Qualitative Physics based on exact physical principles*†

Charles Chiu

Department of Physics and Artificial Intelligence Laboratory

University of Texas 78712

February 8, 1987

[Submitted to Workshop on Qualitative Physics; Foundational research in qualitative physics.]

1 Introduction.

Consider a two-body collision process. Raiman[1986] observed that the conventional qualitative approach to this problem amounts to suppressing the mass information, which becomes inadequate when one wants to determine the direction of the velocity of an object after a collision. He resolved this by going back to the original algebraic equations, which have all the mass factors. He then proceeded with order of magnitude reasoning. There is at least one weak point in Raiman's approach, i.e. intuitively the direction of concerned should be associated with some threshold phenomena. But thresholds are absent in his work. Consequently, he is only able to predict the direction in the region substantially beyond the expected threshold.

Our general point of view is that, physical phenomena which we encounter daily are governed by a number of relatively simple physical principles. It is often possible to define a physical system or a process in terms of a collection of intuitive and yet exact statements. Complexities arise when we try to see how different relationships work together. We believe that Raiman's semi-quantitative approach is a step in the right direction. However, for those problems which can be solved algebraically, we would like to go one step further to work with the exact equations and

*This paper is essentially a condensed version of [Chiu 87a].

†This research was supported in part by the National Science Foundation through grants MCS-8303640, DCR-8417934, and DCR-8512779.

solve them algebraically. Below we will concentrate on the qualitative phyusics domain, in which problems can be solved algebraically. Within this domain, we advocate following approach.

- We state physics principles and definitions in terms of intuitive and exact statements. A complete set of these statements arranged in a specific order constitutes a model.
- Coupled with an algebraic package such as Macsyma, the program "solves" the model.
- Finally we do qualitative physics.
  - **Handling intuitions and partial knowledge.** We introduce a "monotonic function abstractor" and an "order-of-magnitude estimator" to extract qualitative knowledge from the solution. This enables the program to reproduce commonsense intuitions and to generate further predictions based on available partial knowledge.
  - **Qualitative description of the solution behavior.** We introduce a qualitative "solution descriptor" which evaluates intermediate steps of the model and offers some "intuitive scenario" to describe how the final result comes about.

2 Model for two-body collisions.

a. Collisions between asteroids and super-cannon balls. Imagine we are inside a gigantic skylab in the outer space. There is a school of tiny asteroids approaching us with some identical positive velocity u_a, and identical mass m_a. We want to do qualitative reasoning about the head-on collision phenomena between these asteriods and super-cannon balls with various masses and velocities shot from the skylab. Figure 1 illustrates five sample cases.

— Figure 1. Five collision cases. near here. —

We are mainly interested in asteroid's the final velocity v_a. See Fig. 1 for velocity labels.

b. Physical principles. The underlying principles to deduce the outcome of collisions for the initial setups of those cases in Fig. 1 and of other cases are as follows:

- *The center of mass velocity of the two-body system is constant throughout a collision,*

$$u^* = \frac{m_a u_a + m_b u_b}{m_a + m_b} = \frac{m_a v_a + m_b v_b}{m_a + m_b}$$

- *When the center of mass is at rest, the collision may be viewed a "brick wall bouncing" process with velocities of each object before and after the bounce being equal in magnitude and opposite in direction. See e.g. case 2 in Fig. 1.*
- *A velocity in the cm frame u' is related to that in the lab frame u by $u' = u - u^*$.*

The model. The collision model may now be defined in terms of a set of easy-to-understand and yet exact statements.

$$\begin{aligned}
&Center\ of\ mass\ velocity\ in\ lab\ frame: & u^* &= (m_a u_a + m_b u_b)/(m_a + m_b).\\
&From\ lab\ to\ cm,\ velocity\ of\ a\ becomes: & u'_a &= u_a - u^*.\\
&From\ lab\ to\ cm,\ velocity\ of\ b\ becomes: & u'_b &= u_b - u^*.\\
&After\ collision\ velocity\ of\ a\ in\ cm\ becomes: & v'_a &= -u'_a.\\
&After\ collision\ velocity\ of\ b\ in\ cm\ becomes: & v'_b &= -u'_b.\\
&From\ cm\ to\ lab,\ velocity\ of\ a\ becomes: & v_a &= v'_a + u^*.\\
&From\ cm\ to\ lab,\ velocity\ of\ b\ becomes: & v_b &= v'_b + u^*.
\end{aligned}$$

Here the statements are arranged in the time sequence as we visualize it, of the unfolding of the process. This arrangment is loosely referred to as in a "temporal order". Then the Macsyma function SOLVE "solves" the model. For instance, it expresses all the statements in terms of masses m_a, m_b and initial velocities u_a , u_b.

3 Extracting qualitative knowledge from algebraic expressions.

a. Monotonic function abstractor. First we observe that in our skylab setup, based on commonsense one expects several intuitive tendencies. For instance if our goal is to make the recoil velocity of the asteroid v_a to be more negative, intuitively we know that this can be achieved either by increasing the mass m_b, or by making the velocity u_b to be more negative. So, there are two qualitative monotonic functional relations: *for fixed* u_b, $M_-(m_b, v_a)$; and *for fixed* m_b, $M_+(u_b, v_a)$. Now if we change our goal to maintaining the same recoil velocity of the asteroid, then the decrease in the mass of the ball may be compensated by making the ball velocity to be more negative. So there is also the relation: *for fixed* v_a, $M_+(m_b, u_b)$. We have introduced an "M-function abstractor" program which explores various monotonic functional relationships. Applying our abstractor program to the Macsyma solution, we obtained a set of monotonic functions including the ones mentioned above.

b. The order of magnitude estimator. Our discussion of order of magnitude estimates is mainly to develop appropriate semantics for handling problems where only an order of magnitude information is given. Let me illustrate the use of our "order of magnitude estimator" program with an example. Consider case 3 of Figure 1. Suppose we are told that the mass of the ball is comparable to that of the asteroid, but the ball's speed is overwhelmingly larger than the asteroid's speed. One may ask which way is more effective in changing the final recoil velocity: whether by varying ball mass or by varying ball velocity. If there were an automatic agent on board of the skylab equipped with our order of magnitude estimator program, the agent could proceed with the estimate and conclude that the variation of the ball mass is a more effective to increase the recoil speed of the asteroid.

4 Qualitative description of the solution.

Within our approach after Macsyma solves the model our "solution descriptor" program goes back to evaluate and display the intermediate steps involved leading to the final results, which provides an explanatory description of the process.

Again, we use the examples in Fig. 1 to illustrate this. Notice for case 1 where $m_a = m_b$ and $u_b = 0$, Macsyma solution implies that the center of mass velocity equals to the critical value $u_a/2$, and the final asteroid velocity $v_a = 0$. Our solution description takes this case as the *reference* case. For any of the other cases in Fig. 1, a qualitative comparison is made between each parameter value and the corresponding reference value in case 1, to determine whether the difference is $+$, 0, or $-$. For instance for case 5, the text description for the asteroid could go as follows:

[Here the ball is lighter and the center of mass velocity should be greater than that for the equal mass reference case. Consequently the velocity of the asteroid in the center of mass frame is less than the corresponding reference value. After the "brick wall bounce", the asteroid velocity in the center of mass frame is not as negative as that of the reference case. In turn in the lab frame, the final asteroid velocity is greater than the reference value 0. So it is positive.]

We see the comparative description of this type provides the user with an "intuitive scenario" explaining how the final qualitative results come about. One sees that the very assignment of the reference case allows us to discuss a positive or a negative deviations from a reference value, which correspond to above or below the corresponding threshold.

5 Discussion.

Within our framework, we found that commonsense intuitions: such as those in connection with the monotonic functional relations, order of magnitude reasoning and threshold phenomena can be successfully generated. In addition to the elastic collision work here, we have applied the present approach to inelastic collision and explosion phenomena[see Chiu 87b]. We are pursuing other test cases. In a larger context, we are also considering the possibility of making use of the formalism of Sacks [1985], which involves a detail qualitative reasoning on algebraic expressions.

In the format of the present approach, the laborious mathematical tasks are separated out from the statements of a physical model. When studying a physics model, through the use of various tools, the user may interact with the machine at an intuitive level. We find this format to be appealing. After it is fully developed, it could have a profound impact in teaching physics.

The underlying algebraic framework of the present approach plays an important role in bridging the gaps between commonsense, qualitative and quantitative approaches. Given that there are many physics problems which have already been solved algebraically, and that there are also many engineering problems and problems in other areas, where closed-form algebraic expressions are utilized, we believe that there will be much opportunity to apply the present approach in the future.

6 References

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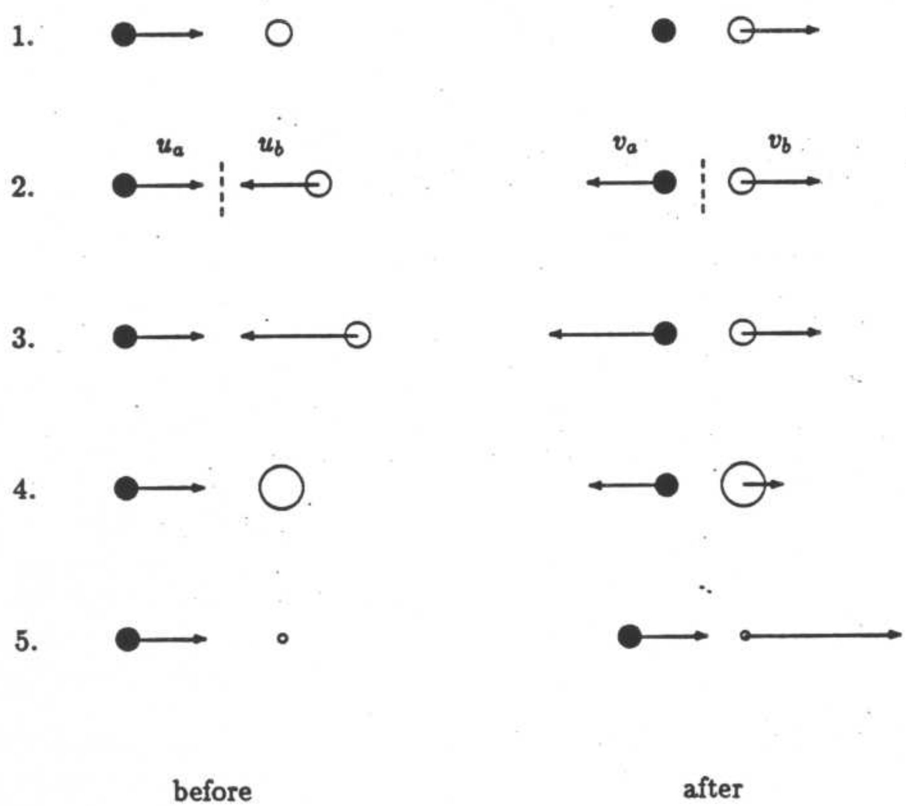

Figure 1: Collisions between asteroids and super-cannon balls.

- Asteroids (solid circles): with identical masses and initial velocities.
- Super-cannon balls (open circles): with variable masses and velocities.