Integrating qualitatively Time and Topology for Spatial Reasoning.

Lledó Museros* and M. Teresa Escrig.**
*Alicer (Asociación para la Promoción del Diseño Cerámico), Avda. del Mar 42,
E-12003 Castellón (Spain)
e-mail: lledo@alicer.es
phone: +34 964 220312
fax: +34 964 230414
**Universitat Jaume I, Engineering and Computer Science Department, Campus Riu Sec
E-12071 Castellón (Spain)
e-mail: escrigm@icc.uji.es
phone: +34 964 728303
fax: +34 964 728486

Abstract

In this paper we present a qualitative representational model and the corresponding reasoning process for integrating qualitatively time and topological information. In the calculus presented topological information in function of the point of the time in which it is true is represented as an instance of the Constraint Satisfaction Problem. That representation together with the implementation of the reasoning process by means of Constraint Logic programming extended with Constraint Handling Rules allow us the integration of both aspects (time and topology) with other spatial aspects such as orientation, distances and cardinal directions, The resulting method can be applied to qualitative navigation of autonomous agents. The model presented in this paper will help during the path-planning task by describing the sequence of topological situations that the agent should find during its way to the target objective.

Introduction

An autonomous mobile robot should be able to understand and reason with spatial aspects such as orientation, named distances, compared distances, cardinal directions, topology and so on, in such a straightforward way as humans do. Spatial information that humans obtain through perception is coarse and imprecise, thus qualitative models which reason with distinguishing characteristics rather than with exact measures seems to be more appropriated to deal with this kind of knowledge.
Several qualitative models have been developed for dealing with spatial concepts such as orientation [Guesguen 89], [Mukerje & Joe 90], [Freksa 91, Freksa & Zimmerman 92], [Hernández 94], named distances [Zimmermann 93], [Clementini et al. 95], [Escrig & Toledo 00], cardinal directions [Frank 92], and so on.

Our aim is to formalize the intuitive notion of spatio-temporal continuity for a qualitative theory of motion. As motion can be seen as a form of spatio-temporal change, the paper presents a qualitative representation model for integrating qualitative time and topological information for modeling motion and reasoning about dynamic worlds in which spatial relations between regions may change with time.

Moreover, we want to integrate the concept of motion (topology and time), with other spatial aspects, such as orientation, distances and cardinal directions.
The bases for the integration in the spatial reasoning field of different spatial aspects, have been inspired in the temporal reasoning field, where the integration of point algebra, interval algebra and metric information has been successfully accomplished [Meiri 1991]. In order to accomplish the task of integrating different spatial aspects in the same model, the next three steps are defined:

- the representation of the spatial aspect to be integrated
- the definition of the Basic Step of the Inference Process (BSIP). It is defined such as: given the spatial relationship between objects A and B, and the spatial relationship between objects B and C, the BSIP consists of obtaining the spatial relationship between A and C.
- the definition of the Complete Inference Process (CIP), that consists of repeating the BSIP as many times as possible, with the initial information and the information provided by previous steps of the BSIP, until no more information can be inferred.

The concepts of orientation, cardinal directions, and absolute and relative distances have been integrated in the same model thanks to consider the representation and the reasoning process of each aspect as an instance of the Constraint Satisfaction Problem [Escrig & Toledo 01].

In this paper, topological together with time information will also be integrated in the previous mentioned model, following the same idea.

Bases for the Integration of Several Temporal Aspects

As we have mentioned in the introduction, the integration of several temporal aspects has been accomplished by considering them as instances of the CSP. The best-known solution to the CSP problem has not polinomic temporal complexity, however, there exist algorithms which approximate the solution. These algorithms approximate the complete propagation process by local constraint propagation, as path consistency. If the constraint graph is complete (that is, there is a pair of arcs, one in each direction, between every pair of nodes) it suffices to repeatedly compute paths of two steps in length at most. This means that for each group of three nodes (i,k,j) we repeatedly compute the following operation until a fix point is reached [Fruehwirth 1994]:

$$c_{ij} := c_{ij} \oplus c_{ik} \otimes c_{kj} \qquad (1)$$

This operation computes the composition of constraints (⊗) between nodes *ik* and *kj*, and the intersection (⊕) of the result with constraints between nodes *ij*. The complexity of this algorithm is $O(n^3)$, where *n* is the number of nodes in the constraint graph (that is, the number of objects involved in the reasoning process) [Kumar 92; Mackworth & Freuder 85].

Constraint Handling Rules (CHRs) are a tool which helps to write the above algorithm. They are an extension of the Constraint Logic Programming (CLP) which facilitate the definition of constraint theories and algorithms which solve them. They facilitate the prototyping, extensions, specialization and combination of CSs [Fruehwirth 94]. There exist mainly two types of CHRs: propagation and simplification. *Propagation* CHRs add new constraints which are logically redundant but may cause further simplification. A *propagation* CHR is of the from:

$$H_1,\cdots,H_i \Longrightarrow G_1,\cdots,G_j \mid B_1,\cdots,B_k \qquad (i>0,\ j\geq 0,\ k\geq 0)$$

The propagation from user-defined constraints, H’, means the addition of the set of constraints B to the initial set of constraints if H’ matches the head (H) of a propagation rule and G is satisfied. This kind of rules are used to compute the part ‘⊗’ of formula (1).

Simplification CHRs replace constraints by simpler constraints preserving logical equivalence. A *simplification* CHRs is of the form:

$$H_1,\cdots,H_i \Longleftrightarrow G_1,\cdots,G_j \mid B_1,\cdots,B_k \qquad (i>0,\ j\geq 0,\ k\geq 0)$$

The multi-head $(H_1,\cdots,H_i)$ is a conjunction of user-defined constraints and the guard $(G_1,\cdots,G_i)$ is a conjunction of literals. To *simplify* the user-defined constraints H’ means to replace them by B if H’ matches the head (H) of a simplification rule ant the guard G is satisfied. This kind of rules are used to compute the part ‘⊕’ of formula (1).

Overview of the Topological Calculus

To make this paper self-contained we now give a concise summary of the topological calculus selected to get the integration of topology and time. A fuller explanation can be found in [Isli, Museros et al. 00]. We have developed this topological calculus because it is presented as an algebra alike to Allen’s [Allen 83] temporal interval algebra and it allows us to reason about point-like, linear and areal entities, which will allow us the use of different granularities of the same map. The calculus defines 9 topological relations, which are described bellow, that are mutually exclusive, that is, given 2 entities, the relation between them must be one and only one of the 9 relations defined.

A topological relation r between two entities h1 and h2, denoted by (h_1,r,h_2), is defined on the right hand side of the equivalence sign in the form of a point-set expression. The definitions of each topological relation are described bellow in table 1.

Integrating Topology and Time

The representational model

Our aim is to propose a constraint-based approach to integrate the topological calculus developed in [Isli, Museros et al. 00] and time. The topological relations of the calculus in [Isli, Museros et al. 00] has been described in the previous section. But we still have to describe the time algebra that we have chosen for the integration. We are going to define a temporal algebra, in which variables represent time points and there are five primitive constraints: prev, next, <<, >>, ==, which are defined as follows:

Definition1. Given two time points, t and t’, t **==** t’ iff has not occurred a change between t and t’ (or between t’ and t) on any relation.

Definition2. Given two time points, t an t’, t’ **next** t iff t’ > t and some relation or relations have changed to a neighbor relation between t and t’.

Definition3. Given two time points, t and t’, t’ **prev** t iff t’ < t and some relation or relations have changed to a neighbor relation between t and t’.

Definition4. Given two time points, t and t’, t’ >> t iff t’ > t and a relation has changed strictly more than once to a neighbor relation.

Definition5. Given two time points, t and t’, t’ >> t iff t’ < t and a relation has changed strictly more than once to a neighbor relation.

According to this, time is represented by disjunctive binary constraints of the form $X\{r_1, ..., r_n\}Y$, where each r_i is a relation that is applicable to X and Y. $X\{r_1, ..., r_n\}Y$ is a disjunction of the way $(Xr_1Y) \vee .... \vee (Xr_nY)$ and r_i is also called primitive constraints.

We have chosen this type of qualitative time constraints because we are only interested in the point of the time in which one region is transformed into its topological neighborhood.

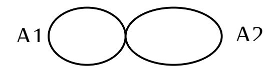
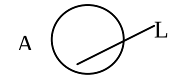
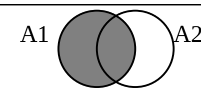
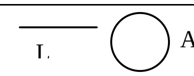
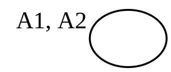
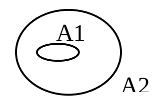
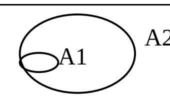
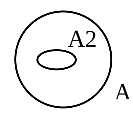
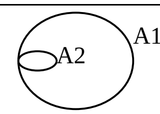
Relation	Definition	Graphic Example
touch	$(h_1, \text{touch}, h_2) \leftrightarrow h°_1 \cap h°_2 = \varnothing \wedge h_1 \cap h_2 \neq \varnothing$	
cross	$(h_1,\text{cross},h_2) \leftrightarrow \dim(h°_1 \cap h°_2) = \max(\dim(h°_1),\dim(h°_2)) - 1 \wedge h_1 \cap h_2 \neq h_1 \wedge h_1 \cap h_2 \neq h_2$	
overlap	$(h_1, \text{overlap}, h_2) \leftrightarrow \dim(h°_1) = \dim(h°_2) = \dim(h°_1 \cap h°_2) \wedge h_1 \cap h_2 \neq h_1 \wedge h_1 \cap h_2 \neq h_2$	
disjoint	$(h_1, \text{disjoint}, h_2) \leftrightarrow h_1 \cap h_2 = \varnothing$	
equal	Given that $(h_1, \text{in}, h_2) \leftrightarrow h_1 \cap h_2 = h_1 \wedge h°_1 \cap h°_2 \neq \varnothing$: if (h_2, in, h_1) then (h_1, equal, h_2)	
completely-inside	Given that (h_1, in, h_2) and not (h_1, equal, h_2): if $h_1 \cap \delta h_2 \neq \varnothing$ then $(h_1, \text{touching-from-inside}, h_2)$	
touching-from-inside	If (h_1, in, h_2), not (h_1, equal, h_2) and not then $(h_1, \text{touching-from-inside}, h_2)$ then: $(h_1, \text{completely-inside}, h_2)$	
completely-inside$_i$	$(h_1, \text{completely-inside}_i, h_2) \leftrightarrow (h_2, \text{completely-inside}, h_1)$	
touching-from-inside$_i$	$(h_1, \text{touching-from-inside}_i, h_2) \leftrightarrow (h_2, \text{touching-from-inside}, h_1)$	

Table 1. Topological relations of the calculus.

The topological neighborhood of a region is that region to which the original region can be transformed to by a process of gradual, continuous change which does not involve passage through any third region.

To reason about these temporal constraints we need to define the converse and composition operations and construct the converse and composition tables.

First of all we need to define what we understand as a general relation of the calculus because we are going to define the converse and composition operation in terms of general relations.

Definition6. *A* ***general relation*** *R of the calculus is any subset of the set of all atomic relations.*

Definition7. *The* ***converse*** *of a general relation R, called $R^{\smile}$ is defined as:*

$$\forall (X,Y)\ ((X,R,Y) \Leftrightarrow (Y,R^{\smile},X)) \qquad (2)$$

Definition8. *The* ***composition*** *$R1 \otimes R2$ of two general relations R1 and R2 is the most specific relation R such that:*

$$\forall (h_1, h_2, h_3)\ ((h_1, R1, h_2) \wedge (h_2, R2, h_3) \Rightarrow (h_1, R, h_3) \qquad (3)$$

Below, we find table 2 and 3 which are the converse table and composition table respectively for the time algebra.

r	**$r^{\smile}$**
==	==
<<	>>
>>	>>
next	*prev*
prev	*next*

Table 2. The converse table for the time algebra.

r_2 \ r_1	<<	**prev**	==	**Next**	>>
<<	{<<}	{<<}	{<<}	{prev,<<}	{<<,prev, ==,next,>>}
prev	{<<}	{<<,prev}	{prev}	{==,prev,next}	{next,>>}
==	{<<}	{prev}	{==}	{next}	{>>}
next	{<<,prev}	{prev,==,next}	{next}	{>>,next}	{>>}
>>	{<<,prev, ==,next,>>}	{>>,next}	{>>}	{>>}	{>>}

Table 3. The composition table for the time algebra.

The composition and converse tables for topological relations can be found in [Isli, Museros et al. 00].

The first step to define the framework to reason with several spatial aspects including motion is to create the representational model of topology and qualitative time points. The representational model follows the formalism used by Allen for temporal interval algebra [Allen 83].

The Allen style formalism will provide to our approach the possibility of reasoning with topology in dynamic worlds by applying the Allen's constraint propagations algorithm.

As we have mentioned the representational model uses the topological calculus developed in [Isli, Museros et al. 00] and the time algebra described above.

The binary relations between two objects, which can be points, lines or areas, h_1 and h_2 of the representation model in a point of time t are defined as tertiary constraints or propositions where the topological relation r between h_1 and h_2 in the point of time t is denoted by $(h_1,r,h_2)_t$. From this definition we define a **general relation R** of the algebra during time t as:

$$\forall(h_1,h_2)\ ((h_1,R,h_2)_t \Leftrightarrow U_{r\in R}\ (h_1,r,h_2)_t) \qquad (4)$$

Definition9. *The* ***converse*** *of a general relation R in time t, denoted as* $R^{\cup}$*, in defined as follows:*

$$\forall(h_1,h_2)\ ((h_1,R,h_2)_t \Leftrightarrow (Y,R^{\cup},X)_t) \qquad (5)$$

From this definition we observe that the converse of he algebra defined including topology and time is the same as the converse defined only for topological relations because the converse is calculated in the same point of time, therefore time does not affect to the converse operation. The spatio-temporal converse table is depicted in table 4.

r	$r^{\cup}$
touch	*touch*
cross	*cross*
overlap	*overlap*
disjoint	*disjoint*
equal	*equal*
completely-inside	*completely-inside$_i$*
touching-from-inside	*touching-from-inside$_i$*
completely-inside$_i$	*completely-inside*
touching-from-inside$_i$	*touching-from-inside*

Table 4. The converse table for the spatio-temporal algebra

The Basic Step of the Inference Process

The BSIP for topological information and time consists of: *"given three objects A,B, C, if the topological relationships in time between A and B and B and C are known, it is possible to obtain the topological relationship in time between objects A and C"*. To infer such topological relationship in time we are going to define the *composition* operation for two general relations R1 and R2.

The composition for the model including topology and time has to be defined to include all the possibilities in three different ways as follows:

Definition10*. The resulting general relation R obtained from the* ***composition*** *(⊗) operation could be calculated as:*

a) $(A,R1,B)t_0 \otimes (B,R2,C)t_0 \Rightarrow (A,R,C)t_0$
b) $(A,R1,B)t_0 \otimes (t_0, Reltime, t_1) \Rightarrow (A,R,B)t_1$
c) $(A,R1,B)t_0 \otimes (B,R2,C)t_1 \,/\, (t_0, Reltime, t_1) \Rightarrow ((A,R1,B)t_0 \otimes (t_0, Reltime, t_1)) \otimes (B,R2,C)t_1 \Rightarrow (A,R',B)t_1 \otimes (B,R2,C)t_1 \Rightarrow (A,R,C)t_1$

The first type of composition (*Definition10.a*) is the composition of the topological relations between three regions A, B and C, in the same point of time, where A, B, C belong to {point, line, area}. Then it is the usual topological composition, the time does not affect. To calculate this composition we will use the 18 composition tables and the converse table defined in [Isli, Museros et al. 00]. The tables can be found in [Isli, Museros et al. 00].

The second type of composition (*Definition10.b*) is the composition which implements Freksa's conceptual neighborhood notion. It looks for the possible topological relations which will appear between two regions as time changes. To reason about this we need to construct 6 composition tables that will be referred to as XY_t–table where the regions X and Y belong to {point (P), line (L), area (A)} and t represents the dimension of time of the algebra. We would need 9 composition tables (3^2) if we consider all possibilities with X and Y being a point-like, a linear or an areal entity. However, we construct only 6 tables from which the other 3 tables can be obtained using the converse operation. We construct the AA_t-table, LA_t-table, PA_t-table, LL_t-table, PL_t-table and the PP_t-table, which are depicted in tables 5 to 10 respectively. Note: due to limitation restrictions the topological relation are represented in the next way: *touch* is represented by T, *cross* by C, *overlap* by O, *disjoint* by D, *completely-inside* by CI, *touching-from-inside* by TFI, *equal* by E, *touching-from-inside*$_i$ by TFI_i and *completely-inside*$_i$ by CI_i. And due to limitation space we have depicted in a common column the case for "next" and "prev" and a common column for the case of "<<" and ">>" because their entries are the same.

From the tables we can also infer that the = time relation represents the identity.

Reltime / RelTop	next or prev	<< or >>	==
T	{D,O,T}	{T,E,TFI, CI,TFI, CIi,TFIi}	{T}
O	{T,TFI,O}	{O,D,E, CI,TFIi,CIi}	{O}
D	{T,D}	{D,O,E, TFI,CI, TFIi,CIi, TFIi}	{D}
E	{O,E,}	{E,T,D, TFI,CI, TFIi,CIi}	{E}
TFI	{O,CI,TFI}	{TFI,T,	{TFI}

		D,E,CI, TFIi, CIi}	
CI	{TFI,CI}	{CI,T,O, D,E, TFIi,CIi}	{CI}
TFIi	{O,CIi,TFi}	{TFIi,T,D,E, CI,TFI}	{TFIi}
CIi	{TFIi,CIi}	{CIi,T,O,D,E, TFI,CI}	{CIi}

Table 5. AA_t-table

Reltime **Reltop**	**next or prev**	**<< or >>**	**==**
T	{C,D,T}	{T,TFI,CI}	{T}
C	{D,TFI,C}	{C,T,CI}	{C}
D	{T,D}	{D,C,TFI,CI}	{D}
TFI	{C,CI,TFI}	{TFI,T,D}	{TFI}
CI	{TFI,CI}	{CI,T,C,D}	{CI}

Table 6. LA_t-table

Reltime **Reltop**	**next or prev**	**<< or >>**	**==**
T	{D,CI,T}	{T}	{T}
D	{T,D}	{D,CI}	{D}
CI	{T,CI}	{CI,D}	{CI}

Table 7. PA_t-table

Reltime **Reltop**	**next or prev**	**<< or >>**	**==**
T	{D,O,C,T}	{T,E,TFI,CI, TFi,CIi}	{T}
D	{T,C,D}	{D,O,E,TFI,CI, TFIi,CIi}	{D}
O	{T,C,O}	{O,D,E,TFI,CI, TFIi, CIi}	{O}
C	{T,D,C}	{C,O,E,TFI,CI, TFIi, CIi}	{C}
E	{T,O,E}	{E,D,C,TFI,CI, TFIi,CIi}	{E}
TFI	{C,CI,T,TFI}	{TFI,D,O,E, TFIi,CIi}	{TFI}
CI	{TFI,C,CI}	{CI,T,D,O,E, TFIi,CIi}	{CI}
TFIi	{T,C,CIi,TFi}	{TFIi,D,O,E, TFI,CI}	{TFIi}
CIi	{C,TFIi,CIi}	{CIi,T,D,O,E, TFI,CI}	{CIi}

Table 8. LL_t-table

Reltime **Reltop**	**next or prev**	**<< or >>**	**==**
T	{D,CI,T}	{T}	{T}
D	{T,CI,D}	{D}	{D}
CI	{T,D,CI}	{CI}	{CI}

Table 9. PL_t-table

Reltime **Reltop**	**next or prev**	**<< or >>**	**==**
E	{D,E}	{E}	{E}
D	{E,D}	{D}	{D}

Table 10. PP_t-table

As a relation t prev t' corresponds to a change of some topological relation to a neighbour relation, the tables always keep the possibility that a relation has not changed between time t and t', this situation model the fact that the time changes from t to t' because other topological relationship has changed and the relationship between X and Y (RelTop) has not changed.

The 3 tables not constructed can be obtained by applying the converse operation. For example, the AL_t-table is not constructed but we can get any of its entries using the LA_t-table. This means that we have to find the most specific relation R such that, if X and Y are an areal and a linear entity respectively:

$$(X, Reltop, Y)_{t0} \otimes (t_0, Reltime, t_1) \Rightarrow (X, R, Y)_{t1} \quad (6)$$

From the LA_t-table and using the converse operation we will get the relation R as follows:

$$(Y, Reltop^{\cup}, X)_{t0} \otimes (t_1, Reltime^{\cup}, t_0) \Rightarrow (Y, R', X) \quad (7)$$

Then the relation R that we are looking for is $R=(R')^{\cup}$.

For the third case of composition (*Definition10.c*) we want to infer the composition R in time t_1 between 3 regions, X, Y and Z having the topological relation in time t_0 between X and Y, the topological relation in time t_1 between Y and Z and the qualitative time relation between times t_0 and t_1. To get the composition relation R, first we have to obtain the topological relations that can appear between X and Y in time t_1 using the composition tables defined for the case of *Definition10.b* above described. Then we have the general relation R' which appear between X and Y during t_1, this together with the general relation R2 between Y and Z in t_1 is a case suitable to apply the usual composition tables as explained for the case of *Definition10.a* and we will get the general composition relation R.

The Full Inference Process

For computing the Full inference process (FIP) of topological and time information we consider that:

1) each topological relationship between two objects in time t is seen as a constraint;
2) the set of topological relationships in time forms a constraint graph, where the nodes are spatial objects (points, lines and areas) and the arcs are the binary constraints between objects. This constraint graph is not complete at the beginning, that is, all the nodes are not bi-directional connected, because there is no initial topological relationship in time between all the objects in the space;
3) the fact of propagating the constraints for making explicit the topological relationships between all the nodes in the graph is seen as an instance of the CSP.

The formula (1), which approximated the solution for temporal objects, is rewritten for topological relations between spatial objects in a point of time in three formulas for each of the definition of composition given for the BSIP, as follows:

Case 1: $c_{a,c,t} := c_{a,c,t} \oplus c_{a,b,t} \otimes c_{b,c,t}$ (8)

Case 2: $c_{a,b,t1} := c_{a,b,t1} \oplus c_{a,b,t0} \otimes c_{t0,t1}$ (9)

Case 3: $c_{a,b,t1} := c_{a,c,t} \oplus c_{a,b,t0} \otimes c_{b,c,t1}$ (10)

In our approach, the constraint $c_{a,b,t}$ (which represents the topological relationship holding between objects a, b in time t) is represented by the PROLOG predicate ctr_comp_top(TB,TA,A,B,Rel,t), where A and B are the spatial objects which holds the set of atomic topological relationships included in the set *Rel* in the point of time t, TB and TA represents the types of the objects A and B, which can be point (*p*), line (l) or area (a). And the constraint $c_{to,t1}$ represents the time constraint between points of time t_0 and t_1, (t_0, Rtime,t_1), and is represented by the PROLOG predicated ctr_comp_time(t0,t1,Rtime).

We have implemented a PROLOG algorithm for the FIP, which is not included in this paper due to limitation restrictions, and in this algorithm the part of the intersection ($c_{a,b}\oplus$....) of (8,9 and 10) is implemented by a *simplification CHR* and the part of the composition ($c_{a,b} \otimes$), which corresponds to the BSIP defined in the previous section, is implemented by *propagation CHRs*.
In the algorithm, which implements the FIP, no queue of modified constraints is needed because the new constraint goal itself will trigger new applications of the propagation CHRs.

The complexity of this algorithm is $O(n^3)$, where *n* is the number of nodes in the constraint graph (that is, the number of objects involved in the reasoning process).

Conclusions and Future Work

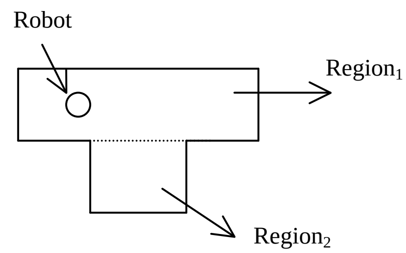
The major contribution of the work presented in this paper is the definition of an approach for integrating topological aspects together with time with other spatial aspects (orientation, cardinal directions and named distance) in the same model. To make possible the integration it is necessary to define (1) its representation; (2) the basic step of the inference process; and (3) the full inference process. A uniformity of implementation of these three parts for each spatial aspect allows us the integration. It is achieved by using constraint logic programming extended with constraint handling rules (CLP+CHR) as tool. The paradigm CLP+CHR is used to implement a constraint solver which solves in a straightforward way the complete inference process for each aspect of the space to be integrated. Therefore CLP+CHR provides a suitable tool for the integration.

Although only the topological together with time model has been described in this paper, qualitative orientation, cardinal direction and named and compared distances have also been integrated into the same model following the same steps described here with the topological and time information ([Escrig & Toledo 01]).

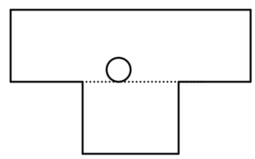
The framework presented here could help us to reason about the sequence of topological situations that an autonomous robot should find during its way from a starting region to a target objective. It can also help to detect situations in which the robot is loosing its direction of movement. This is our future work, that is the application of this representational model to the autonomous robot navigation problem. For instance, if we have a situation as the one depicted in figure 1 in time t_0, and we want that the robot goes from $region_1$ to $region_2$, we know that the sequence of topological relations between the robot (interpreted as a mobile region) and the origin region, called $region_1$, and the target region, called $region_2$, is the next one:

$(Robot,CI_i,Region_1)_{t0}$ and $(Robot,D,Region_2)_{t0}$,
$(Robot,TFI_i,Region_1)_{t1}$ and $(Robot,T,Region_2)_{t1}$,
$(Robot,O,Region_1)_{t2}$ and $(Robot,O,Region_2)_{t2}$,
$(Robot,T,Region_1)_{t3}$ and $(Robot,TFI_i,Region_2)_{t3}$,
$(Robot,D,Region_1)_{t4}$ and $(Robot,CI_i,Region_2)_{t4}$
where t0 prev t1 prev t2 prev t3 prev t4.

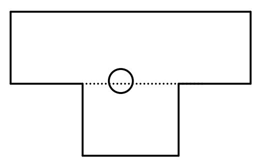
Note that we have used the same notation for the topological relations as the one used for the composition tables.



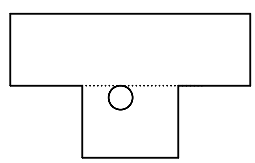
a. Initial situation



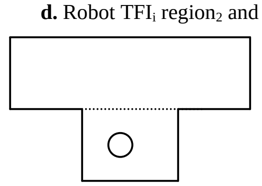
b. Robot TFI_i $region_1$ and T $region_2$.



c. Robot overlapping both regions



d. Robot TFI_i $region_2$ and T $region_1$



e. Final situation

Figure 1. Graphical Sequence Situations of the example given in conclusions.

If during the robot's way until the target objective we find a situation which does not follow the sequence, for instance we find $(Robot,TFI_i,Region_1)_{t1}$ and

(Robot,D,Region$_2$)$_{t1}$, the robot is losing its direction of movement. Therefore we want to use this knowledge to the navigation of an autonomous robot integrating this knowledge to other qualitative spatial information such as orientation, distance and cardinal directions in the same way as it has been done in [Escrig & Toledo 01]. A preliminary result of that application has been obtained by using qualitative representation of such spatial aspects for the autonomous simulated navigation of a Nomad-200 robot, on a structured environment of an easy corridor (with offices in only one side) in a building [Museros and Escrig 02].

Acknowledgments

This work has been partially supported by CICYT under grant number TAP99-0590-002-02 and by Alicer, Ceramic Design Technology Instutitute.

REFERENCES

[Allen 83] Allen, J. Maintaining Knowledge about Temporal Intervals. *Communications of the ACM* , Vol. 26, No. 11. pp. 832-843, 1983.

[Clementini et al. 95] Clementini, E. Di Felice, P., Hernández, D. Qualitative Representation of Positional Information. Technical Report TR FKI-208-95, Technische Universität München, 1995.

[Escrig & Toledo 01] Escrig, M.T and Toledo, F. Autonomous Robot Navigation using human spatial concepts. International Journal of Intelligent Systems, Vol.15, No.3, pp.165-196, 2000, March 2000

[Frank 92] Frank, A. Qualitative Spatial Reasoning about Distance and Directions in Geographic Space. In *Journal of Visual Languages and Computing 3*, pp. 343-373, 1992.

[Freksa 91] Freksa, C. Conceptual neighborhood and its role in temporal and spatial reasoning. Technical Report Tech. RPt. FK1-146-91, Institute of Information, University of Munich, 1991.

[Freksa & Zimmerman 92] Freksa, C. and Zimmerman, K. On the Utilization of Spatial Structures for Cognitively Plausible and Efficient Reasoning. In Prooc. Of the IEEE International Conference on Systems, Man and Cybernetics, pp. 18-21. 1992

[Fruehwirth 1994] Fruehwirth, T. Constraint Handling Rules. In A. Podelski (ed.), Constraint Programming: Basic and Trends, volume 910 of Lecture Notes in Computer Science, Springer, pp. 90-107, 1994.

[Guesguen 89] Guesguen, H.W. Spatial reasoning based on Allen's temporal logic, Technical Report TR-89-049, International Computer Science Institute, Berkeley, 1989.

[Hernández 94] Hernández, D. Qualitative Representation of Spatial Knowledge, In Volume 804 in Lecture notes in AI. Eds: Springer-Verlag, 1994.

[Isli, Museros et al. 00] Isli, A., Museros, Ll., Barkowsky, T. and Reinhard, M. "A Topological Calculus for Cartographic Entities". In Lecture notes in Artificial Intelligence 1849. Spatial Cognition II. Integrating Abstract Theories, Empirical Studies, Formal Methods and Practical Applications. Spatial Cognition. Springer-Verlag, pp. 225 – 239, 2000

[Kumar 1992] Kumar, V. Algorithms for constraint satisfaction problems: A survey. *AI Magazine*, 13(1), pp. 32.44, 1992.

[Mackworth and Freuder 1985] Mackworth, A.K. and Freuder, E.C. The complexity of some Polynomial networks consistency algorithms for constraint satisfaction problems. *Journal of Artificial Intelligence* 25:65-74, 1985.

[Museros and Escrig 02] L. Museros and M.T. Escrig. "Combining Qualitative Spatial Information: The Integration of Topology, Time, Orientation and Distances for Spatial Reasoning", accepted to ECAI 2002 Workshop on Spatial and Temporal Reasoning.

[Meiri 91] Meiri, I., "Combining qualitative and quantitative constraints in temporal reasoning", in Proc. of the 7th National Conference on Artificial Intelligence, pp. 260-267, 1991.

[Mukerjee & Joe 90] Mukerjee, A. and Joe, G. A Qualitative Model for Space. In 8th-AAAI, pg. 721-727, 1990.

[Zimmerman 93] Zimmerman, K. Enhancing Qualitative Spatial Reasoning Combining Orientation and Distance. In Frank, A.U. and Campari, I. Eds. Spatial Information Theory. A Theoretical Basis for GIS. European Conference, COSIT'93, pp. 69-76, Marciana Marina, Italy, Vol. 716 of Lecture notes in Computer Science. Springer, Berlin.