Controlling Qualitative Resolution

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Extended abstract. Submitted to the Paris QP Workshop.

An extended version of this paper has been submitted to AAAI88.

Assembling a device

We proposed in [5] a new way of reasoning on a device, we called assembling a device. We sum up here what is the nature of this method.

The qualitative resolution rule

The qualitative resolution rule states under which conditions it is correct to combine two confluences and produce a more global one:

Qualitative resolution rule: Let $\mathbf{x} + E_1 \quad a$ *and* $-\mathbf{x} + E_2 \quad b$ *be two confluences, where* $\mathbf{x}$ *is a variable and* E_1 *and* E_2 *have no variable with opposite coefficients in common. Then* $E_3 \quad a + b$ *is a valid confluence, where* E_3 *is the same expression as* $E_1 + E_2$*, but with no repeated variable.*

The resolution rule can be understood as a "qualitative gaussian elimination":

A variable may be eliminated by adding or subtracting two confluences, provided that no other variable is eliminated at the same time.

An example

Consider the well-known pressure regulator (Fig. 1).

The resolution rule provides the following set of global relations:

$[dP_2]$	$[dP_1]+[dP_5]$	(A1)
$[dP_4]$	$[dP_1]+[dP_5]$	(A2)
$[dQ]$	$[dP_1]-[dP_5]$	(A3)
$[dA]$	$-[dP_1]-[dP_5]$	(A4)
$[dP_3]$	$[dP_1]+?[dP_5]$	(A5)

We call such a set of relations an *assemblage*.

$[dP_2]$	$[dP_1]+[dP_5]$	(A1)

For example, the first relation (A1) is deduced using the resolution rule in four steps:

$$
\begin{array}{lll}
[dP_2]-[dP_3]-[dQ]-[dP_4] & \simeq 0 & (6)=(2)-(5) \\
[dP_2]-[dP_4]-[dQ] & \simeq 0 & (7)=(6)+(3) \\
[dP_2]-[dP_5]-[dQ] & \simeq 0 & (8)=(7)+(4) \\
[dP_1]-[dP_2]+[dP_5] & \simeq 0 & (9)=(1)-(8)
\end{array}
$$

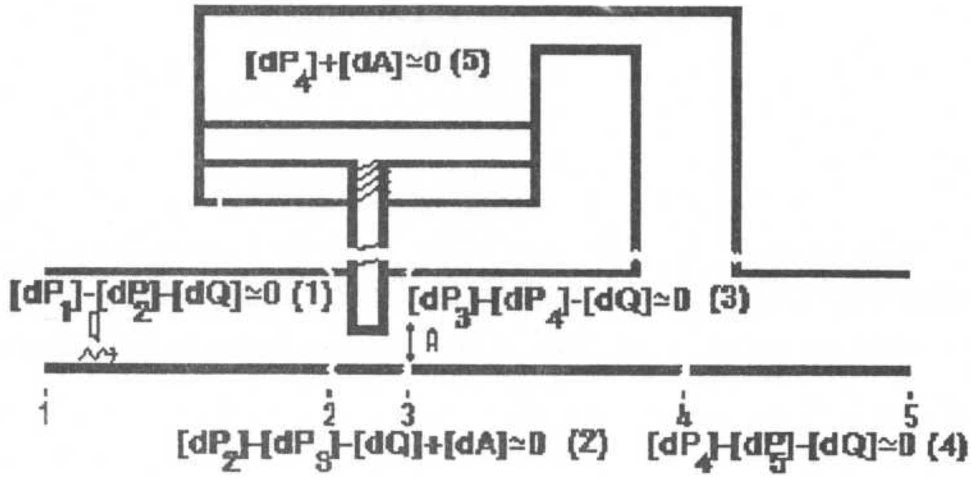


Fig. 1: The pressure regulator

The meaning of the qualitative resolution rule

Initial confluences describe the links between the physical variables involved in the elementary components. The inferred confluences describe the consequences of connecting these components: they are specific properties of the composite device. The qualitative resolution rule discovers global relations starting from local ones.

But the resolution rule has another advantage: performing simulation is straightforward as soon as an assemblage is known and requires using only the two basic propagation rules:

PR1: If the value of a variable x *is known, then replace* x *by its value in all the confluences mentioning* x*.*
PR2: If an equation mentions exactly one variable, then deduce its value.

For example:

$$
\begin{array}{lll}
[dP_1]=0,\ [dP_5]=0 & \Longrightarrow & [dP_2]=[dP_3]=[dP_4]=[dQ]=+,\ [dA]=- \\
[dP_1]=+,\ [dP_5]=+ & \Longrightarrow & [dP_2]=[dP_4]=+,\ [dA]=- \\
 & & [dP_3] \text{ and } [dQ] \text{ remain ambiguous}
\end{array}
$$

Splitting simulation into two steps

What has been written suggests to split simulation into two steps:

- in the first step the resolution rule *assembles the device*, i.e. discovers global relations directly linking internal variables to the selected ones. We mention that we have proved a completeness result concerning this step.

- in the second step, the values of the reference variables are propagated into these global laws using only the basic propagation rules.

Splitting simulation into these two steps is fundamental. The first step is N-P complete, but it is done once and for all. It can be viewed as *compiling* the description of the device. The second step is performed each time one wants to predict the behavior of the device for some particular values of the reference variables. But it is known to be polynomial. The initial set of confluences is not *re-interpreted*. The second step can be coded as a very simple and efficient program.

We may eventually expect to perform on-line simulations or observations on large-scale systems having multiple input variables.

The joining rule

Controlling the resolution rule

Unfortunately, the number of potential applications of the resolution rule (probably) exponentially increases as it is being fired. For example, the resolution rule can be fired in hundreds of different ways in the pressure regulator example. This behavior is to be related to the N-P completeness of the problem consisting of solving confluences. However, as shown in the examples, discovering an assemblage requires few steps. The resolution rule needs to be controlled. Some heuristic rules must choose between its potential applications.

Consolidation

Consider a component-based model of a device, and let C_1, C_2 and C_3 be three mutually interacting components. If we denote C_{12} the composed component $C_{12}=\{C_1,C_2\}$, the interactions between C_1 and C_2 define how C_{12} works. Indeed, they are of no interest to C_3: from C_3's point of view, the collection made up of C_1 and C_2 is equivalent to C_{12}. C_3 cannot distinguish C_1 and C_2 from each other. Hence, it should be possible to draw a model for C_{12} from models for C_1 and C_2 such that they are equivalent for C_3. Joining local models together in order to provide more global ones is what has been called *consolidation* [7]. The trouble lays on giving concrete expression to this idea. In particular, some rules for choosing at any inference step the couple of components to be consolidated must be stated: it certainly cannot be randomly selected.

The resolution rule under the microscope

In a confluence-based model, C_1 and C_2 interact through their common variables. Hence, building a model for C_{12} means providing confluences by eliminating them. Consider a variable involved in both C_1 and C_2 models. If it is involved in some other component model as well, then it must appear in a model for C_{12} (like variable **y** in Fig. 2). But if it is not, then it must be completely eliminated (like variable **x** in Fig. 2). The resolution rule seems to tackle this problem, but we have to go more precisely into what it accomplishes.

Consider a simple case (but this case happens more often than not), when both C_1 and C_2 models are made up of a single confluence, say respectively E_1 and E_2. Let **x** be a variable involved in both, and assume that the resolution rule applies to E_1 and E_2 and so eliminates **x**. Then we get a new confluence, say E_{12}, which is global to C_{12}. Any other variable involved in E_1 or E_2, or both, will belong to E_{12} as well. But it is not true in general that E_{12} is equivalent to E_1 and E_2. For instance, if another component C_3 is concerned by

x, then E_{12} is certainly not a proper model for C_{12}. But if x is involved exclusively in E_1 and E_2, then E_{12} should be one.

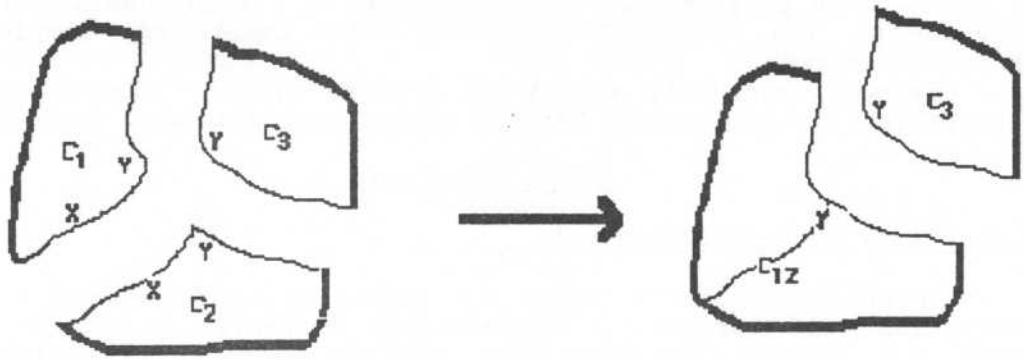


Fig. 2: Joining two components

Joining two components

Previous remarks provide a heuristic rule for choosing at each inference step between the potential applications of the resolution rule:

Joining rule. Let (E) be a set of confluences corresponding to a component-based description of a device. If the resolution rule applies to confluences E_1 and E_2 by eliminating variable x, and if x is involved exclusively in E_1 and E_2, then choose this application. A model equivalent for the variables different from x is obtained by substituting confluence E_{12} produced in this way for confluences E_1 and E_2.

If E_1 and E_2 are the respective models for components C_1 and C_2, then E_{12} is a proper model for C_{12}. C_1 and C_2 are *joined*.

This rule can apply recursively. Indeed, a variable y different from x and involved solely in E_1, E_2 and a third confluence belongs to exactly two confluences after the resolution rule has been fired. Therefore, the joining rule might choose to eliminate it at a next step. This means that a compound component can be joined in turn to another atomic or itself compound component.

A mathematical justification

The choice heuristic contained in the joining rule conditions has been justified by some commonsense arguments. It needs no mathematical proof. But the conclusion needs one:

A model equivalent for the variables different from x is obtained by substituting confluence E_{12} produced in this way for confluences E_1 and E_2.

We have proved that this is true for square systems, i.e. when the number of confluences is equal to the number of internal variables. Indeed, it can be proved in this case that, starting from task-oriented confluences, all the pieces of task-oriented assemblages (concerning variables different from x) that can be drawn from the initial model can be drawn after the joining rule has been fired as well. We do not give the proof here, because it is too long and requires mathematical notions which are beyond the scope of this paper. It can be found in [6].

We have proved more:

Let (E) be a <u>*non decomposable*</u> *set of confluences, and* x *a variable involved in exactly two confluences, say* E_1 *and* E_2*. If the resolution rule does not apply to* E_1 *and* E_2 *by eliminating* x*, then no piece of assemblage concerning a variable different from* x *can be drawn from (E).*

A set of confluences (E) is said to be decomposable if it contains a subset (E') involving variables that are not mentioned in (E)-(E'). In a practical way, if (E) happens to be decomposable, then one studies first (E'). This is what is called by Iwasaki and Simon *causal ordering* [10]. The problem comes down to studying non decomposable sets of confluences. In concrete terms, a "loop of components" is not decomposable. Efficient algorithms have been described for decomposing a set of equations (see for example [11]).

This second property is important: it states what happens when two components are about to be joined, but ultimately cannot. The conclusion looks natural: finding out a piece of assemblage for a variable different from x requires eliminating x at some step. This property can be viewed as the "negative part" of the joining rule (it states when joining is not possible).

However, it must be pointed out that this second property never applies when the qualitative model is "good". What a "good" qualitative model based on confluences is can formally be defined, but this is not the topic discussed here. Both examples presented in this paper are "good" models.

<u>*Implementation*</u>

We can now build our heuristic machinery for controlling qualitative resolution:

Let (E_0) *be the qualitative model to be assembled. Perform step 0.*

<u>*Choice, step i*</u>*: Select within the current set of confluences* (E_i) *a variable* x *such that:*

- x is involved in exactly two confluences of (E_i)*,*

- x has not been yet selected at step i,

- there is a variable different from x *involved in* (E_i) *which has not yet been assembled.*

<u>*Joining rule (JR), step i*</u>*: Let* x *be the selected variable, and* E_1 *and* E_2 *the confluences involving* x*. Then, eliminate* x *by mean of the resolution rule. This produces confluence* E_{12}*. Set* $(E_{i+1}) \longleftarrow (E_i)-\{E_1,E_2\}\cup\{E_{12}\}$*. Perform step i+1.*

<u>*Backtracking, step i*</u>*: Make a new* <u>*choice, step i*</u>*. If no such choice is possible, and if i is different from 0, then go back to step i-1.*

In addition, as soon as a confluence involving a single variable is produced, the corresponding piece of assemblage is kept. The "negative part" of the joining rule may be added, too.

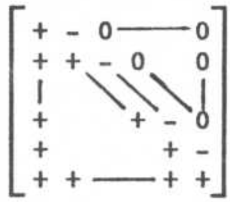
Though the joining heuristic is self-sufficient, some rules can be added in order to speed up the assembling step. <*This part is present in the extended version of this paper*>.

When can the joining rule fail?

The system presented here has been tried in examples stemming from different physical areas, from electronic circuits to thermodynamic systems (e.g., the pressurizer of a PWR nuclear power plant). It never failed to yield an assemblage in a straightforward way. So, it is justified to wonder whether this method is complete, i.e. always discovers an assemblage. It is obvious that this would require any model which can be assembled to involve at least one variable belonging to exactly two confluences.

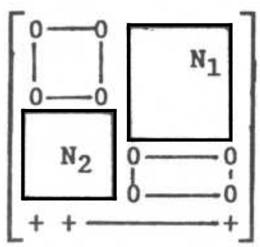
Indeed, the joining rule may fail. There exist models which can be assembled, but having no variable belonging to less than 3 confluences. We cannot go into the underlying mathematics, but previous work related to this question has to be mentioned.

Similar issues were studied more than twenty five years ago by mathematical economists. They led to many mistakes. Lancaster [12] claimed that the matrix of any square system having a determinate value turns to be deducible from the form:



A system having a determinate value is a particular case of a system which can be assembled. This result would imply that the joining rule is complete in the square case.

Two years later, Gorman [13] showed that this is wrong by producing the following counter-examples:



N_1 and N_2 are square matrices. They have a single line in common. They are themselves supposed to be Lancaster's or Gorman's matrices.

Gorman claimed in a footnote that he had proved that all the determinate matrices are deducible from this generic form. Unfortunately, this is wrong, too, as shown by the counter-example:

$$\begin{pmatrix} 0 & + & + & + \\ + & 0 & - & + \\ + & + & 0 & - \\ + & - & + & 0 \end{pmatrix}$$

It can be shown that Lancaster's and Gorman's forms, plus this last form, are the only generic forms of 4x4 matrices. There are 6 basic forms of 5x5 matrices, and we do not know how many there are for nxn matrices with n>5. A generalized control for qualitative resolution is strongly related to these topics.

Let's go back to the real world. The fact that the joining rule works without trouble within a *physical* model can be justified by a commonsense argument: there must be a variable linking two components, but not involved in the interaction with any other component.

Conclusion

If not controlled, qualitative resolution leads to combinatorial explosion. But the fact that qualitative models stem from real-world devices prevents it from meeting the fate of resolution in logics. The heuristic control presented here is strongly related to the structural properties of a sane device.

We have tried our system in examples corresponding to different physical areas. But they were all *small* devices. However, we think that the assembling technique, controlled by the joining heuristic, could assemble some larger artefacts. The next step will be to get a model for a large-scale plant.

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