

# Qualitative Modeling and Simulation of a Coupled Bioeconomic System\*

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## Abstract

The understanding and management of marine processes and resources is of great economic and social importance. Marine resources have been greatly exploited, which has led to a thriving but unadopted fishery infrastructure and brought about a number of negative environmental impacts. However, the main reason for the non-prosperous rationing of marine resources is the lack of knowledge about certain processes as well as the non-availability of adequate steering instruments. This paper addresses the lack of conceptualization and proposes a qualitative dynamical model approach for an improved decision support under the premise of vague knowledge. This approach makes it necessary to reduce and classify the large solution set supplied by a qualitative differential equation. The derivation of a focus graph, which displays the behavior of variables of interest, allows to extend previous approaches in bioeconomic modeling fundamentally and also illustrates the general impacts of an overcapitalization of fleets on a renewable marine resource.

## Introduction

The last decade has seen growing concern about the development of strategies allowing a sustainable resource utilization. Especially, marine resources are of major interest because they play an important role in the worldwide food security. However, the absence or failure of adequate policies has led more than ever to the situation, that we can observe an almost inevitably severe overexploitation and/or depletion of marine resources in certain regions of the world (FAO 2001; Caddy 1999). Mace (1996) states five fundamental problems that limit the ability to manage marine fish resources under a precautionary principle: Policy has suffered from a lack of long-term planning and uncoordinated regulatory frameworks. Institutions are unable to involve stakeholders; poor communication and a large set of institutional uncertainties affect commercial fisheries (Young

1998). Management goals like the expansion of offshore fishing fleets compromises other appropriate measurements. Finally, management suffers from inadequate methods for stock assessment, forecasting and modeling (Caddy 1999). Especially, the crisis of commercial fisheries perfectly illustrates the range of questions and problems encountered if we are confronted with a transsectoral question. Taking into account all aspects, it is the highly interrelated entirety of such problems combined with the lack of data which constitutes the complexity of the investigated system.

In this contribution, we apply the concept of qualitative differential equations (Kuipers 1994) to a specific question: the overexploitation of marine resources under consideration of the socioeconomic impacts, especially overcapitalization. This pattern is a prominent and ubiquitous effect of a non-sustainable utilization of natural resources and causes severe impacts on the marine ecosystem as well as on the socioeconomic inventory via a cascade of significant processes. We further address the problem of the usually large solution set of a qualitative differential equation (QDE) by using a graph theoretic approach to form equivalence classes of qualitative states and state transitions.

The paper is organized as follows: In the first part the general pattern of the overexploitation of a marine resource and some previous approaches are briefly introduced. The second part outlines extended model hypotheses and the implementation of the model by a QDE, as well as the efforts to reduce the solution set. The third part describes obtained results and our experiences in using QDEs in the domain of bioeconomics and environmental system analysis.

## Previous Bioeconomic Model Approaches

Using expert knowledge and information from field studies, an archetypical pattern of fishery under overcapitalization can be tentatively and inductively defined as follows (Mace 1996; Munro 1999; Hatcher 2000):

Overexploitation of fish stocks leads to high short-term benefits but in the long run, it threatens the marine resources and consequently, the economic basis of the employees in commercial fisheries.

- Incipiently we observe an intensive exploitation of marine fish stocks and a build-up of harvesting infrastructure.

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- Despite of the decline of the fishing grounds the harvesting capacities are maintained and fishery is, notwithstanding greatly increasing costs, enlarged.
- Fish stocks as food resource as well as source of earnings for the fishermen are rapidly decreasing.

A lot of prevailing models address this typical pattern under various points of view. The first (static) approach was introduced by Gordon (1954) and emphasizes the equilibrium case of stable fish stock and harvest. More recent examples try to combine economic as well as biological system parts in a very specific way (Pezzey, Roberts, & Urdal 2000; McDonald, Parslow, & Davidson 2001).

Early attempts utilizing the maximum principle of Pontryagin (1962) are the models of Clark et al. (1979), and McKelvey (1985; 1986), who first introduced applied capital in a bioeconomic model. It is the goal of the maximum principle to find functions for state variables (e.g., harvest, investment) that maximize an objective function (e.g., a profit function) whilst dynamic constraints have to be satisfied. The most recent model of McKelvey (1986) can only be solved numerically for singular sets of parameters. Due to undetermined parameters/functions and/or some additional simplifications, the latter models are of limited use, e.g. simplification to linear relations between variables lead to unrealistic consequences, such as the occurrence of pulse investment (investment with a rate of infinity, i.e., investment takes no time).

## Model Derivation and Extended Hypotheses

A more genteel way to overcome the above mentioned difficulties can be provided if one is using qualitative differential equations. The QDE approach takes into account, that integrated research efforts often suffers from sparse data and other uncertainties inherently based on some properties of the system, e.g., nondisclosure due to competition.

Our investigations run along three lines: First, despite of the qualitative model formulation we use the model of McKelvey (1986) as a basis. Second, this approach is extended in the following points:

- Common assumptions about linearities of functions are eased. This concerns the price of investment (to eliminate pulse investment), as well as earned profits depending on harvest, fish stock and applied capital.
- A straightforward *dynamical model* is developed, taking explicitly into account certain kinds of uncertainties on functions or parameters.

Third, these foundations are used to establish an analytic business-as-usual (BAU) model in order to investigate short-term rational choices of economically acting fishermen on the long-term system behavior. Briefly, the model assumptions can be described as follows:

It is assumed, that  $N$  factory units compete for a common property resource  $x$ , and behave in the same way. Capital  $C$  is regarded as a not explicitly operationalized aggregation of fishing gear and capacity, cannot be resold, and can only be reduced by depreciation. Accordingly, the time behavior

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$\begin{bmatrix} \pi_x \\ \pi_C \end{bmatrix} = \begin{bmatrix} + \\ + \end{bmatrix}$	$\begin{bmatrix} \pi_{xx} \\ \pi_{CC} \\ \pi_{hh} \end{bmatrix} = \begin{bmatrix} - \\ + \\ - \end{bmatrix}$
$\begin{bmatrix} c_I \\ h_x^* \\ h_C^* \end{bmatrix} = \begin{bmatrix} + \\ + \\ + \end{bmatrix}$	$c_{II} > 0, const.$
$x < xmsy \Rightarrow \begin{bmatrix} R_x \\ R_x \end{bmatrix} = \begin{bmatrix} + \\ - \end{bmatrix}$	$\begin{bmatrix} \pi_{CC}^* \\ \pi_{Cx}^* \end{bmatrix} = \begin{bmatrix} + \\ + \end{bmatrix}$

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Figure 1: Assumptions on monotony and convexity of the functions in the model. For sake of readability, indices denote partial derivatives and brackets  $[\ ]$  map a real number to its sign.

of the capital is modeled by the differential equation  $\dot{C} = I - \delta \cdot C$ , where  $\delta$  refers to the depreciation rate and  $I \geq 0$  holds for the investment.

The profit  $\pi$  of each firm depends on harvest  $h$ , utilized capital  $C$  and fish stock  $x$ , which is combined to the so-called profit function  $\pi(h, C, x)$ . Due to economic reasons ('law of diminishing returns', economics of scale, cmp. Eisenack and Kropp (2001)), some assumptions about the signs of the derivatives of  $\pi$  can be made (fig. 1). Profits increase concave in harvest and fish stock, but convex in capital. The efficiency of capital increases, if high-technology equipment (e.g., advanced electronic sonar and radar techniques) only operates efficiently when particular preconditions are given, e.g., longliner or freezer vessel fleets. Investment costs  $c(I)$  are implemented as a function in the amount of investment  $I$  (for assumptions about monotony see table 1). Its second derivative  $c_{II}$  is assumed to be constant and always positive<sup>1</sup>. This reflects the situation that a fast or a large investment induces higher costs.

The dynamics of a stock  $x$  can be expressed by  $\dot{x} = R(x) - N \cdot h$ , where  $R$  refers to the recruitment. The function has its maximum  $rmsy$  (maximum sustainable yield) at  $xmsy$ . For a totally exploited stock, the recruitment vanishes. If the stock resides above  $xmsy$ ,  $R$  is decreasing due to intraspecific competition and approaches zero if the natural carrying capacity  $q$  is attained.

The economically optimal long-term strategy for commercial fisheries can be achieved, if each factory tries to maximize the present value profit

$$\max_{h, I} \Pi(h, I) = \int_J e^{-r \cdot t} (\pi(h, C, x) - c(I)) dt, \quad (1)$$

which has to be distinguished from the short-term profit  $\pi$ . The parameter  $r$  refers to the discount rate and  $J$  represents a planning interval. This approach defines a dynamical optimization problem which can be embraced by the maximum

<sup>1</sup>For sake of readability  $X_z$  labels the first and  $X_{zz}$  the second derivative of a function  $X$  with respect to  $z$ .

principle of (Pontryagin 1962) and which finally leads to a system of ODEs

$$\dot{x} = R(x) - N \cdot h, \quad (2)$$

$$\dot{C} = I - \delta \cdot C, \quad (3)$$

$$c_{II} \dot{I} = (r + \delta) \cdot c_I - \pi_C, \quad (4)$$

$$\dot{\pi}_h = (r - R_x) \pi_h + N \pi_x. \quad (5)$$

In general, eq. (2)–(5) model an economical optimal acting fisherman. He is interested in a precautionary resource utilization because he needs the natural resource as an economical basis in the future. However, it is our major goal to concentrate on typical pattern observable in an unadopted and overcapitalized fish industry. Thus, the above ODE system has to be modified in some details. It is assumed that each factory selects (i) a well-based and long-term oriented investment decision and (ii) a short-term fishing perspective (Asche 1999).

Because marine fishing grounds are common property resources, it seems - from various reasons - to be a better decision for a fishing company to leave the optimal long-term harvest path and to increase the catch: (i) It assumes that the own behavior has no or only a neglectable influence on the stock. (ii) It also supposes that other companies are ordering their vessels to do whatever it take to fill their holds (e.g., also *pirate fishing*)<sup>2</sup>. And (iii) as an direct outcome of such a competition situation a single fisherman assumes that he himself achieves a similar profit if he imitates the behavior of the others. Regarding to eq. (5) this signifies that an individual firm assumes  $\pi_x$  to be zero.

Taking into account this kind of prisoner's dilemma (Hardin 1968), an entrepreneur tries to obtain a yield  $h$  so that the short-term profit  $\pi(h, C, x)$  becomes maximal at each time. The optimum of  $\pi$  is given by the condition  $\pi_h(h^*, C, x) = 0$  for  $h^*$ . The solution provides the optimal harvest function  $h^*(C, x)$  which leads to the so-called optimal profit function

$$\pi^*(C, x) := \pi(h^*(C, x), C, x). \quad (6)$$

This equation models the short-term attitudes of commercial fisheries. For this reasons, in eq. (4)  $\pi_C$  can be replaced by the function for the marginal profit in the short-term optimum,  $\pi_C^*$ , and  $h$  in eq. (5) by the optimal harvest function  $h^*$ . This results in the following ODE system:

$$\dot{x} = R(x) - N \cdot h^*(C, x), \quad (7)$$

$$\dot{C} = I - \delta \cdot C, \quad (8)$$

$$\dot{I} = \frac{r + \delta}{c_{II}} \cdot c_I(I) - \frac{1}{c_{II}} \cdot \pi_C^*(C, x). \quad (9)$$

It can be shown from the extended model assumptions, that  $h^*$  and  $\pi_C^*$  increase in  $C$  and  $x$  (fig. 1). Equations (7)–(9) are used as basis for a qualitative model approach. Again, they can only be solved numerically if explicit assumptions

<sup>2</sup>This is equivalent to a situation where the individual actor ignore the costs that his own decisions impose on others.

```
(quantity-spaces
 (stock (0 xmsy q inf))
 (capital (0 inf))
 (invest (0 inf))
 (recruit (minf 0 rmsy))
 (yield (0 ymsy inf))
 (mprofit (0 inf))
 (dstk (minf 0 inf))
 (dcap (minf 0 inf))
 (dinv (minf 0 inf)))
(constraints
 ((D/DT stock dstk))
 ((D/DT capital dcap))
 ((D/DT invest dinv))
 ((ADD dcap capital invest))
 ((ADD dstk yield recruit)
 (0 ymsy rmsy))
 ((ADD mprofit dinv invest))
 ((U- stock recruit (xmsy rmsy))
 (0 0) (q 0))
 ((MULT stock capital yield))
 ((MULT stock capital mprofit))
 ((MULT capital mprofit yield)))
```

Figure 2: Qualitative variables and constraints of the bioeconomic model as QSIM code. This QDE describes all possible behaviors of the ODE formulated in eq. (7)–(9).

on functions and parameters are made. Due to uncertainties in bioeconomic systems, this would be more an ad-hoc decision instead of a systematic view. Moreover, the outcome is restricted to one special case and we are not able to analyze the patterns of overexploitation/overcapitalization in general.

## Qualitative Model Implementation

In order to realize a qualitative model of the bioeconomic interrelations, we made use of the QSIM simulation package which is distributed by the University of Texas at Austin (for the detailed definitions of the various concepts and their mathematical foundation, see Kuipers (1994)). Henceforth, we consider the entities involved in the model as qualitative variables, which are characterized in terms of *quantity spaces* (fig. 2). Here, **stock** refers to the size of the fish stock  $x$ , **capital** to the utilized capital  $C$ , **invest** to the investment  $I$ , **recruit** to the recruitment  $R$ , **yield** to the harvest  $h$  and **mprofit** labels the marginal profit  $\pi_C^*$ . Additionally, qualitative variables for the derivatives of **stock**, **capital**, and **invest** are introduced. The landmark  $ymsy$  denotes the maximum sustainable yield and  $rmsy$  the maximal recruitment. The size of the fish stock, where maximal recruitment takes place (and maximum sustainable yield is possible) is labeled by  $xmsy$ . The landmark  $q$  indicates the carrying capacity of the ecosystem.

Using certain constraints to abstract the interactions between the variables of the analytic model (for details see, Kuipers (1994)), we define the qualitative bioeconomic model as presented in fig. 2. The recruitment is modeled

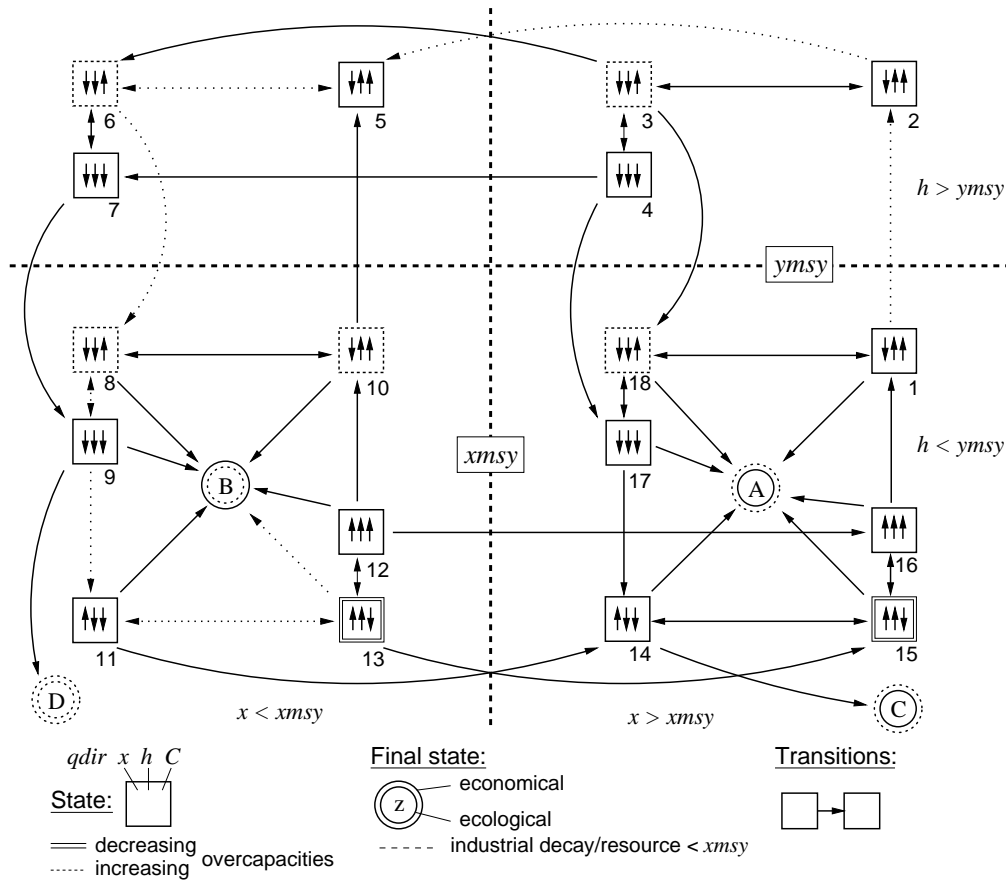


Figure 3: Focus graph of the qualitative bioeconomic model for the variables stock, yield and capital. The time development of the whaling industry is reconstructed by the path through vertices 1–2–5–6–8–9–11–13, indicated by dotted arrows.

by a constraint for so-called concave-down U-shaped functions. Strictly monotonic functions are replaced by their argument, if they vanish at zero. The total yield  $N \cdot h^*(C, x)$  is qualitative equivalent to  $h^*(C, x)$ , for example, and  $c_I(I)$  is equivalent to  $I$ . The optimal harvest function is abstracted by the usage of the multiplicative constraint *MULT*. This constraint generalizes so-called Cobb-Douglas functions:  $Z = p \cdot X^\alpha \cdot Y^\beta$  if  $p, \alpha, \beta, X, Y > 0$ . Functions of this type are widely used in economic models. The same holds for the marginal profit  $\pi_C$ . All these abstractions are consistent with the required properties of the functions (7)–(9) (cmp. fig. 1). The other constraints just restate the ODEs for the variables  $x$ ,  $C$  and  $I$ . The multiplicative constraint for capital, marginal profit and yield is motivated by some further (weak) assumptions on the functional form of  $h^*$  and  $\pi^*$ . This also leads to a reduced set of solutions (see next section).

During the qualitative simulation runs, we check the status of the investments. If  $\text{invest}(t) = \langle 0, \downarrow \rangle$  holds, we have implemented a transition function which models the crossover from the BAU-QDE to a DECLINE-QDE. This refers to a situation, where irreversibility of investment becomes the dominating effect. In such a situation it would be optimal to disinvest, but by assumption capital can be only

reduced by depreciation. Therefore, DECLINE-QDE fixes investment to zero and for the other variables, the resulting simplifications of BAU-QDE are used.

### Reduction of the Solution Set

Starting with the initial conditions

$$\begin{aligned} \text{Stock} &= \langle q, \downarrow \rangle, \\ \text{Capital} &= \langle (0, inf), \uparrow \rangle, \\ \text{Invest} &= \langle (0, inf), \uparrow \rangle, \\ \text{Yield} &= \langle (0, xmsy), \uparrow \rangle, \end{aligned} \quad (10)$$

QSIM computes a set of 347 different qualitative trajectories as potential solutions, even when differences in the second derivatives of  $x$  and  $C$  are ignored (in order to reduce so-called “chatter”). Consequently, a detailed investigation of the solution set is necessary in order to obtain deeper insights on the dynamics of the bioeconomic system. This analysis shows that many differences in the behaviors are characterized by (i) the time points at which certain variables approach a given landmark (*occurrence branching*, see Clancy and Kuipers (1993) for details) or (ii) the ambiguities of qualitative addition (e.g.  $[+] + [-] = [?]$ , see Williams (1991)). However, the current version of the QSIM package generates the solution set as a tree, which branches

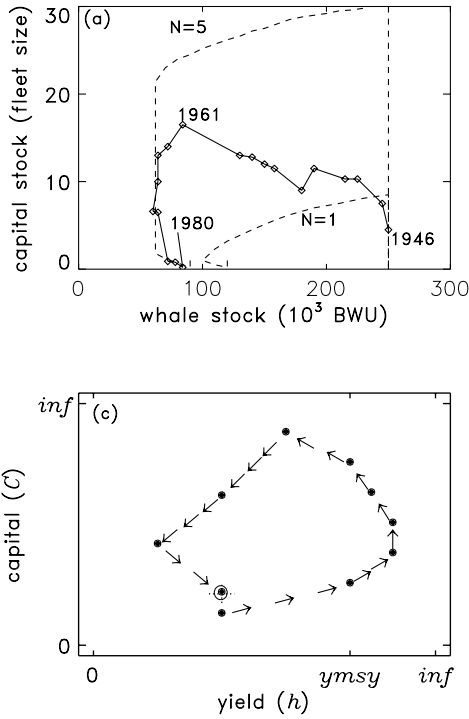


Figure 4: (a) Comparison of the time development of the capital stock versus blue whale stock (BWU = blue whale units, solid line) and model results obtained by the usage of the McKelvey (1986) model (dashed line,  $N$  refers to the number of firms). Phase plots for the qualitative reconstruction by means of the focus graph (cmp. fig. 3): (b) capital versus stock and (c) capital versus yield.

at each time step where different possible successors of a system state are consistent with the constraints defining the QDE. On the other hand many solutions only differ in the dynamics of some auxiliary variables. In such a situation Mallory (1996) suggests to focus only on important variables. This can be performed by generation of so-called *variable views* (Clancy 1997), which are a projection of the entire solution tree on the state space of these variables. The variable view for stock, capital and yield supplies a tree with 110 behaviors. However, a closer look reveals that the variable view is not a tree in the graph theoretic sense. This is due to the occurrence of so-called cross-edge transitions between different behaviors. They are inevitable, because in the projection different solutions can be identical in several sections (not only in a sequence beginning with the initial state).

Therefore, we propose to use *focus graphs*. A focus graph is a digraph, where each vertex represents a class of qualitative states consistent with the underlying QDE, and which are equivalent in the focus variables. For further simplification only states for time intervals and equilibrium states are regarded. An edge from a vertex  $a$  to a vertex  $b$  is introduced if there exists at least one solution with a qualitative time interval state in the class represented by  $a$ , and for which the next time interval state is in  $b$ . However, the focus graph can be further simplified by omitting vertices and edges corresponding to non-generic states. These are states where several variables become stable or approach a landmark. The resulting graph is shown in fig. 3.

## Results and Discussion

Figure 4 shows the development of the whaling industry in the last century, which can also be characterized by a path in the focus graph (dotted edges in fig. 3). In general, the QDE model allows a qualitative reconstruction of measured data.

Additionally, the graph can be used as an aid for the development of management options for the bioeconomic system, since critical points in its evolution can be identified. Examples are the vertices 9 and 14 (fig. 3). Here, the system can evolve to vertices D or C, respectively, which are economic and/or environmental precarious situations. At vertex C the fishing industry declines; at vertex D the fish population vanishes as well as the industry. At least at these points management measures should be imposed. Another critical factor is the occurrence of over-capacities, which is a major motivation to introduce applied capital in a bioeconomic model. States with increasing or decreasing over capacities are marked in the focus graph (over capacities are assumed to increase, if harvest decreases while further capital is applied). If the system starts from a situation with high stocks and low but increasing capital and yield (vertex 1 in fig. 3), over capacities are inevitable. There exists no path from vertex 1 that avoids transitions to vertices 3, 6 and 18, except a direct transition to the stable state A. This state is economically uninteresting, because a transition to the stable state B would allow additional yield, and thus it is a candidate for spurious behavior (see model assumptions).

This pessimistic analysis of the system reveals the potential risks of missing management strategies: (i) an irreversible decline of the resource and (ii) high costs (or

even a decline) in the industry induced by over capacities. Also further questions on the management of a marine resource are imposed. If management only adjusts parameters (e.g., taxes, investment costs), only the likelihood of certain transitions changes, but not the general structure of focus graph. Therefore, further investigations on management options should concentrate on structural changes in the model in order to enable development paths which avoid over capacities.

## Conclusion

In this contribution, we have tried to present a qualitative approach to a complex and prominent question: the socio-economic impacts of an over-developed fish industry on marine resources. This enables a more systematic view on the causes and effects of overcapitalization. Models of this type are a useful platform to discuss general mechanisms of a system and potential development paths, especially if uncertainties have to be faced. Hence, they are valuable for political guidance strategies since also decision makers normally do not dispose of the complete information about the system to manage.

However, solution trees strongly increase if larger models have to be solved. Thus, there is an urgent need for the development of techniques for an automatic classification of the solutions supplied by a QDE. The representation of the solution set as a tree has disadvantages, because it is rather difficult to compare the solutions. We have tried to overcome this by using the focus graph method. Further work will concentrate on the development of techniques which allow to generate such graphs automatically. Beside these difficulties we believe that qualitative models greatly facilitate a progress in scientific reasoning about environmental and economic systems.

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