Consolidating Pipes

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1 Introduction

Consolidation is inferring the behavioral description of a device by composing the behavioral descriptions of its components. The intent is to infer the behavioral laws of the device without assuming a scenario (i.e., initial state and external interactions). Consolidation differs from qualitative simulation (Bobrow, 1985) in that qualitative simulation assumes such a scenario (Bylander, 1988a). Consolidation differs from envisioning, i.e., the generation of all possible qualitative states and state transitions of a device, because the result of consolidation is an intensional description.

In previous work, we proposed a conceptual representation and reasoning process for performing consolidation (Bylander and Chandrasekaran, 1985). This proposal is primarily based on predicating paths within the components with their conceptual behavior (e.g., allow, pump, move) and inferring the conceptual behavior of path combinations. Other work has showed how qualitative differential equations (QDEs) can be composed (Dormoy and Raiman, 1988; Williams, 1988). However, little work has been done to show how the conceptual level can be related to the QDE level.

The main motivation for our work is computational complexity. *Because solving QDEs is in general computationally intractable (Davis, 1987), any theory that relies on QDEs without imposing sufficient restrictions on them does not explain how qualitative physics problems can be tractably solved.*[1] As a consequence, the practicality of such theories on specific problems cannot be determined in advance of implementation. Our hope is that a conceptual analysis can provide sufficient restrictions on QDEs.

So far our results are: (1) a general schema for pipes that can be instantiated with any number of ports and that fully supports the intuition that two pipes connected together behave like a single pipe, and (2) the conditions under which the behavioral description of a component also describes the consolidation of the component with one connection to a pipe. It is tractable to perform consolidation of a configuration of pipes in the first case and a pipe connected to a component in the second case. This partially confirms the "allow" conceptual behavior and the inferences using "allow" in our previous paper (Bylander and Chandrasekaran, 1985). These results are briefly described below. See Bylander (1988b) for more details.

To describe these results, we adopt the notation of Q1, the qualitative algebra proposed by Williams (1988), with the following variations in notation. We add $\approx$ to denote "qualitative equality." That is, for all $s_1, s_2 \in \{[-],[0],[+],[?]\}$, $s_1 \approx s_2$ iff $s_1 = s_2$ or $s_1 = [?]$ or $s_2 = [?]$. Instead of $d/dt(x)$, we use ∂x.

2 Pipes

Figure 1 is the schema for pipes with n ports, $n \geq 1$. Q_i represents the rate of flow into the pipe through $port_i$; Q_i is negative if the substance is flowing out through $port_i$. P_i represents the "pressure" of the substance at $port_i$. Semantics of connection are as follows. Each port can be connected to at most one other port. Both ports of a connection must be for the same type of substance. If $port_i$ of one component is connected to $port_j$ of another component, then $Q_i = -Q_j$ and $P_i = P_j$.

The first constraint in Figure 1 states that the sum of the rates is zero. In essence, it says that substance is conserved. The second constraint, which follows from the first, applies to the first derivatives of the rates. The third line specifies a group of n constraints, relating each Q_i to the pressures. It says that the direction of flow for any $port_i$ (the sign of Q_i), corresponds to the "sum" of pressure differences (the sign summation

*This work has been supported by the Air Force Office of Scientific Research through grant AFOSR-87-0090.

[1] It is worthwhile noting that exceptions to this generalization such as Weld's comparative analysis (Weld, 1988) usually require the results of a qualitative simulation or envisionment.

ports:	$port_1, \ldots, port_n$
quantities:	$Q_1, P_1, \ldots, Q_n, P_n$
constraints:	$\sum_{i=1}^{n} Q_i = 0$
	$\sum_{i=1}^{n} \partial Q_i = 0$
	$[Q_i] \approx \bigoplus_{j=1}^{n}[P_i - P_j], \quad 1 \leq i \leq n$
	$[\partial Q_i] \approx \bigoplus_{j=1}^{n}[\partial P_i - \partial P_j], \quad 1 \leq i \leq n$

Figure 1: General Pipe Schema

of P_i minus other pressures). The fourth line specifies similar constraints for the first derivatives.

At first glance, the schema appears to be highly ambiguous. For example, in the case of 4 ports, the constraint for Q_1 would be (after a simplification):

$$[Q_1] \approx [P_1 - P_2] \oplus [P_1 - P_3] \oplus [P_1 - P_4]$$

From this QDE, the sign of Q_1 can be inferred only if P_1 is the maximum or the minimum of the 4 pressures. Applying qualitative simulation to constraints like these would result in a combinatorial number of possible state sequences. Nevertheless, we have demonstrated the following:

> **Theorem 1** If two components can be described by the general-pipe schema and they are connected, then the general-pipe schema describes their consolidation.

That is, if a pipe with m ports has k connections to a pipe with n ports ($k \leq m$ and $k \leq n$), their consolidation can be described as a pipe with $m+n-2k$ ports. Consequently, the consolidation of any configuration of pipes can be done very efficiently.

Also, we would expect that the consolidation of a component with a single connection to a pipe would be about the same as the original component and thus, also very efficient. We have determined a sufficient and necessary condition under which this inference can be performed.

Consider a two-ended pipe α connected to a component β. For convenience, assume $port_2^\alpha$ is connected to $port_1^\beta$. From the behavioral description of the pipe, it is clear that $Q_1^\alpha = Q_1^\beta$ and $\partial Q_1^\alpha = \partial Q_1^\beta$, so Q_1^α and ∂Q_1^α can be substituted for Q_1^β and ∂Q_1^β respectively in β's behavioral description. However, P_1^α and ∂P_1^α will be different from P_1^β and ∂P_1^β, e.g., $Q_1^\beta > 0$ implies $P_1^\alpha > P_1^\beta$. Hence, if P_1^α and ∂P_1^α are to be substituted for P_1^β and ∂P_1^β, then β's behavioral description must be insensitive to the differences between these quantities implied by the pipe's behavioral description. This leads to the following:

> **Theorem 2** Let $port_i$ of a two-ended pipe α be connected to $port_j$ of a component β. Let $\mathcal{Q}$ be the set of β's quantities (including any derivatives mentioned in β's behavioral description). Let $\mathcal{V}: \mathcal{Q} \rightarrow \Re$ denote an assignment of values to β's quantities. Consider the behavioral description derived by substituting the quantities of α's other port for the quantities of $port_j^\beta$. Then this behavioral description describes the consolidation of α and β if and only if the following condition (the "pipe axiom") holds:

$$\begin{aligned} \forall \mathcal{V}, \mathcal{V}' \; ((\mathcal{V} \text{ satisfies } \beta\text{'s behavioral description} \\ \wedge \forall q \in \mathcal{Q} \backslash \{P_j^\beta, \partial P_j^\beta\} \; (\mathcal{V}'(q) = \mathcal{V}(q)) \\ \wedge [\mathcal{V}(Q_j^\beta)] \approx [\mathcal{V}'(P_j^\beta) - \mathcal{V}(P_j^\beta)] \\ \wedge [\mathcal{V}(\partial Q_j^\beta)] \approx [\mathcal{V}'(\partial P_j^\beta) - \mathcal{V}(\partial P_j^\beta)]) \\ \Rightarrow \mathcal{V}' \text{ satisfies } \beta\text{'s behavioral description}) \end{aligned}$$

Thus, if the pipe axiom is true for each of a component's ports, then the component can be consolidated with two-ended pipes without, in essence, any changes in its behavioral description.

What happens if the pipe has more than two ports? If the pipe axiom holds for the connected port of the component, then the following rules can be employed to derive a description of the consolidation (for convenience, we assume $port_1^\alpha$ of the pipe is connected to $port_1^\beta$ and α has n ports):[2]

1. If $[Q_1^\beta] \approx e$, where e is any qualitative expression, then the following set of constraints are implied:

   $$[Q_i^\alpha] \approx e_i' \oplus \bigoplus_{j=2}^{n}[P_i^\alpha - P_j^\alpha], \quad 2 \leq i \leq n$$

   where e_i' is derived by substituting P_1^β with P_i^α.

2. Substitute Q_1^β with $\sum_{j=2}^{n} Q_j^\alpha$.

3. Substitute any expression $[e]$ that contains P_1^β with $\bigoplus_{j=2}^{n}[e_j']$, where e_j' is derived by substituting P_1^β with P_j^α.

4. The above rules also apply to the first derivatives.

Thus, Theorem 2 can be generalized to encompass the general-pipe schema without changing the pipe axiom, even though the pipe axiom is derived from the two-ended case.

[2] These rules are not "complete"—they do not handle all possibilities, e.g., substitution of P_1^β cannot be performed unless it is contained in a sign coercion expression.

3 Remarks

Our approach of using schemas and schema composition rules to simplify and analyze devices is similar to the "slices" of Sussman & Steele (1980) and the "teleological rules" of de Kleer (1984). One major difference in our analysis is the generality of the schemas and rules; both Sussman's and de Kleer's rules only apply to electrical circuits. Another is that our schemas and rules are firmly grounded in qualitative algebra.

We have shown that the behavior of pipes can be qualitatively described so that consolidation of pipes and of components with pipes can be performed tractably. Whether our approach can be extended to additional types of components (e.g., containers, pumps, transformers) is the subject of further investigation.

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