The Use of the Metric Diagram in Qualitative Kinematics

Boi Faltings
Laboratoire d'Intelligence Artificielle
Departement d'Informatique
Swiss Federal Institute of Technology (EPFL)
MA-Ecublens
1015 Lausanne
Switzerland

Tel. +41-21-47-2738
E-Mail: faltings@CLSEPF51.BITNET

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Abstract

A promising approach to qualitative reasoning about mechanism kinematics is the theory of *place vocabularies*. In this paper, we propose a new method of computing the place vocabulary representation. The computation is split into the combinatorial manipulation of the symbolic information, and the evaluation of predicates on the metric dimensions by an abstract device, the *metric diagram*. This abstraction allows reasoning about mechanism behavior without reference to an explicit representation of the numeric dimensions of the parts.

When the metric information is incomplete, the solution may contain ambiguities. When no metric information is available, the result is an ambiguous, but purely symbolic, place vocabulary. This provides an existance proof that purely symbolic spatial reasoning is possible.

Figure 1: *A ratchet.*

1 Introduction

An important problem in Artificial Intelligence is spatial reasoning about physical objects. A solution to this problem has long eluded researchers and its lack is a major obstacle to the application of AI to problems in the physical world. Important examples of spatial reasoning problems are encountered when analyzing mechanism kinematics. For example, understanding of the function of a ratchet, shown in Figure 1, requires sophisticated spatial reasoning to predict the interactions of its parts. The ratchet consists of two parts, a wheel and a lever, both hinged at a fixed axis. The geometries of these parts restrict their relative motion and achieve the desired behavior of the ratchet. The goal of *qualitative kinematics* is a qualitative description of behavior based on the geometries of parts.

In earlier work ([FNF87,FALT86,FALT87a,FALT87b,FALT88a]), we have developed the theory of *place vocabularies* for qualitative reasoning about mechanism kinematics. A mechanism's place vocabulary is a graph that represents the set of its possible kinematic states and transitions between them. Currently, the place vocabulary theory is restricted to the analysis of two-dimensional higher kinematic pairs ([FALT87a,FALT87b,REU75,REU76]). A higher kinematic pair is a pair of two objects, each hinged so that they have only a single degree of freedom. All results described in this paper are for this restricted domain; their generalization has not yet

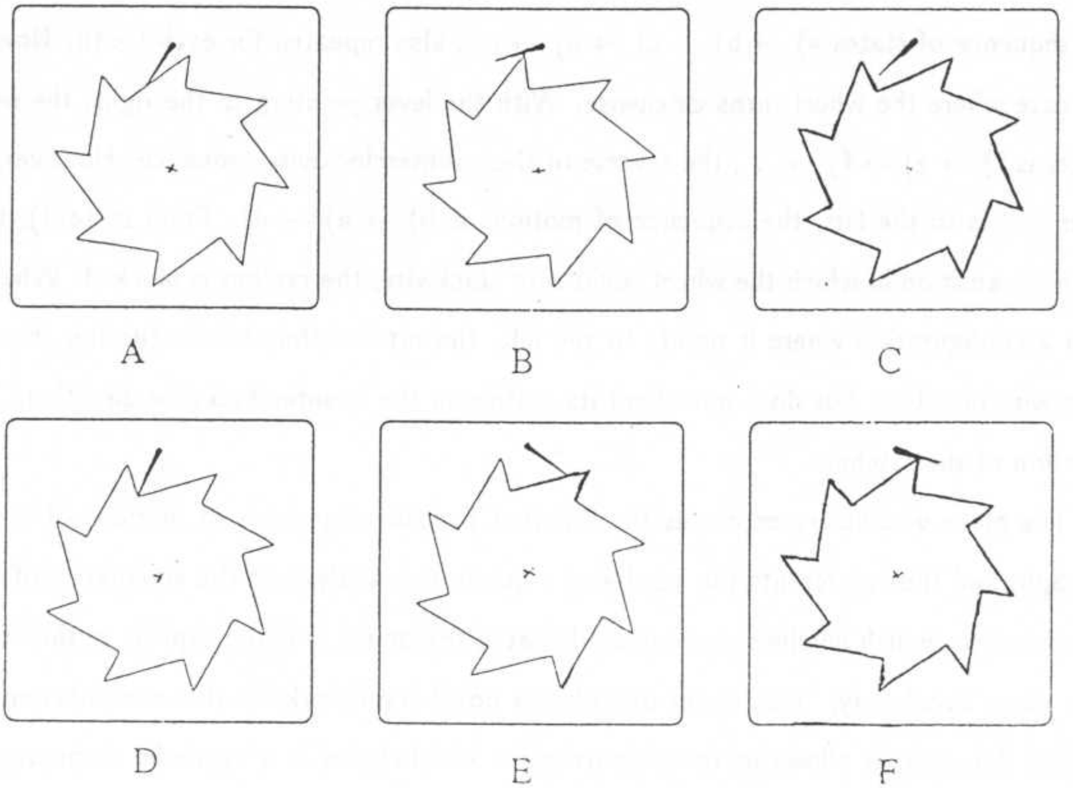

Figure 2: *Examples of possible states of the ratchet.*

been investigated.

As an example of a place vocabulary, consider how it can be used to express the behavior of the ratchet example. An explanation based on sequences of kinematic states, such as shown in Figure 2, is natural and intuitive to people. The kinematic states shown in Figure 2 are elements of the ratchet's place vocabulary, augmented by specifying the directions of motion of the elements.

Under the influence of gravity, the lever is pulled toward the position where it hangs straight down. We find that there are two stable states where the lever points either to the left or to the right, supported by the wheel. First, consider the behavior when the wheel turns counterclockwise. When the lever points to the right, we find the sequence of states e) → f) → e) → ..., repeated for each tooth. When the lever points to the left, the behavior of the ratchet is characterized by the sequence of states a) → b) → c) → a) → ..., also repeated for each tooth. Now, consider the case where the wheel turns clockwise. With the lever pointing to the right, the sequence of states is f) → e) → f) → ..., the reverse of the counterclockwise sequence. However, when the lever points to the left, the sequence of motions is b) → a) → d). From state d), there is no further transition in which the wheel could turn clockwise; the ratchet is blocked. When the lever is in a configuration where it points to the left, the ratchet thus blocks turning the wheel in a clockwise direction, but does not affect its motion in the counterclockwise direction. This is the function of the ratchet.

The place vocabulary expresses the different possible sequences of motions of the device as a graph, and thus represents the results of a qualitative analysis of the kinematics of the device. This analysis, which involves sophisticated spatial reasoning, is made explicit as the computation of a place vocabulary. This paper describes a novel framework for this computation, that of a *metric diagram*. It allows us to compute place vocabularies in a symbolic manner, and closely models human problem-solving behavior. In the next section, we define the metric diagram model and discuss how it can be applied to the computation of place vocabularies. We then show how it can be used to solve problems such as mechanism design and variable modeling of devices.

2 The Metric Diagram Model

During the initial development of the place vocabulary theory, place vocabularies were computed using rather involved methods of computational geometry. However, such methods do not allow reasoning about the aspects of the particular shapes that are important for achieving a particular behavior. People are very good at determining the effects of changes of particular features on the behavior of a device. For example, when presented with a pair of gearwheels, people can readily state conditions that the distance between their centers must satisfy in order for the gears to mesh, as shown in Figure 3. This type of reasoning is important in many spatial reasoning

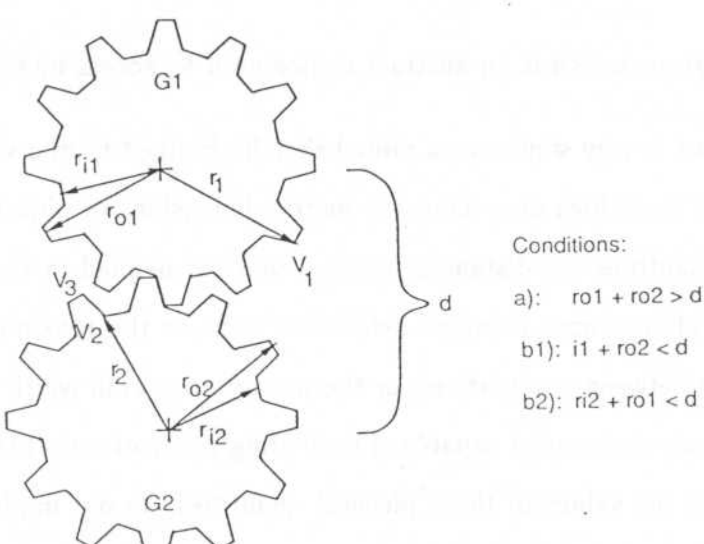

Figure 3: *A pair of gears meshes only when the distance d between their centers is a) small enough for the gears to touch, and b) large enough so that the teeth fit into each other.*

problems including mechanical design, troubleshooting, learning new physical phenomena, and reasoning under uncertainty.

In this paper, we propose a computation model based on the idea of a *metric diagram*. In this model, the input information is divided into 2 parts:

- a *symbolic* part, containing a purely symbolic description of the structure of the objects in terms of vertices and smooth boundary segments.
- a *metric diagram*, which is an abstract device used to access metric information.

The symbolic part is very similar to a primal sketch obtained from a vision system. It defines a *language* in which quantities describing the metric dimensions of objects can be defined. The simplest of these quantities are distances between vertices defined in the symbolic description. The language also allows more complex definitions such as the maximum width of an object orthogonal to a line between two vertices, or the maximum overall width of an object.

The metric diagram is assumed capable of evaluating *predicates* that test the sign of algebraic expressions involving the values of these physical quantities. In our implementation, the metric diagram consists of tests on exact metric representations of objects. In general, the metric diagram is an abstract device and may be implemented by a combination of internal and external processes. The internal implementation uses only information that is already represented in the system. For example, the sizes of the objects may be globally categorized in some interval-based system, and this information may be sufficient for many predicate evaluations. The external implementation is responsible for measurements to obtain missing information. For example, distances can be compared by placing objects against each other, or by drawing them on paper. For our system, the metric diagram fulfills the function that a diagram does for a human problem solver: to provide measurements of quantities that are not adequately represented in the mind.

2.1 The Metric Diagram and Human Problem Solving

The metric diagram models very well human problem solving behavior in the mechanism domain. When a certain predicate is very clearly decided by the metric dimensions, it can be evaluated by the internal implementation of the metric diagram. In this case, people consider the resulting structure as "obvious" and may even fail to notice the existance of a condition. For example, in the ratchet example shown in Figure 1, the angle at the tip of the lever has to be smaller than the opening angle of the teeth, for otherwise the lever will not fit between the teeth properly. This condition is so obviously satisfied that it is very hard for the human observer to notice it. On the other hand, in Figure 1 it is very hard to tell if there is sufficient distance between the lever and the wheel to allow it to pass under the lever at all. The reader may feel the urge to take additional measurements on the objects. This corresponds to using an external implementation of the metric diagram.

2.2 The Metric Diagram in Qualitative Kinematics

The tests that the metric diagram needs to carry out for the computation of place vocabularies are described in detail in [FALT87b,FALT88b]. In this paper, we can only give an example of the predicates that the metric diagram evaluates during the place vocabulary computation.

Consider the two gears $\mathcal{G}_1$ and $\mathcal{G}_2$ in Figure 3. A particular pair of vertices $\mathcal{V}_1$ on $\mathcal{G}_1$ and $\mathcal{V}_2$ on $\mathcal{G}_2$ can touch only if their distances from the centers of rotation, r_1 and r_2 satisfy the following relation with respect to the distance d between the two centers of rotation:

$$d < r_1 + r_2 \ and \ d > |r_1 - r_2|$$

Similarly, a touch between a $\mathcal{V}_1$ and the boundary between $\mathcal{V}_2$ and $\mathcal{V}_3$ is only possible if there exists at least one point on this boundary which satisfies the above condition. In this case, this can be expressed by the condition that at least one of $\mathcal{V}_2$ and $\mathcal{V}_3$ satisfies the condition for a possible touch with $\mathcal{V}_1$.

When a vertex $\mathcal{V}_1$ touches a boundary segment $\mathcal{B}_1$ on the other object, the two objects can move in a coordinated manner while maintaining the contact. In Figure 3, turning the gear $\mathcal{G}_1$ in the clockwise direction will cause $\mathcal{G}_2$ to turn counterclockwise. In the qualitative analysis, we need to infer the direction of motion of $\mathcal{G}_2$ from the direction of motion of $\mathcal{G}_1$. However, this direction may not be the same for all configurations where this type of contact occurs. Such direction changes must be distinguished in the place vocabulary, and this may require additional subdivisions. Whether such a subdivision is required or not is also determined by evaluating predicates using the metric diagram, as described in [FALT87b,FALT88b]).

3 Kinematics and Qualitative Reasoning

In this section, we show how using the metric diagram allows us to integrate qualitative kinematics with the principles of established qualitative reasoning theories, and illustrate possible applications, such as mechanical design and variable modelling. For the purposes of this discussion, we treat the place vocabulary as a complete specification of the mechanism kinematics. A complete envisionment of a mechanism's actual behavior has to take into account forces on the objects and is obtained by a dynamic analysis based on the place vocabulary. This is the topic of current research by Paul Nielsen ([NIE88]).

3.1 Integration with Qualitative Physics Theories

The distinction we have made between symbolic and metric information can be applied to most qualitative reasoning problems. In many domains, such as circuit analysis, the symbolic information defines the situation precisely enough so that quite accurate predictions of behavior can be made without information about the metric parameters. Certain qualitative reasoning methodologies, such as qualitative process theory ([FOR84]), allow the specification of *quantity conditions* on metric parameters to decide between ambiguous predictions. The predicates in our computation model could be stated as quantity conditions in such a framework. However, the number of resulting quantity conditions would be so large that the approach does not seem practical. Even in the simple example of the ratchet in Figure 1, more than 10000 predicates were evaluated to compute the place vocabulary.

Note, however, that making each metric predicate a quantity condition would in principle allow reasoning about kinematics without *any* knowledge of the metric dimensions of the objects, with the unknown quantity conditions causing ambiguities. This is an existance proof of a purely symbolic kinematics. We use the word "symbolic" instead of "qualitative" because the predicates may contain complicated algebraic expressions of quantities. The stronger classification "qualitative" should be reserved for systems that use only inequalities between quantities themselves.

3.2 Perturbation Analysis

The prediction of the effects of changing a parameter in a given device is an important problem, particularly in problem-solving applications, where the behavior of a device is to be modified by parameter changes. We call the analysis to find a suitable change *perturbation analysis* of the device. In the place vocabulary computation, we mark each element of the place vocabulary with all the predicates whose value has contributed to its existance and particular form. The set of parameters that can influence the element is given as the set of parameters whose values were used in the computation of the predicates. It is left to the application using the place vocabulary to decide which parameter should be varied, as this must be carried out by domain-dependent heuristics.

When a parameter to vary is picked, the system is faced with the problem of determining what new value it should be changed to. To determine a suitable value, we find the *landmark values* of the parameter where the predicates under consideration change their values. As each

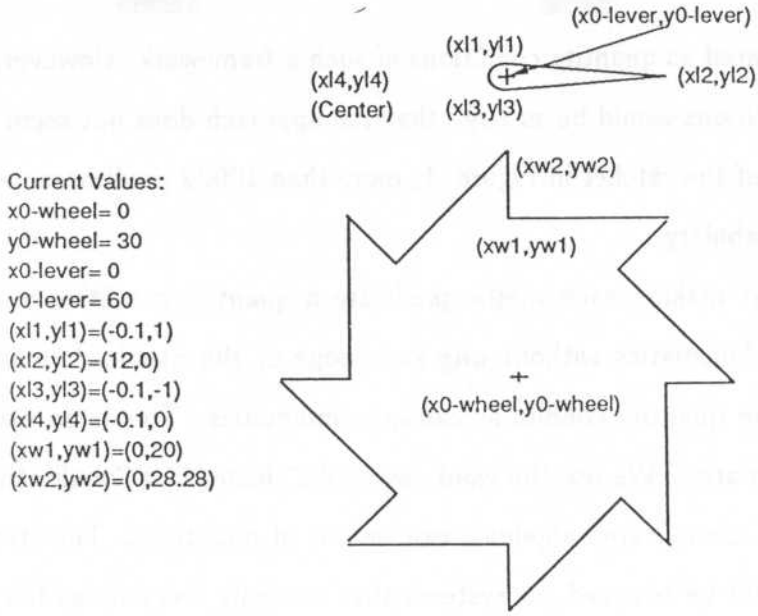

Figure 4: *A ratchet labelled with variable parameters. The actual values are indicated on the side.*

predicate is defined as the test of the sign of an algebraic expression, the roots of this expression in the parameter define the desired landmark values. To change the behavior, the parameter value must be changed beyond this landmark value.

As an example, consider the ratchet shown in Figure 4. In this figure, we have explicitly labeled the parameters that may be varied, such as the coordinates of the vertices of the boundaries in the local coordinate systems of the objects, or the coordinates of the centers of rotation in the global coordinate system. The particular choice of values in Figure 4 results in a place vocabulary containing a situation where the lever can push the wheel in the clockwise direction, as shown in Figure 5. If the wheel is connected to a transmission of gears, this may cause rattle and unnecessary wear. Suppose that we would like to modify the design to eliminate this behavior.

The state shown in Figure 5 is represented as a place in the place vocabulary. In the computation, it has been marked with two conditions for its existance. The first condition is for the existance of this type of contact itself and turns out not to be useful for our purposes,

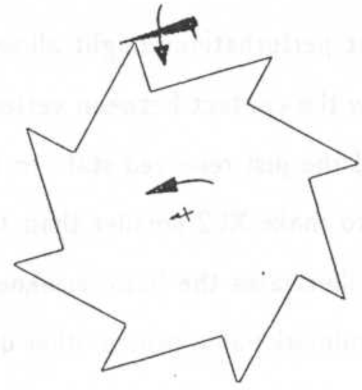

Figure 5: *In this state, pushing down on the lever pushes the wheel in a clockwise direction.*

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⇨ (places::explore-condition cseg-17 T)
Element CSEG-17 has conditions for (EXIST), choose one by number:0
There are 2 conditions, choose one by number:1
The expression contains the following variables:
 (Y0-WHEEL Y0-LEVER X0-WHEEL X0-LEVER YW2 XW2 XL3 YL3 YL2 XL2)
 Select one:y0-lever

The condition is
 (+ 12.0d0
    (* -0.28699899069702527d0
       (SQRT (+ -10391.823929839276d0 (* 12.141251994749l4d0 (ABS (+ -30 Y0-LEVER)) (ABS (+ -30 Y0-LEVER)))))) >0
 Variable Y0-LEVER has the value 60.
Lower bound on variable variation?:50
Upper bound on variable variation?:70

 Interval for Y0-LEVER actually used: 59.255962f0 .. 70.0f0
 landmark-value found in interval 50..70 is 61.621359970334524d0
NIL
```

Figure 6: *The program output for analyzing variation of XL2.*

as changing its value also renders the ratchet disfunctional. The second condition is dependent on the following parameters: XO-WHEEL, X0-LEVER, Y0-WHEEL, Y0-LEVER, XW2, YW2, XL2, YL2, XL3 and YL3. One way to fix the design is to shorten the lever, so let us try to vary the parameter XL2. The program transforms the condition into an expression in XL2, as shown in Figure 6. A landmark value for XL2 is a value where this expression changes sign. Because the program finds this value by binary search, we have to provide an initial interval. The landmark value found for XL2 is 6.6975, exactly the maximum value for XL2 that will eliminate the undesirable state. If we had chosen to vary the position of the center of rotation of the lever, we would have obtained a landmark value at 61.62136. If we had chosen the height of the teeth of the wheel and varied YW2, we would have obtained a landmark value at 26.5248. By similar analysis, we can also determine what perturbations might allow us to add certain elements to the place vocabulary, such as to allow the contact between vertex (XL1, YL1) and the tip of the wheel, (XW2, YW2), or to again add the just removed state to the place vocabulary.

However, note that if we chose to make XL2 smaller than the landmark value, we wind up with a non-functional ratchet. This illustrates the basic weakness of the perturbation analysis: we do not know if a change past a landmark value causes other undesirable changes as well. This problem can be solved by enumeration analysis, discussed in the next section.

3.3 Enumeration Analysis

For a given parameter, the set of predicates in which it occurs defines a complete set of landmark values. A change in a parameter can influence the place vocabulary only if it passes one of the landmark values of the parameter. The landmark values can be ordered on the real axis, so that within each interval between two landmark values, the place vocabulary is the same for all values of the parameter. By computing the place vocabulary for a representative value of the parameter in each interval, we can find a complete list of all possible place vocabularies that can be achieved by varying the parameter. We call the computation of such a list an *enumeration analysis*.

In design problems, enumeration analysis is useful when the perturbation analysis has indicated which parameter is to be varied and it must be determined whether its variation also causes other, perhaps undesirable, changes. It is also necessary for the analysis of mechanisms with switches, such as the device shown in Figure 7. The position of the large lever varies the qualitative behavior of the device. The different dynamic behaviors allowed by the different positions can be represented by a graph of models ([PEN87,ADA87]). By enumeration analysis, we can find the distinct models in this graph and the conditions for their selection.

In this example, we vary the distance between the centers of rotation of the wheel and the ratchet lever, a variation of Y0-LEVER in Figure 4. The enumeration analysis of our program

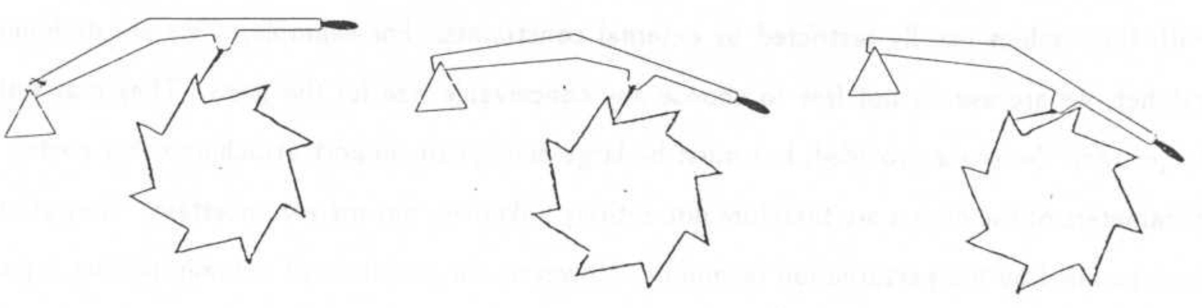

Figure 7: *The ratchet is either completely blocked, engaged or disengaged, depending on the position of the large lever.*

finds a total of 14 landmark values, at 51.0045, 52.479, 53.1375, 58.30565, 59.0998, 59.2725, 59.28925, 59.38427, 60.72458, 61.62136, 62.0, 67.7359 and 70.28427. The same landmark values exist for choices of Y0-LEVER below the wheel, but are not considered because only an interval of 30 to 80 was given for the binary searches that determine the landmark values.

Below the landmark value at 51.0045, there exist no legal configurations of the objects, and beyond the landmark value at 70.28427, no contact between the objects is possible. The most significant changes in behavior occur at 59.38427, where it becomes possible for the wheel to turn, and at 62.0, where the wheel can turn freely in both directions. Choices of Y0-LEVER in the interval between 59.38427 and 62.0 result in a functioning ratchet. The other landmark values reflect changes in possible contact relationships and breakups of places.

The enumeration analysis thus provides us all possible models that can be achieved by varying the position of the large lever in Figure 7. It also shows us a proper solution to the problem we investigated in the previous section: the undesirable state could be eliminated by choosing

Y0-LEVER at a value greater than 61.62136, and a choice between 61.62136 and 62.0 will give us the desired ratchet design.

In most applications of qualitative kinematics, there will be several unknown parameters, with their values usually restricted by external constraints. For example, if we are designing a ratchet, we are usually not free to choose any conceivable size for the parts. They may not be larger than the space provided, but must be large enough to support attaching other parts. The parameters of the design are therefore not entirely unknown, but merely uncertain. They are thus best handled by the perturbation technique. However, the problems of actually picking a proper parameter value, as well as those of variation of the modeling of a device seem to require some sort of enumeration analysis. A practical application must be based on a combination of the two techniques.

4 Conclusions

In this paper, we have shown two major results. First, we have proposed a plausible symbolic computation model for the place vocabulary theory of qualitative kinematics in mechanisms. Second, we have shown how this computation model allows us to deal with problems of mechanical design, uncertain information, and variable modelling.

Using the metric diagram model introduces an explicit distinction between dynamic parameters, which change as part of the operation of a device, and metric parameters of the shapes of the objects, which usually do not change in normal operation. The number of static metric parameters is usually far too large for their variation to be modeled explicitly in a device's envisionment. Modeling their variation by means of landmark values, as described in this paper, allows us to predict the effects of changing arbitrary parameters without having to construct a model for their dynamic variation. However, analyzing the simultaneous variation of several design parameters is very difficult in the metric diagram model.

The metric diagram model was inspired by observation of human problem-solving behavior. People are very good at stating the conditions under which a certain behavior is possible. For example, when we predict the effect of changes in the shapes of mechanism parts, we seem to use these conditions to determine landmark values where the variation changes the behavior. Similar to the prediction of the metric diagram model, we are very bad at predicting what happens when many parameters are changed simultaneously. The metric diagram thus closely models human behavior.

The metric diagram model is applicable not only to mechanism kinematics, but also to other domains where the values of metric parameters are important. In many domains, quantitative information is needed to make the qualitative analysis manageable and avoid intractable ambiguities. As our research focusses on spatial reasoning, we have not yet investigated other applications of the model.

4.1 Acknowledgements

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