Qualitative Systems Identification for Linear Time Invariant Dynamic Systems

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Abstract

The problem of Systems Identification starts with a time-series of observed data and tries to determine the simplest model capable of exhibiting the observed behavior. This optimization problem searches the model from a space of possible models. In traditional methods, the search space is the set of numerical values to be assigned to parameters. In our approach we are constrained, and therefore limit the search space, to Linear Time-Invariant models. In this paper, we present the theory and algorithms to perform Qualitative Systems Identification for Linear Time Invariant Dynamic Systems. The methods described here are based on successive elimination of the components of the system's response. Sinusoidals of high frequencies are eliminated first, then their carrying waves. We continue with the process until we obtain a non-oscillatory carrier. At that point, we determine the order of the carrier. This procedure allows us to determine how many sinusoidal components, and how many exponential components are found in the impulse response of the system under study. The number of components determines the order of the system. The paper is composed of two important parts, the statement of some mathematical properties of the responses of Linear Time Invariant Dynamic Systems, and the proposal of a set of filters that allows us to implement the recognition algorithm.

Introduction

The problem of Systems Identification starts with a time-series of observed data and tries to determine the simplest model capable of exhibiting the observed behavior. This optimization problem searches the model from a space of possible models. In traditional methods, the process of structural identification has received less attention than the parametrization of the model. In most of cases the structural estimation is not generally made. The user selects the model of a defined group of possibilities. The choice of an appropiate model structure is most crucial for a successful identification application. This choice must be based both on an understanding of the identification procedure and on insights and knowledge about the system to be identified. An alternating way to infer a suitable structure, guided by system knowledge and the collected data set, is presented in this paper. In our approach we limit the search space to Linear Time-Invariant models.

The problem we are assessing in this paper is that of Qualitative Systems Identification for Linear Time Invariant (LTI) Dynamic Systems. This kind of systems can be represented by Linear ordinary Differential Equations with Constant Coefficients. Although this problem may seem over-constrained, there are many important problems in engineering and physics that can be expressed in mathematical terms by this kind of differential equations. We could even assess time-varying or even non-linear systems, if we consider them as piece-wise decomposed by linear approximations of the original systems. So, this is a limited yet interesting domain to start with.

The Systems Identification process can be decomposed in two steps: the first step called structural (or qualitative) identification, involves determining the qualitative features of the mechanism, i.e. the qualitative form of the system inside the black box (Kay, Rinner, & Kuipers 2000; Bradley & Stolle 1996; Bradley, Easley, & Stolle 2001; Bellazzi, Guglielmann, & Ironi 1999); once we know the nature of the mechanism, in the second step we proceed to determine the numerical value of the parameters of the model determined in the first step.

This second part can be done by any optimization process (e.g. minimum square error (Ljung 1987; P. 1989), genetic algorithms (Goldberg 1998; Haupt & Haupt 1998; Hunt 1993; Pastor 2000; Zhang Zibo 1987), etc.). Some algorithms use an optimization process to determine both faces at the same time (e.g. genetic algorithms (Downing *et al.* 1996; Kristinsson & Dumont 1992)).

Section 2 (Linear Time-Invariant Dynamic System) describes some properties of Linear Time-Invariant Dynamic Systems. These properties will be used in section 3 (QSI Algorith) to define an algorithm capable of determining the structure of such a system. The process of determination of the structure of a system is performed by repeated elimination of components of the impulse response of the system. Finally, in section 6 (Conclusions), we conclude our work and present the limitations and future work.

Linear Time-Invariant Dynamic Systems

As mentioned before, linear ODEs with constant coefficients are the most studied kind of differential equations; they have complete analytical solutions. Also, there is a good number of problems that can be described by these kind of equations, and more complicated cases can be reduced to one or several of these equations.

In this section, the theory of solution of linear ODEs with constant coefficients, and a qualitative interpretation is presented. The facts presented in this section are the basis for the framework developed and presented in the next section.

Consider the homogeneous nth-order ODE given in Equation 1.

$$a_n \frac{d^n x(t)}{dt^n} + a_{n-1} \frac{d^{n-1} x(t)}{dt^{n-1}} + \cdots + a_1 \frac{dx(t)}{dt} + a_0 x(t) = 0 \quad (1)$$

where $a_n, \ldots, a_0$ are real constants.

The solution represents the behavior of the system in response to the forcing function and initial conditions $X(0), X'(0), ..., X^{(n)}(0)$.

It is quite natural to think of an exponential function as a candidate solution to that equation. Substituting $x = e^{rt}$ in Equation 1 and factoring e^{rt} yields Equation 2.

$$e^{rt}(a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r + a_0) = e^{rt} Z(r) \quad (2)$$

$x(t)$ is a solution of Equation 2, for those values of r that satisfy the characteristic equation. i.e. the roots of polynomial $Z(r)$. The general solution of Equation 1 is of the form:

$$x(t) = c_1 e^{r_1 t} + \cdots + c_n e^{r_n t} \quad (3)$$

We can see that the natural response of an nth-order system, is the sum of n exponential terms. One for each root of the characteristic equation of the ODE. If it has positive roots, the system is unstable, otherwise, it is stable. If the roots of the characteristic equation of the ODE are all real, the system's response is non-cyclic. If the characteristic equation has complex roots, they come in conjugate pairs, in which case, the general solution is still of the form of Equation 3, only that each pair of complex roots ($r \pm i\omega$) becomes an exponential sinusoidal function. This property is known as Euler's identity.

$$C_1 e^{(r+i\omega)t} + C_2 e^{(r-i\omega)t} = e^{rt}(A_1 \cos \omega t + A_2 \sin \omega t) \quad (4)$$

So, if we restrict the kind of systems we are to analyze to those that can be expressed by an nth-order ordinary differential equation with constant coefficients, we know the kind of responses we are to get. We can express the behavior of a system in terms of the exponential and sinusoidal components in the response. We define

$$E_m(t) = \sum_{1 \le i \le m} a_i e^{r_i t} \quad (5)$$

as a summation with at most n exponential terms, and

$$ES_m(t) = \sum_{1 \le i \le m} a_i e^{r_i t} \sin \omega_i t \quad (6)$$

as a summation of exponentially decreasing sinusoidal functions. Note that we are not interested in giving analytical solutions to the differential equation, but a qualitative description of its behaviors. That is, all possible different qualitative forms of the solution to Equation 1.

Theorem 1 *Given a system of order n, the response can be expressed as in Equation 7.*

$$X(t) = E_{n_1}(t) + ES_{n_2}(t) \quad (7)$$

where $n_1 + 2n_2 = n$. This result is evident from Equation 3, Equation 4, and the definitions of Equations 5 and 6.

We see that if the second term of Equation 7 does not exist, the response will be acyclic. Otherwise, it is a sinusoidal wave, where $E_{n_1}(t)$ represents its axis or attractor, and $ES_{n_2}(t)$ its sinusoidal components.

Note that if we include a forcing function, the system's response would be decomposed into Natural (the solution to the homogeneous equation) and Forced responses. If we restrict the forcing functions to be of the form $e^{\alpha t} \sin \beta t$ (i.e. constant, exponential, or sinusoidal), the forced response always has the same qualitative form as the forcing function (Boyce & DiPrima 1969). This would preserve the qualitative form of the response, and only add one more exponential or sinusoidal term to the response.

Let us analyze the qualitative form of the responses, as expressed by eq. 7. This qualitative form can be derived from the qualitative form of its exponential and sinusoidal components.

Exponential Components

The qualitative behavior of the exponential part of the response is characterized by Theorem 2.

Theorem 2 $X(t) = E_n(t) = \sum_{1 \le i \le n} a_i e^{r_i t}$ *exhibits at most* n *extrema (maxima or minima), including the one when* $t \to \infty$.

We will assume, without loss of generality, that $r_1 > r_2 > \ldots > r_n$. Theorem 2 is equivalent to saying that the derivative $X'(t)$ has at most n different zeroes. That is,

$$\begin{aligned} X'(t) &= -r_1 a_1 e^{-r_1 t} - \ldots - r_n a_n e^{-r_n t} = 0 \\ &= r_1 a_1 e^{-r_1 t} + \ldots + r_n a_n e^{-r_n t} = 0 \end{aligned} \quad (8)$$

Performing the variable change $z = e^{-t}$, Equation 8 becomes

$$-X'(t) = r_1 a_1 z^{r_1} + \ldots + r_n a_n z^{r_n} = 0 \quad (9)$$

Equation 9 is a polynomial in z of degree r_1. Based on Descartes' theorem (Kurosch 1977; Rees, Sparks, & Reeds 1991), the number of roots of a polynomial are determined by the sign changes in the polynomial coefficients. Since we have n terms in polynomial 8, we can have at most $n-1$ changes of sign, and therefore $n-1$ roots. There is one more root placed where the function becomes zero, and that is when $t \to \infty$.

Given we have at most n roots for Equation 8, we can therefore have at most n extrema for $E_n(t)$.

Sinusoidal Components

As mentioned in (Flores & Farley 1995), if the frequencies of the sinusoidal components of Equation 7 are equal, their shapes are reduced to one. If their frequencies are different, they can be seen as the faster sinusoidal mounted on the slower one. If two frequencies of the sinusoidal components are very close together, the resulting wave beats.

The results presented in this section fully characterizes all possible responses of a LTI Dynamic System. In the next section we will describe how to use these results to produce a framework for performing Systems Identification at the qualitative level.

QSI Algorithm

The identification algorithm presented in this section is based on the fact (see Equation 7) that the response of a LTI system can be decomposed in a sumation of exponential terms. If some of those exponential terms are complex, in which case they are conjugate complex pairs, each pair forms a sinusoidal. If we can think of an algorithm capable of separating the terms of Equation 7 we can determine the structure or qualitative form of the system exhibiting the observed behavior. Separating the terms of the system's response can be performed by a filtering process.

Assume the observation of the system includes a number of samples large and frequent enough to show all details of the system's behavior. If this assumption does not hold, we can miss important events that would not let us identify the system properly.

Also, assume the terms of Equation 7 are sorted in order of increasing frequencies (i.e. $w_0 < w_1 < \ldots < w_n$), where the first n_1 of those terms are non-oscillatory exponentials, and are equivalent to a sinusoid of zero frequency. The filtering process eliminates each component at a time, starting by the component with the highest frequency. Each time we eliminate one sinusoidal component, the remainder $X^*(t)$, contains the summation of all the previous components except the eliminated one. After the elimination of j sinusoidal components, the remainder is:

$$X^*(t) = E_{n_1-1}(t) \qquad (10)$$

The elimination of components continues until the rest of the signal is non-oscillatory. Figure 1 shows the QSI algorithm. QSI determines the order of the system by adding the order of all eliminated components Function Filtering (see next section) eliminates one component and returns the order of the eliminated component and the remainder signal.

Figure 2illustrates the application of QSI to a sample signal.

Filtering eliminates the fastest sinusoidal component, then the slower one, and then determines the order of the non-oscillatory component. QSI has determined that the simplest LTI system capable of exhibiting the observed behavior is of order 5. The resulting model is shown in Equation 11

$$a_5 \frac{d^5x(t)}{dt^5} + a_4 \frac{d^4x(t)}{dt^4} + \cdots + a_1 \frac{dx(t)}{dt} + a_0 x(t) = 0 \qquad (11)$$

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QSI(X)
    degree=0
    repeat
        (k,X)=TPAFilter(X)
        degree+=k
    until k=0
    return degree
```

Figure 1: QSI Algorithm

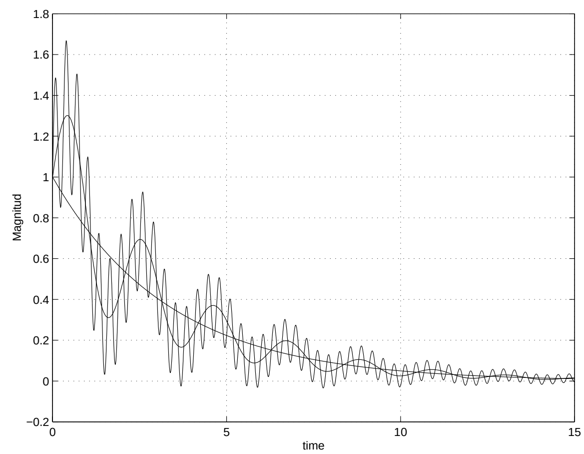

Figure 2: Component Elimination Process

Filters

Digital filters have been used mainly for two general purposes: signal separation, for signals that have been previously mixed, and signal restoration, for signals that have been distorted. In our application, we use filters to separate the different components that form the impulse response of a LTI system.

Preprocessing data

When the data has been collected from the identification experiment, it is not in a usable form for the identification algorithms. There are several possible deficiencies in the data. The most common problem is the high-frequency disturbances due to noise; such noise is typically of frequencies above the natural frequencies of the system under study. This problem can be solved applying a low pass filter to the data. In this approach a Windowed-Sinc and Kalman filters were used for preprocessing data.

Two-Point Average Filter

A modification to a two-point difference filter (Smith 1999) is proposed as an alternative to perform the first step in the filtering process. This first step of the filtering process detects the points where the derivative of the observed time series changes. The derivative can be computed using Equation 12

$$x'(t) = \frac{x(t) - x(t-1)}{\Delta t} \qquad (12)$$

where Δt is the sampling period, $x'(t)$ is the derivative of x at point t, and $x(t)$ is the input signal at time point t.

Using the approximation given by Equation 12 we determine the time points where the sign of the derivative changes, i.e., we are detecting the extrema of x.

Once the vector of extrema has been determined, we proceed to compute the average values between adjacent extrema using Equation 13

$$m_k = \frac{x(t_k) - x(t_{k-1})}{2} \tag{13}$$

This is a simple implementation of a low-pass filter. Using this filter, the midpoint vector is computed.

Vector m represents the remainder $X^*(t)$ defined in Equation 10. A spline approximation is used to smoothen the form of the carrier and has proven to provide better results than the bare filtered data.

Figure 3 shows the Two-Point Average Filter Algorithm (TPAFilter).

TPAFilter(X)
    Xtr=determine extrema of X
    if oscillatory(X)
        k=2
        Compute m, middle point vector
        X^*=Spline(m)
    else
        k=sizeof(Xtr)
        X^*=0
    return (k, X)

Figure 3: TPAFilter Algorithm

Figure 3illustrates the procedure. Extrema are marked with plus signs and the computed middle points with stars. The signal formed by the middle points constitutes the carrier of the original wave (i.e. the remainder).

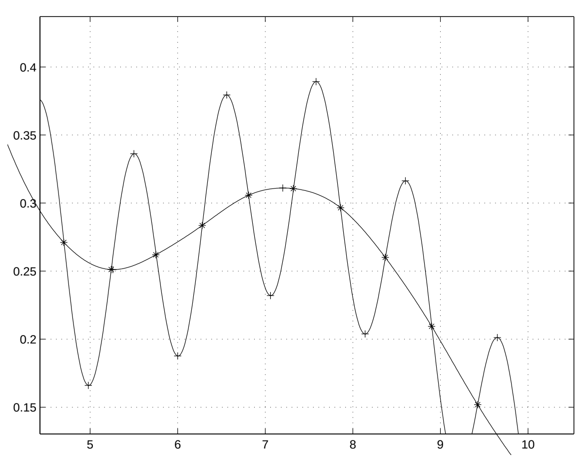

Figure 4: Average Filtering

Gabor Filters

An alternative way to filtering of frequencies is by Gabor Filters.These filters are defined by a harmonic function, modulated by a gaussian distribution. In the frequency domain, the filtering function is given by Equation 14;

$$h(\omega, \mu) = g(\omega, \mu, 1)\, e^{-j\left(\frac{2\pi\mu}{N}\omega\right)} \tag{14}$$

where

$$g(z, \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\left(\frac{z^2}{2\sigma^2}\right)} \tag{15}$$

μ and σ are the mean and standar deviation of the gaussian distribution, ω is the frequency we want to preserve, and N is the number of points in the sample. The gaussian distribution determines the likelihood of components of frequencies near to ω to pass the filter.

Figure 4shows the filter profile in the frequency domain.

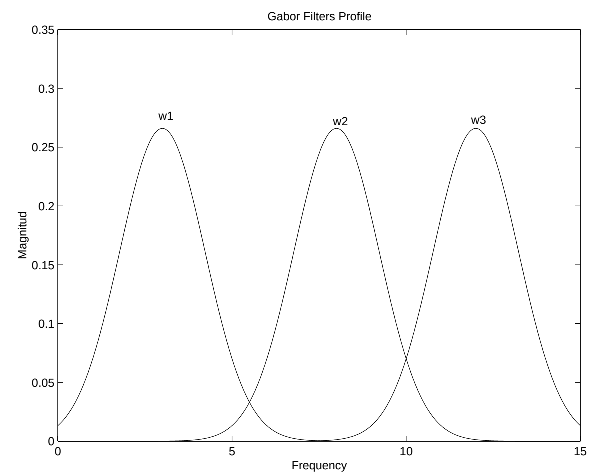

Figure 5: Gabor Filter Profile

In the time domain, the filtering function is defined by Equation 16.

$$\begin{aligned} h(t, \mu) &= g(t, \mu, 1)\left(e^{j\left(\frac{2\pi\mu}{N}t\right)} + e^{-j\left(\frac{2\pi\mu}{N}t\right)}\right) \\ h(t, \mu) &= g(t, \mu, 1)\cos\left(\frac{2\pi\mu}{N}t\right) \end{aligned} \tag{16}$$

The Gabor filters are considered as band-pass filters. The filtering process is done by a convolution operation of the original signal $X(t)$, and the filtering signal $h(t)$.

The first step in applying a Gabor Filter is to determine the frequency components of the signal by fourier analysis. Figure 4 shows the fourier spectrum of the observed data $x(t)$. The fourier spectrum shows all frequency components, which are used to tune the filters at the target frequencies. Figure 6shows a plot of the filter's response tunned at $\omega = 0$, Figure 7shows the output at $\omega = 3$, and Figure 8shows the output at $\omega = 20$.

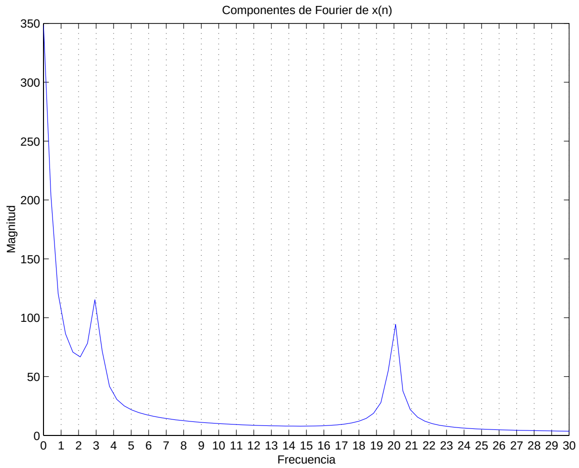

Figure 6: Fourier Spectrum

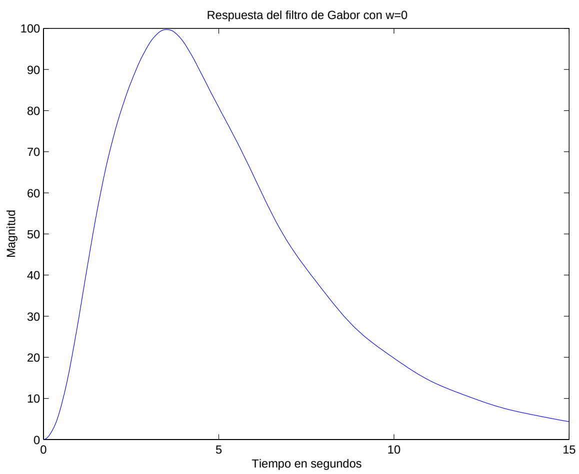

Figure 7: Response of Gabor Filter at ω=0

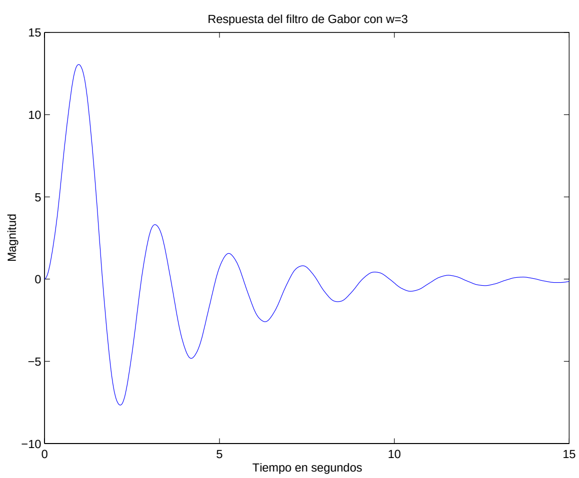

Figure 8: Response of Gabor Filter at ω=3

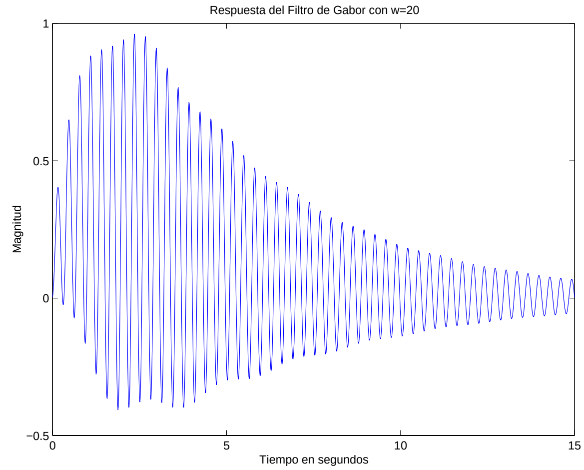

Figure 9: Response of Gabor Filter at ω=20

Related Work

Systems identification plays an important role in Electrical Engineering, specially in control theory. Aiming in that direction, abundant results have arisen in what we call classical systems identification (Ljung 1987). Classical systems identification focuses in what we know as paremetric identification. There are several methods to determine the parameters of a given function that makes that function best fit the stream of observed data. Among others, we have least squares and maximum likelihood estimators (Clarke & Kempson 1997)

Within the area of Artificial Intelligence, several researchers have developed methods to perform qualitative, quantitative, and semi-quantitative system identification, with applications in different disciplines.

Key et al (Kay, Rinner, & Kuipers 2000) developed SQUID, a system for system identification which searches in the space of semi-quantitative diferential equations. Given a stream of observations, WQUID tries to find the most specific envelop functions for monotonic constraints in the SQDE, as well as determining intervals for the model's parameters. SQUID presents statistical methods to determine overall qualitative trends on intervals of data. This characteristic makes it tolerant to noise. Instead, we are experimenting with different filters, which will provide a similare tolerance to noise.

SQUID is more general than our approach, but it needs a pre-identified model in the form of a SQDE, and refines its functions and parameters as much as possible. Our approach is more limited in scope (only linear, time invariant systems), but all it needs is the raw observation data.

Bradley and Stolle (Bradley & Stolle 1996) developed a system known as PRET, which constructs an ODE-based model of the black box, based on the user's hypothesis, observations, specifications, and physical measurements. PRET construct a set of candidate models applying domain rules to the user's hypothesis. Models are filtered using symbolic techniques and checking against specifications and observations. If a model fails to pass those tests, it is refined introducing another hypothesis. Process repeats until either a successful model is found or the process fails. In the latter case, power-series expansion methods are used to determine a model from scratch.

Conclusions

We have presented the analysis, algorithms, and implementation of a framework to perform Qualitative Systems Identification for Linear Time-Invariant Systems.

Once identified the structure of the system we send it to a parametric identification module to complete the identification process. The parametric identification part has been implemented using genetic algorithms, which have proven to be efficient.

The system has been successful in determining the sinusoidal components of different test systems we have fed it with. Nonetheless, the filters still need some work on the detection of non-oscillatory components. Since the middle points of a damped sinusoidal do not exactly match the carrier, when we join them, they present small oscillations even when we have removed all the oscilatory components. This fact has lead us to higher the order of the system by two in some cases.

The current status on implementation is the following. The Systems Identification algorithm, including filters have been implemented using Matlab. In our implementation, the input is the observed signal, and the output is the structure of the black-box system.

It is obvious that the limitation of this system identification technique is related to the availability of good data and good model structures. Without a reasonable data record not much can be done, and there are several reasons why such a record cannot be obtained in certain applications. Therefore, a bad model structure cannot offer a good model, regardless of the amount and quality of the available data.

An expression for the time complexity is also needed. We need to compare how our algorithm behaves with respect to traditional approaches.

Acknowlegements

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