# Exploring Naive Topology: modelling the force pump.

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## 1. Introduction

The importance of representation within a formal framework has always been a central topic for discussion within AI. This has been particularly apparent since Hayes' [1979, 1985a] call for the development of formalisms supporting a clean semantics that can be used to represent and reason about non-trivial domains. In the spirit of Hayes' Naive Physics Programme, we present an outline of a concise rigorous formalism currently being developed [Randell and Cohn, 1989], and illustrate this with a partial axiomatisation of a force pump. The formalism exploits topological and metrical information inherent in descriptions and explanations encountered in everyday discourse about the world and covers both the representation of and reasoning about spatial and temporal topological and metrical information. Noting the recent controversy surrounding Hayes' programme [Levesque et al, 1987] we are encouraged that this approach actually bodes well for the central ideas Hayes advocated.

Many explanations of phenomena and descriptions of the relationship between objects in informal discourse appeal to topological and metrical information. Much of this appears to be done unconsciously, but a little reflection on our use of language (particularly the use given to prepositions and prepositional phrases) reveals how important this information is. While we do not assume that linguistic descriptions necessarily uncover those entities represented and exploited by the brain in all such activity (eg. in the encoding and representation of perceptual information), it is interesting to note that many of the underlying concepts captured with our formalism are surprisingly intuitive. Emphasis on 'naive' notions supposedly underlying our conceptual framework is not stressed. Instead we emphasise the need to build a rigorous axiomatic theory that can support these intuitive notions.

The structure of the rest of this paper is as follows. Section 2 discusses related work. Section 3 is a synopsis of the formalism (covered in more detail in Randell and Cohn [1989]). Section 3.1 develops the temporal parts of the formalism, and section 4 illustrates the use of the formalism with a partial axiomatisation of a force pump.

## 2. Related work

Hayes [1979, 1985a] and Welham and Hayes [1984] discuss the importance of building large scale formalisms encoding topological and metrical information. Part of the motivation is centred on the assumption that experiential knowledge needs to be exploited if AI is to be able to tackle non-trivial domain problems. A clean semantics is secured by the preferred use of first order predicate logic both as a representation language into which theories should be capable of being translated and as a reference language by which competing formalisms and theories can be compared. 'Ontology for Liquids' [Hayes, 1985b] still stands as Hayes' main contribution to applied Naive Physics.

A significant collection of papers dealing with commonsense knowledge and reasoning appears in Hobbs and Moore [1985] and Hobbs et al [1985]. The latter includes work by Kautz who discusses a formalism for the representation of spatial descriptions and concepts. Hager tackles the representation of properties of materials, and Shoham the representation of kinematics and shape. Many of these papers suffer to some extent from the free use of "axioms" without demonstrating the worth of their respective formalisms and theories in terms of interesting deductions. Leyton's [1988] process grammar represents an alternative approach. Changes in measures of curvature extrema are exploited and provide the basic geometry by which processes can be modelled. Transformation of shape is covered but the modelling is restricted to capturing the geometrical properties of homogeneous entities. Allen [1981, 1983] and Allen and Hayes [1985, 1987] develop a theory of time that exploits topological information, but do not consider the possibility of a unitary formalism that captures both spatial and temporal topological information. Davis [1988] describes a very expressive first order formalism for reasoning about solid object behaviour; the whole being illustrated by modelling the possibility and passage of a die passing through a funnel. Davis exploits several concepts that also appear in our formalism although the basic set of primitives in the compared formalisms differ. Other related work can be found in Kuipers [1988, 1989], Kuipers and Levitt [1988] and Kuipers and Yung-Tai Byun [1988].

### 3. The formalism

The formalism [Randell and Cohn, 1989] is expressed in a many sorted logic (msl) allowing arbitrary ad hoc polymorphism on predicates and functions [Cohn, 1987] and is rooted in a calculus of individuals developed by Clarke [1981, 1985]. It should be noted that variables derive their sortal restrictions implicitly from the argument positions they occur in. Space does not allow a detailed description of the logic or the sortal apparatus used, but important sortal restrictions will be highlighted.

Our formalism differs from Clarke's with the following important conditions and additions: a) our formalism is strictly first order (while Clarke allows second and even third order variables); b) we add additional primitives, relations, functions and predicates; c) partial functions implicit in Clarke are made explicit in a msl and made total on their domains by introducing a new sort (called Null) – see later; and d) continuity constraint information (discussed below) is added linking temporal periods over which some particular property remains invariant. This forms the basis upon which processes are constructed.

The domain of discourse includes regions as basic entities upon which the formalism is built. Informally these regions may be thought to be potentially infinite in number allowing any degree of overlapping. Each region coincides with a set of incident points and every region is contained in a special region called the universe.

Two primitive relations are introduced: '$C(x,y)$' read as 'x connects with y' and '$B(t_1,t_2)$' as '$t_1$ is wholly before $t_2$'. '$C(x,y)$' is reflexive and symmetric, '$B(t_1,t_2)$' is irreflexive, asymmetric and transitive. In terms of points incident in regions, $C(x,y)$ holds when regions x and y share a common point, and $B(t_1,t_2)$ when all the points incident in $t_1$ are wholly before (temporally speaking) all the points incident in $t_2$. $C(x,y)$ covers cases of connection from contact or 'touch' to all degrees of mutual penetration between regions including mutual total overlap or identity. $B(t_1,t_2)$ excludes the case of temporal abutment. First we will outline that part of the formalism which can be given a purely spatial interpretation; the discussion of the temporal aspects appear in section 3.1. The basic relations are built up as follows:

| | |
|---|---|
| $C(x,y)$ (primitive) | |
| $DC(x,y) \equiv_{def.} \neg C(x,y)$ | $DR(x,y) \equiv_{def.} \neg O(x,y)$ |
| $P(x,y) \equiv_{def.} \forall z[C(z,x) \rightarrow C(z,y)]$ | $EC(x,y) \equiv_{def.} C(x,y) \wedge \neg O(x,y)$ |
| $O(x,y) \equiv_{def.} \exists z[P(z,x) \wedge P(z,y)]$ | $TP(x,y) \equiv_{def.} P(x,y) \wedge \exists z[EC(z,x) \wedge EC(z,y)]$ |
| $PP(x,y) \equiv_{def.} P(x,y) \wedge \neg P(y,x)$ | $NTP(x,y) \equiv_{def.} P(x,y) \wedge \neg \exists z[EC(z,x) \wedge EC(z,y)]$ |
| $\forall x[C(x,x) \wedge \forall y[C(x,y) \rightarrow C(y,x)]]$ | $\forall xy[\forall z[C(z,x) \leftrightarrow C(z,y)] \rightarrow x=y]$ |

'$DC(x,y)$' is read as 'x is disconnected from y'; '$P(x,y)$' as 'x is a part of y'; '$O(x,y)$' as 'x overlaps y'; '$PP(x,y)$' as 'x is a proper part of y'; '$DR(x,y)$' as 'x and y are discrete'; '$EC(x,y)$' as 'x is externally connected to y'; '$TP(x,y)$' as 'x is a tangential part of y'; and '$NTP(x,y)$' as 'x is a non-tangential part of y'.

A set of configurations satisfying these (and other relations defined below) are given in Figure 1. A lattice ordering the implication strength of the basic set of relations appears in Figure 2. The symbols $\top$ and $\perp$ represent tautology and contradiction respectively. Notice is drawn to the fact that $DC(x,y)$ implies $DR(x,y)$ but not vice-versa. The inclusion of $C(x,y)$ enabling $EC(x,y)$ and the tangential, non-tangential (and surround relations to be defined) distinguishes this calculus from the standard set of relations used in other calculi of individuals [eg. Eberle, 1970].

Inverses of all the asymmetrical relations given above and below are named and defined but not given here. Six further relations are:

| | |
|---|---|
| $TPP(x,y) \equiv_{def.} TP(x,y) \wedge \neg P(y,x)$ | $PO(x,y) \equiv_{def.} O(x,y) \wedge \neg P(x,y) \wedge \neg P(y,x)$ |
| $NTPP(x,y) \equiv_{def.} NTP(x,y) \wedge \neg P(y,x)$ | $x=y \equiv_{def.} P(x,y) \wedge P(y,x)$ |
| $TPI(x,y) \equiv_{def.} TP(x,y) \wedge x=y$ | $NTPI(x,y) \equiv_{def.} NTP(x,y) \wedge x=y$ |

'$TPP(x,y)$' is read as 'x is a tangential proper part of y'; '$NTPP(x,y)$' as 'x is a non-tangential proper part of y'; '$PO(x,y)$' as 'x partially overlaps y'; and '$x=y$' as 'x is identical with y'; '$TPI(x,y)$' is read as 'x is a tangential part of (and identical with) y' and '$NTPI(x,y)$' read as 'x is a nontangential part of (and identical) with y.'

A particularly useful function conv(x) read as 'the convex-hull of x' is introduced and linked to the formalism. This function takes a region as its argument and maps this to the region of space which corresponds to its convex hull or convex cover. Informally this function generates the region of space that would arise by completely enclosing a body in a taught 'cling film' membrane. This function provides a very intuitive notion for describing objects that may be considered inside, partially inside or outside another object without forming part of that object. The function conv(x) is a primitive of the formalism.

'W-Outside(x,y)' is read as 'x is wholly outside y', 'Outside(x,y)' is read as 'x is outside y', 'P-Inside(x,y)' as 'x is partially inside y' and 'Inside(x,y)' is read as 'x is inside y'. A new relation 'J-Outside(x,y)' read as 'x is just outside y' is added and defined to pick out those regions forming part of the outside of y and connected to y's convex hull:

$$\text{W-Outside}(x,y) \equiv_{def.} DC(x,conv(y)) \qquad \text{P-Inside}(x,y) \equiv_{def.} PO(x,conv(y)) \wedge \neg P(x,y)$$
$$\text{J-Outside}(x,y) \equiv_{def.} \text{Outside}(x,y) \wedge \neg \text{W-Outside}(x,y) \qquad \text{Inside}(x,y) \equiv_{def.} P(x,conv(y)) \wedge \neg P(x,y)$$
$$\text{Outside}(x,y) \equiv_{def.} DR(x,conv(y)) \wedge \neg P(x,y)$$

Functions are defined which yield the inside and outside of a region. The inside is defined as the sum fusion of all those regions that are inside x; the outside as the sum fusion of all those regions outside x.

$$\text{inside}(x) =_{def.} \iota y[\forall z[C(z,y) \leftrightarrow \exists w[\text{Inside}(w,x) \wedge C(z,w)]]]$$
$$\text{outside}(x) =_{def.} \iota y[\forall z[C(z,y) \leftrightarrow \exists w[\text{Outside}(w,x) \wedge C(z,w)]]]$$

Atomic regions (atoms) are provided, and are defined to be regions having no sub-parts. If atoms are allowed to have interiors, then they become open regions. This generates an interesting deductive result, since if two regions are externally connected, no atoms externally connect. This casts some light on the conundrum of how objects can 'touch' even though no constituent atoms touch. Every non-atomic region has an atom as a part. We then define a closed atom as the closure of an atom:

$$\text{Atom}(x) \equiv_{def.} \forall y[P(y,x) \rightarrow y=x]$$
$$\forall x[\exists y[\text{Atom}(y) \wedge Pyx]]$$
$$\text{C-Atom}(x) \equiv_{def.} \exists y[x=cl(y) \wedge \text{Atom}(y)]$$

The formalism[1] includes analogues of the standard Boolean operators: 'sum(x,y)' read as 'the sum of x and y', 'compl(x)' read as 'the complement of x', the constant 'U' denoting the universal region, the predicate 'Null(x)' read as 'x is null' (denoting the set of objects that are not regions, i.e. objects having no incident points or having incident points, have no interior points), 'prod(x,y)'[2]: read as 'the product (i.e. intersection) of x and y' and 'diff(x,y)' read as the difference (i.e. the relative complement) between x and y' For details of the topological operators: the interior, closure and exterior operators, and the closed-complement operator discussed below, see Randell and Cohn [1989].

$$\text{sum}(x,y) =_{def.} \iota z[\forall w[C(w,z) \leftrightarrow [C(w,x) \vee C(w,y)]]]$$
$$\text{compl}(x) =_{def.} \iota y[\forall z[C(z,y) \leftrightarrow \neg P(z,x)]]$$
$$U =_{def.} \iota x[\forall y[C(y,x)]]$$
$$\text{prod}(x,y) =_{def.} \iota z[\forall w[C(w,z) \leftrightarrow \exists v[P(v,x) \wedge P(v,y) \wedge C(w,v)]]]$$
$$\text{Null}(x) \equiv_{def.} \forall yz[\text{prod}(y,z)=x \leftrightarrow DR(x,y)]$$
$$\text{diff}(x,y) =_{def.} \text{prod}(x,\text{compl}(y))$$

The closed complement operator 'c-compl(x)' is used to define a set of relations that function as the 'surround' analogues of TPP(x,y) and NTPP(x,y), where x is inside y but forms no part of y. The important difference between the standard complement operator and the closed complement operator is that c-compl(x) when x is closed returns a region that includes x's boundary, while the complement of x excludes x's boundary. c-compl(x), when x is closed, also guarantees that the surrounding region is externally connected to x. c-compl(x) also enables the tangential relations eg. TPP(x,y) to be satisfied with the existence of another region z that externally connects both x and y.

'TS(x,y)' is read as 'x is tangentially surrounded by y' and 'NTS(x,y)' is read as 'x is nontangentially surrounded by y'. Because no metric is assumed and the degree of connection between x and y in TPP(x,y) is not restricted, 'inversion' between x and y can arise, i.e. the proper part relations do not preserve asymmetry with respect to a surrounding region (see Figure 3). We ensure asymmetry by imposing an extra condition on TPP(x,y)

[1] With respect to defined functions, when we write $\alpha(\bar{x}) =_{def.} \iota y[\Phi(y)]$ we mean $\forall x \Phi[\alpha(\bar{x})]$ – this convention clarifies the ambiguity inherent in the notation used in Randell and Cohn [1989].

[2] The definition used for prod(x,y) used here corrects the definition that arises in Randell and Cohn [1989] and in Clarke [1981]. The function prod(x,y) is only defined over overlapping regions; also, whereas in Randell and Cohn [1989] there was a unique null object, now Null(x) is true of the intersection of any pair of non overlapping regions. In contrast with the ontology proposed in Randell and Cohn [1989] we dispense with the explicit representation of points and boundary-like objects. This departure enables us to simplify the ontology without detracting from the overall useful expressiveness of the formalism.

by restricting the degree of connection between x, y and the externally connected region z to a single atom[3] The relation 'TPP$_a$(x,y)' and its surround dual 'TS$_a$(x,y)' are defined[4] as follows (clear linguistic readings for these relations are not readily formulated):

$$TS(x,y) \equiv_{def.} \exists z[TPP(x,z) \wedge y = prod(c\text{-}compl(x),z)]$$
$$NTS(x,y) \equiv_{def.} \exists z[NTPP(x,z) \wedge y = prod(c\text{-}compl(x),z)]$$
$$TPP_a(x,y) \equiv_{def.} PP(x,y) \wedge \exists z[[EC(z,x) \wedge EC(z,y) \wedge \exists! u[C\text{-}Atom(u) \wedge P(u,x) \wedge P(u,y) \wedge EC(u,z)]]$$
$$TS_a(x,y) =_{def.} \exists z[[TPP(x,z) \wedge y = prod(c\text{-}compl(x),z)] \wedge \exists! u[C\text{-}Atom(u) \wedge P(u,x) \wedge P(u,y) \wedge EC(u,z)]]$$

Open and closed regions are defined as are connected (one-piece) or disconnected (fragmented) regions. The latter property allows a simple definition of a hollow region to be defined – simply being a region with a disconnected complement.

The explicit notion of an interior (which is an open region) enables one to attribute properties to some region in question while refusing to attribute those properties to the interior. In this way 'wet' objects can be described (cf. Hayes' 1985b problems). Gaseous objects bear some of the formal properties attributed to open regions, eg. lacking distinct boundaries. The concept of an exterior of a region has a natural interpretation as an environment.

A primitive predicate 'SCON(x)' (read as 'x is simply connected') is used to define multiply connected (or 'holed' regions). This forms the topological basis for modelling a filter (being a multiply connected region). We also add the primitive 'C-Ball(x)' read as 'x is a closed ball' and use this to define a arbitrarily thin sectioned sphere and incorporate it into a set of axioms that enable a nest of concentric balls to be generated. This provides the geometric basis of modelling phenomena that reflect the inverse square law [5].

One other concept needs to be introduced, that of a quasi-manifold. A manifold proper (eg. in 3-space) is a connected (ie. one piece) surface such that sufficiently near to each point, the surface is topologically equivalent to an open disk; ie. for each point incident in the surface, all the points sufficiently near to that point form a set equivalent to an open disc. The definition ensures that any region that has point connected parts is not a manifold eg. as would arise in any vertex connected composite object viewed as a whole. We define a quasi-manifold as a region that has a connected interior, remembering that a quasi-manifold need not be a manifold. 'Manifold(x)' is read as 'x is a (quasi-) manifold':

$$Manifold(x) \equiv_{def.} CON(int(x))$$

Given two regions that externally connect, the definition of a quasi-manifold if true for that configuration ensures that the composite region must be at least edge connected (in 2-space) or share a 'fused' surface (in 3-space). The use of this concept will be made clear below.

### 3.1. The Temporal Formalism

We now give a brief summary of the temporal ontology used. In Randell and Cohn [1989] a spatial interpretation of the formalism was adopted. But the partial axiomatisation of the working force pump introduced below requires temporal information to be made explicit. For the temporal ontology we only require open (temporal) regions which we name periods. These are sub-divided into intervals and moments which will be discussed below. The basic primitive temporal relation 'B($t_1$,$t_2$)' ('$t_1$ is wholly before $t_2$') mentioned earlier is irreflexive and transitive; and has the property that for any period wholly before another interval, any part of the first is likewise before any part of the second:

$$\forall t_1[\neg B(t_1,t_1) \wedge \forall t_2 t_3[[B(t_1,t_2) \wedge B(t_2,t_3)] \rightarrow B(t_1,t_3)]]$$
$$\forall t_1 t_2[B(t_1,t_2) \rightarrow [\neg C(t_1,t_2) \wedge \forall t_3 t_4[[P(t_3,t_1) \wedge P(t_4,t_2)] \rightarrow B(t_3,t_4)]]]$$

Although we could go on to define all the analogues of the 13 mutually exclusive and exhaustive dyadic relations common to 'interval' logics [eg. Allen and Hayes, 1985], in fact all we shall need here is the relation 'Meets(x,y)' which is irreflexive, asymmetric and intransitive:

[3] A metric is implicitly assumed here. Strictly speaking just restricting the degree of connection to single atoms (unlike single points) does not preserve the intended asymmetry with respect to the proper part relations, for without a metric defined on regions, atoms can assume any size, and inversion follows.

[4] When we write $\exists! x[\Phi(x)]$ we mean $\exists x[\forall y[\Phi(y) \leftrightarrow y=x]]$. Note the change in these definitions as compared to those in Randell and Cohn [1989] as points are dispensed with and atoms introduced.

[5] Since we have dispensed with points and boundaries, 'Sphere(x)' defined in Randell and Cohn [1989] ceases to be defined. In its place we introduce a spherical object that is not surface only but has a definite thickness - not unlike the space taken up by a soap bubble.

$\boxed{\text{Meets}(t_1,t_2) \equiv_{def.} B(t_1,t_2)\wedge\neg\exists t_3[B(t_3,t_2)\wedge B(t_1,t_3)]}$

We isolate a special kind of period known as a moment, which as the name suggests is intended to denote a very small period of time. A moment is defined as an period that has no (temporal) part before any other (temporal) part; 'Moment(x)' is read as 'x is a moment', remembering that every period has at least one moment as a part (in the case of a moment which is defined to be atomic, this part is itself; in the case of an interval, intervals have at least two parts ):

$\boxed{\text{Moment}(t_1) \equiv_{def.} \forall t_2t_3[[P(t_2,t_1)\wedge P(t_3,t_1)] \rightarrow \neg B(t_2,t_3)]}$

Temporal points have no function other than supplying an appropriate intuitive semantics for the formalism. Moments have a point like status and are the minimal periods of time over which some proposition is indexed. We do not index propositions to points (in time). Periods which are not moments we call intervals: intervals and moments are mutually exclusive.

The distinction we draw between what we identify as states-of-affairs (or states) and events comes from Galton [1984]. States are stipulated to be periods of time where some description deemed to be true over the duration of the state also holds over any sub-period of the temporal period (this property is typically known as being dissective). In contrast an event entails a change in a truth value over the period under consideration. We construe processes as a series of linked periods which have the minimal condition that a process is an interval with at least two moments, ie. having an initial and final moment. (If the initial and final moment *meet* then the process is essentially 'instantaneous'). Implicit in this stipulation is an assumption that processes are finite. We recognise that some processes do not have a recognised initial or final moment, but for the force pump example to be discussed below, all processes are taken to be finite and have an identifiable initial and final moment. An immediate consequence of this analysis is that while moments are states, states are not necessarily processes. Only some processes can be viewed as a state of affairs. In terms of the evolution of a process, a moment coincides with a static spatial description given for a set of entities under consideration. No change arises during a moment. Processes can be divided up into those that are dissective, and those that are not.

Our axiomatisation guarantees that every period has at least one moment as a part, and that periods are open regions. Both intervals and moments are periods and every period is either an interval or a moment. Moments are stipulated to be are atomic regions. Moreover we stipulate that the precedence relation '$B(t_1,t_2)$' defined over moments is connected, and that the interval time line is infinite in both directions.

$$\boxed{\begin{array}{l}
\forall t_1[\text{Period}(t_1)\rightarrow\exists t_2[\text{Moment}(t_2)\wedge P(t_2,t_1)]] \\
\forall t[\text{Period}(t)\rightarrow\text{Open}(t)] \\
\forall t[\text{Period}(t)\leftrightarrow[\text{Interval}(t)\vee\text{Moment}(t)]] \\
\forall t[\text{Moment}(t)\leftrightarrow\neg\text{Interval}(t)] \\
\forall t[\text{Moment}(t)\rightarrow\text{Atom}(t)] \\
\forall t_1t_2[[\text{Moment}(t_1)\wedge\text{Moment}(t_2)]\rightarrow[t_1=t_2\vee B(t_1,t_2)\vee B(t_2,t_1)]] \\
\forall t_1[\text{Moment}(t_1)\rightarrow\exists y[\text{Moment}(t_2)\wedge\text{Meets}(t_2,t_1)]] \\
\forall t_1[\text{Moment}(t_1)\rightarrow\exists y[\text{Moment}(t_2)\wedge\text{Meets}(t_1,t_2)]]
\end{array}}$$

Our ontology of time is very similar to that of Allen and Hayes [1985, 1987a] although we actually used material from Carnap [1958] and Woodger [1937] when building the formalism. One difference between our formalism and that of Allen and Hayes is that we explicitly axiomatise periods as open regions and moments as atomic. There is also a difference between the interpretation given to both formalisms. While Allen and Hayes consider beginnings and endings of (decomposable) moments [Allen and Hayes, 1985, p. 531] we say moments do not have beginnings and endings but that beginnings and endings *are* moments; and that a moment can only be individuated with respect to other moments that meet it and it meets, not by points. By keeping the role of the intuitive semantics clear and separate, and by refusing to introduce temporal points and making periods open, we avoid many problems that can occur when attempting to index predicates to temporal entities.[6] We now add three temporal functions that generate the initial, final moments for any interval and the next moment in time for any period. 'initial($t_1$)' is read as 'the initial moment of $t_1$', 'final($t_1$)' read as 'the final moment of $t_1$', and 'next($t_1$)' as 'the next moment (in time) after $t_1$':

[6] Allen and Hayes [1987b] suggest a revision of their earlier paper in which moments can be mapped to points through a granularity change. Their ontology is rather rather different to the present one; for example their moments cannot meet.

$$\begin{array}{|l|}\hline \text{initial}(t_1) =_{\text{def.}} \iota t_2[\text{Interval}(t_1)\wedge\text{Moment}(t_2)\wedge PP(t_2,t_1)\wedge\neg\exists t_3[\text{Moment}(t_3)\wedge PP(t_3,t1)\wedge B(t_3,t_2)]] \\ \text{final}(t_1) =_{\text{def.}} \iota t_2[\text{Interval}(t_1)\wedge\text{Moment}(t_2)\wedge PP(t_2,t_1)\wedge\neg\exists t_3[\text{Moment}(t_3)\wedge PP(t_3,t1)\wedge B(t_2,t_3)]] \\ \text{next}(t_1) =_{\text{def.}} \iota t_2[\text{Moment}(t_2)\wedge \text{Meets}(t_1,t_2)] \\ \hline\end{array}$$

This choice is motivated by a desire to provide a formal semantics for such intuitive notions such as "...the next moment...". Moreover the inclusion of moments as well as intervals into the ontology allows one to use the real number line as a model for the underlying intuitive semantics without incurring a well known difficulty of characterising change when the temporal ontology only assumes points and periods as sets of points. In order to abbreviate formulae, we will write $\overleftarrow{t}$ instead of initial(t) and $\overrightarrow{t}$ instead of final(t).

Increasing, decreasing and constancy measures over time are treated as follows. A metalogical n-ary function '$\phi(\bar{x})$' used below is to be understood as being replaced with appropriate functions, eg. 'pressure(x)' read as 'the pressure of x'. 'volume(x)' read as the volume of x' and 'distance(x,y)' as 'the distance between x and y'. The function 'at($\phi(\bar{x})$,y)' read as '$\phi(x)$ at y' has the intended meaning that $\phi(x)$ holds at the moment y. The symbols '<','>','≤','≥', and '=' carry their standard meaning. The relation 'Increase(x,y)' is read as 'x increases over y','Decrease(x,y)' as 'x decreases over y' and Constant(x,y)' as 'x is constant over y':

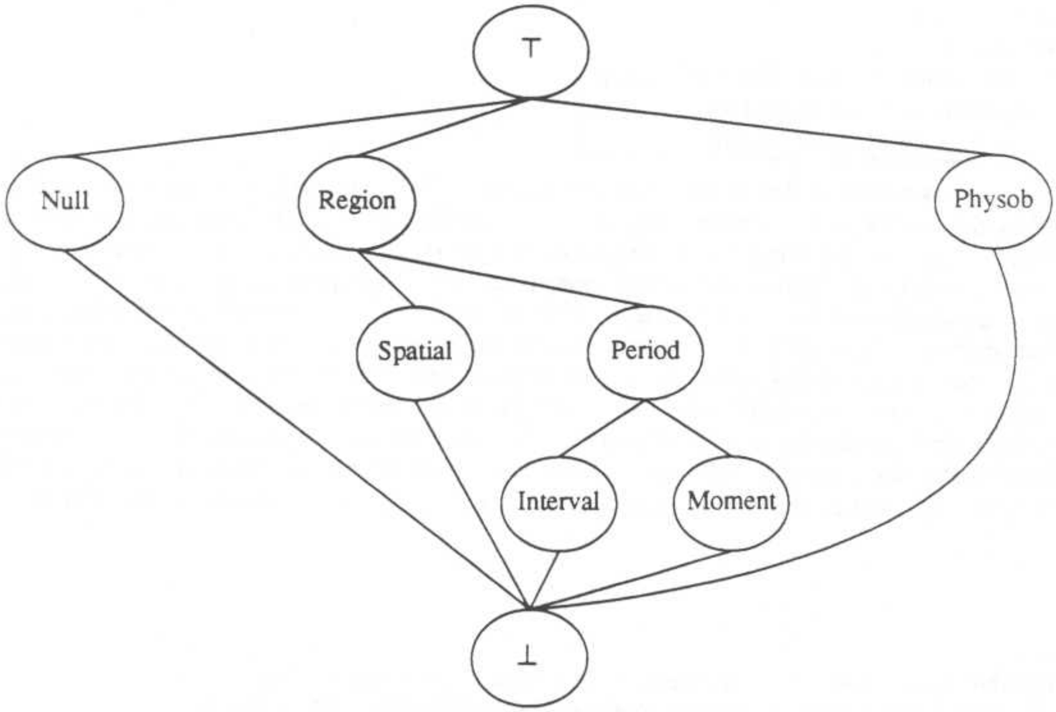
$$\begin{array}{|l|}\hline \text{Increase}(\phi(\bar{x}),t_1) \equiv_{\text{def.}} \\ \qquad \text{at}(\phi(\bar{x}),\overleftarrow{t_1})<\text{at}(\phi(\bar{x}),\overrightarrow{t_1}) \wedge \forall t_2t_3[[P(t_2,t_1)\wedge P(t_3,t_1)\wedge B(t_2,t_3)]\rightarrow(\text{at}(\phi(\bar{x}),t_2)\leq \text{at}(\phi(\bar{x}),t_3))]] \\ \text{Decrease}(\phi(\bar{x}),t_1) \equiv_{\text{def.}} \\ \qquad \text{at}(\phi(\bar{x}),\overleftarrow{t_1})>\text{at}(\phi(\bar{x}),\overrightarrow{t_1}) \wedge \forall t_2t_3[[P(t_2,t_1)\wedge P(t_3,t_1)\wedge B(t_2,t_3)]\rightarrow(\text{at}(\phi(\bar{x}),t_2)\geq \text{at}(\phi(\bar{x}),t_3))]] \\ \text{Constant}(\phi(\bar{x}),t_1) =_{\text{def.}} \forall t_2t_3[[P(t_2,t_1)\wedge P(t_3,t_1)]\rightarrow(\text{at}(\phi(\bar{x}),t2)=\text{at}(\phi(\bar{x}),t_3))] \\ \hline\end{array}$$

The following axiom links together the defined relations:

$$\boxed{\forall xt[\text{Constant}(x,t)\leftrightarrow[\neg\text{Decrease}(x,t)\wedge\neg\text{Increasing}(x,t)]}$$

### 3.2. Integrating temporal and spatial information

Clarke presents his calculus as uninterpreted. Of course, in reasoning about the physical world, we want to reason about physical objects over time. The approach we adopt here is to take two 'copies' of (our extension of) his formalism: one which is purely temporal and one which is purely spatial. Physical objects are mapped to spatial regions via a two place space function (cf Hayes' [1985b] one place space function): space(x,t) which yields the spatial region occupied by x at the moment t, or is of sort Null if x does not exist at t. For brevity we will usually adopt an alternative syntax and write x|t instead of space(x,t). It may help to picture the relevant portion of the sort hierarchy at this juncture:




(Lines indicate a subsort relation. A greatest lower bound of ⊥ indicates that the sorts are mutually disjoint.)

## 4. A sample modelling problem and generation of processes

The descriptive power of this formalism can be appreciated with the following challenge problem: how to describe and model a working force pump. A force pump is illustrated in Figure 4. For simplicity, we assume the pump is primed and that the reservoir feeding the inlet pipe is always full of liquid. The pump has two valves, $valve_1$ and $valve_2$ which open by doors, $door_1$ and $door_2$. The doors are hinged to the pump body closing portals $portal_1$ and $portal_2$ respectively.[7] On the upstroke, $valve_1$ is not shut, while $valve_2$ is shut. This arises because the upthrust pressure of the liquid acting upon $door_1$ is greater than the downthrust forces acting on that door. The pressure difference opens the valve door and allows the liquid to pass from the inlet pipe into the main chamber. During this process the door of $valve_2$ remains closed, sealing $valve_2$. In this case atmospheric pressure acting on the door plus that arising from any liquid in the outlet pipe, thrusts the door into the portal effecting a seal. A similar chain of processes arises with the downstroke of the piston. In this instance $valve_1$ shuts and valve opens and the liquid passes from inside the pump body out into the outlet pipe. The cycle is then repeated.

Fig. 5 represents a pictorial series of topological state descriptions coinciding with one cycle of the operation of the pump. Three basic states of the piston are assumed, moving up, moving down, and stationary. For simplicity we have assumed that when the piston is either at the nadir of its upward or downward motion, the next moment in time coincides with both valves being shut. In actual fact this would not arise in a primed and working force pump, eg. $valve_1$ would remain open for a few moments as the piston travelled on its downward path. Other strong assumptions implicit in the description of the working pump are covered below.

First of all we build up the pump from a library of parts. A partial formal description of the assembly of the pump is given for Figures 6a through to 6c. The pump body is a multiply connected object with three portals, $portal_1$, $portal_2$ and $portal_3$ which are proper parts of its inside. (Figure 6a). Note that the portals are represented as membrane like regions inside the pump body whose outer surface typically coincides with the exterior surface of the pump body. This makes a portal distinct from any passageway that might link the outside of a body from some inner chamber that might exist (as in our example). Portals are specifically defined not to be surface only or having zero-thickness.

The definition of a portal proceeds as follows. A portal x of a region y is defined as a part of the inside of x and composed of the sum fusion of closed atoms forming part of the inside of y and connecting with the outside of y. The last conjunct in the definition ensures that the portal/outside interface is not point-like. 'Portal(x,y)' is read as 'region x is a portal of region y' and 'Portal(x)' as 'x is a portal':

$$\begin{aligned} \mathrm{Portal}(x,y) \equiv_{\mathrm{def.}}\ & P(x,\mathrm{inside}(y)) \wedge \\ & \forall z[C(z,x) \leftrightarrow \exists w[C\text{-}\mathrm{atom}(w) \wedge P(w,\mathrm{inside}(x)) \wedge C(w,\mathrm{outside}(x)) \wedge C(z,w)]] \wedge \\ & \mathrm{Manifold}(\mathrm{sum}(x,\mathrm{outside}(y))) \\ \forall x[\exists y[\mathrm{Portal}(x,y)& \leftrightarrow \mathrm{Portal}(x)]] \end{aligned}$$

By making portals regions and not parts of the boundary interface between the inside and outside of bodies, properties that can be ascribed to regions can also be ascribed to portals. In particular, if the space taken up by a portal's door seals a passageway between the interior (in the non-topological sense) of a pump body and its outside (hence filling in the portal), we can infer that the portal is sealed. We will also find it useful to be able to predicate that a region is a portal of physical object during some time t.

$$\mathrm{Portal}(p,x,t) \equiv_{\mathrm{def.}} \forall t_1 P(t_1,t) \rightarrow \mathrm{Portal}(p,x|t_1)$$

A piston with plunger attached and two pipes, an inlet and outlet pipe, are added (Figure 6b). Since the piston always forms a seal with the inner wall of the pump body, adding the piston means that two disconnected chambers are created, the main and top chamber. It is worth pointing out that our formalism can make this relationship explicit.

It would be useful to be able to pick out that region of the pump that functions as the main chamber. This region is delineated by first of all taking the sum region of the pump-body and its inside, and then taking the difference between this composite region and the piston and plunger. This results in a disconnected region consisting of the top and bottom 'chambers'. We now choose that region connected to $portal_1$. Finally the target region can be isolated by taking the maximally convex region that fits 'inside' the region in question[8]. In effect this

[7] Portals are of course pure spatial regions rather than physical objects. In this paper we will just name portals, though they are of course definable (within the formalism) in terms of the space occupied by physical objects that they are portals of.

[8] It should be clear here that this English description can be readily expressed within the formalism.

is tantamount to defining a convex kernal (cf. convex hull), but unlike the convex hull, a convex kernal is non-unique (eg. think of a body with a regular cruciform shaped interior – it would contain two such convex regions) and hence cannot be defined as a function. However this limitation hides an important fact about pumps of the type given. Given the function of sliding piston in a pump body (and the fact that pistons and pump bodies are typically rigid objects) some regularity in the interior shape of the inside of the pump is ensured. The piston always forms a good seal with the inside wall of the pump body and one would not expect to find component parts of the pump body that act as protrusions into this working space. Hence despite the fact that no general definition of this region can be given (although in many cases it can be adequately described) a certain utility in picking it out can be argued for. For example, we might want to be able to reason that if the side of the working pump got indented, that the piston would jam, or the pump would loose its efficiency.

Valves are created by adding hinged doors to the pump body which can seal their respective portals. We represent valves as a two place functor valve(x,y) whose argument sorts are Portal, Door and whose result sort is Valve. The complete pump is illustrated in Figure 6c.

Axioms given below establish a relationship between the pump's valves, the regions that straddle them and the possibility of liquid flow through the valves. The first axiom states that a valve is shut iff the that valve's portal is filled by (part of) the (solid) valve door. The predicate 'Shut(x,t)' read as 'x is shut during t' has the obvious intended meaning that x has a sealed aperture; while 'Solid(x)' read as 'x is solid' denotes the empirical notion of solidity or inpenetrability. The definition of Sealed(p,t) states that a portal is sealed iff it is not part of anything solid.

$$\forall pyt[\mathrm{Shut}(\mathrm{valve}(p,y),t) \leftrightarrow \forall t_1 P(t_1,t) \rightarrow P(p,y|t_1)$$
$$\mathrm{Sealed}(p,t) \equiv_{def.} \forall t_1 P(t_1,t) \rightarrow \exists x[P(p,x|t_1) \wedge \mathrm{Solid}(x)]$$
$$\forall x[\mathrm{Door}(x) \rightarrow \mathrm{Solid}(x)]$$

For example, given the following description:

$\mathrm{valve}_1=\mathrm{valve}(\mathrm{portal}_1,\mathrm{door}_1)$

we can see that if for some moment in time t, $\mathrm{valve}_1$ is shut, $\mathrm{door}_1$ seals $\mathrm{portal}_1$ making it a solid region and hence $\mathrm{Sealed}(\mathrm{portal}_1,t)$; and that conversely if $\mathrm{valve}_1$ is not shut, $\mathrm{portal}_1$ is not sealed by $\mathrm{door}_1$ and hence (by a closed world assumption) $\mathrm{portal}_1$ is open (ie. not sealed).

Additional axioms give functional definitions of both liquid outflow, inflow and static flow with respect to a portal. Note the use put to the part-whole relation "$P(w|\overleftarrow{t_1},x|\overleftarrow{t_1},)$". Here we use the relation to capture the idea of some quantity of a liquid body moving eg. outside the portal over time.

$$\mathrm{Outflowing}(x,p,t_1) \equiv_{def.} \mathrm{Liquid}(x) \wedge$$
$$\exists zw[\mathrm{Portal}(p,z,t_1) \wedge \neg \mathrm{Sealed}(p,t_1) \wedge P(w|\overleftarrow{t_1},x|\overleftarrow{t_1}) \wedge$$
$$P(w|\overleftarrow{t_1},\mathrm{inside}(z|\overleftarrow{t_1})) \wedge O(w|\overleftarrow{t_1},p) \wedge$$
$$\text{J–Outside}(w|\overrightarrow{t_1},z|\overrightarrow{t_1}) \wedge C(w|\overrightarrow{t_1},p)]$$

e.g. given the description:

$$\mathrm{Outflowing}(\mathrm{liquid}_1,\mathrm{portal}_4,t_1) \equiv_{def.} \mathrm{Liquid}(\mathrm{liquid}_1) \wedge$$
$$[\mathrm{Portal}(\mathrm{portal}_4,\text{inlet–pipe},t_1) \wedge \neg \mathrm{Sealed}(\mathrm{portal}_4,t_1) \wedge P(\mathrm{liquid}_2|\overleftarrow{t_1},\mathrm{liquid}_1|\overleftarrow{t_1}) \wedge$$
$$P(\mathrm{liquid}_2|\overleftarrow{t_1},\mathrm{inside}(\text{inlet–pipe}|\overleftarrow{t_1})) \wedge O(\mathrm{liquid}_2|\overrightarrow{t_1},\mathrm{portal}_4) \wedge$$
$$\text{J–Outside}(\mathrm{liquid}_2|\overrightarrow{t_1},\text{inlet–pipe}|\overrightarrow{t_1}) \wedge C(\mathrm{liquid}_2|\overrightarrow{t_1},\mathrm{portal}_4)]$$

we can see that during an outflowing of liquid from $\mathrm{portal}_4$ i.e. out from the portal of the inlet pipe (and into $\mathrm{portal}_1$) at time $t_1$ a quantity of liquid overlapping $\mathrm{portal}_4$ moves to be just outside the inlet-pipe and (with the last condition) just outside $\mathrm{portal}_4$.

$$\mathrm{Inflowing}(x,p,t_1) \equiv_{def.} \mathrm{Liquid}(x) \wedge$$
$$\exists zw[\mathrm{Portal}(p,z,t_1) \wedge \neg \mathrm{Sealed}(p,t_1) \wedge P(w|\overleftarrow{t_1},x|\overleftarrow{t_1}) \wedge$$
$$\text{J–Outside}(w|\overleftarrow{t_1},z|\overleftarrow{t_1}) \wedge C(w|\overleftarrow{t_1},p) \wedge$$
$$P(w|\overrightarrow{t_1},\mathrm{inside}(z|\overrightarrow{t_1})) \wedge \; O(w|\overrightarrow{t_1},p)]$$

$$\mathrm{Static}(x,y,t) \equiv_{def.} [\neg \mathrm{Outflowing}(x,y,t) \wedge \neg \mathrm{Inflowing}(x,y,t)]$$

The processes just defined are non-dissective: if Outflowing(x,y,t) is true it is not necessarily true that Outflowing(x,y,t') where t' is a subinterval of t. We can easily define dissective versions of these processes if desired; for example here is a dissective Outflowing:

$\text{Dissective–Outflowing}(x,y,t) \equiv_{def.} \forall t_1[\text{Moment}(t_1) \wedge B(\overleftarrow{t},\text{next}(t_1)) \wedge B(t_1,\overrightarrow{t}) \rightarrow \text{Outflowing}(x,y,\text{sum}(t_1,\text{next}(t_1)))]$

A relation for connected portals (where 'Connected-Portal(x,y)' is read as 'x and y are connected (ie. adjacent) portals' is defined; and an axiom that states that for any two connected portals, outflow from one coincides with an inflow into the other is given:

$$\text{Connected–Portal}(x,y) \equiv_{def.} \text{Portal}(x) \wedge \text{Portal}(y) \wedge \text{Manifold}(\text{sum}(x,y)) \wedge$$
$$\forall z[[P(z,x) \wedge \text{C–Atom}(z)] \rightarrow C(z,y)] \wedge \forall w[[P(w,x) \wedge \text{C–Atom}(w)] \rightarrow C(w,x)]$$
$$\forall xyt[\text{Connected–Portal}(x,y) \rightarrow [\text{Outflowing}(z,x,t) \leftrightarrow \text{Inflowing}(z,y,t)]$$

e.g.

$$\text{Connected–Portal}(\text{portal}_1,\text{portal}_4) \rightarrow [\text{Outflowing}(\text{liquid}_1,\text{portal}_4,t) \leftrightarrow \text{Inflowing}(\text{liquid}_1,\text{portal}_1,t)]$$

The definition of connected portals ensures that the connection between them is not point-like (use of Manifold) and that they are totally aligned.

The axioms and definitions given above are sufficient to make the following deductions. Suppose $\text{valve}_1$ is shut, and $\text{portal}_1$ and $\text{portal}_4$ are connected. We can then infer that since $\text{portal}_1$ is part of $\text{door}_1$ (ie. occupied by the door), the portal is not open (because implicitly the door has been construed as a solid region). We can then deduce that no inflowing or outflowing can arise through the portal (or between the connected portals). Hence the liquid within the pump is static with respect to $\text{portal}_1$. With the converse case when $\text{valve}_1$ is not shut either an inflowing or outflowing may arise across the connected portals.

Directionality of liquid flow through the valves in the example pump is fixed (eschewing the realistic case where e.g. $\text{portal}_1$ would actually experience bi-directionality of flow over time as the piston commenced on its downstroke and the valve was closing). Appropriate axioms fixing the directionality of the flow (actually fixing the direction in which the valve doors open) could be done as follows:

$$\text{In–valve}(\text{valve}(x,y)) \equiv_{def.} \forall t_1 f\, \neg\text{Outflowing}(f,x,t_1)$$
$$\text{Out–valve}(\text{valve}(x,y)) \equiv_{def.} \forall t_1 f\, \neg\text{Inflowing}(f,x,t_1)$$

We can now state that $\text{valve}_1$ is an in-valve and $\text{valve}_2$ is an out-valve i.e

$\text{In–valve}(\text{valve}_1)$
$\text{Out–valve}(\text{valve}_2)$.

We recognise some strong assumptions underlying the use of these biconditionals used in the axiomatisation, e.g. that at all times no foreign body blocks the valve (even though the portal may remain open) and that the liquid doesn't undergo any change of state. This has been done to simplify the example, but to date we see no indication that such restrictions are a by-product of the formalism and its underlying ontology.

As yet, no information has been given covering either causal factors or the initial conditions required for fluid to flow through the pump. But it is not that difficult to see what could be added and exploited. For example we could state that for the the inlet pipe must be filled with liquid in order for the liquid to pass into the pump body on the upstroke of the piston. Given the simple case of a primed pump this fact is easily expressed in the formalism:

$$\exists x \forall t[\text{Liquid}(x) \wedge P(\text{inside}(\text{inlet–pipe}|t),x|t)]$$

i.e. that the inside of the inlet pipe is part of a liquid body - which is to say the pipe is (in this instance always) full of liquid! The fact that liquid can be drawn up into the inlet pipe and into the pump body ie. that the condition given above need not hold to get liquid into the pump could be expressed in the formalism reasonably easily. In this case it would be useful to add an axiom abstracting out the direction of inequalities expressed by Boyle's Law which states that at constant temperature the pressure of a given mass of gas is inversely proportional to its volume.

$$\forall xt[[\text{Gas}(x) \wedge \text{Constant}(\text{temp}(x),t)] \rightarrow [\text{Increase}(\text{pressure}(x),t) \leftrightarrow \text{Decrease}(\text{vol}(x),t)]$$

Given this information we could reason that when the pump is started (and with the inlet pipe placed in a reservoir filled with liquid) the act of pulling up the piston would coincide with the trapped air in the pump (constant mass) increasing in volume. Assuming $\text{portal}_1$ was not sealed this would mean that forces arising from the atmospheric pressure acting on the reservoir fluid would propogate through the liquid. This would force the liquid into the inlet pipe and eventually into the pump body. Indeed we could adopt a rather naive view of suction by stipulating that at all times a pocket of air exists between the bottom face of the piston and the liquid in the pump body; and that when the piston moves upward, the volume of the air pocket increases, its pressure drops, and the liquid fills the vacuum formed. In the downstroke process the trapped air would decrease in volume resulting in its internal pressure increasing which would force the liquid down and out through the outlet pipe.

### 5. Further Work

A complete axiomatisation of the pump has yet to be done. However we outline how we see this could be tackled. In the first place it would be useful to be able to pick out those surfaces of the liquid that come into contact with the surfaces of the valve doors, the piston and the surface of the air/liquid interface (known as the free surface). By doing this the action of an external force on such bodies (or impressed force of liquid on an object) could be set up using an appropriate function. We already have definitions that pick out the region that may be construed as the outside 'surface' or 'skin' of an object. More than one definition has been constructed and the choice depends upon whether either an open or closed object is being considered. Below we give the definition for the skin of a closed region:[9]

$$\text{skin}(x) =_{\text{def.}} \iota y[\forall z[C(z,y)\leftrightarrow\exists w[C\text{–}Atom(w)\wedge P(w,x)\wedge C(w,c\text{–}compl(x))\wedge C(z,w)]]]$$

The skin of a region x is the sum fusion of all its closed atoms that connect with the closure of the complement of x (see Randell and Cohn [1989] for details of the undefined Boolean and topological operators used here). Given this function it is relatively easy to see how to define the free surface of a liquid region:

$$\text{free–surface}(x,t) =_{\text{def.}} \iota y[\forall z\exists v[Air(v)\wedge[C(z,y)\leftrightarrow\exists[C\text{–}Atom(w)\wedge P(w,x|t)\wedge C(w,v|t)\wedge C(z,w)]]]]$$

It is worth mentioning here that the 'skin' function provides an alternative and arguably more intuitive concept of the surface of an object than that suggested in Randell and Cohn [1989] and in section 3 for modelling objects with wetted surfaces, i.e. taking a closed region and attributing the property of wetness to the region but denying it with respect to its interior.

With the free surface defined one could reason that if the surface of the liquid inside the reservoir were at or below the level of the bottom of the inlet pipe (e.g. coincident with the pipe's portal), no further liquid could be pumped through the system.

Motion of the piston would require one to pick out interior surfaces within the pump body so as to introduce the distance function. Varying rates of volume of the liquid in the pump body would then be linked to the position of the piston in the chamber. In turn this would be linked with differences in pressure between bodies of liquid and whether or not valves were shut. We need to be able to reason that when the piston is drawn up, the downthrust force of the piston acting on the contained fluid is less than the atmospheric pressure acting on the free surface of the fluid in the reservoir. The downthrust force of the atmospheric pressure propogating a force through the liquid results in an upthrust force on the piston/liquid surface interface (if we assume no pocket of air between the two, or between the liquid/air and air/piston surface interfaces if we do). The pressure differences serve to force the liquid through the inlet pipe and into the main chamber. $\text{Valve}_1$ opens because the external force of the liquid impressed on its underside is less than the sum of the forces acting on the other side of the door, and so on.

Additional empirical information could be added and linked with the basic formalism, e.g. rigidity in a body would mean that deformability could not arise (e.g. no change in the convex hull over time), physical solid objects would not overlap if originally discrete. Liquids could be construed as incompressible (having constant volume with respect to compressive forces) connected regions which can deform (changing their convex hull) and pass into and fill insides of regions. In contrast gaseous objects (connected open regions (?)) would have the property of filling and occupying the inside of some sealed container; below we define the relation of being sealed within and the property of being a container. We recognise that the latter definition is for a generalised container and that containers vary according to the material being considered is to be contained, question of gravity and orientation and so on.

$$\text{Sealed–Inside}(x,y,t) \equiv_{\text{def.}} Container(y,t)\wedge\exists z[P(z,y|t)\wedge Inside(x|t,z)\wedge\forall p[Portal(p,z)\rightarrow Sealed(p,t)]$$
$$Container(x,t) \equiv_{\text{def.}} [Solid(x)\wedge Hollow(x)]\vee\exists p[Portal(p,x|t)]$$

The modelled domain contains numerous sorts. This becomes particularly noticeable as empirical information is added. For example the monadic predicates Liquid, Fluid, Deformable, and Incompressible can all be sorts and the axioms:

$$\forall x[Liquid(x)\rightarrow Fluid(x)]$$
$$\forall x[Fluid(x)\rightarrow Deformable(x)]$$
$$\forall x[Fluid(x)\rightarrow Incompressible(x)]$$

can all be represented implicitly by subsort relationships in the sort structure. The sorts are in turn all subsorts of Physob.[10]

[9] We are particularly indebted to a discussion with Ian Gent which finally led to the definition given here.

[10] Occurrences of sort literals predicating variables appearing in any axiom can be deleted when they are encoded in the

One final point worth making is that we have blurred two separate notions of 'part' in the above. The predicate 'P' of the topological logic is not the same as a component part (e.g. when we talk of the valve being part of the pump). The former is defined only on regions whilst the latter is only defined on physical objects. We can make this relationship clear by adding the axiom:

$\boxed{\forall xyt[\text{Component-part}(x,y) \rightarrow P(x|t,y|t)]}$

We can then attribute characteristics to component parts which might not be true of a topological part, e.g.:[11]

$\boxed{\forall x[\text{Force-pump}(x) \rightarrow \forall y[\text{Component-part}(y,x) \rightarrow \text{Rigid}(x)]}$

Empirical information in the domain (such as the rigidity or deformability of particular objects) can be exploited to control unnecessary inference. The calculus has a natural sort structure (for example open and closed, and connected and fragmented regions) and further deductive control will arise through the use of a many sorted logic in an implementation of the formalism.

Additional modelling problems that have been explored with this calculus include the process of filtration, and phagocytosis [Randell and Cohn 1989].

## 6. Final Comments

We have concentrated upon spatial topological descriptions to illustrate the descriptive power of the formalism though we have started to develop the temporal and process related aspects of the formalism (which had been ignored in Randell and Cohn [1989]). Our approach is that certain processes can be adequately modelled by linking and ordering appropriate state descriptions over which some topological or metrical property remains invariant. Although not exploited in this paper, continuity constraints on the transformation of regions over time can be defined. Part of this transition network is illustrated in Figure 7. Directions of permitted transitions are indicated with the entry '$\rightarrow$'.[12] Re-description of asymmetrical part/whole relations is also given so that a region can either be described as a part of a whole or as being surrounded by another region. This is indicated in the table by the entry '$\Rightarrow$'. This feature is used in our work modelling phagocytosis [Randell and Cohn 1989] where a food particle can either be regarded as a part of a cell or surrounded by a cell but forming no part of it.

We are confident that the underlying richness of the concepts captured in this formalism makes it especially attractive as a representation language where topological and metrical information is in abundance.

## 7. Acknowledgements

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many sorted logic LLAMA [Cohn 1987].

[11] In LLAMA this axiom could actually be represented by a sortal declaration for Component-part if desired.

[12] In Randell and Cohn [1989] entries for equality differ from those given here. We are indebted to Ben Kuipers who first pointed out the advantage in separating out the purely topological information from the empirical. This amendment reflects this point; ie. we now allow (in the sample table) $= \rightarrow$ PO.

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Figure 1.

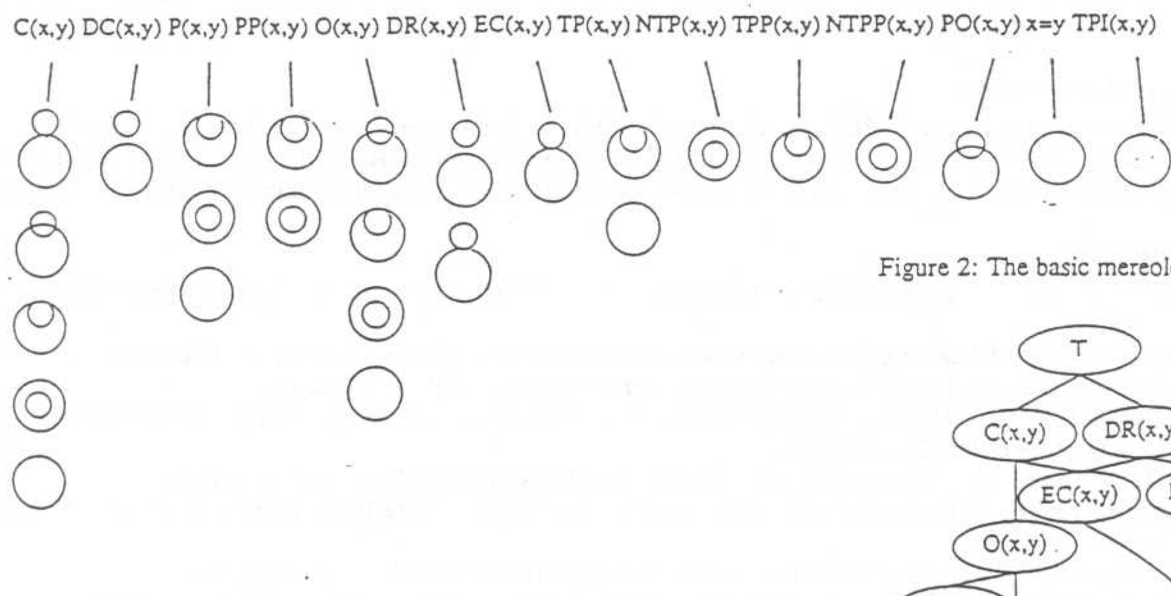



Figure 2: The basic mereological lattice

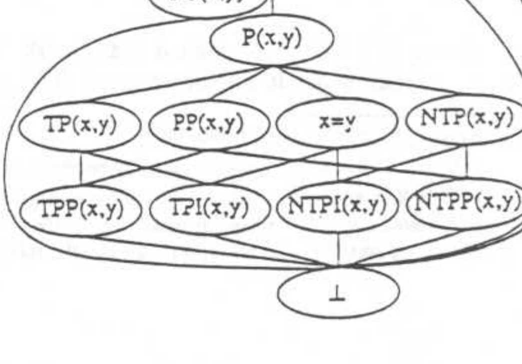



Figure 3: Inversion of TPP(x,y) (region y is the whole figure)

Figure 4: Section through a force pump

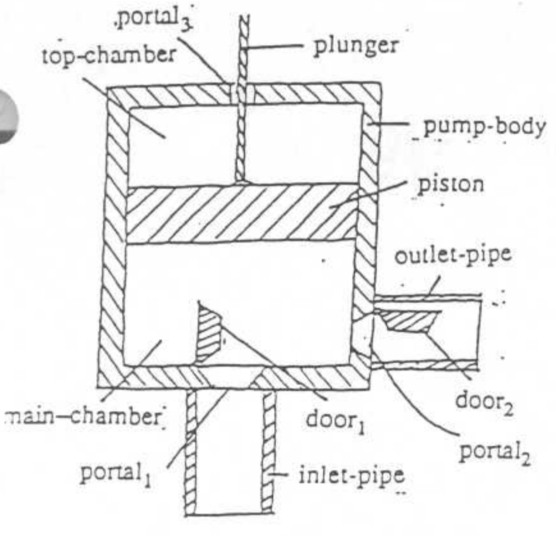
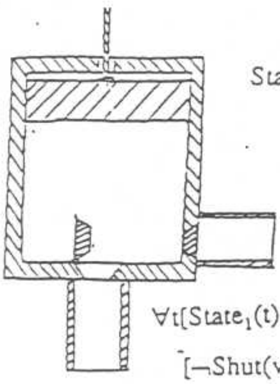
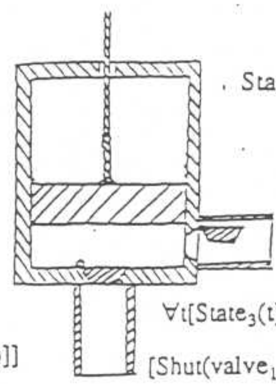



Figure 5: Cycle of the force pump



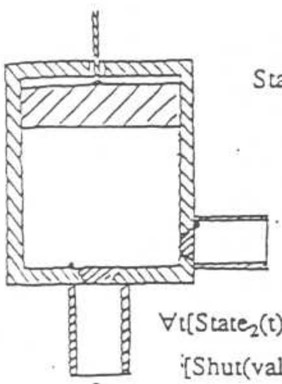
State$_1$

$\forall t[State_1(t) \rightarrow$
$[\neg Shut(valve_1, t) \wedge Shut(valve_2, t)]]$



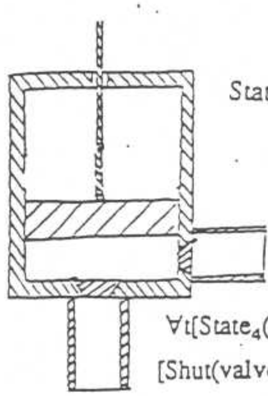
State$_3$

$\forall t[State_3(t) \rightarrow$
$[Shut(valve_1, t) \wedge \neg Shut(valve_2, t)]]$



State$_2$

$\forall t[State_2(t) \rightarrow$
$[Shut(valve_1, t) \wedge Shut(valve_2, t)]]$



State$_4$

$\forall t[State_4(t) \rightarrow$
$[Shut(valve_1, t) \wedge Shut(valve_2, t)]]$

Figure 6: The pump assembly: descriptions inherited from top figure(s).

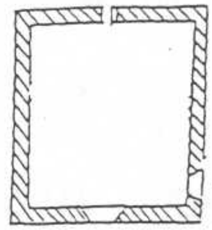


Figure 6a

$\forall t[MCON(pump\text{-}body|t)]$
$\forall t[PP(portal_1, inside(pump\text{-}body|t))$
$\forall t[PP(portal_2, inside(pump\text{-}body|t))$
$\forall t[PP(portal_3, inside(pump\text{-}body|t))$

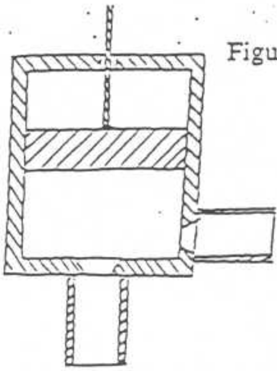


Figure 6b

$\forall t[Inside(piston|t, pump\text{-}body|t)]$
$\forall t[EC(plunger|t, pump\text{-}body|t)]$
$\forall t[P\text{-}Inside(plunger|t, pump\text{-}body|t)]$
$\forall t[\neg Con(diff(inside(pump\text{-}body|t), piston|t))]$
$\forall t[DC(portal_1, portal_2)]$
$\forall t[DC(sum(top\text{-}chamber, portal_3),$
$sum(portal_1, portal_2, main\text{-}chamber))]$
$\forall t[J\text{-}Outside(inlet\text{-}pipe|t, pump\text{-}body|t)]$
$\forall t[EC(inlet\text{-}pipe|t, pump\text{-}body|t)]$
$\forall t[J\text{-}Outside(outlet\text{-}pipe|t, pump\text{-}body|t)]$
$\forall t[EC(outlet\text{-}pipe|t, pump\text{-}body|t)]$
$\forall t[State_2(t) \vee State_4(t) \rightarrow$
$[NTS(main\text{-}chamber,$
$sum(piston|t, pump\text{-}body|t, door_1|t, door_2|t))]]$

$valve_1 = valve(portal_1, door_1|t)$
$valve_2 = valve(portal_2, door_2|t)$

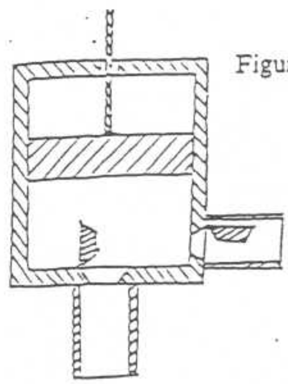


Figure 6c

Figure 5: Part of the continuity/redescription table.

| | DC | EC | PO | = |
|---|---|---|---|---|
| DC | | → | | |
| EC | → | | → | |
| PO | | → | | → |
| = | | | → | |

| | $TS_1$ | $TPP_1$ | NTS | NTPP |
|---|---|---|---|---|
| $TS_1$ | | ⇒ | → | |
| $TPP_1$ | ⇒ | | | → |
| NTS | → | | | ⇒ |
| NTPP | | → | ⇒ | |