Analogical Learning of Visual/Conceptual Relationships in Sketches

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Abstract

This paper explores the use of analogy to learn about properties of sketches. Sketches often convey conceptual relationships between entities via the visual relationships between their depictions in the sketch. Understanding these conventions is an important part of adapting to a user. This paper describes how learning by accumulating examples can be used to make suggestions about such relationships in new sketches. We describe how sketches are being used in Companion Cognitive Systems to illustrate one context in which this problem arises. We describe how existing cognitive simulations of analogical matching and retrieval are used to generate suggestions for new sketches based on analogies with prior sketches. Two experiments provide evidence as to the accuracy and coverage of this technique.

1. Introduction

Analogy is a powerful learning mechanism. Learning from examples is a crucial way of adapting based on experience. Analogies often include visual and spatial information as well as conceptual information. For example, we learn many things from diagrams and sketches, as well as from direct experience with the physical world. Being able to reason by analogy from sketches, and thus learn by accumulating examples, is an important capability for making flexible learning systems that capture more of the breadth of human processing.

This paper describes some results on analogical learning by accumulating examples, focusing on the problem of learning how visual relationships between entities in a sketch suggest conceptual relationships. We start with describing the context in which learning occurs in Section 2. Section 3 briefly reviews the theory of analogy and the simulations that this work builds on, and how they are used to generate suggestions for users about what relationships might be relevant for what they have sketched. Section 4 describes some experimental results obtained using this implemented system. Section 5 describes related work, and Section 6 summarizes and discusses future work.

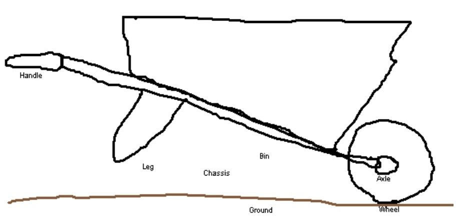

Figure 1: A wheelbarrow

2. The Context

Sketches depict relationships as well as objects. Often these relationships are implicit in the visual relationships of the objects being depicted. Consider Figure 1, which shows a sketch of a wheelbarrow. Seeing that the axle is depicted as being inside the wheel suggests that there is some relationship between them. People who know how a wheelbarrow works assume that the wheel is attached to the axle in a way that allows it to rotate. Similarly, the fact that the chassis touches the bin suggests that they, too, are connected physically. On the other hand, if you were explaining how a wheelbarrow worked to someone who had never seen one, you would have to provide these relationships in your explanation. Knowledge about how physical, causal, and other conceptual relationships can be inferred from visual relationships is part of the knowledge that one accumulates, part of the conventions used in interpreting a sketch.

Analogy is an excellent method of learning conventions for interpreting sketches because of their variability. For example, when considering the shopping cart of Figure 2, one might conjecture (by analogy with the wheelbarrow of Figure 1) that a similar relationship might hold between its

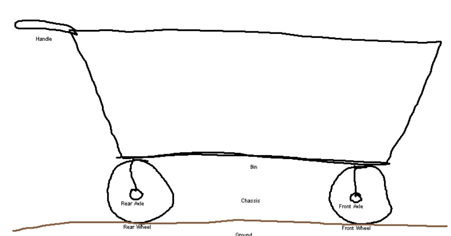

Figure 2: A shopping cart

wheels and axles. Some conventions are highly formalized, standardized, and used in a uniform way across an entire community of practice. Other conventions are short-lived and *ad hoc*, introduced for, and used only within, a single conversation. Groups working together even for a short time often evolve local conventions (Markman & Makin, 1998). Thus learning by accumulating examples seems to be a natural way to adapt to changing circumstances.

The experimental setting we use to explore this task involves using sketching to learn to solve problems, as part research on a new cognitive architecture, *Companion cognitive systems* (Forbus & Hinrichs, 2004). Companions are intended to work with, and learn from, human collaborators. The major learning mechanism used in Companions is analogy, exploring the conjecture (Forbus & Gentner, 1997) that analogical reasoning is crucial to the flexibility and breadth of human common sense reasoning and learning. One of the domains being explored is learning everyday physical reasoning, using the kinds of questions that are found on the Bennett Mechanical Comprehension Test (BMCT), an examination used for over 50 years to classify candidates for technical jobs and also by cognitive psychologists as an independent measure of spatial ability. Problems on this test are posed in the form of diagrams, making it perfect for sketched input.

The sketching Knowledge Entry Associate (sKEA, Forbus & Usher, 2002) is used to handle user interaction via sketching for these problems. sKEA is the first *open-domain* sketch understanding system. It provides conceptual labeling facilities in its interface that avoid the reliance on recognition that limits other sketching systems to tightly circumscribed domains (cf. Cohen *et al* 1997; Alvarado & Davis, 2001). Anything that can be described in terms of sKEA's knowledge base can be used in a sketch. sKEA's knowledge base consists of a 1.2 million fact subset of Cycorp's Cyc KB[1], which includes over 38,000 concepts, over 8,000 relations, and over 5,000 logical functions. We have added to that our own representations of qualitative physics, visual properties and relationships, spatial knowledge, and representations to support analogical reasoning, but the vast majority of the content that we deal with was independently developed.

sKEA enables users to fill in information about visual/conceptual relationships as they sketch, using a hypertext form. As described elsewhere, sKEA computes qualitative topological relationships as part of its visual processing, using Cohn's (1996) RCC8 vocabulary. When sKEA finds two glyphs that are either touching (PO or EC, in RCC8) it asserts a very high-level relationship, `atOrOverlapsInSketch`, between the two entities depicted by those glyphs. Similarly, when one glyph is inside another (TPP or NTPP), it asserts `insideInSketch` between the entities depicted. These two relationships form the bridge between the visual properties of the sketch and the conceptual understanding of what is depicted. `atOrOverlapsInSketch` and `insideInSketch` are tied into the rest of the KB by asserting `genlPreds` statements that link them into the relationship lattice. (Like most KB's, the Cyc KB organizes predicates into a generalization lattice. The relation (`genlPreds` *<subPred>* *<superPred>*) indicates that the relationship *<superPred>* is a generalization of the relationship *<subPred>*, in that whenever *<subPred>* holds for a collection of entities, that implies *<superPred>* holds of them as well.) Specifically, `atOrOverlapsInSketch` is asserted to be a generalization of `aligned`, `touches`, and `connectedTo`. These predicates in turn have more specialized versions, leading to a total of 204 potential ways to specialize `atOrOverlapsInSketch`. Not all of these specializations are relevant to every situation. For example, there are type restrictions placed on the arguments to every relationship, using the 38,000+ collections defined in the KB. For example, over 60 collections are used to restrict the arguments to the specializations of `atOrOverlapsInSketch`, ranging from the very general (e.g., `SpatialThing`) to the very specific (e.g., `City`, `Nucleotide`). Similarly, `insideInSketch` is a direct generalization of `inRegion` and `spatiallyIncludes`, which lead to a total of 150 distinct specializations that can be chosen, with a similar variation in argument type restrictions.

When sKEA's visual system asserts one of these bridge relationships, the system considers what more concrete relationships might make sense in this situation. The initial candidate set is generated by considering all specializations via a query (e.g., `(genlPreds ?r insideInSketch)`), and eliminates those relationships whose argument type restrictions are not satisfied by what is currently known about the entities depicted. The remaining candidates are then presented to the user, who is invited to select one or more of them as appropriate for that pair of entities. There tend to be a lot of candidates: In the worst case, there are 150 specializations of `insideInSketch`, and 204 for `atOrOverlapsInSketch`.

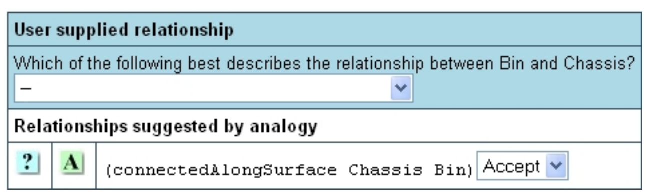

Figure 3: Using analogy to suggest a relationship

In Companions, this sKEA-level mechanism is augmented by analogy, to attempt to make things easier for the user. This is a worthwhile problem since the number of candidates is so large: In one corpus, there were on average of four such questions per sketch, with an average of 122 candidates to consider per question. We use analogy to suggest relationships that they used previously in a prior similar sketch. Figure 3 illustrates one such suggestion for the shopping cart of Figure 2, given the

[1] The conventions used in this KB are documented at www.opencyc.org, although our KB includes a larger subset of Cyc than OpenCyc does.

retrieved analog of the wheelbarrow of Figure 1. (In this case, there were 109 candidate relationships.) The ? button enables users to see why this particular suggestion was made, and the A button lists the assumptions underlying the suggestion.

3. The Analogical Learning Model

Here we describe how the analogical reasoning is carried out, starting with a brief review of structure-mapping and associated simulations to provide context.

3.1 Brief review of Structure-Mapping

The approach we use is based on Gentner's (1983) structure-mapping theory. Structure-mapping defines analogy and similarity in terms of comparisons involving structured representations, the *base* and the *target*. The matching process produces *mappings*, which consist of three parts. The *correspondences* of a mapping describe "what goes with what", e.g., how items in the base (entities and statements) align with items in the target. The *structural evaluation score* of a mapping is an expression of overall match quality. The *candidate inferences* of a mapping are surmises about the target, given the correspondences and other structure in the base. That is, they are projections of non-matched base structure that is suggested by the match into the target. Candidate inferences sometimes introduce new entities into the target, when they mention an entity for which there is no known correspondent in the target. These new entities are called *analogy skolems*. The principles that govern the mappings people compute are described in (Gentner, 1983).

There is now considerable empirical support for structure-mapping theory as a model of human analogy and similarity (cf. Markman & Gentner, 2000). Moreover, this theory has guided the development of cognitive simulations of analogical processing. Two of these simulations, used in this system, are SME (Falkenhainer *et al* 1986) and MAC/FAC (Forbus *et al* 1994b). SME models analogical matching. It operates in polynomial time, using a greedy merge algorithm to compute up to three alternate mappings, which can be extended incrementally as new information is provided (Forbus *et al* 1994a). MAC/FAC models similarity-based retrieval. The first stage (MAC) uses a special-purpose feature vector representation that is automatically constructed from structured descriptions, so that the dot product of two vectors is an estimate of the score that SME would compute for the best possible mapping from them. This provides a scalable first stage filter, operating in parallel over a case library, that provides the best cases (up to three) for consideration in the second stage. The second stage (FAC) uses SME to compare the structured descriptions for each MAC candidate against the probe. The best reminding (or up to three, if they are very close) are returned.

Both SME and MAC/FAC have been used to successfully model a variety of psychological findings, and have been used to make predictions that have been subsequently borne out in psychological experiments (Forbus, 2001). Moreover, they have both been used in performance systems (*ibid.*). This makes them suitable for use in this task, since the notion of similarity used in Companions needs to be compatible with their users' notion of similarity.

Interactions between analogical processing and first-principles reasoning are handled using an analogy ontology (Forbus *et al* 2002), which provides relationships that are implemented via procedural attachment to SME and MAC/FAC, enabling them to be run and their results inspected as simply another kind of inference. In addition to specifying cases by explicitly storing sets of facts, cases can be constructed dynamically from the knowledge base, or existing cases can be filtered for particular purposes (Mostek *et al* 2000).

3.2 Baseline analogical learning model

Human analogical learning is extremely flexible. It is important when studying a complex phenomenon to establish a baseline, a model that is easily understood and against which the gain provided by specific improvements can be measured. In Companions the baseline analogical learning method used is example-based, since we can then later measure how much improvement is provided by models of analogical generalization (e.g., SEQL (Kuehne *et al* 2000)). Given a problem, we use MAC/FAC to produce a set of remindings. The candidate inferences provided by these remindings are then inspected for information relevant to the problem at hand. There is evidence that people will attempt to find other remindings when their first set is insufficient, and that they will try alternate mappings when the first does not work out. We use neither of those techniques here; whatever suggestions are provided by the first remindings are the only answers provided.

3.3 Generating visual/conceptual relationship suggestions via analogy

Given a library of prior sketches stored in the knowledge base (denoted by `(CaseLibraryFn sKEA-VCM-CaseLibrary)`), the visual/conceptual relationships interface looks for suggestions based on prior experience by using this Prolog-style rule:

```
(<== (analogySuggestionFor
      (visualInterpretationRelationSuggestion
       ?reln ?o1 ?o2)
      ?probe ?mapping)
;; Get reminding(s) based on the whole thing
 (reminding ?probe
   (CaseLibraryFn sKEA-VCM-CaseLibrary)
    ?case ?original-match)
;; Focus only on conceptual knowledge
 (matchBetween ?case
  (ConceptualFactsOfCaseFn ?probe)
  (TheSet) ?match)
 (bestMapping ?match ?mapping)
;; Find relevant candidate inferences
 (candidateInferenceOf ?ci ?mapping)
 (candidateInferenceContent ?ci ?formula)
 (unifies
  (userAcceptsBinaryRelationSuggestion
    (AnalogySkolemFn ?reln) ?o1 ?o2)
  ?formula)
;; Possibly relevant, but might it be valid?
;; See if it has been made as a suggestion.
 (visualInterpretationRelationSuggestion
    ?reln ?o1 ?o2)
;; Last two improve the explanations
 (correspondsInMapping ?mapping ?bo1 ?o1)
 (correspondsInMapping ?mapping ?bo2 ?o2))
```

Here `?probe` is the current sketch, with `?reln` being the relationship suggested between entities `?o1` and `?o2`. The `reminding` query causes MAC/FAC to be run, using the entire set of information about the sketch to find potentially relevant prior sketches. This term provides a set of bindings for each reminding, with `?case` being the prior example and `?original-match` being the comparison of the prior example with the current sketch.

While appearance information is useful in finding relevant precedents, we found through experimentation that focusing only on conceptual information led to better candidate inferences. The reason is that conceptual information (i.e., the categories of the entities) is crucial to accurate candidate inferences, but it can be overwhelmed by the larger quantity of visual information. Therefore instead of simply using the match constructed by MAC/FAC, we match the remindings against just the conceptual facts of the sketch. This is done by the `matchBetween` term of the query, which invokes SME directly. The user's current sketch is filtered using the case constructor `ConceptualFactsOfCaseFn`, which removes any facts that mention glyphs from the case. Importantly, spatial relationships between entities that were inferred as a consequence of visual relationships between glyphs remain as part of the case. It is only the glyphs themselves and their properties that are removed. The third argument to `matchBetween` is the set of constraints under which the mapping is to be done (here, empty), and the results are bound to `?match`.

The `bestMapping` term extracts the best mapping from the match. SME produces up to three alternate mappings, if the descriptions warrant, but we only use the best one here. The next three terms in the rule identify relevant candidate inferences, by finding suggestions that the user accepted in the retrieved sketch. Notice the use of `?o1` and `?o2` in the inference content, versus `(AnalogySkolemFn ?reln)` for the relation. We want `?o1` and `?o2` to be constrained by the mapping, otherwise we cannot tell to what the relationship is being applied to. On the other hand, `?reln` isn't aligned in the mapping, so it will be treated as an individual hypothesized to exist in the target. This is a simple case of *skolem resolution*, an important problem in analogical reasoning. While skolem resolution in general can require sophisticated constraint solving (cf. (Forbus *et al* 2003)), here we simply import the exact same relationship by using the value of `?reln` without further processing.

Recall that candidate inferences are not necessarily valid; to check that this suggestion is in fact valid, we exploit the reasoning done by the standard visual/conceptual relationship suggestion generation routines, looking to see if this suggestion was in fact among those generated. If it is, then it must be valid, and otherwise, it cannot be, since that inference process is complete. The last two terms of the query are not needed to generate suggestions. Their purpose is to make explanations more understandable, by making explicit which entities in the retrieved sketch were used in creating the suggestion.

The retrieved suggestions are displayed as illustrated in Figure 2. Since MAC/FAC can produce between zero and three remindings, this process can produce multiple suggestions for the same pair of entities, due to different remindings. When multiple suggestions are generated for the same relationship, all are displayed for the user.

4. Experiments

We carried out two experiments with this system to evaluate its performance. We describe each in turn.

4.1 Visual/Conceptual Relations in Problem Solving

The first experiment used sketches generated as part of a larger experiment, where Companion software was tested against 13 problems drawn from the BMCT[1]. Three graduate students acting as knowledge enterers (KEs) drew 18 sketches, each specified by a phrase (e.g., "A crane lifting a load", "A wheelbarrow, with a rock in it."). Their goal was to provide a causal explanation for whatever principle(s) they thought applied in that situation. As part of the sketching process, they specified collections and relationships so that when the system's qualitative reasoning facilities were applied, the appropriate model fragments would be instantiated. The starting endowment of qualitative knowledge for the system is qualitative mechanics (Kim, 1993; Nielsen 1988) and the ontology of qualitative process theory (Forbus, 1984). They could also annotate the sketch with additional, situation-specific causal information, using a concept map system linked to sKEA. For example, they could specify that the stability

[1] This is roughly 19% of the exam.

of a ladder was qualitatively proportional to the width of its base, using the sketch to indicate what they meant by the width of the base, and the concept map to introduce the stability parameter and the qualitative relationship. The visual/conceptual relationship interface was used to enter some of the relationships needed for appropriate qualitative modeling. KEs were encouraged to answer as many visual/conceptual relationship questions as possible. The sketches were then used as a case library to solve problems specified via sketches drawn by a fourth graduate student, as described in (Klenk *et al* 2005).

We used the library of 54 sketches from that experiment to investigate the performance of the visual/conceptual relationship suggestions method described in Section 3. Our method was to draw each case out of the library in turn, filter out the answers to its visual/conceptual relationship questions, and use the other 53 cases to make suggestions about it. This "round robin" strategy yielded 181 visual/conceptual relationship questions which the KEs had answered, and thus against which we could test the generation of suggestions via analogy. We scored answers using the following rubric: If the suggestion had the correct relationship and the arguments in the right order, it received a score of 1.0. If the arguments were flipped (and the relationship wasn't symmetric), it received a score of 0.5, on the grounds that it is easier to recognize that arguments need to be reversed than to find the right relationship out of over a hundred candidates. Similarly, if the predicate is one step away from the correct relationship within the `genlPreds` lattice, we give it a score of 0.5, and within two of the correct relationship, a score of 0.25, since these are still in a usefully close neighborhood of the correct answer. Any other answer is scored 0.0.

This is a harsh scoring rubric: The expected value for the 181 questions is just 24.2, if relationships from the set of suggestions were picked at random[1]. Importantly, the system's score on these questions was much better, 74.25. This is statistically significant ($P << 10^{-5}$), indicating that the system is providing very reasonable suggestions. Is this due to partial credit? Not really: The score for 64 questions was 1.0, the score for 19 questions was 0.5, and only three questions were scored at 0.25.

The coverage of the system is a weak point. We define coverage as the ratio of the number of problems where the system provides an answer to the number of problems. It tackled only 97 out of the 181 problems (54% coverage), but was able to provide reasonable answers for 86 of those 97 (87%). Given the baseline analogical model described above, the limitation in coverage is not too surprising. Consider again the wheelbarrow-shopping cart comparison. The wheel/axle combination of the wheelbarrow could in principle be used to make suggestions about both the front and rear wheels and axles of the shopping cart, but because mappings must be 1:1 and we are only working with a single mapping, a suggestion is made about only one of them. Examining multiple mappings, and even remapping to aggressively generate additional hypotheses by specifying additional match constraints, for example, might significantly expand coverage.

4.2 Open-ended tasking

One possible explanation for the good performance above is that the constraint of generating qualitative models might have simplified the task, since relationships that allowed the system to infer qualitative models were preferred. To rule out this explanation, it is useful to look at a similar situation where no such restrictions were imposed. A list of ten entities drawn from the BMCT was selected at random, covering a larger range of phenomena than the experiment above (e.g., "a boat moving in water", "a bicycle"). Two graduate students sketched each system, being told only that they should draw them in enough detail to illustrate what they thought were the relevant principles in their operation. The students were encouraged to answer as many of the visual/conceptual relationships that the system asked as they could, but only when they were sure of the answers. Again, the round-robin methodology was used, drawing each case in turn out of the library of twenty cases, stripping the user choices from it and using the other 19 cases to construct suggestions for its visual/conceptual relationships via analogy. This yielded a set of 138 questions.

The system generated suggestions for 63 out of the 138 questions (46% coverage). Using the same scoring rubric as the first experiment, the system's score was 21.75, which is statistically significant ($P < 10^{-7}$). Unlike the previous experiment, where the system nailed the correct relationship most of the time, here partial credit was very important: It only suggested the exact relationship 6 times, or 10% of the time. However, the overall accuracy of the system is still quite reasonable. Interestingly, there is a small drop in coverage (46% versus 54%), but this is still quite respectable.

5. Related Work

Some have argued (cf. Davies & Goel, 2001, 2003) that visual analogy requires special-purpose mechanisms. Their Galatea model uses a hand-crafted special-purpose ontology, and appears to have only been tested with a few examples. By contrast, the system described here draws upon off-the-shelf cognitive simulations and an independently-derived large knowledge base, using only a few new relationships to provide a bridge between visual and conceptual information. In this regard our work is more like Ferguson's (1994; Ferguson & Forbus 2000), which uses representations and processing models inspired by psychological results whenever available, and off-the-shelf components and representations otherwise.

[1] For accuracy, we explicitly calculated the distribution of possible scores, rather than relying on an approximation.

Competing cognitive simulations of analogical processing mostly fall into two categories. Domain-specific models (e.g., Mitchell, 1993; French 1995) are hard-wired in their matching and representation construction to work only in a pre-defined microworld, and hence are inapplicable to the kinds of problems tackled here. Connectionist simulations (cf. Eliasmith & Thagard, 2001; Hummel & Holyoak, 1997; Larkey & Love, 2003) attempt to explain how similar computations can be expressed given assumptions about neural hardware. Unfortunately, all of these schemes to date are limited to very small descriptions, typically 2-3 relations, and cannot to scale to the size of descriptions used here (i.e., sketches involve hundreds of propositions). We know of no other existing simulations of analogical mapping or retrieval that have been successfully tested with the number of domains and with the size of examples (and size of case libraries) that SME and MAC/FAC have been used with.

Case-based reasoning systems (cf. Kolodner 1994; Leake 1996) tend to use special-purpose matchers and memory indexing systems. By contrast, SME and MAC/FAC operate in multiple domains, and MAC/FAC does not require any indexing, which simplifies the incorporation of new examples into memory.

6. Future Work

We have shown that an important kind of knowledge about sketches, conventions for interpreting visual relationships between elements of a sketch to suggest conceptual representations between the entities depicted, can be learned by accumulating examples and reasoning by analogy over those examples. This is a hard problem, as illustrated by the number of possible relationships that participants in sketching must consider when providing answers. The ability for a straightforward analogical reasoning process, using off-the-shelf cognitive simulation models, to handle this task is very encouraging.

This work raises a number of interesting lines of future investigation. First, we know from cognitive psychology that there are a variety of ways that people use analogy in solving similar problems that are not captured by our baseline model. These include using *rerepresentation* (Yan *et al* 2003) to improve alignment of retrieved cases, using multiple mappings on a single problem, and retrieving additional cases as the problem representation is augmented. We plan on implementing all of these, and measure quantitatively how much each of them improves coverage and accuracy over the baseline model described here. Second, we plan on experimenting with SEQL (Kuehne *et al* 2000), a model of generalization based on analogical processing of structured representations, to refine examples into more abstract, rule-like descriptions, to see if that improves learning. Third, the experiments described above were carried out off-line. We plan to do on-line experiments, where users can give the system feedback about its suggestions (good/bad) and about the precedents it retrieved to generate them (including selecting something it should have retrieved, using a sketch browsing interface to scan the case library). This feedback will then be used to help the system improve its methods for encoding sketches, and thus facilitate future retrievals and mappings.

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