Reasoning about Kinematic Topology

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Abstract

Reasoning about kinematics is an important aspect of common sense physics. In earlier work, we have developed the place vocabulary theory of qualitative kinematics in mechanisms, a formal theory for representing the kinematic behavior of two-dimensional mechanisms. The computation of a place vocabulary is very complex because it takes into account the details of object shapes. In this paper, we present a representation which is much more abstract than a place vocabulary, the *kinematic topology*. Kinematic topology does not define qualitative inference rules, but provides a characterization of the topology of legal configurations. For example, the kinematic topology of a pair of gears is one or several doubly connected regions, whose shape in configuration space indicates the relative speeds of the two gears. For many applications, reasoning about kinematics at this level is sufficient.

Kinematic topology can be computed in a qualitative manner and thus gives an existance proof that a purely qualitative kinematics is possible. Like in other qualitative reasoning applications, the qualitative computation has the effect that the result is almost always ambiguous. On the other hand, a kinematic topology can be given even for mechanisms whose designs are only imprecise sketches, and can be generalized to arbitrary object shapes, several degrees of freedom, and three dimensions. We hope that such generalizations of kinematic topology can provide the basis for efficiently computing place vocabularies, and reasoning about general kinematic interactions.

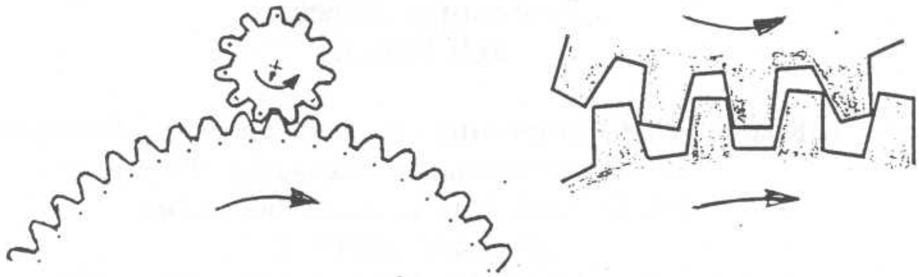

Figure 1: *A pair of gearwheels. The drawing on the left shows an actually working device, while the one on the right is only a sketch that will not work as shown.*

1 Kinematic Topology

Reasoning about kinematic behavior is an important problem in commonsense physics. A large proportion of physical systems involve some form of kinematic interaction, and few methodologies are known for first-principles modeling of kinematics. In earlier work, we have developed the *place vocabulary* theory for the special case of mechanism kinematics. It provides a general first-principles formalism capable of describing the behavior of complex device such as a mechanical clock [FALT87b,FALT87a,NIE88].

The place vocabulary describes the kinematic behavior of a device as a state graph of different contact relationships. Each different pair of object parts which can be in contact forms a distinct state. There are aspects of human reasoning where this representation is overly detailed. Consider the example of a pair of gearwheels, shown in Figure 1. A mechanism made precisely to the dimensions shown in the drawing on the left will actually work. Its behavior can be analyzed by precise computation on the given data, resulting in an unambiguous place vocabulary. However, the sketch on the right is far from a functional gear, and its precise analysis will certainly not reveal a gear function. Yet people are capable of predicting that the gear function is possible, given that the dimensions are adjusted properly.

The metric diagram computation model developed for the place vocabulary theory [FALT88b,FALT88a] provides one solution to this problem. It allows us to make complete lists of all possible place vocabularies, and thus all possible behaviors, which may be achieved by variation of the dimensions of the parts. However, this list will be unecessarily big, distinguishing all the different ways that the teeth could mesh or not mesh. A much more appropriate level of analysis would distinguish only five different cases, each of which corresponds to a different topology of the set of legal configurations[1]:

1. The device is impossible to contruct, because the parts overlap each other in all possible configurations: the set of legal configurations is empty.
2. The gears block each other, and both wheels can only turn a small amount: several simply connected sets.
3. The teeth mesh properly: one or more doubly connected sets.
4. The teeth do not mesh, and the wheels can turn independently of each other: a multiply (> 2) connected set.
5. No contact between the parts is possible: a simply connected set containing all imaginable configurations.

We call such a description the device's *kinematic topology*. Kinematic topology expresses the connectedness of configuration space and the form of its regions. For example, when the gears mesh properly, the doubly connected regions extend n_1 times across the motion parameter of the first gear, and n_2 times across the motion parameter of the second gear, where the ratio n_1/n_2 is the ratio of the number of teeth. A description at this level is sufficient for many applications of reasoning about kinematics.

Extracting the kinematic topology from place vocabularies or configuration space is not very promising, however, as it presupposes that these stronger descriptions have already been computed. The main point of this paper is that the possible kinematic topologies can be determined directly based on only a symbolic description of the objects, and qualitative information about their relative dimensions. Note the qualification: without metric information, only the possibilities can be listed. Determining the actual kinematic topology in general is not significantly easier than computing the device's complete place vocabulary. To see why this is the case, consider how intricately the meshing of the teeth depends on their precise shape! The existance of such ambiguities is a necessary consequence of the qualitative nature of the representation.

The fact that kinematic topologies can be computed in a purely qualitative manner is a contradiction to the poverty conjecture made earlier ([FNF87]), which states that no purely qualitative kinematics is possible. Note, however, that kinematic topology is not strong enough to compute an envisionment of the device's behavior.

[1] Assuming that the periodicity of the parts is given

Kinematic topologies are of interest not only as a qualitative description, but they can also form the basis for a much more efficient computation of place vocabularies. Preliminary tests indicate that the resulting algorithm, called the bubble algorithm, is about 100 times faster than the earlier implementation which was based on configuration space computation. More importantly, kinematic topology can be determined not only for problems with few degrees of freedom, but also for much more complicated ones where the computational cost of computing with a very high dimensional configuration space is prohibitive. It can provide the basis on which efficient generalizations of the place vocabulary theory can be built.

The qualitative nature of kinematic topology also allows it to be computed for arbitrary object shapes, eliminating restrictions on the type of boundary curves allowed. For lack of space, we describe first its derivation for the case of polygonal objects with rotational freedom, and then indicate how it is generalized.

2 Computing Kinematic Topology

The kinematic behavior of a pair of objects is determined by the condition that they may not overlap each other. This condition defines a set of illegal configurations, the *blocked space*. Kinematic behavior, on the other hand, refers to the set of legal configurations, the *free space*, which, by its definition as the absence of overlaps, can only be determined as the complement of blocked space. Both spaces can be represented as regions in the space of possible configurations of the objects, the *configuration space* (as in [FALT87b, FALT87a]).

A configuration is part of blocked space if there exists a pair of object parts which would overlap in the configuration. Pairs of object parts are therefore the elementary building blocks for computing a description of blocked space.

Object Description In the following discussion, we refer to the pairwise interaction of two polygonal objects A and B, each of which has freedom of rotation only. We assume that the boundary of each of the objects is described as a sequence of *pieces* and *cavities*. A piece is centered around a convex vertex and consists of the vertex and the two boundary segments which are joined there (extending to infinity). A cavity is the complement of a piece, centered either around a concave vertex or a minimum of the radius to the center of rotation, and the two boundaries adjoining it[2]. A sample decomposition is shown in Figure 2.

[2] In the case of a radius minimum, the two halves of the boundary segment.

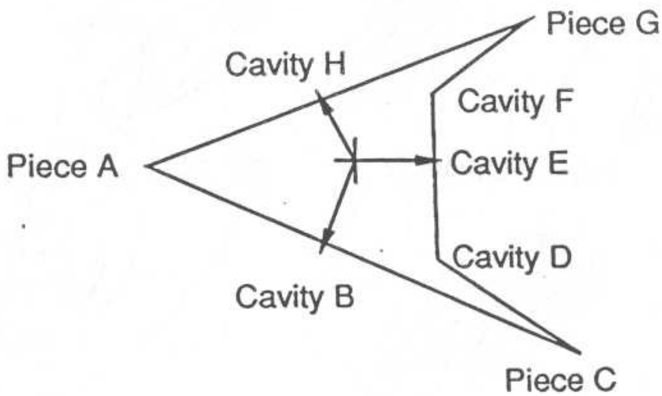

Figure 2: *The decomposition of an object boundary into pieces and cavities.*

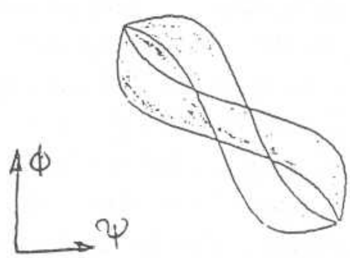

Figure 3: *Example of an obstacle.*

Formulating the non-overlap condition Since the two objects A and B are rotating around a fixed center, the configuration space of such a pair is two-dimensional, spanned by the orientations ϕ and ψ of A and B. The fact that two pieces P_A and P_B on objects A and B can not overlap yields the condition that none of the line segments bounding P_A and P_B can intersect each other, and none of the two can be entirely inside the other. For each pair of object pieces, this defines an *obstacle*, the set of configurations which violate the condition. The boundary of the obstacle is given as the envelope of four constraint curves, which consist of the configurations where a vertex of one piece touches one of the two boundary segments belonging to the other piece. An example of such an obstacle is shown in Figure 3. An obstacle contains two *touchpoints* where all four bounding constraints intersect each other. They are configurations where the vertices of the two objects touch each other. When the sizes of the object pieces generating the obstacles is very different, there are cases where the obstacle is in fact a doubly connected set, enclosing a region of potential free space.

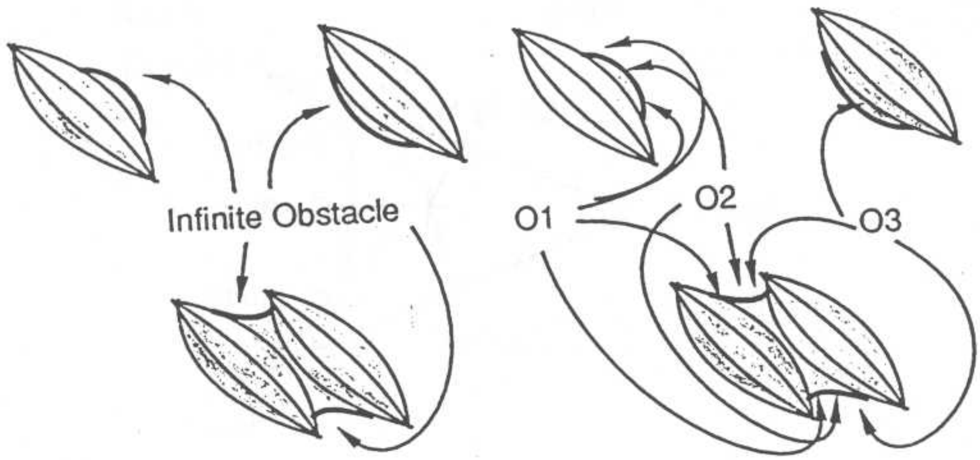

Figure 4: *Infinite obstacles connect or modify obstacle boundaries.*

Because object pieces are assumed to extend to infinity, the obstacles often also contain many configurations where no overlap exists, and we call these parts of the obstacle invalid. The importance of the obstacles lies in the fact that any boundary between free and blocked space is part of the boundary of some obstacle (see [FALT87a,BLP83]). Our algorithm composes the correct boundary between free and blocked space by composing only the boundary elements of the valid parts of the obstacles. The obstacles serve as *tokens* which permit an efficient organization of these elements.

In the case of several adjacent pieces, the obstacles they generated have to be intersected to form the true region of blocked space. For kinematic topology, this means that these obstacles can be combined into a single token of a blocked space area.

The initial topology graph Between the obstacles thus centered in configuration space lie regions of potential free space, described by *bubbles*. Intuitively, a bubble describes the potentially legal configurations where a piece on one object falls within a cavity on the other. It is a token which stands for the interaction of either a piece and a cavity or two cavities, and takes its shape from the surrounding obstacles and bubbles.

When two cavities follow each other on an object boundary, they enclose between them a boundary segment which does not belong to any piece. This segment can also generate a boundary between free and blocked space, which is represented by a token called an *infinite obstacle*. An infinite obstacle can not exist independently, but only modify or link together obstacles to which it becomes adjacent, as shown in Figure 4.

The first approximation of the kinematic topology computed by the algorithm is an array of alternating bubbles and obstacles, where the sequence of

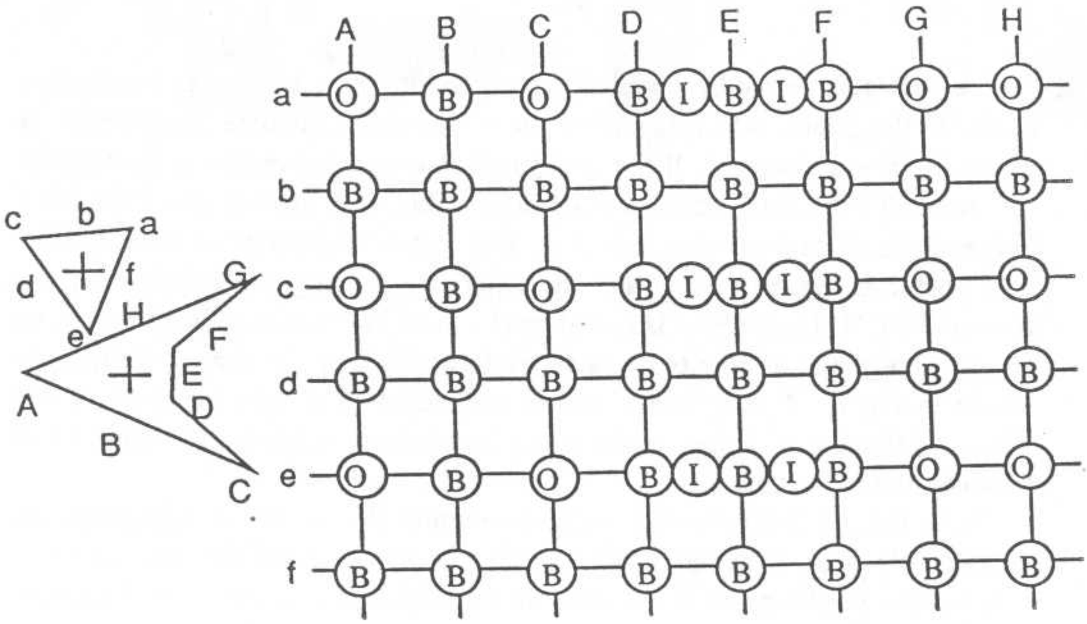

Figure 5: *An example of the initial topology graph.*

bubbles and obstacles in each dimension reflects the sequence of pieces and cavities along the object boundary. An example of such a graph is shown in Figure 5.

The weak topology graph The initial topology graph can be computed without any metric tests whatsoever, but does not yet say very much about kinematic topology. A much more expressive version, the *weak topology graph*, can be obtained by a modifications based on the tests involving information only about the distance of the centers of cavities and pieces from the center of object rotation, and the distance between the centers of rotation of the objects. This classification involves strictly linear distance comparisons and can be handled via quantity spaces or even order-of-magnitude reasoning.

First, if a pair of object parts P_A and P_B are too far apart or too close together to touch each other, there is no configuration that falls within the corresponding obstacle, and it is marked inactive and ignored in further processing. If we let r_P^{min} and r_P^{max} denote the minimum and maximum radius (distance from the center of rotation) for any point on piece P, and d be the distance between the two centers of rotation, this condition can be expressed as:

$$\begin{aligned} |r_{P_A}^{min} - r_{P_B}^{min}| &> d \text{ too close together, or} \\ r_{P_A}^{max} + r_{P_B}^{max} &< d \text{ too far apart} \end{aligned}$$

These linear distance comparisons can in general be carried out with only very rough information about object dimensions. In the case where not enough information is available, a case split results.

Obstacles separated by a single bubble often intersect each other and destroy or divide the bubble which separates them. Consider a bubble generated by a cavity C_A and a piece P_B. If the configuration where the vertex of P_B touches the point of minimum radius of C_A is achievable, this point is an intersection between the obstacles generated by P_B and each of the pieces adjacent to C_A. The obstacles are marked as directly adjacent, dividing the bubble between them in half. This condition is tested by the same radius comparisons described above. The shape of infinite obstacles is determined by the same criterion. As shown in Figure 4, they can either be connected to a finite obstacle on one side only, forming a "bulge", or be connected on both sides, thus forming a link between the finite obstacles.

Note that all the tests necessary to compute the weak topology graph are quite simple distance comparisons. We will discuss the use of the weak topology graph after finishing the description of the algorithms to compute kinematic topology.

The full topology graph In the weak topology graph, we have incorporated those aspects of the metric dimensions which can be easily captured by qualitative representations. However, this can not be enough for a complete description of kinematic topology. For example, deciding whether the teeth of a pair of gearwheels mesh requires evaluation of nonlinear expressions for which we have not even found a closed-form solution. The reason why computing the full topology is hard is that is must take into account the occurence of *free subsumptions*, subsumptions between obstacles which involve two distinct points of contact.

Devices in which free subsumptions are important are difficult to understand for people also, and we can do so only in very limited cases, such as that of successive teeth touching each other in gearwheels.

However, the situation is not hopeless, because the set of possible subsumptions can often be bounded. The most powerful criterion for bounding the set of possible subsumptions is the extent of an obstacle in configuration space. The valid portions of an obstacle can be enclosed within two possibly overlapping *bounding rectangles*, which enclose all valid portions of obstacle boundaries. Only pairs of obstacles whose bounding rectangles intersect are candidates for free subsumptions.

In general, testing whether the subsumption actually occurs requires a detailed (and expensive) analysis of the precise dimensions of the objects' shapes. In many practical cases, however, it can be determined that a subsumption *must* occur, by finding a valid point on an obstacle which falls within the valid region of another. For this purpose, we test whether the configurations corresponding to the touchpoints enclosed by the intersecting bounding rectangles violate the non-intersection constraints of the other obstacle. In practical cases, such as gearwheels, this test has proven very powerful and reliable.

Both the computation of bounding rectangles and the tests of non-intersection involve the nonlinear function relating radius to angle of touch. They can either be approximated, or computed by manipulating an analogical representation, for example in the vision system.

A free subsumption is installed in the topology graph by establishing a direct connection between the obstacles involved in the subsumption, dividing in half all intervening bubbles. As the individual bubbles do not have precise boundaries in configuration space, the exact shape of the path is of no importance - in our implementation, we chose the shortest one (the one that requires modifying the fewest bubbles).

The resulting structure now correctly describes the kinematic topology of the device. Every free subsumption candidate whose validity could not be determined means that the existance of the corresponding adjacency in the topology graph could not be decided, and the solution is ambiguous. The bubbles and their connectivity represent the topology of the free space. Additionally, each bubble can be associated with the interaction (or rather closeness) of particular object features. The conditions which have resulted in the transformation of the initial topology graph to the full topology graph are a causal explanation for those aspects of the final result. In the topology graph, we mark adjacencies where the actual positions correspond to zero-crossings of the respective orientation parameter. The shape of each region in configuration space - for example, the number of rotations of each gear required to traverse the doubly connected regions for a pair of gearwheels - is determined by counting the traversals of marked links on a path around the region boundary.

3 Applications and Extensions

Kinematic topology by itself is often a sufficient description for reasoning about kinematics. For example, the kinematic topology of gearwheels describes succinctly their behavior as two objects which can turn in coordinated motion. Note, however, that kinematic topology is not a substitute for place vocabularies: the behavior of a clock escapement, for example, is only described as a doubly connected free-space region, which doesn't capture its function.

An interesting aspect is the fact that kinematic topology often contains ambiguities. This reflects the ambiguity which arises in human reasoning based on rough sketches. For problem solving, it can be an advantage because it points out the possibilities which could be achieved given that the details are chosen in the proper way. In the example of a sketch of gearwheel behavior the interpretation of a functioning set of gears requires very particular shapes and dimensions of the gearteeth, which are far more likely to be violated than satisfied. Yet, it is the only one which exhibits a remarkable and useful function

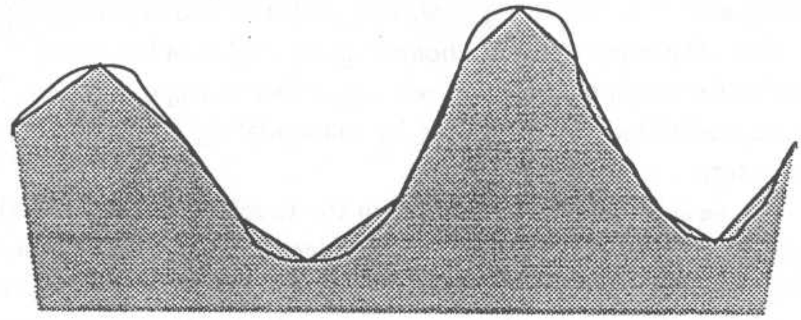

Figure 6: *Convex segments of an arbitrary curve can be approximated by sequences of pieces, and concave segments by sequences of cavities.*

not achieved by most other objects, and is therefore chosen as the desired interpretation.

The algorithm described in this paper has been implemented as part of a research project in automatic mechanism design and has proven robust and efficient. It is between 500 and 5000 times faster than computing a complete place vocabulary, and can work using less precise object descriptions.

Generalizations An important aspect of kinematic topology is that it is easily generalizable. The topological computation can be extended without great difficulty to devices with more than two degrees of freedom. Because bubbles and obstacles are tokens, their nature does not change in higher-dimensional configuration spaces. An important difference, however, lies in the potentially much higher number of ambiguities which may result in such a device.

Another important generalization is that of object shapes. For reasons of conciseness, we have so far assumed objects to be polygons. The same theory applies, however, to arbitrary boundary shapes, where pieces are defined by convex segments of the boundary, and cavities by concave ones. Imagine the boundary approximated by a very fine polygon, as shown in Figure 6. Each convex segment is then a sequence of convex vertices (pieces), and each concave segment one of concave vertices (cavities). The obstacles generated by the adjacent pieces form one contiguous region of blocked space, and the bubbles generated by the adjacent cavities generate a contiguous region of free space, possibly broken in half by the succession of the infinite obstacles between them. As the grain size of the approximation becomes infinitely fine, this becomes the kinematic topology of the complex shapes.

Topologically, both the succession of obstacles or bubbles can equally well be modelled by a single obstacle and a single bubble, generated by pieces and cavities formed by the convex and concave segments of the boundary curve.

The condition for the existance of the combined obstacle is that at least one of its component exists, which is the case if and only if the one at the extremum of the radius exists. If the point of extreme radius is taken as the "vertex" of the combined piece, it will correctly predict the existence of the combined obstacle. An equivalent result holds for combined cavities.

Note that this generalization requires the same division into segments of equal curvature that has already been proposed on independent grounds by vision researchers such as ([BRAS86]).

The Bubble Algorithm The kinematic topology can be used as the basis for an efficient algorithm to compute place vocabularies. The place vocabulary is derived from the kinematic topology by determining the shape of the region boundaries as sequences of different contact relationships, and marking their qualitative directions, which define the inference rules for qualitative analysis (as described in [FALT89?]).

When the kinematic topology is given, the place vocabulary computation can ignore most of the actually impossible contact relationships, as the regions of kinematic topology contain only legal contacts. The place vocabulary can now be computed in time proportional to its actual size. We also do away with an explicit representation of configuration space and all the expensive computation associated with it. Preliminary tests indicate that the bubble algorithm is about 100 times faster than earlier implementations of the place vocabulary theory. More importantly, the topology-based computation can potentially be generalized to more than two simultaneous degrees of freedom, three dimensions, and complex boundary curves.

4 Conclusions

In this paper, we have introduced the concept of kinematic topology as a robust model of commonsense reasoning about kinematics from very approximate information. Kinematic topology is an abstraction of the place vocabulary concept and distinguished by the fact that (i) it is almost always ambiguous, but with a manageable number of possibilities, (ii) requires significantly less information for its computation, but (iii) is not powerful enough to allow an actual envisionment of the behavior. The prime motivation for the development of the concept was the need for a causal analysis of kinematic topology in an ongoing project to develop a system for automatic mechanism design.

Kinematic topology can be computed in a purely qualitative way, and is the first representation of kinematics with this property. It stands in contradiction to the earlier poverty conjecture that no purely qualitative kinematics is possible ([FNF87]), and gives an indication of the extent to which we may succeed in the challenge of disproving this negative prediction.

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