Cognitive Maps for Mobile Robot Navigation: A Hybrid Representation Using Reference Systems[1]

Juan Carlos Peris Broch* and Mª Teresa Escrig Monferrer**
Cognition for Robotics Research Laboratory. Jaume I University.
Campus Riu Sec. Castellón (Spain), E-12071
* jperis@lsi.uji.es ** escrigm@icc.uji.es

Abstract

Extracting useful information from the environment has an important effect over the autonomous robot navigation process.
In this paper, we describe a hybrid representation model for robot navigation in indoor environments, which uses local reference systems as basic elements. We present a model, which integrates quantitative and qualitative information, relating in a natural way the different working scales of an autonomous robot navigation system.
An algorithm for the extraction of local reference systems from the environment is showed. Integration of quantitative and qualitative information, obtaining a hybrid description of the world, is explained. We also describe an algorithm to extract a topological map as a natural extension from the hybrid description.
Finally, we will show how a cognitive map can emerge from the hybrid description and the topological map.

Keywords: cognitive map, reference systems, autonomous robot navigation.

1 Introduction

It is known that to carry out a successful navigation in complex environments, mobile robots must acquiere and maintain internal maps of the environment. This task is not a trivial process and many factors affect the reliability of these world models. Different solutions have been developed attending to the kind of world model managed by the robot. These solutions can be divided into three main strategies: quantitave (metric), qualitative and hybrid approaches.

Quantitative models represent the environment from the metric information obtained by the sensors. The major exponent of this strategy is the grid-based model, introduced by Moravec and Elfes [1985]. This model represents the environment by a grid of cells where each cell has a value representing the probability of occupancy of the corresponding space in the world. Other metric approaches that do not use occupancy grids are [Giralt et al., 1979, Lozano-Pérez, 1981, Brooks, 1982, Crowley, 1985, Kosaka and Kak 2001]. Metric structures are difficult to apply in large environments. They have a high computational cost and the accuracy of the navigation is affected by odometric and sensor errors.

Qualitative models focus on the boundaries of the objects, making divisions of the space more or less detailed, in a manner inspired by cognitive processes used by humans. The qualitative concept of a topological map, which represents the world using nodes (places) and arcs (relations), has been used in several approaches, such as the one introduced by Kuipers [1978, Kuipers and Byun, 1988, Kuipers and Levitt, 1988]. Another model is defined in [Freksa et al., 2000], where schematic maps are used to reason about relative positions and orientations. Other qualitative models have been carried out by [Jungert, 1988, Davis, 1991, Holmes and Jungert, 1992, Zheng and Tsuji, 1992, Schlieder, 1993, Sutherland and Thompson, 1993, Dai and Lawton, 1993, Park, 1994, Escrig and Toledo, 1998]. Qualitative models are robust against incomplete data; however, problems arise when they try to distinguish between different but similar places to localize the robot position in the map.

More recently, hybrid approaches have been used to overcome the problems of the metric and topological models. Hybrid models try to combine the best of each approximation. One of the first models for map building was proposed by Thrun [1998], which combines the occupancy grids with topological maps. Other hybrid models can be found in [Arleo et al., 1999, Zanichelli, 1999, Musto et al., 1999, 2000, Kuipers, 2000, Remolina and Kuipers, 2002, Tomatis et al., 2003].

The work presented in this paper can be viewed as a hybrid approach, since we will relate geometrical information obtained from the environment with the corresponding qualitative representation. The process of extract useful information from the environment has an important effect over the final robot operation. In this paper, we remark the importance of a natural integration between metric and qualitative information. We have also looked for

[1] This work is partly supported by the Spanish CICYT project TIC 2003-07182.

a representation useful to carry out autonomous robot navigation using qualitative spatial reasoning.

In Figure 1, we can see a diagram where we show the model presented here. At the first level, nearest the world, we cope with metric information. However, we do not use a global reference system to merge that metric information to build a map, but we extract local reference systems and join them by means of their relative orientations, constructing a geometrical representation. In the next level, we extract qualitative features, which are used to identify the local reference systems. This level is interrelated with the geometrical representation giving a hybrid description of the environment. This description can be used by a qualitative reasoning process to carry out local path planning and navigation.

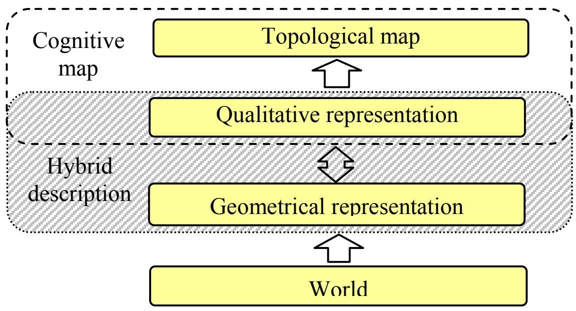


Figure 1. The model presented in this paper.

On top of the hybrid description, we construct a topological map. This topological map addresses global path planning and fixes the mid-term objectives.

Finally, integrating the qualitative representation and the topological map, we will obtain a cognitive map of the world, which relates in a natural way the different working scales of an autonomous robot navigation system.

In the next section, we will describe how we acquire and manage the geometrical information obtained from the world. Then, in section three, we will explain the process used to extract qualitative features from the geometrical data, modelling a hybrid representation of the environment. Afterwards, the creation of the topological map on top of the hybrid description will be showed. Finally, conclusions and future work will be explained.

2 Geometrical information

We are using in our research a commercial mobile robot simulation software, called Webots, developed by Cyberbotics[2] Ltd. We are working with a simulated robot, which has a 360º laser range finder sensor. Throughout this paper, we will restrict ourselves to the interpretation of the data extracted from the simulation by that laser sensor, although part of the work presented here is also being tested in a real robot.

[2] www.cyberbotics.com

We have made three basic assumptions about the kind of data the laser can detect in order to simplify the environment where the robot will navigate:

- The laser is placed at a fixed high from the floor, covering a horizontal line. All the objects existing in the world can be detected in this line.
- Objects do not have interior holes. This assumption is done to avoid the robot hitting with an object not detected.
- Corners are always of 90 degrees. This assumption has been imposed to reduce the complexity of the environments. In future works, we will include corners with different angles.

The laser sensor extracts a vector of distances where we can obtain a geometrical description of the environment (Figure 2). Now, we will formalize this information in order to lay the foundations for later explanations.

- Each sensor reading can be viewed as a pair of elements representing a polar coordinate $<d_i, a_i>$, where d_i is the distance to an obstacle obtained by the laser sensor and a_i the angle of that reading.
- With two sensor readings, i and j, we can obtain the distance between them, d(i,j), and the angles α, β, γ formed by these points and the robot, by means of the triangulation technique. We will use the cosine and sine formulas.

$$d(i,j)2 = di2 + dj2 - 2 \cdot di \cdot dj \cdot cos(\alpha)$$

$$d(i,j) / sin(\alpha) = di / sin(\gamma) = dj / sin(\beta)$$

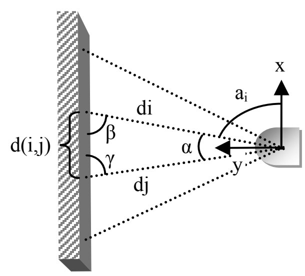


Figure 2. Geometrical information obtained from a laser sensor.

From this geometrical data, we will construct a hybrid spatial representation. In the next section, we will show how we can accomplish this representation.

3 Hybrid Representation

The hybrid representation defined in this paper relates geometrical information with the relevant aspects of the environment in a given qualitative representation. A qualitative representation can be defined as the representation which *makes only as many distinctions as necessary to identify objects, events, situations, etc. in a given context* [Hernández, 1994]. Qualitative representation describes relevant aspects of the environment focusing on the boundaries of objects.

First, we need to provide some definitions needed to relate each qualitative feature with its correspondent geometrical information,

DEFINITION 1. A **distinctive point** *dp* ∈ *DP,* is the bound between two detectable objects in a given context.

Where *DP* is the set of all the distinctive points defined.

DEFINITION 2. The **level of granularity** *G* defines the precision used in the sensor measurements.

In our case where a laser sensor is used, the level of granularity is defined by the angle between two consecutive measures. An increase in the granularity determines a smaller angle. The robot can adjust the granularity adapting the computations to the complexity of the environment or the necessity of the robot to detect specific small features.

DEFINITION 3. Two distinctive points will be **neighbours** if there is not any other distinctive point in the space situated between them,

$dpi, dpj \in DP$ *are neighbours* $\Leftrightarrow \neg \exists\, dp_k \in DP \,/\, i < k < j$

where $i < k < j$ means that a distinctive point dp_k is situated between the distinctive points dp_i and dp_j.

DEFINITION 4. A **reference system** is composed by two distinctive points, which are neighbours,

$RS_{ij} = \{(dp_i, dp_j) \,/\, dp_i, dp_i \in DP \wedge dp_i, dp_j \text{ are neighbours}\}$

The hybrid representation described in this paper uses reference systems as the basic construction blocks. It is by means of reference systems that we combine the geometrical information with the qualitative representation. Now, we shall explain the complete process needed to extract reference systems.

3.1 Extracting Reference Systems

The complete algorithm for the extraction of reference systems is showed in Figure 3,

1.Obtaining the vector of distances from the laser sensor

2.Calculating the vector of differences between adjacent distances

3.Filtering the vector of differences to eliminate spurious errors

4.For each position in the vector of differences

5. Extract distinctive points

6.Creating the current view

7.Extracting reference systems from the current view

Figure 3. Algorithm to obtain the reference systems.

We start from the vector of distances given by the sensor. Then, we calculate the differences between adjacent distances creating a new vector of differences. This vector of differences is filtered in order to eliminate erroneous measures. The filtering process calculates the similarity of each difference with respect to their neighbours, eliminating the discordant values. The number of neighbours examined is given by a window, whose size is determined by the level of granularity. Afterwards, we shall extract, for each position in the vector of differences, the distinctive points, which will be used to define the reference systems.

Form the data provided by the laser sensor used in this work, we can define a qualitative representation, which identifies concave and convex corners as the distinctive points of the environment. The identification process is carried out as follows,

The **Concave corners** can be detected as a change in the sign of two consecutive differences from positive to negative values (Figure 4).

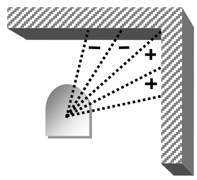
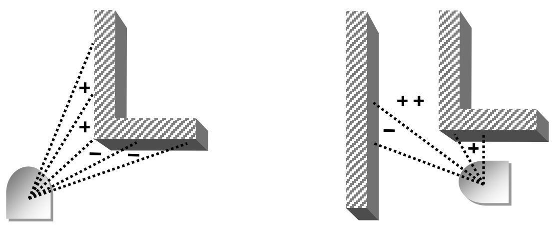


Figure 4. Concave corner extraction.

For the **Convex corners,** we can find two possibilities depending on the position of the robot with respect to the corner (Figure 5). In the Figure 5a, the robot can detect the two sides of the corner, finding a change in the differences from negative to positive values. In Figure 5b, there is a jump between two consecutive differences. Both cases are detected as convex corner.



a) b)

Figure 5. Convex corner extraction.

The example showed in Figure 5a could be confused with the situation appeared when the beam of the laser forms a perpendicular line with an obstacle (Figure 6). In order to differentiate this case from the real convex corner we compute the angles formed by the beam and the obstacle by the triangulation technique. If these angles are different than 90, we will be in the real convex case.

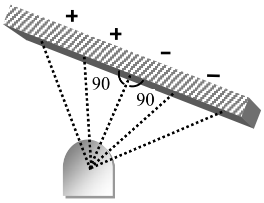


Figure 6. Differentiation between a convex corner and the perpendicular line formed by a laser beam and an object.

At this point, the algorithm has extracted a set of distinctive points from the sensor data. The description of each distinctive point is given by a set of three elements $<DP_i, D_i, A_i>$, where DP_i is the kind of distinctive point and the pair (D_i, A_i) represents the polar coordinate where it has been located.

The next step of the algorithm creates a vector, called the ***current view***, which represents the hybrid information extracted from a particular location of the robot in the environment (Figure 7).

+	+	+	DP_n	-	...	-	DP_1	+	+	+

Figure 7. An example of the vector, which represents the current view of the robot. The DP*i* symbols represents distinctive points and the + and – symbols symbolize positive and negative distance differences, respectively.

Each cell of this vector represents a sensor reading translated into a hybrid representation. We notice that we can add to this view the sign of the differences calculated in the distance difference vector (in those positions where there is not a distinctive point), and we could use this view to identify the current robot position, useful in the navigation process.

The distinctive points extracted from the environment are used directly to define reference systems. A reference systems is composed by two distinctive points which are neighbours, therefore, we will take each pair of consecutive distinctive points extracted from the current view. In Figure 8, we can see an example of an environment, which is explored by the robot. We remark that depending on the level of granularity defined, some characteristics could be missed.

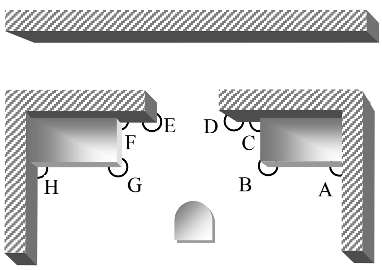


Figure 8. Example of environment with 8 distinctive points.

The set of distinctive points, extracted from the environment showed in Figure 8, will be the next:

{ (A , A_0 , D_0), (B , A_1 , D_1), (C , A_2 , D_2), (D , A_3 ,D_3), (E , A_4 , D_4), (F , A_5 , D_5), (G , A_6 , D_6), (H , A_7 , D_7) }

where,

A, B, C, D, E ,F ,G ,H are distinctive points;

(A_i, D_i) are the polar coordinates of each distinctive point.

We distinguish eight distinctive points, and we will identify seven reference systems composed by the distinctive points *AB, BC, CD, DE, EF, FG, GH*.

In addition, a distinctive point can be on the right, on the left or on the same position with respect to the middle of the vector, which represents the current view. This information is used to obtain the present position of the robot with respect to a reference system looking at the current view the situation of the distinctive points, which forms that reference system (Figure 9b).

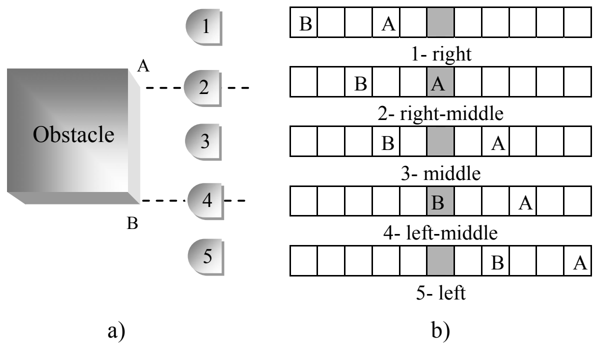


Figure 9. a) Robot positions with respect to the reference system AB extracted from an obstacle; b) current view of the robot where the position of the robot with respect to the reference system AB is obtained (the middle position of the current view is indicated by a shadow cell).

Therefore, the robot can be situated in five different positions with respect to a reference system: right, right-middle, middle and left-middle (Figure 9a). The different positions of the robot with respect to all the reference systems obtained in the current view are calculated to determine the present location of the robot.

3.2 Tracking Reference Systems

Every time the robot moves through the environment, it looks for the new location of the old reference systems. Furthermore, it looks for new reference systems.

The displacement of the reference systems is tracked by the robot using the successive views obtained from the environment. Considering that A and B are two consecutive distinctive points in the current view, therefore if the robot moves towards its right, the distinctive points inside the view will move towards the left in the successive views (Figure 10a), likewise if the robot turns towards its leftt, the distinctive points will move towards the right (Figure 10b).

<<	A		<<	B

a) Right turn

A	>>		B	>>

b) Left turn

Figure 10. Displacement of the distinctive points after a right or left turn of the robot. The symbols << and >> indicate the direction of the displacement.

In Figure 11, displacements corresponding whit a forward or backward movements of the robot are showed. In a forward movement, distinctive points will move towards the extreme of the vector. In a backward movement, distinctive points will move towards the centre of the vector.

<<	A		B	>>

a) Forward

A	>>		<<	B

b) Backward

Figure 11. Displacement of the distinctive points after a forward or backward movement of the robot. The symbols << and >> indicate the direction of the displacement.

The size of the tracking window where the robot looks for the new position of the distinctive points in the successive views is configurable taking into account the velocity of the robot movements and the level of granularity. We can characterize this concept as follows: the size of the tracking window is directly proportional to the product of the velocity of the robot and the granularity.

4 Cognitive Map

In the previous sections, we have showed the extraction of reference systems from the environment using geometrical data and qualitative features. In this section, we will show how we can combine reference systems to construct a map of the world. This map is a hybrid description; however, it contains the bases for the extraction of a topological description.

The final cognitive map will emerge from the integration between the qualitative representation contained in the hybrid description and the topological map. This cognitive map connects in a natural way places, transitions and topological relations.

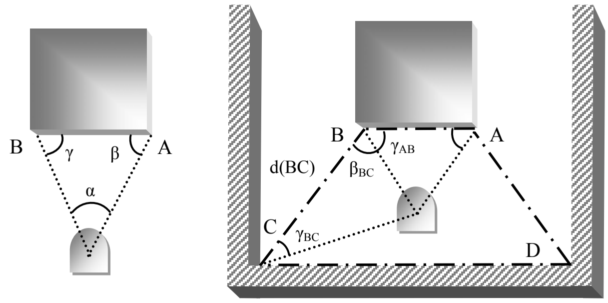
4.1 Relating Reference Systems

While the robot is moving through the world, reference systems are being extracting from the successive views. Therefore, we need to relate them in order to create a world map.

To relate two reference systems they have to be adjacent, that is, they have to share a distinctive point. We will use two measures obtained from the geometrical representation to situate a new reference system with respect to a previous one.

Given two reference systems AB and CD, we connect them by means of (Figure 12),

- The angle between both reference systems, that is, the sum $\gamma_{AB} + \beta_{BC}$
- The distance between the distinctive points B and C.



a) b)

Figure 12. Relating reference systems: a) extracting relevant information from the triangulation technique; b) relating reference system BC with the reference system AB by means of the distance between BC and the angle formed by $\gamma_{AB} + \beta_{BC}$

An important concept, called ***place***, appears when we create the relations between reference systems. Relations between reference systems define places in the map.

DEFINITION 5. Two reference systems will be **related** if they share a distinctive point

DEFINITION 6. A **place** is the space situated inside the area described by the minimum polygon formed by a set of reference systems consecutively related. Inside a **place**, the robot can see all the distinctive points belonging to that place.

While the robot moves through the environment, new information can be acquired and the existing places can be modified (nonmonotonic reasoning). These modifications will be imposed by the observance of the previous definitions during the navigation process.

DEFINITION 7. A reference system is **open** if the robot can pass through it.

DEFINITION 8. A **transition** between two places is done by traversing an open reference system shared by the two places.

As an example, in Figure 13, the reference systems AB, BC, CD and DA define the place ABCD. While the robot pass through the open reference system AB, new distinctive points are found creating a new place AEFB. The polygon AEFBCD is not a place because it has two subsets of related reference systems.

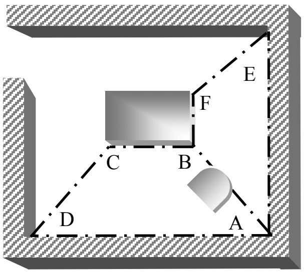


Figure 13. Definition of places.

4.2 Obtaining the Topological Map

The algorithm used for the creation of the topological map is showed in Figure 14. This algorithm works using the reference systems obtained by the algorithm described in the section 3.1.

1.Relating reference systems
2.Creating places
3.Making the correspondence between places and nodes
4.Making the correspondence between transitions and arcs

Figure 14. The algorithm for the extraction of the topological map

The topological map is extracted directly from the places defined in the previous section. Each place corresponds with a node in the topological map. In addition, the transitions between two places define the arcs of the topological map. Each arc stores the relation between the two places, indicating the kind of movement that the robot has to do to traverse them.

In Figure 15, we can see an example of an environment where places have been marked and the topological map is showed. The relation between this places and the topological map can be seen in a clear way.

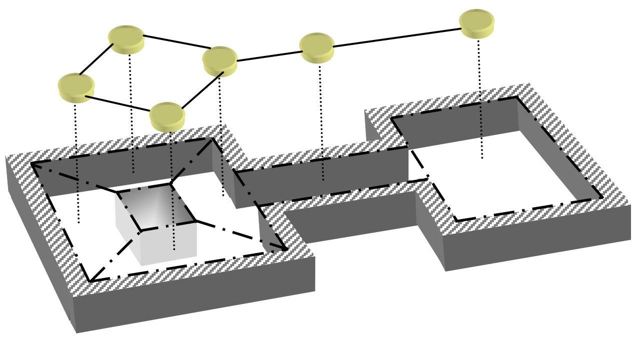


Figure 15. Extraction of the topological map.

With the model described in this paper the number of the places extracted (and the correspondent size of the topological map) is given by the complexity of the environment.

5 Conclusions and Future Work

In this paper, we propose a hybrid representation for autonomous robot navigation in indoor environments, which uses local reference systems as the basic construction blocks. We integrate metric and qualitative information, relating in a natural way the different working scales of an autonomous robot navigation system. We also have described the creation of a topological map as a natural extension from the hybrid description.

Furthermore, we have showed how a cognitive map can emerge from the combination of the hybrid description and the topological map. We have obtained this cognitive map proceeding in two steps: (1) we have extracted a hybrid representation reflected in the successive *current views* observed by the robot, (2) we have extracted a topological map on top of the hybrid representation by means of the relations between local reference systems.

The work reported on this paper is in progress. We are testing this model in a simulation software and in a real robot to extract a complete set of results, as the efficiency or the complexity, about the maps obtained. We are also examining the influence of wrong sensor data in the map creation. In addition, we are studying the application of qualitative spatial reasoning to plan the movements of the robot during its exploration of the environment. Moreover, future works will focus on the integration of several kinds of qualitative representation and qualitative reasoning inside the model presented here.

Acknowledgements

The ideas described in this paper have been discussed with the members of the Cognition for Robotics Research Group of the University Jaume I. We want to specially thank their contribution.

References

[Arleo et al., 1999] A. Arleo, J. del R. Millán and Dario Floreano. *Efficient Learning of Variable-Resolution Cognitive Maps for Autonomous Indoor Navigation.* IEEE Trans. on Robotics and Automation, vol. 15(6), pages 990-1000, 1999.

[Brooks, 1982] R. A. Brooks. *Solving the find-path problem by good representation of free space.* In Proc. AAAI-82.

[Crowley, 1985] J. L. Crowley. *Navigation for an intelligence mobile robot.* IEEE Journal of Robotics and Automation, RA-1(1):31-41, 1985.

[Dai and Lawton, 1993] D. Dai and D. T. Lawton. *Range-free qualitative navigation*. In Proceedings of the IEEE Robotics and Automation, pag783-790, 1993.

[Davis, 1991] E. Davis. *Representing and Acquiring Geographic Knowledge.* Morgan Kaufmann Publishers, Inc., Los Altos, California, 1991.

[Escrig and Toledo, 1998] M. T. Escrig and F. Toledo. *Qualitative Spatial Reasoning: Theory and Practice. Application to Robot Navigation*. IOS Press, Frontiers in

Artificial Intelligence and Applications, ISBN 90 5199 4125, 1998.

[Felip and Escrig, 2003] P. Felip, M. T. Escrig. *Qualitative Spatial Reasoning for Robot Path Planning*. In press for the CCIA'2003.

[Freksa, 1992] C. Freksa. *Using Orientation Information for Qualitative Spatial Reasoning*. A.U. Frank, I. Campari, U. Formentini (eds.), Theories and Methods of Spatio-Temporal Reasoning in Geographic Space, LNCS 639, Springer-Verlag Berlin, 1992.

[Freksa et al., 2000] C. Freksa, R. Moratz and T. Barkowsky. *Schematic maps for robot navigation*. In C. Freksa, W. Brauer, C. Habel, & K. F. Wender (Eds.), Spatial Cognition II - Integrating abstract theories, empirical studies, formal models, and practical applications (pp. 100-114), Berlin: Springer, 2000.

[Giralt et al., 1979] G. Giralt, R. Sobek and R. Chatila. *A multi-level planning and navigation system for a mobile robot*. In IJCAI-79, pag. 335-337, 1979.

[Holmes and Jungert, 1992] P. D. Holmes and R. A. Jungert. *Symbolic and Geometric Connectivity Graph Methods for Route Planning in Digitized Maps*. In IEEE Transactions on Pattern Analysis and Machine Intelligence, vol. 14, No. 5, May 1992.

[Jungert, 1988] E. Jungert. *Extended sysmbolic projections as a knowledge structure for spatial reasoning*. In kittler, J., editor, 4th International Conference on Pattern Recognition, Volume 301 of Lecture Notes in Computer Science, pag. 343-351, Springer, Berlin, 1988.

[Kosaka and Kak, 1992] A. Kosaka and A. C. Kak. *Fast Vision-Guided Mobile Robot Navigation Using Model-Based Reasoning and Prediction of Uncertainties*. Computer Vision, Graphics and Image Processing-Image Understanding, vol.56, no.3, pp.271-329, 1992.

[Kuipers, 1978] B. Kuipers. *Modelling Spatial Knowledge*. Cognitive Science, 2 (1978), 129-153.

[Kuipers and Byun, 1988] B. Kuipers and Y. Byun. *A Robust, Qualitative Method for Robot Spatial Reasoning*. Proceedings of the seventh National Conference on Artificial Intelligence, pag. 774-779, 1988.

[Kuipers and Levitt, 1988] B. Kuipers and T. S. Levitt. *Navigation and Mapping in Large-Scale Space*. AI Magazine, 1988.

[Kuipers, 2000] B. Kuipers. *The Spatial Semantic Hierarchy*. Artificial Intelligence 119 (2000), 191–233.

[Levitt and Lawton, 1990] T. S. Levitt and D. T. Lawton. *Qualitative Navigation for Mobile Robots*. Artificial Intelligence, Vol. 44 (1990), 305-360.

[Lozano-Pérez, 1981] T. Lozano-Pérez. *Automatic Planning of Manipulator Transfer Movements.* IEEE Transactions on Systems Mans and Cybernetics, SMC-11(10):681-698, October 1981.

[Moravec and Elfes, 1985] H. P. Moravec and A. Elfes. *High Resolution Maps from Wide Angle Sonar*. Proc. IEEE Int'l. Conf. Robot. and Automat., St Louis(1985), 116-121.

[Musto et al., 1999] A. Musto et al. *Qualitative and Quantitative representations of Locomotion and their application in robot navigation*. In Proceedings of the IJCAI-99, Morgan Kaufman Publishers, 1999.

[Musto et al., 2000] A. Musto et al. *From motion observation to qualitative representation.* In C. Freksa, C. Habel, C. Hendel (eds.), Spatial Cognition II. LNCS-Series, Springer Berlin, Heidelberg, New York, 2000.

[Park, 1994] Park, II-Pyung. *Qualitative Environmental Navigation: Theory and Practice*. PhD thesis in the Graduate School of Arts and Sciences, Columbia University, 1994.

[Remolina and Kuipers 2002] E. Remolina and B. Kuipers *Towards a general theory of topological maps*. Submitted to Artificial Intelligence Journal, 2002.

[Schlieder, 1993] C. Schlieder. *Representing Visible Locations for Qualitative Navigation*. In Proceedings of the III IMACS International Workshop on Qualittive Reasoning and Decision Technologies —QUARDET'93—, pag. 523-532, 1993. CIMNE, Barcelona.

[Sutherland and Thompson, 1993] K. T. Sutherland and W. B. Thompson. *Inexact Navigation*. In Proceedings of the IEEE Robotics and Automation, pag. 1-7, 1993.

[Thrun, 1998] S. Thrun. *Learning Metric-Topological Maps for Indoor Mobile Robot Navigation*. Artificial Intelligence, Vo. 99, No. 1 (1998), 21-71.

[Thrun, 2002] S Thrun. *Robotic mapping: A survey.* In G. Lakemeyer and B. Nebel, editors, Exploring Artificial Intelligence in the New Millenium. Morgan Kaufmann, 2002.

[Tomatis et al., 2003] N. Tomatis, I. Nourbakhsh and R. Siegwart. *Hybrid simultaneous localization and map building: a natural integration of topological and metric*. Robotics and Autonomous systems 44 (2003) 3-14.

[Zanichelli, 1999] F. Zanichelli. *Topological maps and robust localization for autonomous navigation*, IJCAI workshop on Adaptive spatial representations of dynamic environments, 1999.

[Zheng and Tsuji, 1992] J. Y. Zheng and S. Tsuji *Panoramic Representation for Route Recognition by a mobile robot*. International Journal of Computer Vision, 9(1):55-76, October, 1992.

[Zimmermann and Freksa, 1996] K. Zimmermann and C. Freksa. *Qualitative spatial reasoning using orientation, distance, and path knowledge*. Applied Intelligence, 6, 49-58, 1996.