,Obtuse,N …
- More abstractly, give sequence of relative angle sizes:
  - a1>a2<a3>a4<a5>a6=a7<a7>a8<a1

Shape grammars

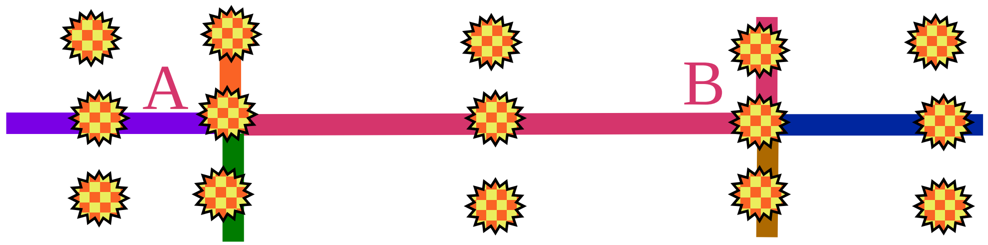
- specify complex shapes from simpler ones
- only certain combinations may be allowable
- applications in, e.g., architecture

Interdependence of distance & orientation (1)

- Distance varies with orientation



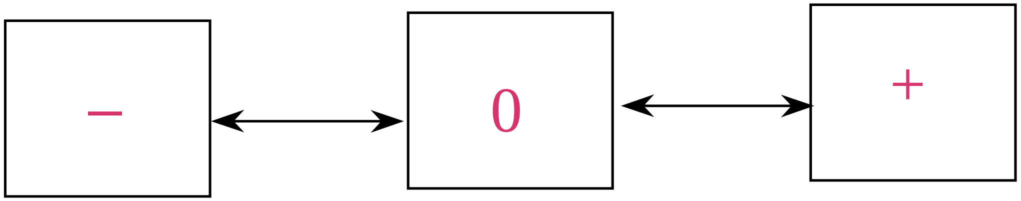
Interdependence of distance & orientation (2)



- Freksa & Zimmerman (93)
- Given the vector AB, there are 15 positions C can be in, w.r.t. A
- Some positions are in same direction but at different distances

Spatial Change

- Want to be able to reason over time
  - ☛continuous deformation, motion
- c.f.. traditional Qualitative simulation (e.g. QSIM: Kuipers, QPE: Forbus,…)



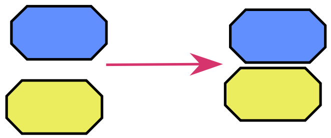
- Equality change law
  - ☛transitions from time point instantaneous
  - ☛transitions to time point non instantaneous

Kinds of spatial change (1)

- Topological changes in ‘single’ spatial entity:
  - change in dimension
    - usually by abstraction/granularity shift
      - e.g. road: 1D $\Rightarrow$ 2D $\Rightarrow$ 3D
  - change in number of topological components
    - e.g. breaking a cup, fusing blobs of mercury
  - change in number of tunnels
    - e.g. drilling through a block of wood
  - change in number of interior cavities
    - e.g. putting lid on container

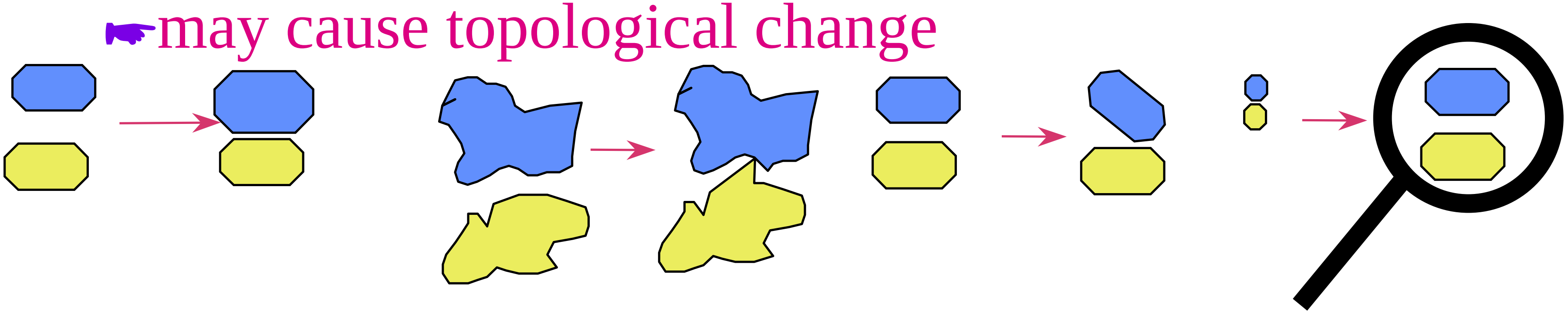
Kinds of spatial change (2)

- Topological changes between spatial entities:
  - ☛e.g. change of RCC/4IM/9IM/… relation



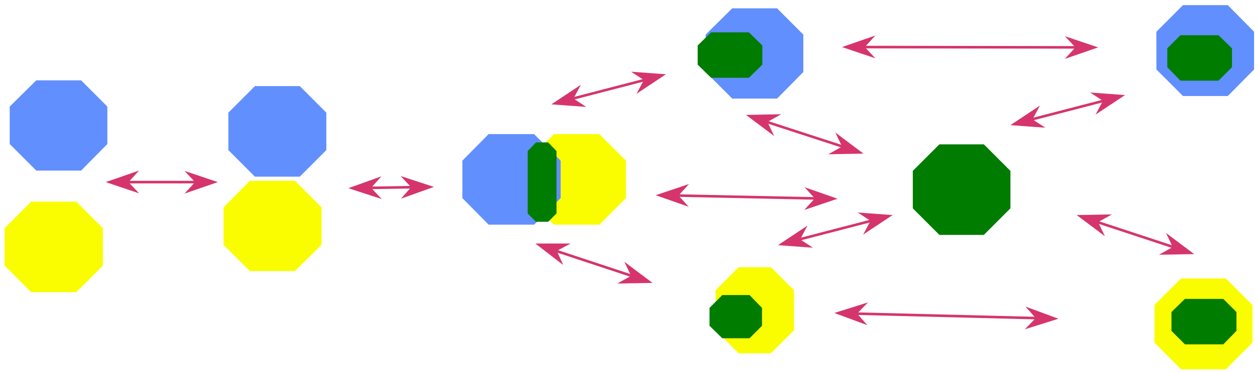
change in position, size, shape, orientation, granularity

- ☛may cause topological change



Continuity Networks/Conceptual Neighbourhoods

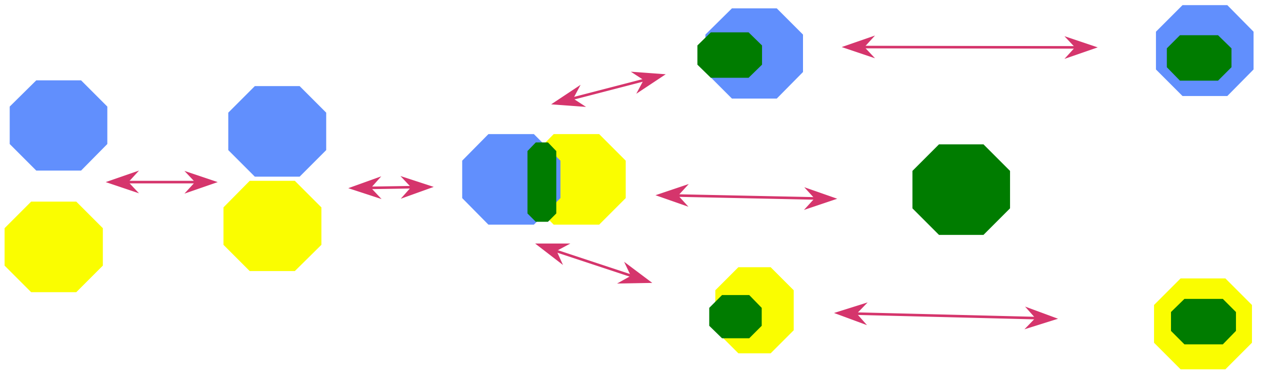
- What are next qualitative relations if entities transform/translate continuously?
  - ☛E.g. RCC-8



- If uncertain about the relation what are the next most likely possibilities?
  - ☛Uncertainty of precise relation will result in connected subgraph (Freksa 91)

Specialising the continuity network

- can delete links given certain constraints
  - e.g. no size change
  - (c.f. Freksa's specialisation of temporal CN)

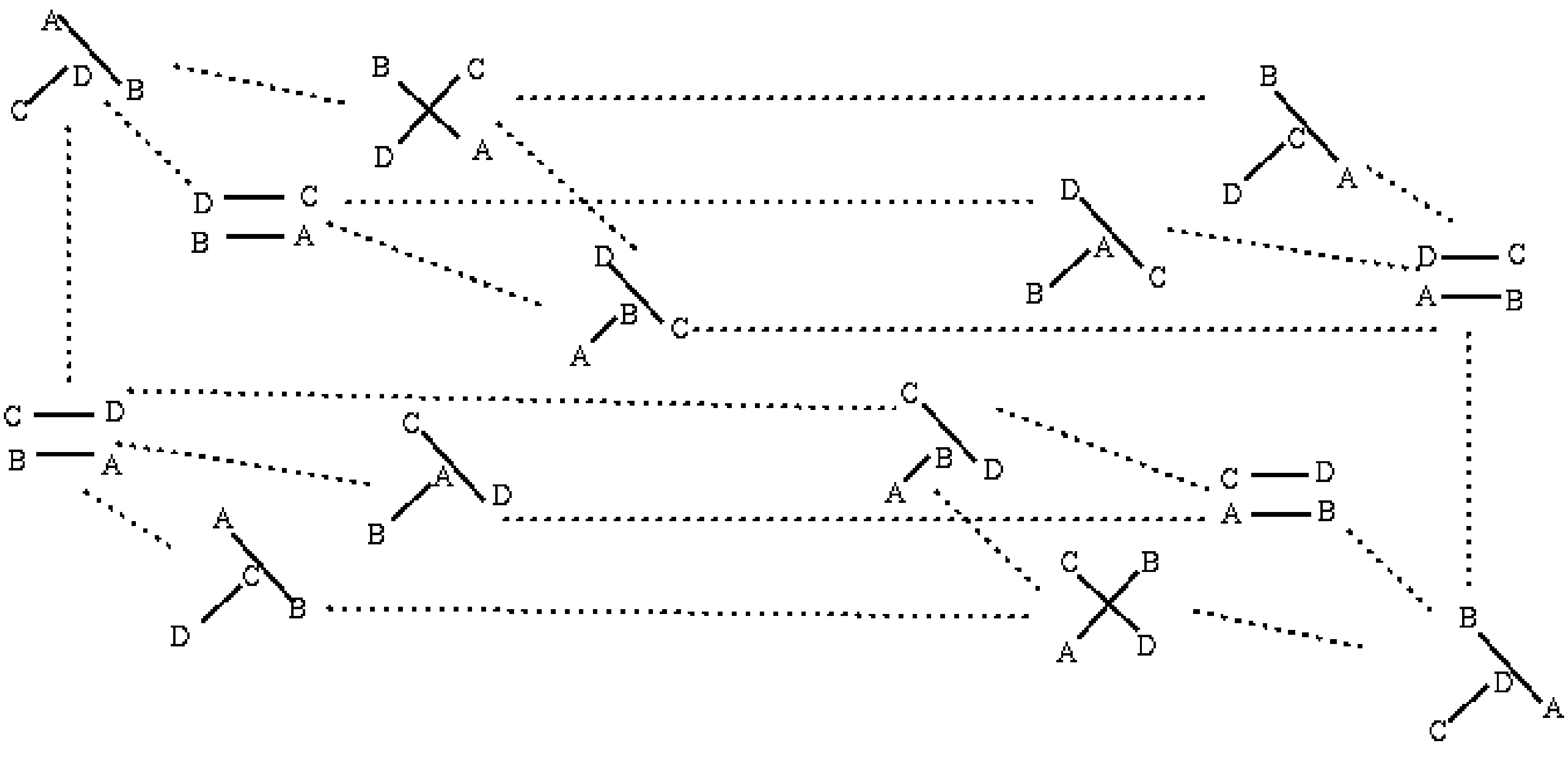


Qualitative simulation (Cui et al 92)

- Can be used as basis of qualitative simulation algorithm
  - initial state set of ground atoms (facts)
  - generate possible successors for each fact
  - form cross product
  - apply any user defined add/delete rules
  - filter using user defined rules
  - check each new state (cross product element) for consistency (using composition table)

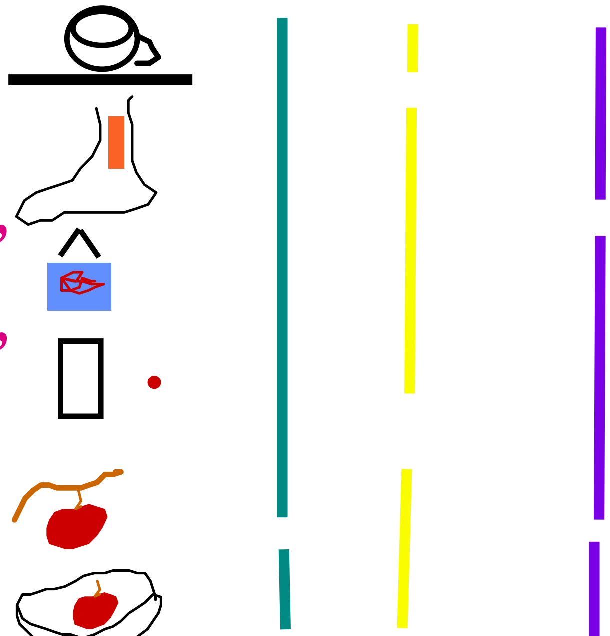
Conceptual Neighbourhoods for other calculi

- Virtually every calculus with a set of JEPD relations has presented a CN.
- E.g.



A linguistic aside

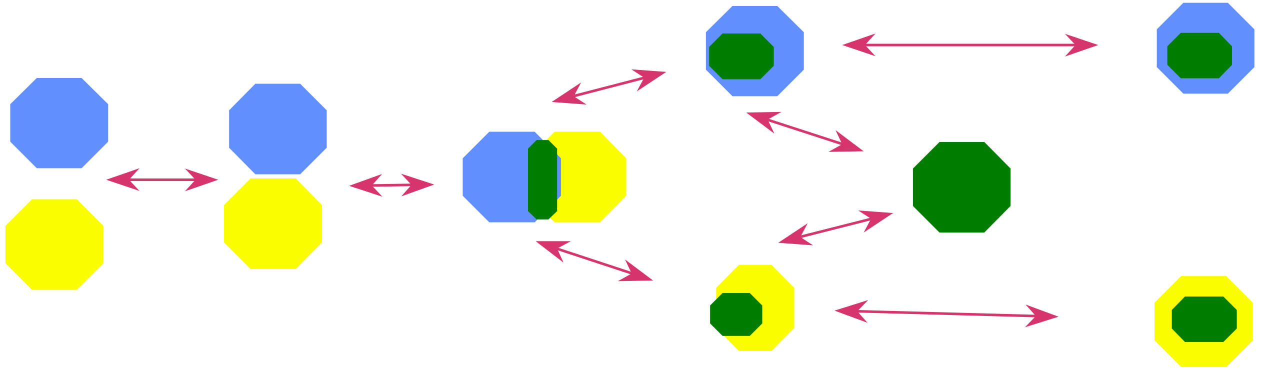
- Spatial prepositions in natural language seem to display a conceptual neighbourhood structure. E.g. consider: “put
  - cup on table”
  - bandaid on leg”
  - picture on wall”
  - handle on door”
  - apple on twig”
  - apple in bowl”



- Different languages group these in different ways but always observing a linear conceptual neighbourhood (Bowerman 97)

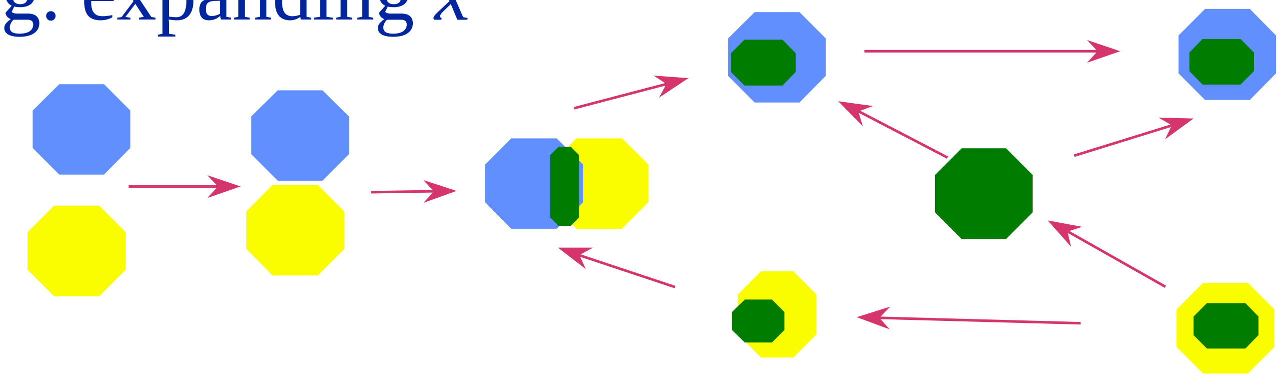
Closest topological distance (Egenhofer & Al-Taha 92)

- For each 4-IM (or 9-IM) matrix, determine which matrices are closest (fewest entries changed)
- Closely related to notion of conceptual neighbourhood
  - ☛3 "missing" links!



Modelling spatial processes (Egenhofer & Al-Taha 92)

- Identify traversals of CN with spatial processes
- E.g. expanding *x*



- Other patterns:
  - ☛reducing in size, rotation, translation

Leyton's (88) Process Grammar

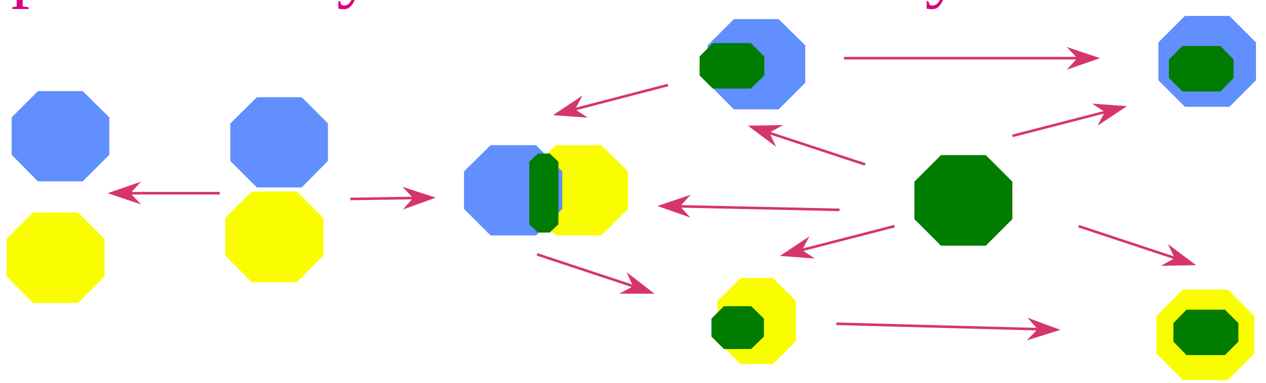
- Each of the maximal/minimal curvatures is produced by a process
  - protrusion
  - resistance
- Given two shapes can infer a process sequence to change one to the other

Lundell (96) Spatial Process on physical fields

- inspired by QPE (Forbus 84)
- processes such as heat flow
- topological model
- qualitative simulation

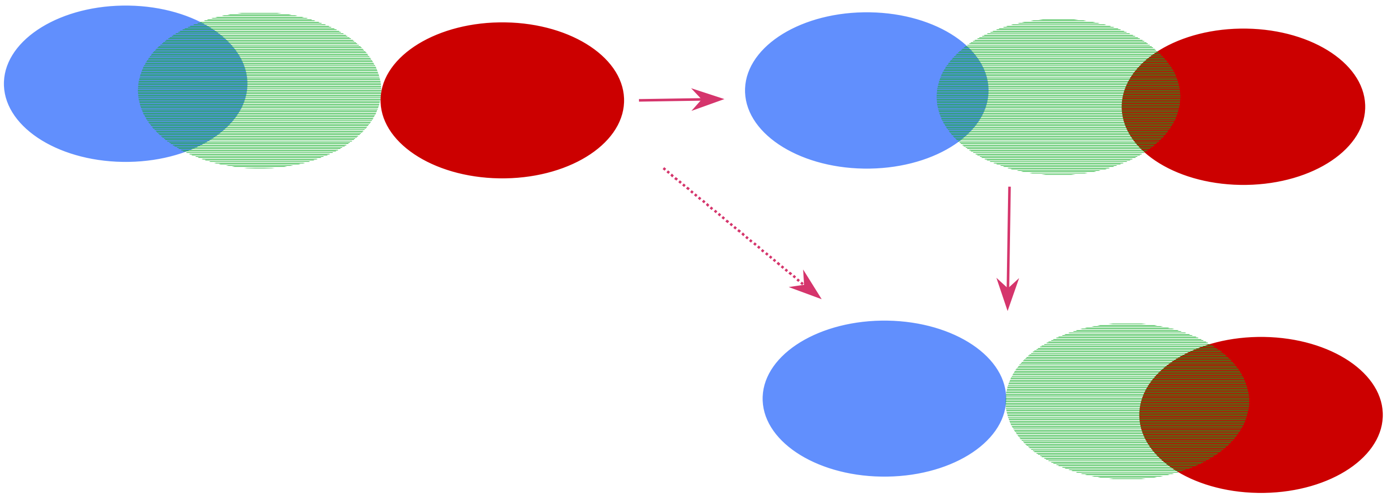
Galton's (95) analysis of spatial change

- Given underlying semantics, can generate continuity networks automatically for a class of relations which may hold at different times
- Moreover, can determine which relations *dominate* each other
  - R1 dominates R2 if R2 can hold over interval followed/preceded by R1 instantaneously
- E.g. RCC8



Using dominance to disambiguate temporal order

- Consider

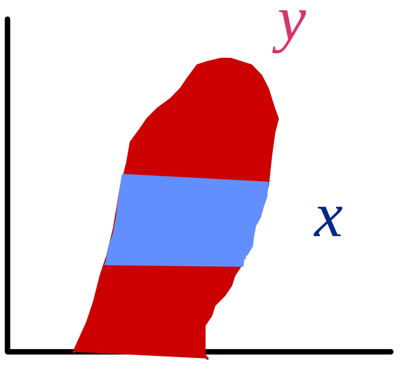


- simple CN will predict ambiguous immediate future
- dominance will forbid dotted arrow
- states of position v. states of motion
- c.f. QR's *equality change law*

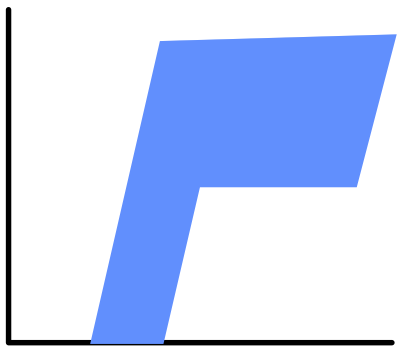
Spatial Change as Spatiotemporal histories (1) (Muller 98)

- Hayes proposed idea in *Naïve Physics Manifesto*
  - ☛(See also: Russell(14), Carnap(58))
- C(*x*,*y*) true iff the n-D spatio-temporal regions *x*,*y* share a point (Clark connection)
- $x < y$ true if spatio-temporal region *x* is temporally before *y*
- $x <> y$ true iff the n-D spatio-temporal regions *x*,*y* are temporally connected
- axiomatised à la Asher/Vieu(95)

Spatial Change as Spatiotemporal histories (2) (Muller 98)

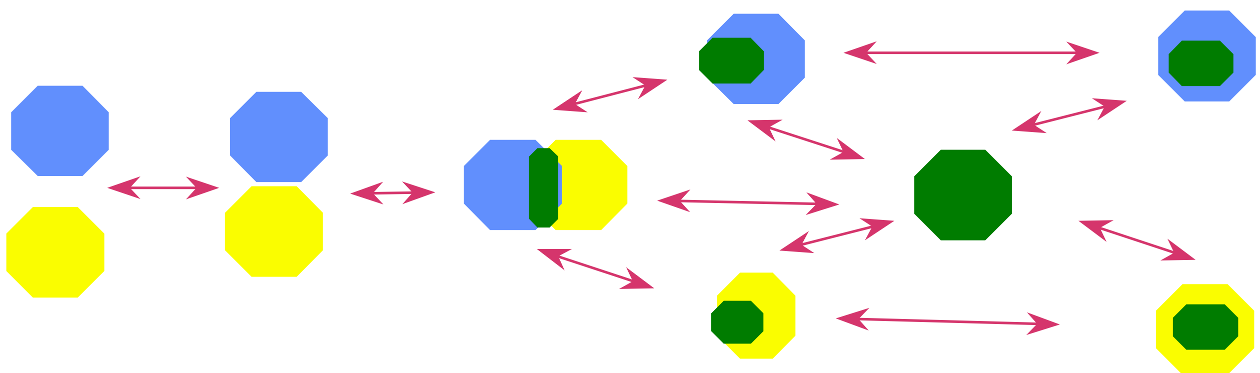


- Defined predicates
  - Con(*x*)
  - TS(*x*,*y*) -- *x* is a "temporal slice" of *y*
    - i.e. maximal part wrt a temporal interval
  - CONTINUOUS(*w*) -- *w* is continuous
    - Con(*w*) and every temporal slice of *w* temporally connected to some part of *w* is connected to that part

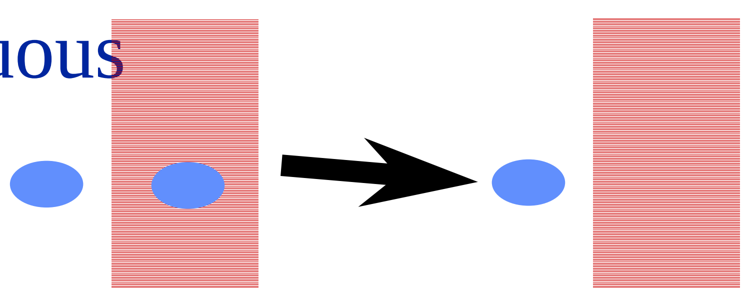


Spatial Change as Spatiotemporal histories (3) (Muller 98)

- All arcs not present in RCC continuity network/conceptual neighbourhood proved to be not CONTINUOUS



- EG DC-PO link is non continuous
  - ☛consider two puddles drying:

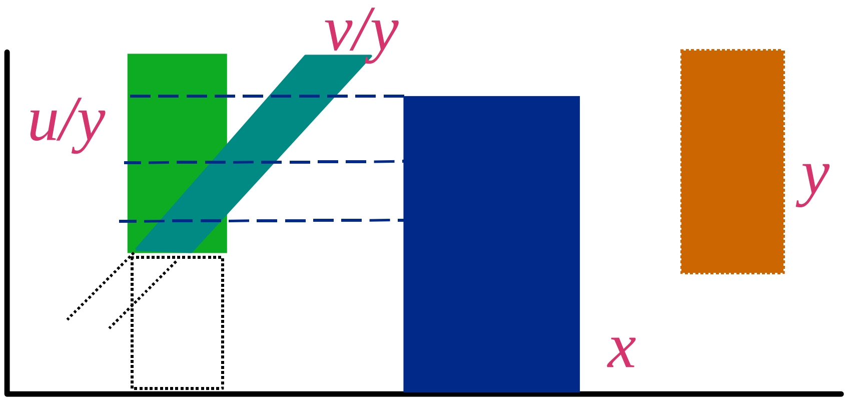


Spatial Change as Spatiotemporal histories (4) (Muller 98)

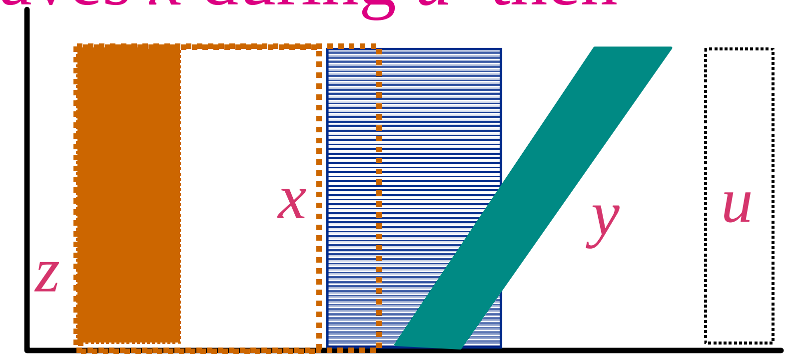
- Taxonomy of motion classes:

Spatial Change as Spatiotemporal histories (4) (Muller 98)

- Composition table combining Motion & temporal k:
  - e.g. if *x* temporally overlaps *y* and *u* Leaves *v* during *y* then {PO,TPP,NTPP}$(u_{/x}, v_{/x})$



- Also, Composition table combining Motion & static k:
  - e.g. if *y* spatially DC *z* and *y* Leaves *x* during *u* then {EC,DC,PO}(x,z)



Is there something special about region based theories?

- 2D Mereotopology: standard 2D point based interpretation is simplest model (prime model)
  - ☛proved under assumptions: Pratt & Lemon (97)
  - ☛only alternative models involve ∞-piece regions
- But: still useful to have region based theories even if always interpretable point set theoretically.

Adequacy Criteria for QSR (Lemon and Pratt 98)

- *Descriptive parsimony:* inability to define metric relations (**Q**SR)
- *Ontological parsimony:* restriction on kinds of spatial entity entertained (e.g. no non regular regions)
- *Correctness:* axioms must be true in intended interpretation
- *Completeness:* consistent sentences should be realizable in a "standard space" (Eg R^2 or R^3)
  - counter examples:
    - Von Wright's logic of near: some consistent sentences have no model
    - consistent sentences involving conv(x) not true in 2D
    - consistent sentence for a non planar graph false in 2D

Some standard metatheoretic notions for a logic

- Complete
  - given a theory ϑ expressed in a language L, then for every wff ϕ: $\phi \in \vartheta$ or $\neg\phi \in \vartheta$
- Decidable
  - terminating procedure to decide theoremhood
- Tractable
  - polynomial time decision procedure

Metatheoretic results: decidability (1)

- Grzegorczyk(51): topological systems not decidable
  - Boolean algebra is decidable
  - *add:* closure operation or EC results in undecidability
    - can encode arbitrary statements of arithmetic
- Dornheim (98) proposes a simple but expressive model of polygonal regions of the plane
  - usual topological relations are provably definable so the model can be taken as a semantics for plane mereotopology
  - proves undecidability of the set of all first-order sentences that hold in this model
  - so no axiom system for this model can exist.

Metatheoretic results: decidability (2)

- Elementary Geometry is decidable
- Are there expressive but decidable region based 1st order theories of space?
- Two approaches:
  - Attempt to construct decision procedure by quantifier elimination
  - Try to make theory complete by adding existence and dimension axioms
    - any complete, recursively axiomatizable theory is decidable
    - achieved by Pratt & Schoop but not in finitary 1st order logic
- Alternatively: use 0 order theory

Metatheoretic results: decidability (3)

- Decidable subsystems?
  - Constraint language of “RCC8” (Bennett 94)
    - (See below)
  - Constraint language of RCC8 + Conv(x)
    - Davis et al (97)

Other decidable systems

- Modal logics of place
  - ◊P: “P is true somewhere else” (von Wright 79)
  - accessibility relation is ≠ (Segeberg 80)
  - generalised to <n>P: “P is true within n steps” (Jansana 92)
  - proved canonical, hence complete
  - have finite model property so decidable

Intuitionistic Encoding of RCC8: (Bennett 94) (1)

- Motivated by problem of generating composition tables
- Zero order logic
  - “Propositional letters” denote (open) regions
  - logical connectives denote spatial operations
    - e.g. $\vee$ is sum
    - e.g. $\Rightarrow$ is P
- *Spatial logic* rather than logical theory of space

Intuitionistic Encoding of RCC8 (2)

- Represent RCC relation by two sets of constraints:
  - "model constraints" "entailment constraints"

	"model constraints"	"entailment constraints"
DC(x,y)	$\sim x \vee \sim y$	$\sim x$, $\sim y$
EC(x,y)	$\sim(x \wedge y)$	$\sim x$, $\sim y$, $\sim x \vee \sim y$
PO(x,y)	---	$\sim x$, $\sim y$, $\sim x \vee y$, $y \Rightarrow x$, $\sim x \vee \sim y$
TPP(x,y)	$x \Rightarrow y$	$\sim x$, $\sim y$, $\sim x \vee y$, $y \Rightarrow x$
NTPP(x,y)	$\sim x \vee y$	$\sim x$, $\sim y$, $y \Rightarrow x$
EQ(x,y)	$x \Leftrightarrow y$	$\sim x$, $\sim y$

Reasoning with Intuitionistic Encoding of RCC8

- Given situation description as set of RCC atoms:
  - for each atom A_i find corresponding 0-order representation $\langle M_i, E_i \rangle$
  - compute $\langle \cup_i M_i, \cup_i E_i \rangle$
  - for each F in $\cup_i E_i$, user intuitionistic theorem prover to determine if $\cup_i M_i \vdash F$ holds
  - if so, then situation description is inconsistent
- Slightly more complicated algorithm determines entailment rather than consistency

Extension to handle conv(x)

- For each region, *r*, in situation description add new region *r'* denoting convex hull of *r*
- Treat axioms for conv(x) as axiom schemas
  - ☛instantiate finitely many times
- carry on as in RCC8
- generated composition table for RCC-23

Alternative formulation in modal logic

- use 0-order modal logic
- modal operators for
  - ☛interior
  - ☛convex hull

Spatiotemporal modal logic (Wolter & Zakharyashev)

- Combine point based temporal logic with RCC8
  - temporal operators: **S**ince**,** **U**ntil
  - can be define: **N**ext (O), Always in the future ®$^{+}$, Sometime in the future ©$^{+}$
  - **ST_0: allow temporal operators on spatial formulae**
  - satisfiability is PSPACE complete
  - **Eg ¬** ®$^{+}$P(Kosovo,Yugoslavia)
    - Kosovo will not always be part of Yugoslavia
  - can express continuity of change (conceptual neighbourhood)
- Can add Boolean operators to region terms

Spatiotemporal modal logic (contd)

- **ST_1: allow O to apply to region variables (iteratively)**
  - **Eg** ® $^+$P(O EU,EU)
    - The EU will never contract
  - satisfiability decidable and NP complete
- ST_2: allow the other temporal operators to apply to region variables (iteratively)
  - finite change/state assumption
  - satisfiability decidable in EXPSPACE
  - P(Russia, © $^+$ EU)
    - all points in Russia will be part of EU (but not necessarily at the same time)

Metatheoretic results: completeness (1)

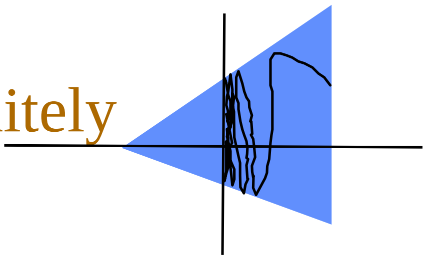
- *Complete*: given a theory ϑ expressed in a language *L*, then for every wff ϕ: $\phi \in \vartheta$ or $\neg\phi \in \vartheta$
- Clarke's system is complete (Biacino & Gerla 91)
  - regular sets of Euclidean space are models
  - Let ϑ be wffs true in such a model, then
  - however, only mereological relations expressible!
    - characterises complete atomless Boolean algebras

Metatheoretic results: completeness (2)

- Asher & Vieu (95) is sound and complete
  - identify a class of models for which the theory RT_0 generated by their axiomatisation is sound and complete
  - Notion of “weak connection” forces non standard model: non dense -- does this matter?

Metatheoretic results: completeness (3)

- Pratt &Schoop (97): complete 2D topological theory
  - 2D finite (polygonal) regions
    - eliminates non regular regions and, e.g., infinitely oscilating boundaries (idealised GIS domain)



  - *primitives*: null and universal regions, +,*,-, CON(*x*)
  - fufills "adequacy Criteria for QSR" (Lemon and Pratt 98)
  - 1st order but requires infinitary rule of inference
    - guarantees existence of models in which every region is sum of finitely many connected regions
    - complete but not decidable

$$\frac{\{\forall x(\beta_n(x) \to \phi(x)) | n \geq 1\}}{\forall x \phi(x)}$$

Complete modal logic of incidence geometry

- Balbiani et al (97) have generalised von Wright's modal logic of place; many modalities:
  - ☛[U] everywhere
  - ☛<U> somewhere
  - ☛[≠] everywhere else
  - ☛<≠> somewhere else
  - ☛[on] everywhere in all lines through the current point
  - ☛[on^{-1}] everywhere in all points on current line
- (consider extensions to projective & affine geometry)

Metatheoretic results: categoricity

- *Categorical*: are all models isomorphic?
  - ☛ $\aleph_0$ *categorical*: all countable models isomorphic
- No 1st order finite axiomatisation of topology can be categorical because it isn't decidable

Geometry from CG/Sphere and P (Bennett et al 2000a,b)

- Given P(x,y), CG(x,y) and Sphere(x) are interdefinable
- Very expressive: all of elementary point geometry can be described
- complete axiom system for a region-based geometry
- undecidable for 2D or higher
- Applications to reasoning about, e.g. robot motion
  - movement in confined spaces
  - pushing obstacles

Metatheoretic results: tractability of satisfiability

- Constraint language of RCC8 (Nebel 1995)
  - classical encoding of intuitionistic calculus
    - can always construct 3 world Kripke counter model
    - all formulae in encoding are in 2CNF, so polynomial (NC)
- Constraint language of 2^{RCC8} not tractable
  - some subsets are tractable (Renz & Nebel 97).
    - exhaustive case analysis identified a maximum tractable subset, $\hat{H}_8$ of 148 relations
      - two other maximal tractable subsets (including base relations) identivied (Renz 99)
    - Jonsson & Drakengren (97) give a complete classification for RCC5
      - 4 maximal tractable subalgebras

Complexity of Topological Inference (Grigni et al 1995)

- 4 resolutions
  - *High:* RCC8
  - *Medium:* DC,=,P,Pi,{PO,EC}
  - *Low:* DR,O
  - *No PO:* DC,=,P,Pi,EC
- 3 calculi:
  - *explicit*: singleton relation for each region pair
  - *conjunctive*: singleton or full set
  - *unrestricted*: arbitrary disjunction of relations

Complexity of relational consistency (Grigni et al 1995)

	High	med	low	No-PO
unrestricted	NP-h	NP-h	P	NP-h
conjunctive	P	P	P	P
explicit	P	P	P	P

Complexity of planar realizability (Grigni et al 1995)

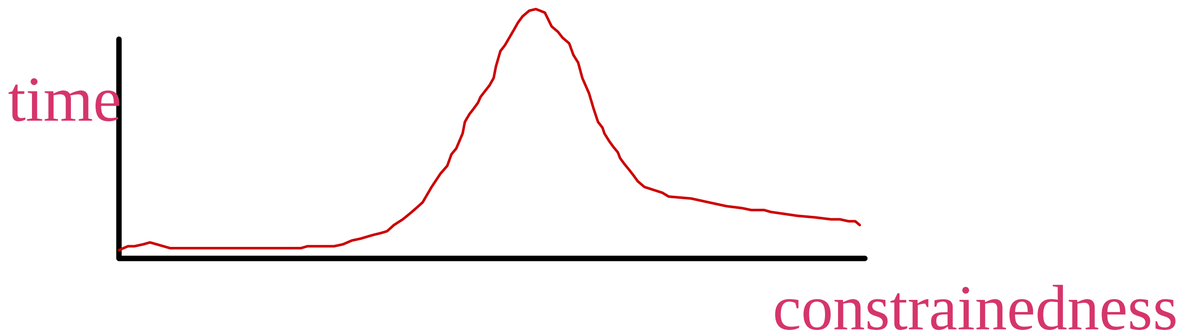
	high	med	low	no-PO
unrestricted	NP-h	NP-h	NP-h	NP-h
conjunctive	NP-h	NP-h	NP-h	?
explicit	NP-h	NP-h	NP-h	P

Complexity of Constraint language of EC(x) + PP(x) + Conv(x)

- intractable (at least as hard as determining whether set of algebraic constraints over reals is consistent
- Davis et al (97)

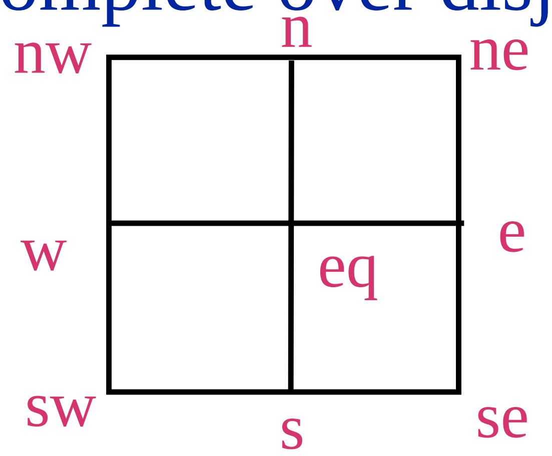
Empirical investigation of RCC8 reasoning (Renz & Nebel 98)

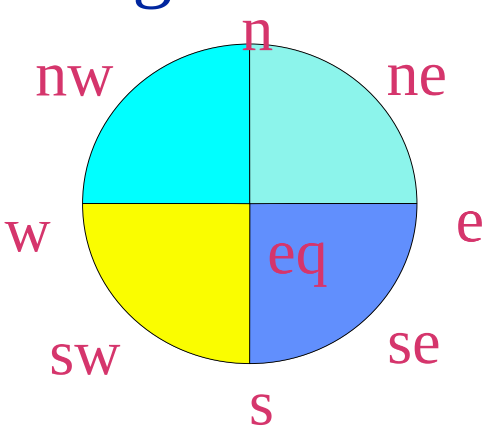
- Checking consistency is NP-hard worst case
- Empirical investigations suggest efficient in practice:
  - all instances up to 80 regions solved in a few seconds
    - random instances; combination of heuristics
  - even in “phase transition region”
  - random generation doesn’t exclude other maximal tractable subsets (Renz 99)



Reasoning with cardinal direction calculus (Ligozat 98)

- general consistency problem for constraint networks is NP complete over disjunctive algebra





- consistency for preconvex relations is polynomial
  - convex relations are intervals in above lattice
  - preconvex relations have closure which is convex
  - path consistency implies consistency
- preconvex relations are maximal tractable subset
  - 141 preconvex relations (~25% of total set of relations)

Reasoning with algebra of ternary orientation relations (Isli & Cohn 98)

- composition table
  - 160 non blank entries (out of 24*24=576) 29.3%
    - 0.36 average relations per cell
- polynomial and complete for base relations
  - path consistency sufficient to determine global consistency
  - also for convex-holed relations
- NP complete for general relations
    - even for PAR ={{oeo,ooe}, {eee,oeo,ooe}, {eee,eoo,ooe},{eee,eoo,oeo,ooe}}
    - also if add universal relation to base relations
- use (Ladkin and Reinefeld 92) algorithm for heuristic search for general relations

Regions with indeterminate boundaries

- “Traffic chaos enveloped central Stockholm today, as the AI community gathered from all parts of the industrialised world”
- traffic chaos?
- central Stockholm?
- industrialised world?

Kinds of Vague Regions

- vagueness through ignorance
  - ⇒e.g.. sample oil well drillings
- intrinsic vagueness
  - ⇒e.g. “southern England”
- vagueness through temporal variation
  - ⇒e.g. tide, flood plain, river changing course
  - ⇒note: temporal vagueness induces spatial vagueness
- vagueness through field variation
  - ⇒e.g. cloud density, percentage of soil type

Two approaches to generalise topological calculi

- Cohn & Gotts(94,…,96)
  - extension of RCC
    - new primitive: *X* is crisper than *Y*
    - "egg-yolk" theory
- Clementini & di Felice (95,96)
  - extension of 9-IM
  - broad boundaries

Limits of Approach

- Imprecision in spatial extent (not position)
- Will not distinguish different kinds of spatial vagueness
  - ⇒assume all types can be handled by a single calculus (at least initially)
- Sceptical about "fuzzy" approaches

Entities vs. Regions?

- Assumption: physical, geographic and other entities are distinct from their spatial extent
  - ⇒mapping function: space(x,t)
- Are spatial regions crisp and vagueness only present through uncertainty in mapping function?
- No, we present here a calculus for representing and reasoning with vague spatial regions
  - ⇒different kinds of entity might be mapped to different kinds of vague region

Basic Notions

- Universe of discourse has:
  - ⇒entities
  - ⇒Crisp regions
  - ⇒NonCrisp (vague) regions
- Given two different OptionallyCrispRegions, how might they be related?
- We will develop calculus from one primitive:
- X < Y: X is crisper than Y

Axioms for <

- A1: asymmetric
  - ⇒hence irreflexive
- A2: transitive
- Thus < is a partial ordering
- Obviously not enough..

Some Definitions

- X and Y are *mutually approximate*

  $MA(X,Y) \equiv \exists Z\, [Z \leq X \wedge Z \leq Y]$

- X is a *crisp* region

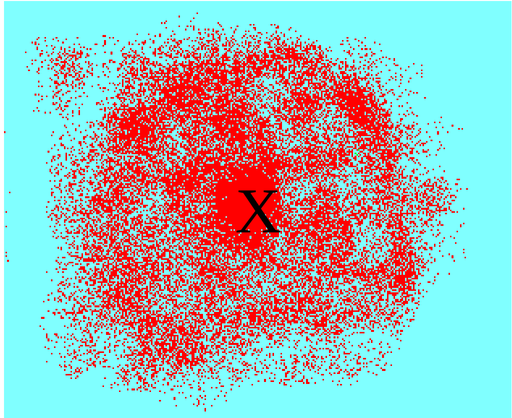
  $crisp(X) \equiv \neg \exists Z\, [Z < X]$

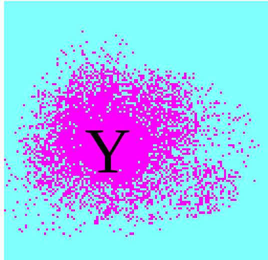
- X is a *completely crisp* version of Y

  $X \ll Y \equiv [X \leq Y \wedge crisp(X)]$

Some Theorems

- If X and Y are not MA, and Z is a crisping of X, it cannot be MA with Y





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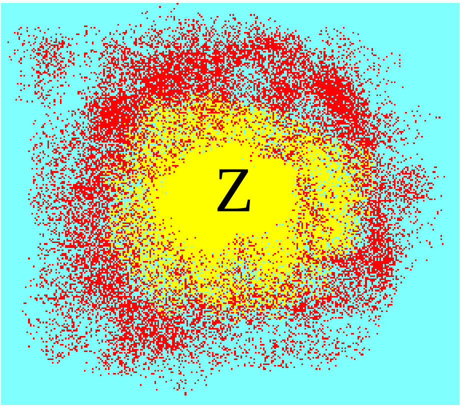
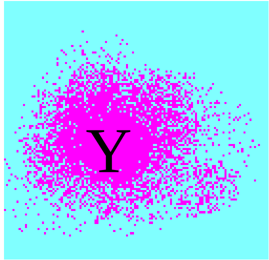




Another Axiom

- There must be alternative crispings

  A3: $\forall (X,Y) [X<Y \rightarrow \exists Z [Z<Y \wedge \neg MA(X,Z)]$
- A1,A2,A3 seem uncontroversial
- Several independent ways of extending the theory
- Explore parallels with a *minimal extensional mereology*

Simons' *minimal extensional mereology*

- Proper part relation: PP(x,y)
  - ⇒Axioms for partial ordering (cf <)
- Axiom: no single proper parts
  - ⇒cf A3: no unique crisping
- Axiom: unique intersections
- various possible axioms for existence of sums
- ....
- which of these carry over to calculus for vague regions? (and thus his theorems too)

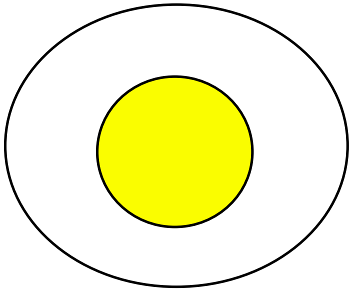
Questions raised by comparison

- Existence of vaguest common crisping (VCC)?
- Existence of vaguest blur sum (BS)?
- Existence of vaguest complete blur?
- Density of crisping relation?
- Existence of crisp regions?
- Identity of vague regions
  - any complete crisping of *X* is a complete crisping of *y* (and vice versa)

Defining other relations

- Can define vague versions of other RCC-like relations such as PP, PO,… by comparing complete crispings
- various versions, depending on usage of quantifiers
- how many relations?
  - ☛relations between complete crispings should be a conceptual neighbourhood?

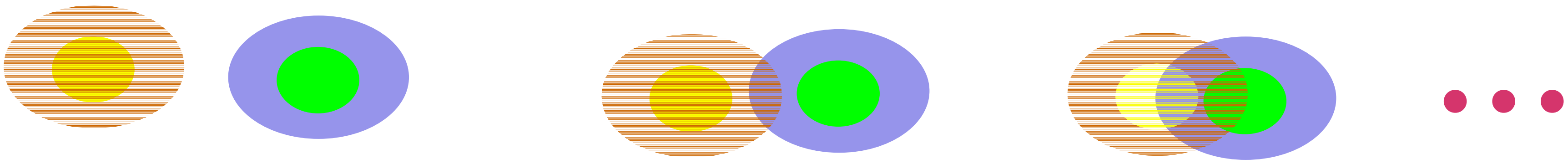
Egg-Yolk Theory



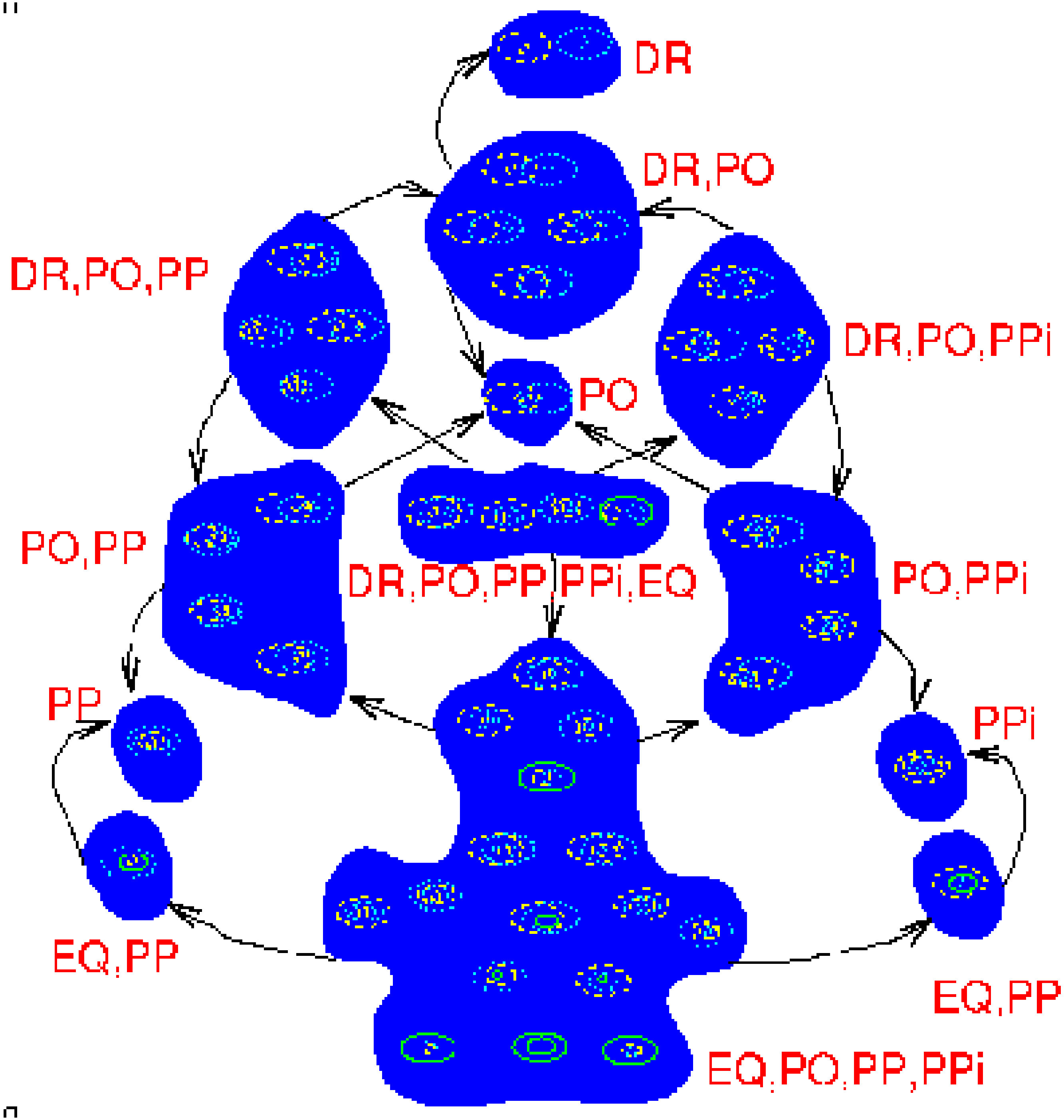
- Given all these possibilities are there any other approaches?
- Exploit *egg-yolk theory*
- Initially based on RCC5
- DR PO PP PPi EQ

- *primitive:* C(x,y): x and y are connected

How many egg yolk configurations?



- In RCC5: 46
- 13 natural clusters
- each configuration in cluster has same set of RCC5 relations between possible CCRs
- each configuration in cluster can be crisped to any other configuration in cluster
- each cluster's complete crispings forms a *conceptual neighbourhood*

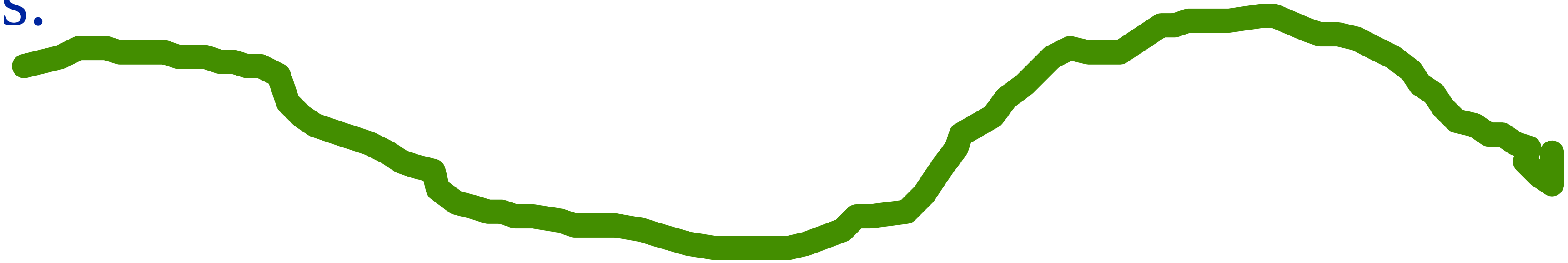

DR
DR,PO
DR,PO,PP
DR,PO,PPi
PO
PO,PP
DR,PO,PP,PPi,EQ
PO,PPi
PP
PPi
EQ,PP
EQ,PP
EQ,PO,PP,PPi

Relating the two theories

- provide (one way) translation from axiomatic theory of < to egg yolk theory
- unidirectionality ensures "higher level" indefiniteness
  - ☛not replacing bipartite by tripartite division of space!
- Can use egg yolk theory to analyse the possible permutations on quantifiers mentioned earlier

Extending the analysis to RCC8

- How many configurations in RCC8: 601
- 252 (assuming don't distinguish whether yolk is TPP or NTPP of its egg
- 40 natural clusters
- Can specify that hill and valley are vague regions which touch, without specifying where the boundary is.

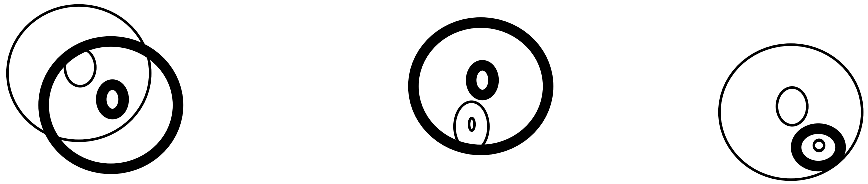


Clementini & di Felice (95,96)

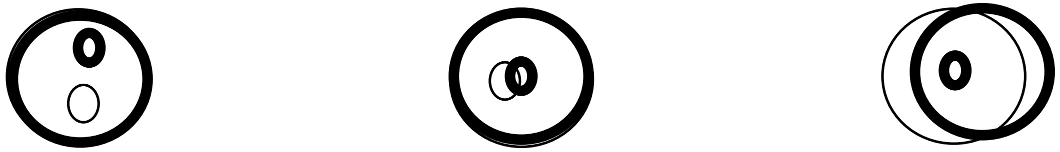
- point set theoretic approach
- similar results
- theory of broad boundaries
- 44 relations rather than 46 because of slightly different analysis of touching
- intuitive clustering into 18 groups

Specialisations of Clementini & di Felice (96)

- small boundaries
  - ☛exclude 4 relations that need thick boundaries and small interiors



- buffer zones
  - ☛exclude 3 cases not realisable fixed width boundaries

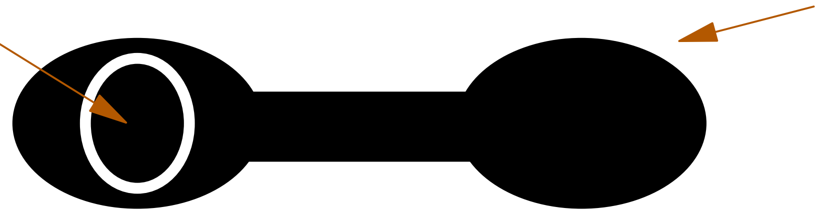


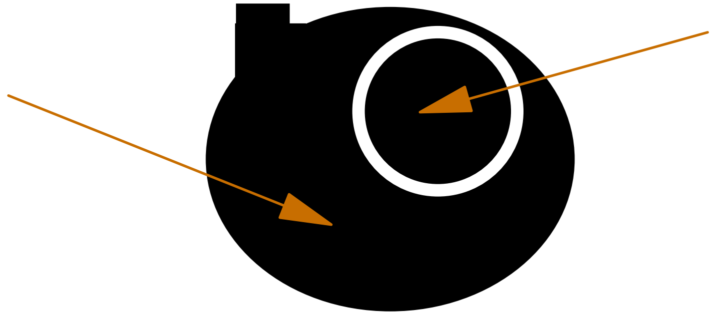
More Specialisations

- minimum bounding rectangles
  - exclude 23 cases (leaving 21)
- convex hull
  - exclude same 23 cases and 1 more
- rasters
  - eliminate 27 cases, leaving 17 (1 more than Egenhofer & Sharma 93) since 1 pixel wide interior allowed

Another interpretation of Egg-Yolk theory: locational uncertainty (Cristani et al 2000)

- The egg represents a spatial environment.
- Both yolk and egg are rigid.
- Location of the yolk is unconstrained within the egg;
  - i.e. the yolk can be anywhere and can move (rigidly) anywhere within the egg.
- 2 primitives: P(x,y), CG(x,y)
- Mobile part





FREYCs

- Free Range Egg-Yolk (FREYC): yolk is mobile part of egg
- FREY-FREYC relationship
  - relate different parts of FREYC using
    - RCC-5
    - MC4
  - identify 24 element subset of RCC-5 which is tractable and which obeys semantic constraints of domain

Other qualitative approaches to uncertainty

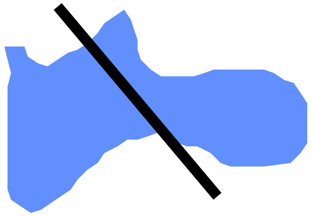
- Tolerance space
  - reflexive, symmetric, intransitive relation
  - Kaufmann (91)
  - Topaloglou (94)

Cognitive Evaluation of QSR

- One motivation claimed for QSR is that and humans use qualitative representations (e.g. spatial expressions in language are qualitative)
- Are the distinctions made in QSR languages cognitively valid?
- Rather little work, but see
  - Mark & Egenhofer (95)
  - Schlieder et al (95, 97)

Mark & Egenhofer 95

- 19 topological relationships 2D area/1D line (9IM)
- 40 drawings (2 or 3 repetitions of each relation)
- “The road goes through the park”, “The road goes into the park” …
- several languages: English, Chinese, German,…
- subjects asked to group drawings according to language description
- largely matched closest topological distance groupings



Tasks

- Spatial Databases
  - consistency
  - redundancy checking
  - retrieval/query
  - update
- Planning, configuration
- Simulation, prediction
- Route finding
- Concept learning
- ...

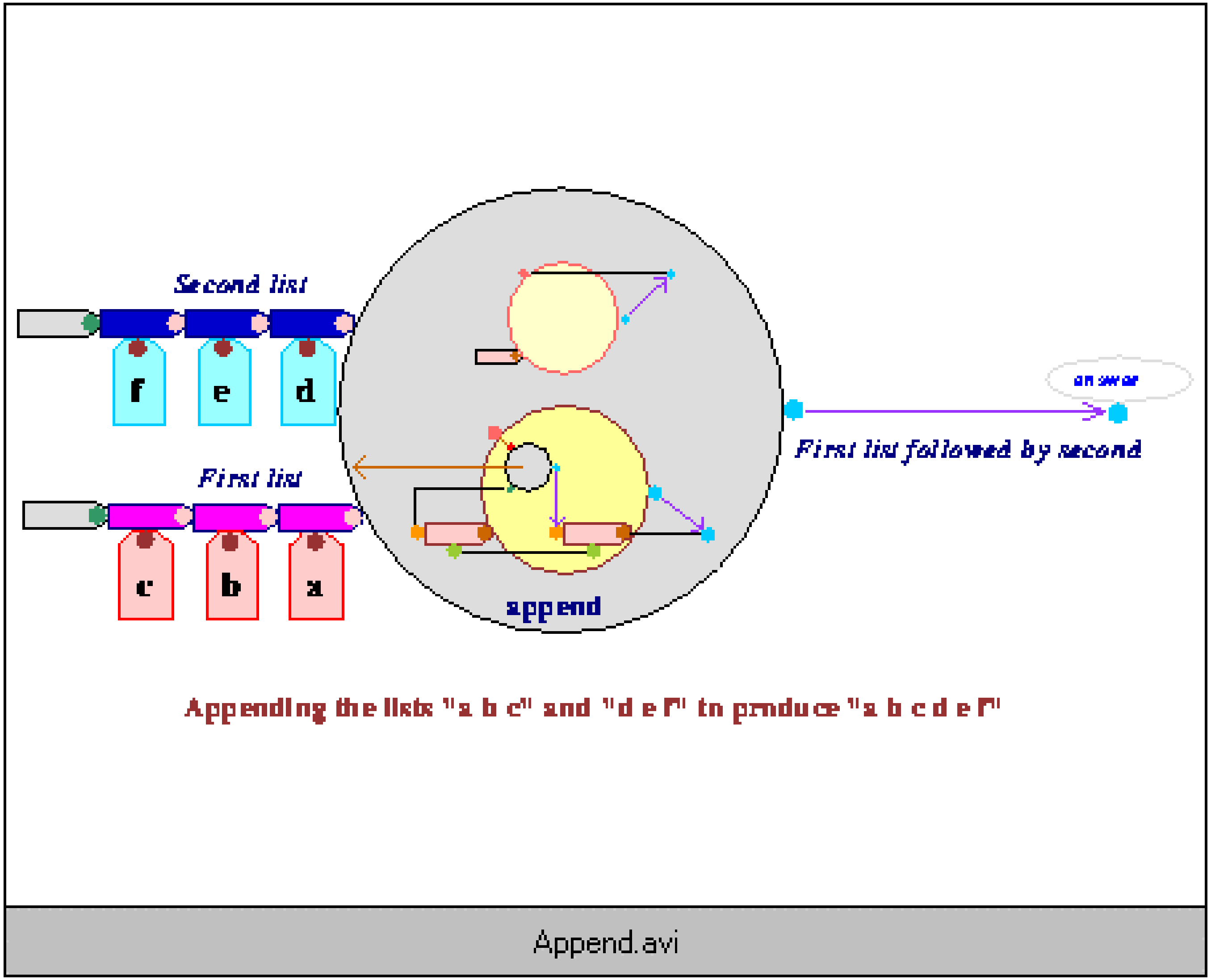
Simple Demonstration of QSR applied to GIS

- Quantitative (vector) DB
- Converted to Qualitative DB (RCC8)
- Additional Qualitative facts
- Queries are expressed in first order RCC representation
- Converted to intuitionistic zero order representation

Visual Programming language analysis

- Many visual programming languages are essentially qualitative in the nature of their syntax
- E.g. Pictorial Janus can be specified almost totally by topological means
- Moreover program execution can be visualised and specified by a qualitative spatio-temporal language
  - ☛Gooday & Cohn (96), Haarslev (96,7)

An example Janus program: appending two lists



Event specification and recognition using QSR

- Given frame by frame data from model based tracking program specifying labelled objects and metric shape information
- Use statistical techniques to:
  - Compute semantically relevant regions
    - Fernyhough et al (96)
  - Learn event types specified finite state machine on a qualitative spatial language
- Recognise instances of specified event types
  - Fernyhough et al (97,98)
  - c.f. e.g. Howarth & Buxton (92,...)

Addresses problem of integration of quantitative and qualitative reasoning

Qualitative Kinematics (Forbus et al, 87,…)

- MD/PV model: need metric diagrams in addition to qualitative representations (for (1) & (2) below)
  - ☛metric diagram: oracle for simple spatial questions
  - ☛place vocabulary: purely symbolic description, grounded in metric diagram
- Connectivity crucial to Kinematics

1) find potential connectivity relationships

   e.g. finding consistent pairwise contacts in rachet mechanism

2) find kinematic states

3) find total states

4) find state transitions

Further Qualitative Kinematics research

- Joskowicz (87)
- Davis (87, book, …)
- Bennett et al (2000)

Rajagopalan (94)

- integrated qualitative/quantitative spatial reasoning
- integrated with QSIM (Kuipers 86) QPC (Crawford 90)
- shape abstraction via bounding box
- applied to magnetic fields problems

Recap

- Surprisingly rich languages for qualitative spatial representation
  - symbolic representations
  - Topology, orientation, distance, ...
  - hundreds of distinctions easily made
- Static reasoning:
  - composition, constraints, 0-order logic
- Dynamic reasoning: continuity networks/conceptual neighbourhood diagrams

Research Issues

- Uncertainty
- Ambiguity
- Spatio-temporal reasoning
- Expressiveness/efficiency tradeoff
- Integration
  - qualitative - qualitative
  - qualitative - quantitative
  - qualitative - analogical
- Cognitive Evaluation
- ...

Where to find out more (1)

- Various conferences
  - Conference on spatial information theory COSIT)
    - biennial, odd years, Springer Verlag
  - Symposium on Spatial Data Handling (SDH)
    - biennial, even years
  - Main AI conferences (IJCAI, ECAI, AAAI, KR)
  - Specialised workshops:
    - QR, Time Space Motion (TSM), …
- Journals
  - AI, Int. J. Geographical Systems/Int J. Geographical Information Science, Geoinformatica, J Visual Languages and Computing, ...

Where to find out more (2)

- Online web bibliographies:
  - ☛http://www.cs.albany.edu/~amit/bib/spatial.html
- Spatial reasoning web pages:
  - ☛http://www.cs.albany.edu/~amit/bib/spatsites.html
  - ☛http://www.cs.aukland.ac.nz/~hans/spacetime/
  - ☛http://www.scs.leeds.ac.uk/spacenet/