

# A Logical Framework for Solid Object Physics

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## 1. Introduction

We have developed a logical framework for qualitative reasoning about solid objects. This framework specifies a first-order language  $L$  in which the physics of solid objects can be described, and defines the semantics of  $L$  in terms of a formal model. We show that interesting problems can be solved qualitatively by inference from plausible axioms expressed in  $L$ . The language  $L$  is more expressive and supports richer inferences than any previous representation scheme in this domain.

In concentrating on the representation and formulation of commonsense knowledge, and postponing questions of algorithms or control structure, we follow Hayes (1978). However, we depart from Hayes' research program in some respects. We do not attempt to model "naive" physics; rather, we have made free use of Newtonian mechanics, including concepts that have no commonsense analogue, such as total mechanical energy. Also, our proofs are lengthy, violating Hayes' dictum that obvious facts should have short proofs.

The mathematics used here is not "qualitative" in the restricted sense of representing quantities purely in terms of order relationships and constants [Bobrow, 85]. Such a representation is too weak to support the inferences needed in this domain.

We focus on variants of one particular problem: predicting what happens when one or two small dice are dropped inside a large funnel with steep sides. Different forms of this problem involve a rich, interconnected body of geometric and physical knowledge for their solution.

## 2. Background

Several previous AI researchers, such as Fahlman (1974), de Kleer (1975), Bundy (1978), Forbus (1979), Funt (1980), Novak (1981), and Shoham (1985) have studied the qualitative physics of solid objects. Despite the significant contributions made by these programs, they suffered from a common fundamental limitation; they were all based almost entirely on extrapolating differential behavior. To make a prediction, the program first determined how each state of the system will tend to change, and then extrapolated these changes to predict a continual trend of change up to the point that the structure of the system changes. This extrapolation could be done qualitatively, as in FROB and NEWTON, or symbolically, as in MECHO, or using point-by-point simulation, as in WHISPER, or by numerical integration, as proposed by McDermott and Bernecky (personal communication).

However extrapolation is done, simulation is inadequate for robust prediction. In this kind of analysis, each different set of boundary conditions is a different system state. Each such state must be separately detected, categorized, and analyzed, and the system's progress through these states must be recorded. Often, however, such a categorization is difficult and pointless. If a die is dropped inside a funnel, it may pass through many different differential states: free-fall; colliding or in continuous contact with the top or bottom part of the funnel on any of eight vertices, twelve sides, or six faces; spinning, sliding, or rolling, up, down, or around the funnel. Intuitively, it is much easier to predict that the die comes out the funnel than to find all these intermediate states. Moreover, the sequence of states traversed depends delicately on the exact shapes, sizes, and physical properties of the die and the funnel, while the conclusion that the die comes out the bottom is robust under small variations in these parameters. Therefore, if the problem is imprecisely specified, simulation

will involve an enormous tree of possibilities. Finally, the complexity of simulation goes up rapidly with the number of interacting objects. If an automobile is driven over a cliff, the number of possible intermediate states is enormous, but general predictions are nonetheless possible.

The programs cited do use some techniques besides simulation. MECHO and NEWTON use energy conservation to prune possible system behaviors. FROB predicts that the system ends in a stable state. We believe that effective qualitative reasoning requires more inferences like these, and less use of simulation.

A knowledge engineering approach would use rules that state the desired prediction, such as "A small object released inside a steep, large-mouthed funnel will fall out the bottom." But rules of this kind are inadequate, and have rightly been rejected by previous researchers. Any single such rule covers only a small class of problems; covering large classes of problems requires many separate disconnected rules.

### 3. Examples and Analysis

We propose to analyze the "die in the funnel" as follows: (i) Due to the funnel's topology, if the die goes from inside it to outside it, the die must exit either the top or the bottom. (ii) Since the die is dropped from rest inside the funnel, it cannot have the energy to exit the top. (iii) There is no stable resting point for the die inside the funnel, since it is smaller than the funnel's mouth, and the funnel's sides are steep. (iv) The die cannot stay forever moving within the funnel, for its kinetic energy will be dissipated. Therefore, the die exits the bottom of the funnel. We claim that most cases where common sense predicts that the die will come out the bottom, can be analysed in this way. Different problems will vary in the justifications of the substeps. The full paper shows how justifications are constructed and combined for many different kinds of geometrical and physical specifications.

Related problems will share parts of the analysis. For instance, in predicting that a die in a small-necked funnel will come to rest at the top of the neck, we may use the identical arguments (i) that the die must either exit the top, exit the bottom, or stay inside; (ii) that it cannot exit the top; and (iv) that it cannot stay inside in a perpetual state of motion. The argument (iii) that it cannot rest stably inside the funnel must be modified to an argument that it can only rest stably at the top of the neck of the funnel; and the additional argument must be made that it cannot exit the bottom of the funnel, since the orifice is too small.

This analysis avoids both problems discussed above. We need not determine the states of motion of the die inside the funnel; we need only determine that the die cannot rest stably inside. Different examples are analyzed in similar ways from general principles.

We have studied in detail how an analysis like this can be formulated for various geometrical specifications. We begin with a simple case. The die is a sphere. The funnel is the surface of revolution about a vertical axis of a planar figure with a convex inner side. The radius of the die is less than the radius of revolution of the funnel. The steps of the argument are easily filled in. (i) The top and bottom of the funnel are the only orifices of free space connecting the inside of the funnel with its outside. Therefore, if the die is to go from inside to outside, it must go through the top or the bottom. (ii) Since the die is spherical, its center of mass is in its interior. Since the top of the funnel is horizontal, and directed upward, if the die were to exit it, each point in the interior of the die would be above the top of the funnel at some time. In particular, the center of mass would be above the top at some time. But the die started out from rest below the top of the funnel, and there is no source of additional energy for the die. Therefore, the die cannot come out the top. (iii) By a geometrical argument, the die can only abut the inside of the funnel in a single point. A uniform sphere can be stably supported at a single point only if the supporting surface is horizontal there. The inner surface of the funnel is nowhere horizontal. Hence there is no resting place for the die inside the funnel.

We now consider how this argument can be generalized and modified. (Further modifications are discussed in [Davis,86].) Part (i), that the die must either exit the top, exit the bottom, or stay inside, is valid as long as the funnel is a tube with only two orifices. We can weaken the condition further, and require only that all orifices other than the top or the bottom be too small to let the die through. Determining whether a die can go through a hole is an easy geometric calculation for various special cases.

Part (ii), the argument from energy conservation that the die cannot come out the top, depends on the die being convex and on the center of mass of the die starting out below any part of the top. Convexity is only used to establish that the center of mass of the object is in its interior. If this can be done otherwise — for example, by exact calculation, or by establishing that the object shape is a small perturbation of a convex shape, — that is sufficient.

A still weaker sufficient condition is that the *ringed-filling-in* of the die contains the center of mass. The ringed-filling-in of a three-dimensional shape  $S$  is defined as follows: Consider any planar cross section of  $S$ . Let  $C$  be any simple closed curve that lies entirely in this cross section. Let  $p$  be a point in the plane in the inside of  $C$ . Then  $p$  is in the ringed-filling-in of  $S$ . It can easily be shown that, if the center of mass of an object is in its ringed-filling-in, and the object goes through a planar surface, then the center of mass also goes through the surface. Thus we can establish step (ii) for such shapes as a torus, a wiffle ball, or a cratered convex shape.

Part (iii), the argument that the die cannot rest inside the funnel, depended in our first example on the strong assertions that the die was a uniform sphere and that it could contact the funnel only in a single point. We can easily generalize to nearly uniform, nearly spherical dice. The following formula holds: let  $\theta$  be the slope of the support; let  $\mu$  be the coefficient of friction; let  $\phi$  be the maximum angle between the line from the center of mass to a point on the surface and the normal to the surface at that point. The ball can stand still only if  $\mu \geq \tan(\theta)$  and  $\phi > \theta$ . Similarly, if one die is a spherical shell containing another die, they rest stably only if the joint center of mass of the two dice is located directly above the contact point of the outer die with its support, and the inner die rests stably inside the outer die.

If the die can contact the funnel in several points with different surface normals, the following rule can be used: Let  $A$  be in contact with  $B$ . Let  $\theta$  be the minimum slope of the surface of  $B$  at a contact point. Consider the horizontal components of the surface normals of  $B$  at the contact points, and assume that there is some direction which lies within some small angle  $\phi$  of all these horizontal components. Let the coefficient of friction be  $\mu$ . If  $\mu < \cos\phi \tan\theta$ , then  $A$  will slide down  $B$ .

Combining all the different ways in which the results (i), (ii), and (iii) may be established, and all the ways in which their geometrical preconditions may be satisfied, gives a rich, interconnected body of results, all with the conclusion, "The die falls out the bottom of the funnel."

#### 4. The Block on the Table

The behavior of the block on the table can be analyzed in a similar way. After the block is released, it will fall to the table, tipple over a bit, and then move along the table in some combination of sliding, bouncing, and rolling. It can be estimated how long it will take for the friction involved in sliding and the inelasticity involved in bouncing to consume all the energy gained in the fall and the tipple, and how far the block can travel during that time. A similar estimation can be made for rolling in some cases. If the surface of the table is uniform, and if these motions will not bring the block off the edge of the table, then it can be predicted that the block will attain a stable state of rest within the estimated time, and within the estimated distance of the point of release.

## 5. The Underlying Knowledge

The arguments in sections 3 and 4 require a rich geometric vocabulary, including topological properties, metric properties, differential properties, special shapes, and convexity. We must also be able to quantify in various ways over irregularities; for example, to say that the funnel has no holes large enough to let the die through. Any adequate geometric language will be strong enough that these, or most of these, can be either expressed directly or inferred.

Our temporal logic follows [McDermott, 82]. A *scene* is an instantaneous snapshot of the universe. In our domain, a scene specifies the positions and velocities of all objects. A *chronicle* is a function from the time line to scenes. Chronicles include all continuous motion of objects through space, not just those that are physically possible.

The "frame" or "persistence" problem of determining what remains true over time ([McCarthy and Hayes, 69] [McDermott, 82]) does not arise. There are two classes of predicates in the domain. The first class includes predicates that depend on position and velocity of objects. These are not assumed to remain constant over any interval unless proven to do so. The second class includes structural predicates, depending only on the shapes and material properties of the objects. These are always constant over the problem, and so are defined atemporally. (The closed world assumption is made explicit through the predicate "isolated( $OO, C$ )", which asserts that, during  $C$ , no mobile object in the set of objects  $OO$  ever comes into contact with any object outside  $OO$ .)

The physical deduction needed include determining whether a set of objects can attain a stable scene while certain geometric conditions hold; finding constraints on the location of the center of mass of an object or a set of objects [Bundy and Byrd, 83]; predicting a collision; predicting the result of a collision; determining whether a chronicle violates a conservation law; and characterizing the open paths available to an object.

## 6. Ontology

The ontology for our language requires a number of sorts of individuals.

*Quantities.* Instants of time, quantities of mass, quantities of energy. These are modelled as real numbers.

*Points and vectors.* These are modelled as elements of  $R^3$ .

*Point sets.* Subsets of  $R^3$ .

*Vector fields.* These are functions from some point sets to the space of vectors. For example, the surface normals to an object in a fixed position, directed outward, form a vector field.

*Rigid mappings.* Mappings from  $R^3$  to  $R^3$  which preserve distance and handedness. These specify a change in position.

*General velocities.* The derivative of a rigid mapping. A general velocity is the composition of a linear velocity and an angular velocity about a specified axis.

*Objects.* These are primitive entities. The shape of an object is the point set that it occupies in some particular standard position. This is assumed to be connected, closed, and normal.

*Scenes.* A scene is a snapshot of the world. Formally, it is a function which maps an object to a pair of a rigid mapping, giving the position of the object, and a general velocity. The place of an object in a scene is image of the object shape under the mapping associated with the object in the scene.

*Pseudo-objects.* These are point sets that "move around" with objects, like the hole of a doughnut, the opening of a bottle, or the center of mass of any object. Formally, a pseudo-object is a pair of a source object and a point set, designating the point set occupied by the pseudo-object when the object is in standard position. The place of a pseudo-object in a scene



is the image of its shape under the mapping associated with its source object in the scene.

**Chronicles.** A chronicle is a function from an interval of time to scenes.

All chronicles are subject to the following constraints:

- i. All scenes in the range of the chronicle have the same objects in their domain.
- ii. Objects move continuously in space.
- iii. Object velocities are continuous from previous times.
- iv. The velocity of an object is the derivative of its position.

Chronicles do not have to be physically possible. We use the predicate "phys-poss( $C$ )" to distinguish chronicles that obey the laws of physics.

## 7. Axioms for Physical Reasoning

Based on the above ontology, we have developed a first-order language  $L$  and a set of axioms adequate to solve the first "die in the funnel" example. The complete analysis is rather lengthy; the language uses about ninety non-logical terms, not including the standard arithmetic operators, and the analysis involves about 140 axioms. Most (over two thirds) of the terms and axioms are purely geometrical; the rest relate to motion and to physics. We give below three sample axioms as illustrations.

**Geometric Axiom:** Smoothness and the value of the surface normal are local properties of the boundary. Specifically, if two bodies share part of their boundary, then, at any interior point of the overlap, one is smooth iff the other is smooth, and their surface normals are either parallel or anti-parallel.

$$\begin{aligned} & [ \text{body}(XX1) \wedge \text{body}(XX2) \wedge XXA \subseteq \text{boundary}(XX1) \cap \text{boundary}(XX2) \wedge \\ & \quad X \in \text{interior}(XXA) \wedge \text{smooth}(XX1, X) ] \Rightarrow \\ & [ \text{smooth}(XX2, X) \wedge \\ & \quad [ \text{surf-norm}(XX1, X) = \text{surf-norm}(XX2, X) \vee \text{surf-norm}(XX1, X) = -\text{surf-norm}(XX2, X) ] ] \end{aligned}$$

**Axiom of Motion:** If an object  $O$  has zero velocity in every scene of a chronicle  $C$ , then it stays in the same place throughout  $C$ .

$$\begin{aligned} & [ \forall S \ S \in \text{scenes}(C) \Rightarrow \text{velocity}(O, S) = 0 ] \Rightarrow \\ & [ \forall S1, S2 \ S1 \in \text{scenes}(C) \wedge S2 \in \text{scenes}(C) \Rightarrow \text{mapping}(S1, O) = \text{mapping}(S2, O) ] \end{aligned}$$

**Physics axiom:** The energy of an isolated set of objects  $OO$  never increases in a physically possible chronicle  $C$ .

$$\begin{aligned} & [ \text{phys-poss}(C) \wedge \text{isolated}(OO, C) \wedge T1 < T2 ] \Rightarrow \\ & \text{energy}(OO, \text{scene}(C, T1)) \geq \text{energy}(OO, \text{scene}(C, T2)) \end{aligned}$$

## 8. Conclusions

The strengths and limitations of this theory are evident. On the positive side: Using pure first-order logic, we give a formal analysis of a class of problems beyond the scope of any previous AI theory. Our analysis suggests that a qualitative physics for solid objects should include the following features, among others:

- A rich geometrical theory, including topological, metric, and differential descriptors, and special shapes.
- An account of the behavior of physical systems over extended intervals of time. Such an account should incorporate constraints placed by one object on another; conservation laws, especially conservation of energy; the principle that a physical system tends towards a stable resting point; and an account of the net effects of collisions over extended time periods.

- The ability to determine the existence of a stable configuration of objects within qualitatively described geometrical constraints.
- The ability to calculate, exactly or qualitatively, important physical parameters such as the center of mass. [14]
- The ability to bound the effect of small perturbations.

On the negative side: We have not shown that this type of analysis is extensible to cover all, or most, qualitative reasoning in this domain. We have not shown that such an extension would be, in the long run, any more parsimonious than simply enumerating special cases, as in the rule-based method rejected in section 2. We have not shown that any effective computational methods can be developed on the basis of this theory. We cannot give a final resolution to these problems until we have implemented a working system, and determined the range of problems that it is adequate to address.

We plan to begin implementation by developing an adequate geometric representation and inference system. Ultimately, we want to implement a physical reasoning system with all the features mentioned above.

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## 10. References

- D. Bobrow, ed. *Qualitative Reasoning about Physical Systems*, MIT Press, 1985
- A. Bundy, "Will it Reach the Top? Prediction in the Mechanics World," *Artificial Intelligence Journal*, Vol. 10, pp. 129-146
- A. Bundy and L. Byrd, "Using the Method of Fibres in Mecho to Calculate Radii of Gyration," in D. Gentner and A. Stevens, *Mental Models*, Erlbaum Associates, 1983
- E. Davis, "Shape and Function of Solid Objects: Some Examples," NYU Tech. Report #137, 1984
- E. Davis, "A Logical Framework for Solid Object Physics," NYU Tech. Report #245, 1986
- J. de Kleer, "Qualitative and Quantitative Knowledge in Classical Mechanics," MIT AI Lab Tech. Report #352, 1975
- S. Fahlman, "A Planning System for Robot Construction Tasks," *Artificial Intelligence Journal*, vol. 5, 1974, pp. 1-49
- K. Forbus, "A Study of Qualitative and Geometric Knowledge in Reasoning about Motion," MIT AI Lab Tech. Report #615, 1979
- B. Funt, "Problem Solving with Diagrammatic Representations," *Artificial Intelligence Journal*, vol. 13, 1980, p. 201-230.
- P. Hayes, "The Naive Physics Manifesto", 1978; revised as "The Second Naive Physics Manifesto", in J. Hobbs and R. Moore, *Formal Theories of the Commonsense World*, ALEX, 1985
- J. McCarthy and P. Hayes, "Some Philosophical Problems from the Standpoint of Artificial Intelligence," in *Machine Intelligence*, B. Meltzer and D. Michie (eds.), Edinburgh University Press, 1969
- D.V. McDermott, "A Temporal Logic for Reasoning about Plans and Processes," *Cognitive Science*, 1982, pp. 101-155
- G. Novak, "Representations of Knowledge in a Program for Solving Physics Problems," *IJCAI 5*, 1977, pp. 286-291
- Y. Shoham, "Naive Kinematics: One Aspect of Shape," *IJCAI 9*, 1985, pp. 436-442