Two Model Abstraction Techniques Based on Temporal Grain Size: Aggregation and Mixed Models

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1. Introduction

Many real world dynamic systems involve such a large number of variables and interconnections that it is difficult to grasp them mentally in their entirety. Abstracting a detailed description to produce a simpler description becomes essential as the complexity of the subject system increases. For example, an abstraction hierarchy of models is necessary to control the combinatorial explosion of envisionment process [4]. The author has investigated two different techniques for generating an abstract model from a detailed, dynamic model of a system. One technique is generation of a model of a coarse temporal grain size from a model of a finer grain size by making assumptions about the relative adjustment speeds of the equations in the model [1]. The other technique is aggregating a dynamic system model to generate an aggregate model when the original model is nearly decomposable [8]. This paper compares the two abstraction techniques to show that they are actually closely related.

The paper also discusses how the notion of causality relates to that of model abstraction. Both abstraction techniques are means of going bottom-up from a detailed description of a system to a description at a higher level of abstraction. There is an alternative, top-down, way of looking at the situation. When we model a complex system, we carry our modeling only down to some level of components. Above that level, structure and the interrelations of components are explicit. Below that level, the components are black boxes with no detailed internal structure. Suppose that we determine the causal structure of the model but decide subsequently that some part of the model must be elaborated in greater detail. Will the new model, incorporating this elaboration, have the same causal structure as the more aggregated model, or do we have to reexamine the causal ordering from the beginning? Section 2 will show that it is not necessary to reexamine the causal ordering of the aggregated model after elaborating a part of an aggregated model.

Kuipers uses abstraction by time-scale in order to control the exponential growth of the number of possible courses of behavior in qualitative simulation [4]. Kuipers has a hierarchy of constraint networks of very fast to very slow mechanisms. When simulating a fast mechanism, variables controlled by slower mechanisms are considered constant, and when simulating a slow mechanism, equilibrium among variables controlled by faster mechanisms is considered to be reached instantaneously. This idea of abstraction by time-scale is similar to the notion of abstraction discussed in this paper. The abstraction techniques discussed in this paper can be used to generate a hierarchy of models of different time-scales.

1.1. Mixed models as abstraction

The first abstraction technique is generating a model of an intermediate temporal grain size from a more detailed, dynamic model [1]. Given a model, represented as a set of differential equations that describe the dynamic behavior of a system, another model consisting of some differential and some algebraic equations can be generated as an abstraction of the former. When a mechanism represented by a differential equation in the dynamic model acts very quickly to restore relative equilibrium, one can regard it to be acting instantaneously when one is not interested in the very short-term dynamics of the system. We call the operation of replacing a dynamic equation by its corresponding equilibrium equation equilibration.

In contrast to variables that adjust to changes in other variables very quickly to restore relative equilibrium, some variables change so slowly in response to changes that they can be regarded as independent of other variables. The equation corresponding to such a variable can be replaced by a constant equation, which amounts to deleting from the system under consideration the slow mechanism through which others influence this variable. We call this operation of replacing a dynamic equation by an exogenous variable equation *exogenization*. More detailed descriptions of these two abstraction operators and their characteristics are found in [1, 2].

In both of these cases, the original model is simplified by replacing a differential equation by the corresponding equilibrium equation or the constant equation. By performing these two operations, one can produce a whole range of models between the completely dynamic model and the completely static model depending on the desired temporal grain size of analysis.

Conceptually, exogenizing is the opposite of equilibrating, because exogenizing a variable assumes it is unaffected by other variables while equilibrating a variable assumes it responds to changes in other variables extremely quickly to restore equilibrium. Exogenizing a variable amounts to deleting a mechanism from the system by placing the mechanism determining the value of the variable outside the scope of the system under consideration, and it is reasonable to do so only when the feedback to the variable from the variables inside the mechanism is negligible.

1.2. Aggregation of nearly decomposable systems

The second abstraction technique studied is aggregation of nearly decomposable dynamic system [8]. The basic idea behind aggregation is this: if variables in a large dynamic system can be partitioned into subsets such that variables in each subset are more strongly connected to each other than to variables in other subsets, one can describe the short-run behavior of each subsystem independently of other subsystems. Furthermore, one can describe the long-run behavior of the entire system in terms of these subsets instead of individual variables, treating each subset as a black box. Simon and Ando proved that this was indeed true for the case of a nearly decomposable dynamic matrix with one significant characteristic root for each subsystem [8].

Consider a self-contained dynamic system M and its matrix P of coefficients such that P is almost block diagonal except for small (less than ε for some small ε) elements outside the diagonal blocks. P looks like,

$$P = \begin{pmatrix} P_1^* & & \\ P_2^* & & \\ & & \\ & & & \\ & & & P_N^* \end{pmatrix}$$

where the elements of P outside of the submatrices are either ε or zero. Then P can be expressed as

 $P=P^*+\varepsilon C,$

where C is an arbitrary $n \times n$ matrix, and P^* is the corresponding block diagonal matrix. A matrix such as P that can be put in this form is called a *nearly completely decomposable* matrix or a *nearly decomposable* matrix.

The system *M* whose matrix *P* is nearly completely decomposable consists of components such that variables within each component interact strongly, but variables from different component interact relatively weakly. The submatrices represent such components and the ε elements outside the submatrices represent weak links among components.

Simon and Ando show that the behavior of such a system may be approximately described in the following four stages [8]:

- 1. short-run dynamics
 - Variables in each subsystem are moving towards their relative equilibrium independently of other subsystems.
- short-run equilibrium
 The most significant root of each subsystem dominates the behavior of the subsystem.
- long-run dynamics
 The variables in each subsystem move together towards over-all equilibrium while maintaining relative equilibrium in each subsystem.
- long-run equilibrium Finally, the most significant root of the entire system dominates.

When the behavior of a large system is approximately described in four stages as above, the goodness of the approximation naturally depends on how small the ε 's are and also how dominant the most significant root of each subsystem is compared to the rest of the roots.

In order to generate an abstract model M' from M, we define an aggregate variable y'_j for each subsystem as a linear combination of all the variables in the subsystem. Then, the entire system can be rewritten in terms of these aggregate variables. The resulting system M' consists N variables and equations, and it is substantially smaller than M. The aggregate system can be used to approximate the middle- to long-term behavior of M. Though the original aggregate variable, we subsequently generalized it to cases where submatrices have any number of aggregate variables.¹

While the actual aggregation procedure is numerical, the relevance of this concept is not limited to cases where numerical information of functional relations among variables is available. Even when only a qualitative model exists, model aggregation is possible and is often performed based on such qualitative

¹See [3, 2] for details.

knowledge as relative strengths of interactions among variables and groups of variables and relative speeds at which groups of variables reach equilibrium through workings of causal mechanisms in the system. The work on aggregation of dynamic system matrices provides justifications and suggests procedures for performing such qualitative abstraction as equilibration and exogenization discussed above.

2. Causal Ordering in an Aggregate Model

Both of these methods are means of going bottom-up from a detailed description of a system to a description at a higher level of abstraction. There is an alternative, top-down, way of looking at the situation. When we model a complex system, we carry our modeling only down to some level of components. Above that level, structure and the interrelations of components are explicit. Below that level, the components are black boxes with no detailed internal structure. Suppose that we determine the causal structure of the model but decide subsequently that some part of the model must be elaborated in greater detail. Will the new model, incorporating this elaboration, have the same causal structure as the more aggregated model, or do we have to reexamine the causal ordering from the beginning? In this section, we will argue that the answer to this question is "no": in other words, the causal structure of a subsystem can be determined independently of that of the global system.

The aggregation procedure described above produces a dynamic model in terms of aggregate variables. Aggregate variables describe the long-term behavior of the subsystems they represent, and the aggregate model describes the long-term behavior of the entire system in terms of the interactions among subsystems. Since the aggregate model produced by the procedure is just another self-contained, dynamic model, its causal structure can be determined based on the theory of causal ordering [1].

Consider the nearly decomposable system, M, consisting of the following equations:

$x'_{1} = -50.000x_1 + 23.000x_2 + (1.0000e-3)x_1$	(1)
$x'_{2} = -1.0000x_{1} - 0.10000x_{2} + (2.0000e-3)x_{2}$	(2)
$x'_{1_{1_{1_{1_{1_{1_{1_{1_{1_{1_{1_{1_{1_$	(3)
$x'_{2_2}^2 = (3.0000e-3)x_{1_1}^2 + 1.0000x_{1_2}^2 - 0.90000x_{2_2}^2$	(4)

Aggregating M produces an aggregated, dynamic model M' consisting of the following equations:

$y'_1 = 0.56526y_1 + 1.1112e - 3y_2$	(5)
$y_2' = 3.8135e - 3y_1 - 1.2718y_2$	 (6)

Figure 2-1 shows the causal ordering in M'.

Suppose that after determining the causal ordering in M', one decides to further elaborate subsystem M_1 . Since M is hierarchical, hence nearly decomposable, the characteristic roots associated with the subsystem to be elaborated will be large in magnitude of their real part compared with the characteristic roots of the aggregated system. If we ignore very short time periods, the structure of the subsystem can be summed up in linear combinations of the elements of the eigenvectors associated with its significant roots as in



Figure 2-1: Causal Ordering in the Aggregated Model, M'

$$y_1 = c_1 \exp(\lambda_1, t)$$

where $-50.000z_{1_1} + 23.000z_{2_1} = \lambda_{1_1}z_{1_1}$, λ_{1_1} is the most significant eigenvalue associated with M_1 , and

 z_1 , and z_2 , are elements of its corresponding eigenvector.

or

 $y_1 = c_1 \exp((-50.000z_1 + 23.000z_2)t / z_1)$

These elements will be constants that, for purposes of examining behavior in the longer run, can be treated as exogenous. They determine the internal relations of the parts of the component with each other. Hence, we can add to M the following constant equations:

$$z_{1_1} = 0.462526$$
 (8)
 $z_{2_2} = 1.0000$ (9)

(7)

Let M" be the model produced by adding equations 7, 8, and 9 to M'. Then, to construct the causal graph of M", we simply add to our causal diagram of M' a new set of causal arrows, one for each element of the eigenvectors, pointing to y1 as shown in Figure 2-2. In all other respects, the causal ordering will remain the same.



Figure 2-2: Causal Ordering in M"

4

Alternatively, if one decides to make explicit the internal causal structure within M_1 , it can be shown as in Figure 2-3. In the figure, the causal structure within M is constructed based on the definition of causal structure in a dynamic model applied to the component in isolation, i.e. M_{*_1} .



Figure 2-3: Causal Ordering in M' with Internal Structure of M1

Note that the additional links (indicated by broken arrows) from the elements of the eigenvector to y_1 in Figure 2-2 represent relations whose nature is somewhat different from that of causal links we have been discussing up to this point. They are abstraction links connecting variables at one level of abstraction to the variable which represents their over-all long-term behavior at the next higher level of abstraction. They are "causal links" only in the sense that behavior at a lower level of abstraction is sometimes said to "cause" a behavior at a higher level of abstraction, and not in the sense of actual causal mechanism, on which structural equations are based. Also, note that in Figure 2-3, the short-term causal links and long-term ones must be carefully distinguished. The short-term ones are the causal links within M_1 , and the long-term ones represent those among components. Introducing these different types of links is necessary in order to show clearly the hierarchical causal structure of a system when causal relations at different abstraction levels are mixed in one diagram.

The above example shows that for hierarchical, nearly-decomposable systems, the causal ordering is not sensitive to the "grain size" of analysis. At any level in the hierarchy, the causal ordering among components is (nearly) independent of the causal ordering that relates to the relative movement of the variables within any single component.

3. Aggregation and Mixed Model as Abstraction Techniques

Aggregation and the techniques for generating mixed models from dynamic models are two ways for abstracting a complex dynamic model to produce a simpler description of the system by ignoring its short-term behavior. In fact, the two techniques are closely related [7]. We summarize the two techniques below;

Aggregation

 Decompose the system into components such that variables within a component strongly interact, quickly restoring relative equilibrium among them, while variables from different components only weakly interact.

- Set up aggregate variables for components and redescribe the long-term behavior of the entire system in terms of the aggregate variables.
- Equilibration
 - Choose the equations representing mechanism that restore equilibrium very quickly so that the equilibrium relations can be regarded to hold instantaneously.
 - Replace these differential equations by corresponding equilibrium equations.

The first step of the both techniques amounts to classifying interactions among variables into two types; strong interactions restoring relative equilibrium among a group of variables very quickly, and weak interactions among different groups that take non-negligible time to reach equilibrium.

When groups of strongly interacting variables can be identified in a given dynamic system, the theory of aggregation provides justification for breaking up the system into subsystems and treating the variables within a subsystem as moving together (always maintaining relative equilibrium among them). This type of decomposition requires only qualitative information, namely the relative strength of interactions among variables. Therefore, the theory of aggregation provides a justification for the qualitative abstraction techniques even in cases where precise numerical values of the coefficients in the dynamic system are not known. Though Simon and Ando only handle linear systems in their theory of aggregation, its usefulness is not limited to linear systems because it is very common to use piece-wise linear models to approximate the behavior of a more complex non-linear systems in engineering problem solving.

When the qualitative technique of mixed model is applied to a nearly decomposable system with a set of assumptions about the relative speeds of mechanisms that are consistent with the way the system is decomposed, the results produced by the two techniques should be consistent with each other. Consider again the dynamic model M in Section 2. Suppose that it is known a priori that the mechanism represented by equation 1, belonging to the subsystem M_1 , acts very quickly to restore relative equilibrium between x_{1_1} and x_{2_1} , and that, likewise, the mechanism represented by equation 3, belonging to M_2 , restores equilibrium between x_{1_2} and x_{2_2} very quickly. Then, ignoring the long-term effects of the ε terms, we can rewrite the two equations as below:

$0 = -50.000x_{1} + 23.000x_{2}$	(1)'
$0 = -47.000x_{10} - 17.000x_{20}$	(3)'

The above two equations, 1' and 3' and equations 2, and 4 shown below form a self-contained mixed model.

$$\begin{aligned} x'_{2_1} &= -1.0000x_{1_1} - 0.10000x_{2_1} + (2.0000e - 3)x_{2_2} \\ x'_{2_2} &= (3.0000e - 3)x_{1_1} - 3.0000x_{1_2} - 0.90000x_{2_2} \end{aligned} \tag{2}$$

Let M" be this mixed model. Figure 3-1 shows the causal ordering in M".

Comparison of the diagrams in Figures 2-1 and 3-1 that they have basically the same causal structure: There is an internal feedback loop in each subsystem, and there is a feedback loop between the two subsystems.



Figure 3-1: Causal Ordering in Mixed Model, M""

3.1. Example : Boiler model

In this section, we present another example to illustrate the close relationship between aggregation and mixed systems. The system modeled here is a boiler.



Figure 3-2: Boiler

Figure 3-2 shows a simplified view of a coal-burning boiler. A boiler has inputs of coal, air, and water. Coal is burned to heat up the input feedwater producing ash, exhaust gas, and super-heated steam. In addition, a considerable amount of energy is lost as heat radiated into the atmosphere.

Following is a mixed model of the boiler. In constructing the model, the equations representing fast mechanisms are equilibrated and others are left as differential equations.

Variables

C

carbon content (as in coal)

E	internal energy						
М	mass .						
Τ	temperature						
P	pressure						
X	steam quality						
R	process efficiency						

Variable subscripts

fwt.in, fwt.out	input feedwater and output steam
air.in	input air
coalin	input coal
gas.out	output gas
rfs.out	output refuse (ash)
cmb	combustion
htl.out	heat lost by radiation into the atmosphere
ld	electricity demand

Equations

Comments

$E_{fwl.in} = f^{*}(T_{fwl.in}, M_{fwl.in}, P_{fwl.in}, X_{fwl.in})$	The internal energy equation for fwt.in.
$P_{fwl.out} = f^+(T_{fwl.out})$	A property of gas. The pressure is a function of the temperature.
$\frac{dM_{fort.out}}{dt} = c_1 \left(M_{fwt.in} - M_{fwt.out} \right)$	The flow equation for the feedwater flow.
$E_{fwl.out} = f^{+}(T_{fwl.out}, M_{fwl.out}, P_{fwl.out})$	The internal energy equation for fwt.out.
$\frac{dE_{fwr.out}}{dt} = c_2 \left(E_{cmb} R_{fwh} + E_{fwr.in} - E_{fwr.out} \right)$	The energy flow equation for fwt.out.
$E_{air.in} = f^+(T_{air.in}, P_{air.in}, M_{air.in})$	The internal energy equation for air in.
$\frac{dM_{gas.out}}{dt} = c_3 \left(M_{coal.in} R_{cmb} + M_{air.in} - M_{gas.out} \right)$	The flow equation for gas.out.
$E_{gas.out} = f^{+}(T_{gas.out}, M_{gas.out}, P_{gas.out})$	The internal energy equation for gas.out.
$\frac{dE_{gas.out}}{dt} = c_4 \left\{ E_{cmb} \left(1 - R_{hll} - R_{fwh} \right) - E_{gas.out} \right\}$	The energy flow equation for gas.out.
$E_{coal.in} = M_{coal.in} h v_c C_{coal.in}$	The internal energy equation for coal.in.
$\frac{dE_{hal.out}}{dt} = c_5 \left(E_{cmb} R_{htl} - E_{htl.out} \right)$	The energy flow equations for E _{htl.out} .
$\frac{dM_{rft.out}}{dt} = c_6 \left\{ M_{coal.in} \left(1 - R_{cmb} \right) - M_{rfs.out} \right\}$	The flow equation for rfs.out.
$E_{rfs.out} = M_{rfs.out} C_{rfs.out} hv_c$	The internal energy equation of the refuse.
$\frac{dE_{comb}}{dt} = c_7 \left(E_{coal.in} \frac{R_{comb}}{C_{coal.in}} + E_{air.in} - E_{cmb} \right)$	Property of a combustion process.
$\frac{dC_{rfunnt}}{dt} = c_g \left(\frac{C_{conlin} - R_{crub}}{1 - R_{crub}} - C_{rfs.oul} \right)$	Property of a combustion process.
$\frac{dM_{coal.in}}{dt} = c_9 \left\{ f^+(E_{ld}) - M_{coal.in} \right\}$	Property of the boiler. The amount of input coal is

8

controlled based on the electrical load.

 $\frac{dM_{fwiin}}{dt} = c_{10} \left\{ f^+(E_{ld}) - M_{fwiin} \right\}$

Likewise, the amount of input feedwater is controlled based on the load.

The following variables are exogenous;

Tair.in, Pair.in, Pgas.out, Xfwt.in, Tfwt.in, Pfwt.in, Eld, Rcmb, Rhut, Rfwh, Mair.in, Ccoal.in

Figure 3-3 shows the matrix associated with the boiler model. The matrix is constructured as follows: Each row and column corresponds to a variable in the model including the exogenous variable equations.² Each row corresponds to the equation which represents the mechanism controlling the variable. For each row, if a variable appears in the equation with non-zero coefficient, 1 or ε is placed in the column of the variable. Since the equations that are equilibrated in the mixed model of the boiler represent strong interactions, the elements in these rows are indicated by 1. In the rest of the equations, the elements are ε 's because these equations represent weak interactions. The matrix shows that the system is nearly decomposable with 12 subsystems marked in the figure with dotted rectangles. Since an exogenous variable is dependent possibly on itself but on no other variables in the model, rows of the exogenous variables have 1 only in the column of the variable itself.

	(1)	(2)	(3)	(4)	(5) ((6) (7	(8)	(9)	(10)	(11) (1	2) (13)	(14) (1	(16) (17)	(18) (1	9) (20) (21) (22)	(23)	(24) (2	25) (24	5) (27)	(28) (29)
(1) 7					!																		
(1) I funcia (2) E	11	1	1	1	1																		
(3) M	1	ε			<u> </u>																	ε	
(4) P for in	1			1	1																		
(5) X for in			-		1				1													1	
(D) P			ε		1	1			1														
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(9) E for and		ε			1			٤	Ĺ		-	е С											
(10) E									11	1 1	1												
(12) M									1	1	- 1												
(13) P									!	5 resources	Li-												
(14) E									3		1	8	-				з						ε
(16) E																							
(17) T													1 1	1	1 1								
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(19) P (20) P													1		_ 1								
(21) E															1	! i		-1-	1				
(22) M and in																1		ε	1				ε
(23) C other in																1			1 1				
(25) M												8				ε			5	E 1		- 1	
(26) C																		c	ε	1 0	ε	1	ε
(27) E -																				1	1	1)	
(28) E																							1
(2) ~																							11

Figure 3-3: Matrix of Coefficients of the Dynamic Model of the Boiler

²Exogenous variable equations are those of the form x = c, where x is an exogenous variable and c is some constant. They represent assumptions that the variable value is determined by factors lying outside the scope of the model.

4. Discussion

This paper discussed causal ordering in aggregate models. For hierarchical, nearly-decomposable systems, the causal ordering is not sensitive to the temporal grain size of analysis. This means that one can determine the causal ordering among components of a system independently of the causal ordering among variables within any single component of the system.

We also discussed the close connection between the concepts of mixed systems and aggregation of nearly decomposable systems. When a nearly-decomposable model is converted into a mixed model using such assumptions about relative speeds of mechanisms that are in accordance with its decomposition, the resulting mixed model and its causal ordering is consistent with the aggregate model produced by the aggregation procedure. Thus, the theory of aggregation provides a theoretical justification for the concept of a mixed system as an abstraction of a dynamic system. By differentiating among long-term, short-term, and middle-term phenomena, attention can be directed to the dynamics of specific subsystems without dealing with the entire system at once, reducing the degree of complexity one must deal with when reasoning about the behavior of a dynamic system.

The original definition of self-contained systems given by Simon required equations in a model (equilibrium or dynamic) to be linear. However, since the procedures for determining causal ordering among variables in a model do not particularly hinge on the fact that the equations are linear, and the definitions of self-containment and of causal ordering are directly extendable to non-linear models, we dropped this requirement in the subsequent development of the theory of causal ordering.

The numerical aggregation procedure, on the other hand, does depend on linearity of a model. However, as demonstrated by the boiler example in Section 3.1, the idea of abstraction based on the relative speed of equilibrium restoration among variables seems to correspond well with the idea of the near decomposability of a matrix. A necessary extension of this work is to devise an analogous aggregation method for non-linear systems since many practical systems in the real world are not linear. Analyzing a system of non-linear differential equations is a difficult problem. Since there is no general technique for analyzing all non-linear systems as there is for a linear case, one would have to restrict oneself to particular classes of non-linear systems, and to develop and aggregation procedure for each class. Sacks worked on a program, PLR, to analyze ordinary differential equations using piecewise linear approximation [5, 6]. His approach may provide a starting point for extending our work on aggregation in this direction.

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