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INTEGRATING COMMON-SENSE AND QUALITATIVE SIMULATION BY THE USE OF FUZZY SETS

Qiang SHEN and Roy LEITCH

Intelligent Automation Laboratory
Department of Electrical and Electronic Engineering
Heriot-Watt University
EDINBURGH
SCOTLAND

Tel: +31 225 6465
Fax: +31 225 1137
Telex: 727918 IOEHWU g
e-mail: sqi@ee.hw.ac.uk
rrl@ee.hw.ac.uk

ABSTRACT

This paper presents a methodology for integrating common-sense and qualitative simulation of physical systems by the use of Fuzzy Sets. This allows a semi-quantitative extension to qualitative simulation that provides three significant advantages over existing techniques. Firstly, it allows a more detailed description of physical variables, through an arbitrary, but finite, discretisation of the quantity space, thereby reducing qualitative ambiguity at source. The adoption of Fuzzy Sets also allows common-sense knowledge to be utilised in defining values through the use of graded membership. Secondly, the fuzzy quantity space allows more detailed description of functional relationships in that both strength and sign information can be represented by fuzzy relations held against two or multi-variables. Thirdly, the quantity space allows ordering information on rates of change to be used to compute temporal durations of system states and the possible transitions. Thus, an ordering of the evolution of the states and the temporal durations is obtained. This knowledge is used to develop effective temporal filters that significantly reduce the number of spurious behaviours. Experimental results with the method are presented and comparison with other recently proposed methods is made.

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1. Introduction

The research strategy for developing qualitative reasoners has as its main aim the development of a qualitative mathematics capable of yielding significant results consistent with the purpose of a system model from a minimum of information. However, the weak description of values, the lack of quantitative information, and no explicit temporal information often leads a qualitative analysis to ambiguities [10, 11, 12, 13, 14], resulting in the generation of many spurious behaviours of the system. Thus, an extension to current qualitative simulation methods based on well-developed mathematical techniques, to make them capable of capturing and using both sign and strength information on the magnitudes and rates of change of the system variables being modelled, would greatly enhance the effectiveness of qualitative reasoning for approaches where such information is available.

This paper presents a methodology for integrating common-sense and qualitative simulation of physical systems based on the use of *Fuzzy Sets* [5, 6, 19]. This gives an arbitrary, but finite, discretisation of the representation of system variables, thereby reducing the qualitative ambiguity at source. The method allows the subjective element of common-sense knowledge to be incorporated within a formal algorithm for generating behaviour from a structural model, hence, combining theoretical knowledge [9], in the form of modelling constraints, and empirical knowledge, in the interpretation of system values. Also, the use of Fuzzy Sets allows a rigorous approach to capturing uncertainty and provides a unifying framework from which other recently developed techniques can be generated as specializations. It synthesizes the previously disparate fields of qualitative simulation and Fuzzy Sets, both of which were introduced to cope with the complexities found in reasoning about the behaviour of physical systems. Further, such a representation provides ordering information on the rates of change of variables, allowing a temporal duration to be calculated. In so doing, it produces an estimate of how long the system remains within a particular state and/or when a state transition occurs. Thus, an ordering of the evolution of system states *and* the temporal durations is obtained. This makes qualitative simulation much more suitable for use within application systems, such as diagnosis, control, and training.

2. Representation of Qualitative Values and Constraints

The choice of representation of physical quantities plays a critical role in qualitative modelling. We exploit fuzzy qualitative values to provide a semi-quantitative extension to the quantity representation of both magnitude and derivative of a system variable. A *fuzzy* *qualitative value* of a system variable is a fuzzy number chosen from a subset of normal convex fuzzy numbers [5]. This subset is generated by an arbitrary but finite discretization of the underlying numeric range of the variable. A set consisting of all the elements of such subsets, for all the variables in the system, is called a *fuzzy qualitative space* and written as Q_F. The real number zero is required to belong to Q_F.

A computationally efficient way to characterise a fuzzy number is to use a parametric representation of its membership function. The membership distribution of a normal convex fuzzy number, expressed by a 4-tuple, $[a, b, \alpha, \beta]$, is defined as

$$\mu_A(x) = \begin{cases} 0 & x < a - \alpha \\ \alpha^{-1}(x - a + \alpha) & x \in [a - \alpha, a] \\ 1 & x \in [a, b] \\ \beta^{-1}(b + \beta - x) & x \in [b, b + \beta] \\ 0 & x > b + \beta. \end{cases}$$

The arithmetic operations for these fuzzy numbers are well-developed [2], and we adopt such a representation to form the Q_F.

As the fuzzy qualitative space Q_F is generated by a finite discretization of the underlying range of each system variable, the variable will, of course, have a finite number of associated qualitative values, and the whole underlying numerical range of interest can be covered by the fuzzy qualitative values. Also, if $x_1, x_2 \in R$ characterize "similar things" or stand for "similar properties" of a variable x, then the relevant qualitative values of x_1 and x_2 will be similar. Hence, we can translate a subset of a numeric range to one qualitative value according to what is needed in a particular modelling process.

Such a representation of qualitative values is more general than ordinary (crisp) interval representations, since it can represent not only the information stated by a well-determined real interval but also the knowledge embedded in the soft boundaries of the interval. This is achieved through the description of a gradual rather than an abrupt change in the degree of membership of which a physical quantity is mapped onto a particular qualitative value. Hence, it is closer to our common sense intuition of the description of a qualitative value. For instance, using the crisp intervals whose characteristic functions are $\mu_B(x)$, $B \in \{zero, small, medium, big\}$, shown in figure 1.b, 40 belongs to [40, 80] named *medium* while 39 does not. Whereas, in the fuzzy qualitative space shown in figure 1.a, 40 belongs to *medium* with a strength or membership equal to 1.0 and 39 belongs to *medium* with a membership 0.9. Notice, however, that 39 also belongs to the set *small* with a membership 0.05. This non-exclusivity of values is an important aspect of fuzzy sets and, again, is important in capturing our common sense intuition.

In common with conventional qualitative simulation, three classes of operations can be performed, i.e., algebraic, derivative, and function relational operations, within a fuzzy qualitative space Q_F. However, the semi-quantitative description of quantities presented by the fuzzy numbers allows a much more flexible method for capturing functional information, allowing strength as well as sign information to be represented if indeed such information is available.

An expression of the form

$$Q(z) = f(Q(x), Q(y)), \qquad Q(x), Q(y), Q(z) \in Q_F,$$

is called a *relevant constraint* on the system variables x, y, and z; where x and y are called *constraining variables, z* is called a *constrained variable*. Both constraining and constrained variables take values from the fuzzy qualitative space Q_F. By a constrained variable, we do not mean that there exists a causality between the variable and the constraining variables in the constraint, but rather that the directionality of solving a fuzzy equation is fixed. In fact, if we know the values of the constraining variables, we can obtain the value which the constrained variable should take by simply performing the operation on the values of the constraining variables. However, we cannot, in general, find an unknown value of a constraining variable by solving the equation as we can do in solving numerical equations. Fortunately, by restricting Q_F to be finite and closed, we can use the constraints to test the consistency of the sets of possible solutions. The approximation principle [1, 19] is, therefore, used to ensure that Q_F remains closed.

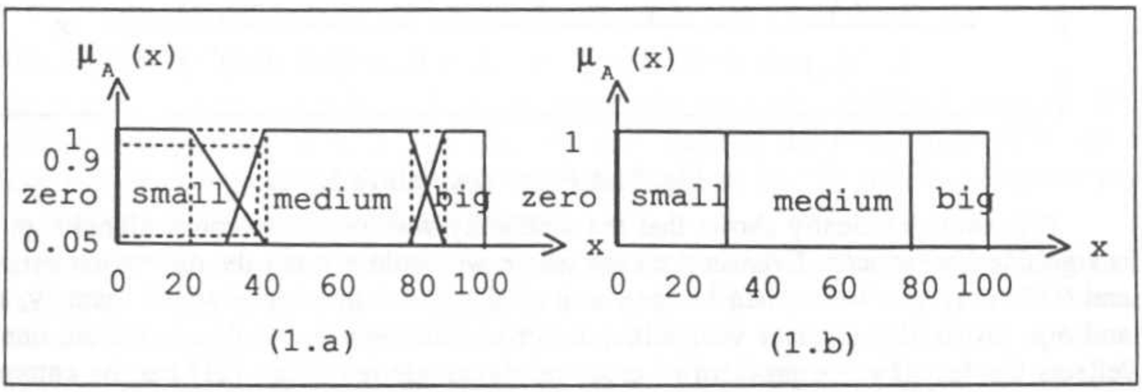

Fig. 1. *Comparison between Fuzzy and Crisp Quantity Spaces*

When f is an algebraic operator, the formulae for the associated arithmetic operations are those in the set of 4-tuple parametric fuzzy numbers [2]. For example, if $m = [a, b, \tau, \beta]$, $n = [c, d, \gamma, \delta]$, then,

$$m + n = [a+c, b+d, \tau+\gamma, \beta+\delta], \qquad m - n = [a-d, b-c, \tau+\delta, \beta+\gamma].$$

A simple example can illustrate how arithmetic constraints serve to limit a set of the possible values that a variable can take. Suppose that a system has three variables, x, y, z, and that they satisfy

$$Q(z) = Q(y) - Q(x).$$

For simplicity, it is assumed that these variables have the same underlying numeric range $[-1, 1]$ and take values from a fuzzy qualitative space, $Q_F = \{-big, -medium, -small, zero, small, medium, big\}$, where each qualitative value is defined by a 4-tuple parametric, normal and convex, fuzzy number $[a, b, \alpha, \beta]$ shown in figure 2. If we know, at the beginning, that the values of the constraining variables x and y are $Q(x) = small$ and $Q(y) = big$, but the constrained variable z might be any value $Q(z)$ in the Q_F, then, through the relevant constraint, we will have

$$Q(z) = [0.9, 1, 0.1, 0] - [0, 0.2, 0, 0.2] = [0.7, 1, 0.3, 0].$$

Since [0.7, 1, 0.3, 0] does not belong to Q_F, the approximation principle is used to ensure Q_F remains closed. The use of this principle consists of two major steps. First, by checking if an element in Q_F intersects with $\hat{A} = [0.7, 1, 0.3, 0]$, the set, ${}_{\hat{A}}Q_F = \{small, medium, big\}$, is generated [12, 13]. Then, distances [1, 13, 14] between $\hat{A}$

and A, $A\ \varepsilon_{\hat{A}}\ Q_F$, are evaluated and the results are: $D = \{1.53, 0.6, 0.63\}$. It follows that

$$big\ -\ small = medium.$$

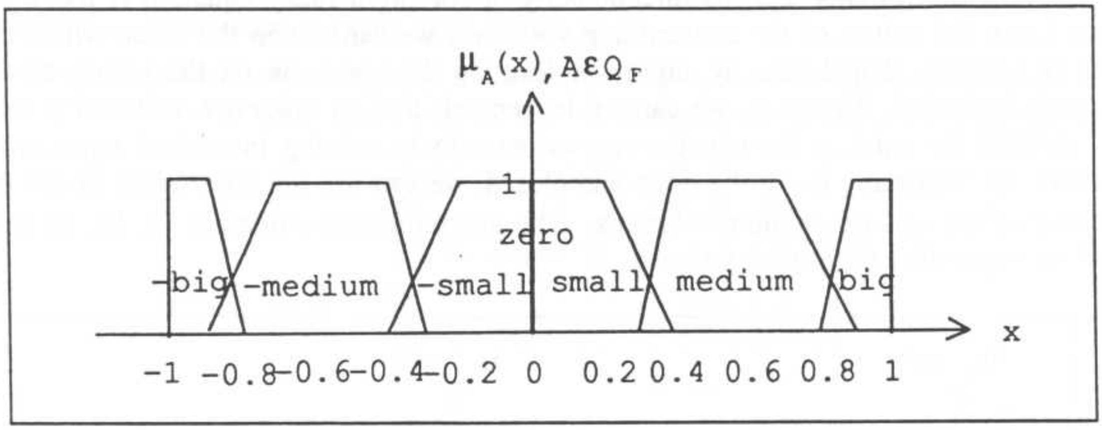

Fig. 2. *A Fuzzy Qualitative Space*

This example clearly shows that the ambiguity with regard to conventional sign algebra is significantly reduced. Even in the case where we could not tell the difference between 0.6 and 0.63, only two values can be the result of the calculation *big − small*, namely, *medium* and *big*. Also, this result is well-suited to our common sense calculus and, from one aspect, reflects the fact that the problem of order of magnitude reasoning [11] can be automatically solved by propagating qualitative values through fuzzy algebraic constraints.

As with any simulation language for dynamic systems, differential operation is essential for determining the transient behaviour of the system. Within the simulation it provides a memory operation that accounts for energy storage in a physical system. A derivative constraint, here, simply represents that the qualitative value of the magnitude of a variable y must be the same as that of the rate of change of a variable x. Thus, the derivative of a variable can take any value on a pre-specified subset of the Q_F, thereby, allowing ordering information on the rates of change to be represented.

Current qualitative simulation techniques model functional dependencies as monotonically increasing or decreasing functions. Also, corresponding values are defined and used to restrict certain values to be related. These functions allow partial knowledge of the relationship between variables to be represented. However, for many applications, these are often too weak as they do not allow any information on the strength of a relationship that is known to be utilised. The use of a fuzzy qualitative space allows qualitative function constraints to be represented as fuzzy relations [5]. This enables partial numerical information to be utilised within functional dependencies, thereby providing a uniform technique capable of representing the range of functional dependencies from complete numerical characterization of constraints to purely qualitative values.

Through fuzzy representation, for example, a rule with the form:

$$\text{if } x \text{ is } A_i, \text{ then if } y \text{ is } B_i, \text{ then } z \text{ is } C_i, \qquad A_i, B_i, C_i\ \varepsilon\ Q_F,$$

can be translated into a fuzzy relation [5, 19]

$$\mu_{L_i}(x, y, z) = \min\,(\mu_{A_i}(x), \mu_{B_i}(y), \mu_{C_i}(z)).$$

When a set of n fuzzy rules is available, the resulting relation L is the union of the n elementary fuzzy relations L_i, $i = 1, 2, \ldots, n$.

$$\mu_L(x, y, z) = \max_{i\ \varepsilon \{1,2,\ldots,n\}} \min(\mu_{A_i}(x), \mu_{B_i}(y), \mu_{C_i}(z)).$$

If the constraining variables x and y take fuzzy values A' and B', respectively, the fuzzy value C' of the constrained variable z is obtained by applying the compositional rule of inference [5, 19]

$$C' = (A' \times B') \circ L.$$

This allows constraining variables to be mapped onto the constrained variables in a semi-quantitative way and makes it possible to represent functional relationships held against multi-variables, making effective use of as much information about the functional dependencies as is available.

3. States and State Transitions

For a given system variable x, the *fuzzy qualitative state* of x within a duration ΔT_p, $QS(x, \Delta T_p)$, is a pair $<A, B>$, $A, B\ \varepsilon\ Q_F$, where A denotes the fuzzy magnitude and B the fuzzy rate of change of x. Following the terminology in [4, 17] ΔT_p is called the *persistence time* of x in this fuzzy qualitative state.

In general, both the extent of the fuzzy magnitude A and the fuzzy rate of change B will indicate the persistence time, which, in principle, should also be fuzzy [15]. However, based on the *resolution identity* [6] a (crisp) interval, within which a persistence time may lie, can be calculated by the following rules [12, 13], where $A = [p_1, p_2, p_3, p_4]$, B_α denotes the α–cut of B [5], and α is a degree of freedom:

(1) If $0\ \bar{\varepsilon}\ B_\alpha$, then $\Delta T_p\ \varepsilon\ \dfrac{p_2 - p_1 + (1-\alpha)(p_3 + p_4)}{|B|_\alpha}$, where $|B| = \begin{cases} B, & B >_\alpha 0, \\ -B, & B <_\alpha 0; \end{cases}$

(2) If $B = 0$, then $\Delta T_p = \infty$;

(3) If $0\ \varepsilon\ B_\alpha$, and $\alpha \neq 1$, then ΔT_p is not well-determined, any length of time may elapse.

In the rule (1) above, the partial order $<_\alpha$ is such defined that for $A, B\ \varepsilon\ Q_F$, $A \neq B$, we say A is α-less than B, $A <_\alpha B$, iff $a < b$, $a\ \varepsilon\ A_\alpha$, $b\ \varepsilon\ B_\alpha$ with A_α and B_α being the α-cuts of A and B, respectively.

For instance, if we know that, in the quantity space given in figure 2, a variable x stays in the state $<$*small*, *medium*$>$, and that $\alpha = 0.5$ is used to calculate the α-cuts, then, the persistence time of x staying at its current state lies in [0.125, 0.286]. Clearly, the persistence time obtained in this way presents a description of the amount of time within which a variable may remain in a particular state, although usually giving only a possible range.

A *state transition* happens when a system variable x changes from state $QS(x, \Delta T_{p1}) = <A_1, B_1>$ to $QS(x, \Delta T_{p2}) = <A_2, B_2>$, $A_1, A_2, B_1, B_2\ \varepsilon\ Q_F$. Four kinds of transitions are possible:

(1) $A_1 = A_2$, $B_1 = B_2$ --- *Null-transition;*

(2) $A_1 \neq A_2$, $B_1 = B_2$ --- *M-transition;*

(3) $A_1 = A_2$, $B_1 \neq B_2$ --- *R-transition;*

(4) $A_1 \neq A_2$, $B_1 \neq B_2$ --- *MR-transition.*

Like previous methods our qualitative simulation, integrating with common-sense, is based on the fundamental assumption that the variables of the physical system being modelled are continuously differentiable functions of time. Thus, possible state transitions, e.g., for a variable x to change from $QS(x, \Delta T_{p,1}) = <A_1, B_1>$ to $QS(x, \Delta T_{p,2}) = <A_2, B_2>$, can be represented by a set of rules -- called the *possible state transition rules*. Within these rules given below, A_1 and A_2 (or B_1 and B_2) are the α adjacent qualitative values of each other. We call A and B the α *adjacent qualitative values* of a variable iff there does not exist a C, which belongs to the subset that the variable takes values from, such that $A <_\alpha C <_\alpha B$ if $A <_\alpha B$.

The following are the possible state transition rules:

(1) If $B_1 >_\alpha 0$ ($B_1 <_\alpha 0$), then
 if $A_1 \varepsilon R$, then $A_2 >_\alpha A_1$ ($A_2 <_\alpha A_1$),
 else $A_2 \geq_\alpha A_1$ ($A_2 \leq_\alpha A_1$);

(2) If $B_1 = 0$, then
 if $A_2 \geq_\alpha A_1$ ($A_2 \leq_\alpha A_1$), then $B_2 >_\alpha 0$ ($B_2 <_\alpha 0$),
 if $A_2 = A_1$, then $B_2 \varepsilon \{0, X, Y\}$, where X and Y, are the α adjacent qualitative values of 0;

(3) If $A_2 = 0$ and $B_2 >_\alpha 0$ ($B_2 <_\alpha 0$), then $A_1 <_\alpha 0$ ($A_1 >_\alpha 0$).

The time that a variable x takes to transition from one qualitative state to another has been called the *arrival time* [17], written as ΔT_a. As with the persistence time, the arrival time should also be a fuzzy number. Consequently, to determine the arrival time we use a similar formula to that previously used for calculating the persistence time [13]:

(1) For a Null-transition or an R-transition, i.e., $<A, B_1>$ to $<A, B_2>$ with $B_1 = B_2$ or $B_1 \neq B_2$ respectively, $\Delta T_a = 0$;

(2) For an M-transition, $<A_1, B>$ to $<A_2, B>$, $A_1 = [p_1, p_2, p_3, p_4]$ and $A_2 = [q_1, q_2, q_3, q_4]$, the rules to calculate ΔT_a are

 (i) If $0 \not\varepsilon B_\alpha$, then $$\Delta T_a \ \varepsilon \ \frac{q_1 - p_2 + (\alpha - 1)(q_3 + p_4)}{|B|_\alpha};$$

 (ii) If $B = 0$, then $\Delta T_a = \infty$;

 (iii) If $0 \varepsilon B_\alpha$, and $\alpha \neq 1$, then ΔT_a is not well-determined, any length of time may elapse.

(3) For an MR-transition, $<A_1, B_1>$ to $<A_2, B_2>$, let $B = B_2 - B_1$, then, the rules to calculate ΔT_a are the same as those in case (2).

4. Filtering

From the possible state transition rules, a set of transitions from one qualitative state description to its possible successors, or next states, can be generated. Further restrictions on the possible successor states can be imposed by checking for consistency with the definition of the constraints and the consistence between constraints which share an argument -- called *constraint filtering*, and information on the rates of change of the system variables held as part of the fuzzy qualitative state -- called *temporal filtering*. In addition, other knowledge about the system being modelled may be used to produce so-called *global filtering* methods.

The Waltz filtering algorithm [16] is used to efficiently realise the constraint filtering. The algorithm entails an operation, called *refine*, on each relevant constraint and each argument of the constraint over and over again until the filtering rule produces no more changes [3]. In general, let $C : C(Q(x_i)), i = 1, 2, 3$, be a relevant constraint among three arguments: $Q(x_i)$, though $Q(x_i)$ may be equal to $Q(x_j), i \neq j$. And, let S_i be the set of qualitative values for the argument $Q(x_i)$. Then, the refine operation is defined by

$$refine(C, Q(x_i)) = \{A_i \in S_i \mid (A_j \in S_j, j = 1, 2, 3, j \neq i); C(A_k), k = 1, 2, 3\}.$$

Thereby, for each variable, the result of the fuzzy constraint filtering is a reduced set of its possible transitions.

The possible successor states survived from the constraint filtering are further checked by using the estimates of the persistence time and the arrival time. The following are the *temporal filtering* criteria:

(1) *Persistence time filtering rule:* for any two system variables, x and y, if the persistence time of $x >_\alpha$ (=, or $<_\alpha$) than the persistence time of y, then, if x is independent of y, x will change (if any) only after (will change at the same time as, or will change before) y does under the condition that prior to the current state, x and y existed for the same amount of time; otherwise, x and y will change at the same time, after the system has remained within current state for the persistence time of y (x).

(2) *Arrival time filtering rule:* for any two system variables, x and y, if the arrival time of x is $>_\alpha$ (=, or $<_\alpha$) than the arrival time of y, then, if x is independent of y, x will arrive (if any) only after (will arrive at the same time as, or will arrive before) y does at the next state under the conditions that prior to the current state x and y existed for the same amount of time and they have the same persistence time in the current state; otherwise, x and y should arrive the next state at the same time, after the system has spent the arrival time of y (x).

In the above filtering rules, a system variable x being independent of another variable y means that, in the physical structure of the system, there is neither any relevant constraint between x and y nor any constraints associating x and y through other system variables to force x to change together with y, or that, although there is a given constraint related x and y, x is controlled by an external object of the system such that it will not affected by the internal variable y.

These temporal filters are rather powerful because they utilize ordering information on the rates of change of system variables. In fact, the order relationships among fuzzy qualitative rates of change reflects additional knowledge about the higher-order (≥ 2) derivatives of the variables. In this way, temporal filtering is one of the most important extensions to current qualitative modelling techniques.

After constraint and temporal filtering, the complete state descriptions are generated. However, the set of next states may still contain a number of spurious behaviours and, therefore, be non-unique. A complete state description is only a mathematical assignment of a possible transition to each variable in the system without conflicting with the constraint and temporal restrictions. Some of these behaviours may be eliminated by knowledge of system theoretic properties of the real behaviour or other, often heuristic, information from external sources.

We utilise two global filtering methods in the simulation by checking for *no-change* and *repeating*. A no-change means that the new state is identical to its immediate predecessor and, therefore, can be deleted in the simulation. By a repeating state we mean the new state is identical to a state of its predecessors but not the immediate previous one, or identical to one of the other successors of its predecessors, which is not in the branch ended with it. When such a state is met the behaviour is marked as repeating and no further next state is generated from it.

In general, we conjecture that the extra information contained within the fuzzy quantity space, i.e., extended state representation, ordering information on rates of change, and strength information on functional dependencies, eliminates many spurious behaviours at source, and, therefore, obviates the need for extensive global filtering beyond the no-change and repeating filters. So far, our empirical results have justified this assertion.

5. Experimental Results

The fuzzy qualitative simulation method has been implemented in the Quintus Prolog language on a SUN 3 workstation with 16*M* bytes of *RAM*, the man-machine interface is realised by using the Prowindows graphics extension. The results given in the following are those obtained by directly running the program.

Consider a simple system, "a mass on a spring", as depicted in the first sub-window within figure 3. The figure is a screendump of the major interface window, automatically produced at the first stage of the simulation. This system consists of three variables: the displacement of the mass from the rest point of the spring, x, the velocity of the mass, v, and the acceleration of the mass, a.

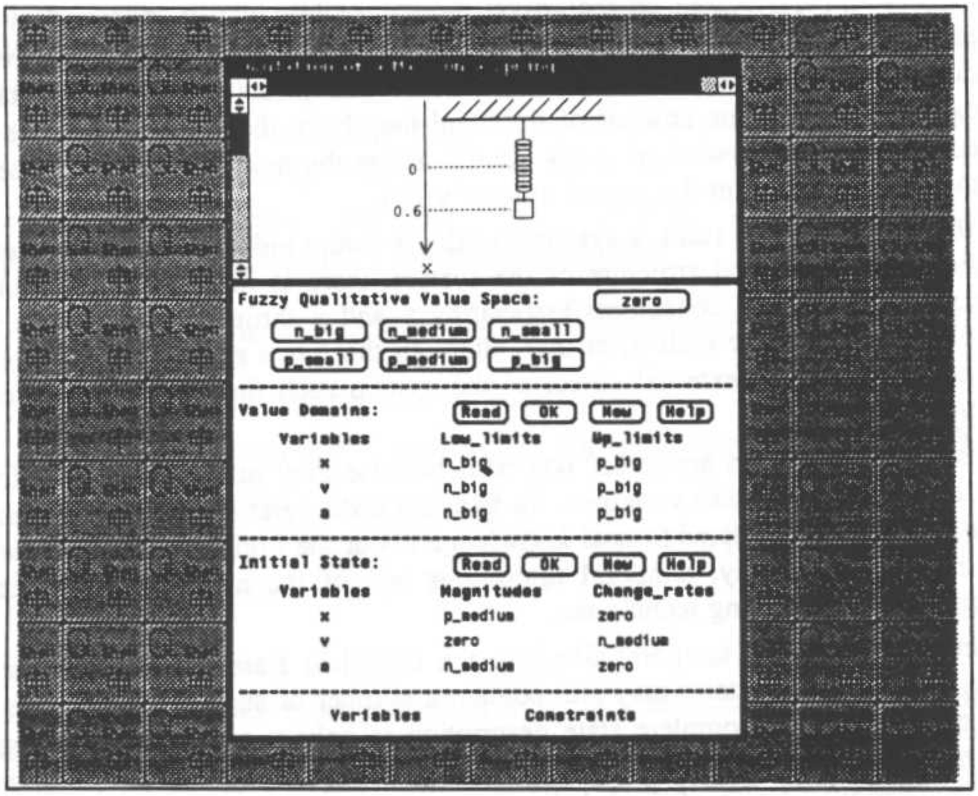

Fig. 3. *The Major Interface Window of the Simulation of a Mass on a Spring*

The following set of 4-tuple parametric fuzzy numbers was chosen to form the fuzzy quantity space:

$$Q_F = \{[-1, -0.7, 0, 0.1], [-0.6, -0.6, 0, 0], [-0.5, -0.1, 0.1, 0.1],$$
$$[0, 0, 0, 0], [0.1, 0.5, 0.1, 0.1], [0.6, 0.6, 0, 0], [0.7, 1, 0.1, 0]\},$$

with respective names given in the second sub-window in figure 3, which corresponds to the perceived meaning. For sake of notational simplicity, the Q_F is represented by

$$Q_F = \{-b, -0.6, -s, 0, s, 0.6, b\}.$$

The physical constraints in the system can be characterised as follows

$$deriv\ x = v, \qquad deriv\ v = a,$$

$$\begin{bmatrix} a \sim x & -b & -0.6 & -s & 0 & s & 0.6 & b \\ -b & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ -0.6 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ -s & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ s & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ b & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The first and second equations establish the ordinary derivative relationships holding amongst the distance, velocity, and acceleration of the mass. The third functional relation between a and x is a very weak form of Hooke's law represented as a degenerated fuzzy relation [12].

The ranges of the magnitudes and the initial conditions of all system variables are shown in figure 3 under the heading "Value Domains" and "Initial State", respectively. The initial state indicates that the mass is moved away from the equilibrium point, $x = 0$, to $x = 0.6 > 0$, and then let go. The simulation programme takes the description of the fuzzy system structure and this initial state as input, and starts the simulation of the system. For this particular system, we are not concerned about the possible behaviour of the rate of change of the variable a. Thus, behaviours only different in the possible values of the rate of change of a can be treated as one. Figure 4 shows one cycle of the unique behaviour for each system variable produced by the simulation. Within the experimental simulation we chose $\alpha = 0.5$, which is used to exhibit how a real number *typically* belongs to a fuzzy qualitative value [5, 12, 13].

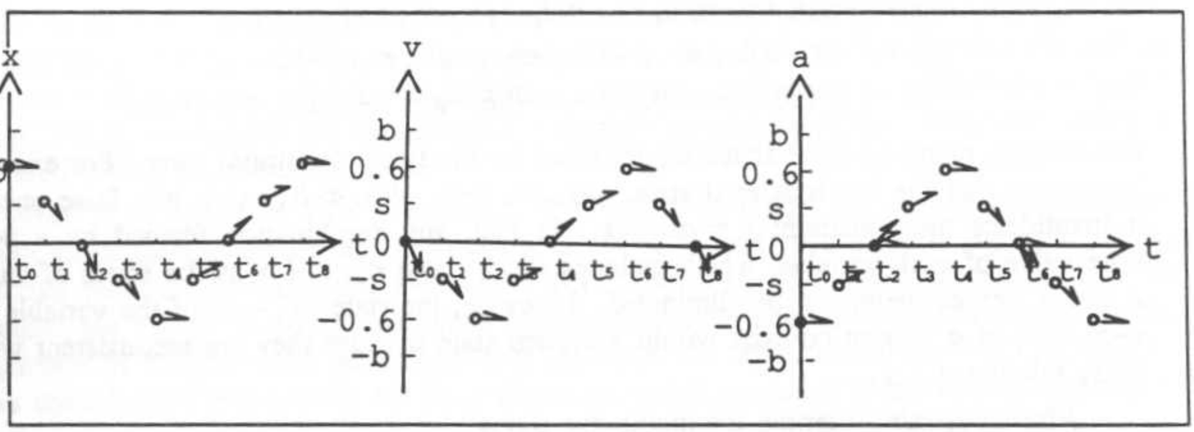

Fig. 4. *A Cycle of the Behaviour of "a Mass on a Spring" System*

It is very important to notice that, in figure 4, the temporal points (t_0, t_1, ..., t_8) satisfy the following expressions [12]:

$$t_0 = 0, \quad t_1 \ \varepsilon \ [0.09, 1], \quad t_2 \ \varepsilon \ [0.63, 2.5], \quad t_3 \ \varepsilon \ [0.72, 3.5] \quad t_4 \ \varepsilon \ [1.26, 5],$$

$$t_5 \ \varepsilon \ [1.35, 6], \quad t_6 \ \varepsilon \ [1.89, 7.5], \quad t_7 \ \varepsilon \ [1.98, 8.5], \quad t_8 \ \varepsilon \ [2.52, 10].$$

Thus, from the fuzzy simulation method, we can determine the time within which system state changes to a new state. In fact, a sequence, $\{\Delta T_{P_i}, \Delta T_{A_i} \mid i = 0, 1, ..., 7\}$, which represents the supremum supersets of the persistence times and arrival times of the system states, respectively, can be generated as follows:

$$\{0, \ [0.09, 1], \ [0.45, 0.5], \ [0.09, 1]\}_i, \qquad i = 1, 2, 3, 4.$$

Consider in two detailed samples. Firstly, the persistence time, determined by the rule of calculating persistence times, for the system (x, v, a) to remain within state $(<s, -s>, <-s, -s>, <-s, s>)$ lies in the range [0.45, 0.5]. For instance, the variable x stays within this state $<s, -s>$ for a duration:

$$\Delta T_{P_1}(x) \ \varepsilon \ \frac{0.5 - 0.1 + (1 - 0.5)(0.1 + 0.1)}{|[-0.5, -0.1, 0.1, 0.1]|_{0.5}} = [0.45, 0.5].$$

Secondly, following the rule to calculate the arrival time, given in section 3, the time cost for the system to transition from this state to its successor, say, $(<0, -0.6>, <-0.6, 0>, <0, s>)$ lies in the range [0.09, 1]. For example, the variable v changes from $<-s, -s>$ to $<-0.6, 0>$, taking the arrival time

$$\Delta T_{A_1}(v) \ \varepsilon \ \frac{-0.5 - (-0.6) + (0.5 - 1)(0.1 + 0)}{|[-0.5, -0.1, 0.1, 0.1]|_{0.5}} = [0.09, 1].$$

This highlights the advantage of our simulation method over previous qualitative simulation techniques.

As an example, here, we give three typical samples to show how the simulation works and produces a single behaviour of the system, (x, v, a), from the initial state to its successor, $(<s, -s>, <-s, -s>, <-s, s>)$.

From the possible state transition rules, each variable will have a respective set of possible successors of the initial state as follows:

$$x \ — \ <0.6, 0>, <0.6, s>, <0.6, -s>, <b, s>, <s, -s>;$$
$$v \ — \ <0, -0.6>, <-s, -0.6>, <-s, -s>, <-s, -b>;$$
$$a \ — \ <-0.6, 0>, <-0.6, s>, <-0.6, -s>, <-s, s>, <-b, -s>.$$

Fortunately, many of these states are removed by the fuzzy constraint filter. For example, it is obvious that the spurious next state, $(<b, s>, <-s, -b>, <-b, -s>)$, is a false one, since it invalidates the constraint $v = deriv \ x$. In fact, any combination formed by a possible state value of each variable, which includes $<b, s>$ and $<-s, -b>$ as the states of variables x and v, respectively, can be eliminated. Likewise, the state $<s, -s>$ of the variable x and $<-b, -s>$ of a cannot co-exist within a system state because they are inconsistent with the fuzzy relation $a \tilde{\ } x$.

After constraint filtering, the qualitative states

$$(<0.6, -s>, <-s, -0.6>, <-0.6, \ s>), \qquad (<0.6, -s>, <-s, -0.6>, <-0.6, \ 0>),$$
$$(<0.6, -s>, <-s, -0.6>, <-0.6, -s>), \qquad (<0.6, -s>, <-s, \ -s>, <-s, \ s>),$$

still remain as the possible next states of the initial one. However, they are removed by the temporal filters. For instance, before the system changes from the initial state to

(<0.6, $-s$>, <$-s$, -0.6>, <-0.6, s>), the variable v persists within the initial state for *no* time, and therefore, the variables x and a remain within the state for the same amount (0) of time, though they would persist there for ever if there were independent variables. While the arrival time for x or a to change is 0 (an R-transition), but the time cost for v to transition from <0, -0.6> to <$-s$, -0.6> is 0.08. Hence, such a transition invalidates the arrival filtering rule and is eliminated.

From the behaviour obtained above, it is clear that the mass will oscillate between the maximum amplitudes +0.6 and -0.6 for ever. This forms a sharp contrast with the results from previous qualitative reasoners, since the conventional qualitative simulation methods, e.g., QSim [7], produce three branches of possible behaviour after one oscillation and cannot describe the sequence of the maxima during the oscillation.

It is worth pointing out that, the simulation of a coupled tanks system has also been done, and again, a unique behaviour of the system was produced. However, because of lack of space, in this paper we give the experimental results from the simulation of "a mass on a spring" system only. More details are given in [13].

6. Conclusion

The work presented herein synthesizes two previously disparate research areas of qualitative simulation and Fuzzy Sets to produce an effective algorithm for the simulation of dynamic systems. The resulting algorithm shows three distinct advantages over previous methods and, in addition, provides a unifying framework from which other recent work on qualitative simulation can be interpreted. The first advantage stems directly from the use of the more detailed description of the quantity space to reduce the qualitative ambiguity. However, the use of graded membership within the fuzzy quantity space, albeit in a parametricised form, allows the subjective element of common-sense knowledge to be incorporated in the basic description of the quantity space. Secondly, the use of fuzzy relations and the fuzzy rule of compositional inference allows semi-quantitative information about the strength, as well as the sign, of functional relationships to be represented. This is an important practical advantage, in that, imprecise and partial numerical information about functional relationships is often known, although not in enough detail to develop a well-posed numerical model. Current techniques do not make use of this knowledge or do so in an ad hoc manner. The third advantage of the method is that by allowing the rate-of-change to be defined on the full fuzzy quantity space, relative rates of change can be represented and temporal durations computed. This results in powerful temporal filters that significantly reduce the number of spurious behaviours generated, and puts qualitative simulation on a firmer ground [18] by describing the behaviour of a dynamic system in its state sequence which is associated with a temporal sequence.

Work is on-going in two directions. The first is to refine and extend the algorithm through application to industrial size case studies. The second, to utilise the algorithm as the predictive function for candidate evaluation within model-based diagnostic systems [8]. In particular, the use of the fuzzy qualitative simulation within diagnostic systems, based on the comparison of the predicted behaviours with the observed behaviour of an evolving system, shows considerable promise for the early detection and diagnosis of faults as they are occurring.

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