Spatial Unification: Qualitative Spatial Reasoning about Steady State Mechanisms An Overview of Current Work

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Abstract

Qualitative simulation is one method for reasoning with incomplete knowledge about mechanisms which change in time. This work adapts that method to reasoning about steady state mechanisms—which change continuously in *space*.

Because flow-paths in mechanisms can loop and self-intersect, spatial behaviors show complexities that temporal behaviors do not. This paper features two extended examples of counter-current heat exchange. Both examples involve such loops—places where a later flow-path segment interacts with an earlier segment.

QSIM has been extended to simulate the behavior of these mechanisms. The extension rules out inconsistent predictions by first assuming them, then following them to a contradiction. The contradictions are detected as inconsistencies between two separate views of the behavior—one view from each side of the interaction.

The two views are cast as the restrictions in a two point boundary value problem. The solutions are generated by instantiating one of the boundaries with all possible qualitative values. The inconsistent predictions manifest as instantiations that have no qualitative solutions at the second boundary.

A formal definition of the process of spatial unification is given in the technical report. [14] This report discusses the technical issues faced and introduces our approach to steady state spatial reasoning.

Chapter 1

Introduction – Temporal and Spatial Reasoning

1.1 Qualitative Simulation

Several authors have advanced methods to do qualitative reasoning [1, 2, 4, 6, 7]. These methods generate descriptions of the behaviors of mechanisms; they are especially useful when the mechanisms cannot be described using numerical or algebraic models. QSIM [8] (Qualitative Simulation) is one of these methods; it has been the basis for the work described in this present paper.

Each method for qualitative reasoning develops some qualitative model. So far, these models have been the qualitative analogue of *lumped parameter* models. In lumped parameter models, properties that are distributed in space or which vary through space are represented by an aggregate scalar value—the distributed parameter is "lumped" at a single point. Many mechanisms show behaviors which cannot be derived from lumped parameter models (for a development, see Weber [15]). These mechanisms must be analyzed with *distributed parameter* models.

This work adapts the QSIM style of Qualitative Simulation to develop a the qualitative analogs of distributed parameter models. This is developed for the class of mechanisms at the steady-state. These have behaviors which vary spatially, not temporally.

Boundary values – QSIM is normally used to solve single point boundary value problems. No boundary conditions of $t > 0$ control the behavior. In the major example discussed in this paper, our extension of QSIM performs spatial reasoning as a two point boundary value problem. In these examples, the mechanisms' boundary values are separated spatially.

Quantitative analysis has methods for addressing two point boundary value problems. When these problems cannot be solved analytically, they can be approximately solved iteratively.

This present work gives a qualitative analog to this method. The simulation is not iterative. Rather, the entire range of qualitative values is assumed, and the qualitatively incorrect values are eliminated by contradictions.

Looped Mechanisms – Because flow-paths in mechanisms can loop and self-intersect, spatial behaviors show complexities that temporal behaviors do not. Oyeleye and Kramer [9, 10] discuss how mechanisms with loops in the flow paths (including mechanisms with feedback control, recycle streams, and streams which interact multiple times) present difficulties for purely qualitative analysis. The qualitative simulations tend to display an untamed branching.

The major example of this paper presents a looped mechanism, where the geometry of a stream's flow-path causes a later part of its behavior to affect an earlier part. The method of *spatial unification* is used to tame the branching of the behaviors.

1.1.1 QSIM Terminology

Within a qualitative simulation, a behavior is represented as a succession of states; the states alternate between time-*points* and time-*intervals*. In each state the parameters of the mechanism are assigned qualitative values (*qval* s). A qval is a (qualitative magnitude / qualitative direction) pair. A qualitative magnitude is either a point value (a *landmark*) or an open interval between a pair of landmarks. The qvals must be consistent with the state's *constraints* and the constraints' *corresponding values* (CVs). The qualitative directions of change (*qdir* s) may be increasing (INC), decreasing (DEC), or steady (STD). For a complete development, see Kuipers.[8][3]

A QSIM model represents a mechanism as a data structure, or set of structures, called REGIONs,[1] Mechanisms that display multiple operating regions are characterized by a different set of constraints in each REGION.

The notation and terminology of this report follow that of Kuipers' *Qualitative Simulation*[8]. Some familiarity with the QSIM program (as described in [3]) is assumed. QSIM is implemented in LISP; some LISP terminology is used [11].

1.1.2 Continuous Variables and Lumped Parameters

The parameters in QSIM models are lumped parameters. That is, parameters are represented as a single scalar quantity. Qualitative simulation cannot simulate a distributed parameter, such as how the pressure profile (the continuous set of values for pressure from the bottom to the top of the tank) changes over time.

QSIM is the qualitative analog of the calculus of ordinary differential equations (ODE). No complete qualitative version of the partial differential calculus is yet available. A full qualitative version of the partial differential calculus would allow us to reason about mechanisms that change in both space and time. This report considers a more restricted class—steady state mechanisms. These mechanisms, which are constant in time, have behaviors which evolve spatially; they vary from one *place* to another.

1.2 Adapting Temporal Reasoning to Spatial Relations

Qualitative simulation of temporally changing mechanisms serves as a basis for reasoning about steady state mechanisms.

1.2.1 The Steady State

We have special knowledge of steady state mechanisms because they do not vary in time. Steady states mechanisms are constrained in ways that changing mechanisms are not. Net forces on elements are zero. Combined flows

[1] similar to QDEs in [8]

at junctions net to zero. The temperature profile along a path is continuous. These properties are not guaranteed in mechanisms that change with time.

The *behavior* of a steady state mechanism is the succession of qualitative states encountered moving along a linear path through the mechanism. Section 2 examines how the geometry of the path places additional types of constraints on the behaviors.

1.3 Some Simple Examples

For the first two examples, only minor changes were introduced into QSIM. We introduce the D/DX constraint. This constraint imposes the same conditions on state transitions as does D/DT; it just has a different interpretation. The *interpretation* of the QSIM output changed—the generated behaviors represent spatial sequences, not temporal ones.

1.3.1 The Garden Hose: Reasoning in Either Direction

Figure 1.1 shows water spewing from a hose, pointed upwards and to the right. The nozzle and water pressure are unchanging; the water traces a fixed arc through the air. This is a non-quiescent steady-state. What can we infer about the shape of the arc?

In this case, although there is flow, the mechanism is at steady-state. We must describe the system in *spatial* constraints—the derivative terms must be w.r.t. X, not T.

Accordingly, we will derive the ODE w.r.t. X, which describes the mechanism's steady state. We begin with the dynamic equations w.r.t. T. Adopting the conventional XY coordinate frame and letting [*term*] be the *sign*(*term*):

$$[dX/dT] = +$$

Since

$$[d^2X/dT^2] = 0 \qquad \text{and} \qquad [d^2Y/dT^2] = -$$

Then

$$dY/dX = \frac{dY/dT}{dX/dT}$$

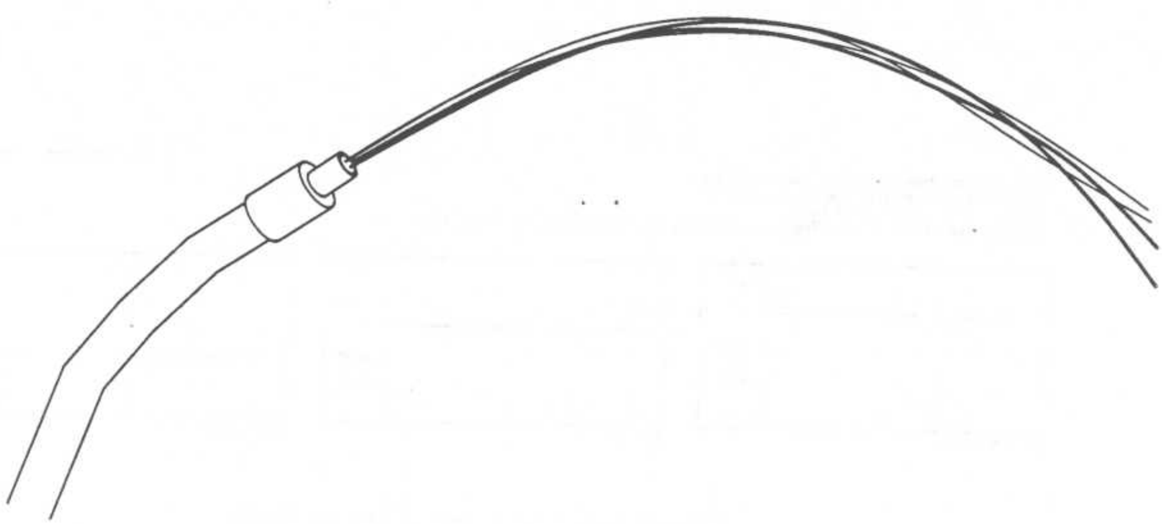


Figure 1.1: Water spewing from a hose, upwards and to the right.

$$d^2Y/dX^2 = \frac{1}{dX/dT}\frac{(dX/dt\ d^2Y/dT^2) - dY/dT\ (d^2X/dT^2)}{(dX/dT)^2} \tag{1.1}$$

$$[d^2Y/dX^2] = \frac{1}{+}\ \frac{(+\ -) - \mathbf{0}}{+^2} = - \tag{1.2}$$

This gives us the system-condition that CURVE (which is Y'', the curvature) is always negative. We initialize with $Y = 0$ at ground level,X_0 to be the place where the water leaves the hose, SLOPE $= +$ and CURVE $= -$. The qdirs are found through propagation, and the simulation yields the behavior shown in Figure 1.2.

Suppose the garden hose is still spewing upwards and to the right, but we where are standing, X_0, is where the water falls to the ground. We take leftward motion to be positive, *the opposite of the usual XY coordinate convention.* Now the water has a negative dX/dT. Equation 1.1 still holds, but equation 1.2 becomes

$$[d^2Y/dX^2] = \frac{1}{-}\ \frac{(-\ -) - \mathbf{0}}{+^2} = -$$

Although the direction the reasoning proceeds in has changed, the sign of CURVE is unchanged. We again initialize at X_0 with the constraints and

Structure: Stream from a garden hose, rising and falling.,
Initialization: Squirt the hose up and to the right (S-0)
Behavior 1 of 1: (S-0 S-1 S-2 S-3 S-4).
Final state: (TRANSITION End of Region, no transitions), (TRANSITION-IDENTITY NIL), NIL.

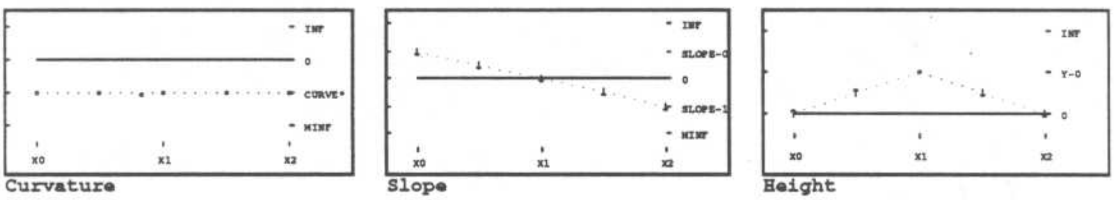


Figure 1.2: The Hose: Flow to Right

Structure: Stream from a garden hose, rising and falling.,
Initialization: Squirt the hose up and to the Left (S-0)
Behavior 1 of 1: (S-0 S-1 S-2 S-3 S-4).
Final state: (TRANSITION End of Region, no transitions), (TRANSITION-IDENTITY NIL), NIL.

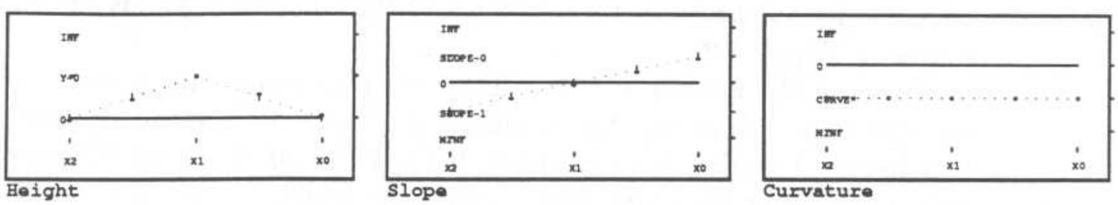


Figure 1.3: The Hose: Flow to Left

deduce the same shape of the arc. Figure 1.3 shows the behavior—it is superficially the same, but note that X_0 is at the right edge, and increasing X_n's are leftward.

Unlike temporal reasoning, which always reasons forward in time, spatial reasoning may start at either end of a behavior.

Reasoning can either go with the flow, or against it. This will be important in reasoning about counter-current flow, where reasoning in the direction of the flow of one stream is, of necessity, against the flow of the other.

Chapter 2

The Need to Unify Multiple Views of a State

In spatial reasoning, a mechanism may have a stream which loops around or is recycled. The same stream parameter, (e.g. temperature) is then present twice in the same place. Changes in the the parameter *later* affect the parameter's value *earlier*. This does not arise in temporal reasoning.

2.1 Example: A Bird's Foot

The next example is of the heat exchange in a bird's leg and foot. Consider the circulation of blood in the foot of a bird perched on a cold wire.

The blood is circulating in the body (BODY1) at the body temperature HSOURCE. The blood is cooled in OUTBOUND, cooled further in FOOT, reheated in INBOUND before returning to the body in BODY2.

Figure 2.1 shows its qualitatively distinct regions of the Bird's Foot example.

2.1.1 Goals for the Qualitative Simulation

When reasoning about this mechanism, we do not know the actual values for the heat-transfer coefficients, either between the bloodstreams nor between the blood and the outside. We do not know the heat-capacities and we may not be able to assume that they are constant over the temperature range.

We cannot then reason quantitatively about this mechanism. Still, we would like to be able to draw certain conclusions about this mechanism –

- The blood can never return to the leg warmer than it left.
- The blood can never be chilled below the outside temperature.
- The blood in the inbound stream can never be warmer than the blood immediately opposite it in the outbound stream.
- The mechanism may exhibit crossover. That is, the blood in the inbound stream may be warmer at the top of the leg than the outbound stream is at the bottom of the leg.

2.2 The Strategy for Finding Consistent Steady-State Behaviors

The CFILTER algorithm (within QSIM) finds the consistent continuations of a behavior by

- generating all legal continuations of each parameter,
- generating all possible combinations of parameter values, (the *tuples*)
- filtering tuples which are inconsistent with the qualitative constraints or with other tests (tests for analytic functions, no-change and so on),
- instantiating these tuples as states, and
- using further tests to filter states that are inconsistent or have no continuation.

The strategy for finding consistent spatial behaviors is similar. At the beginning of the the OUTBOUND region, the temperature in the outbound stream, TEMP, is HSOURCE, the value inherited from the first region. The inbound temperature FACING-TEMP, is not known. FACING-TEMP is instantiated at all possible qualitative values. A state is created corresponding to each of these values (with the values for all of the other parameters following from the values for TEMP and FACING-TEMP).

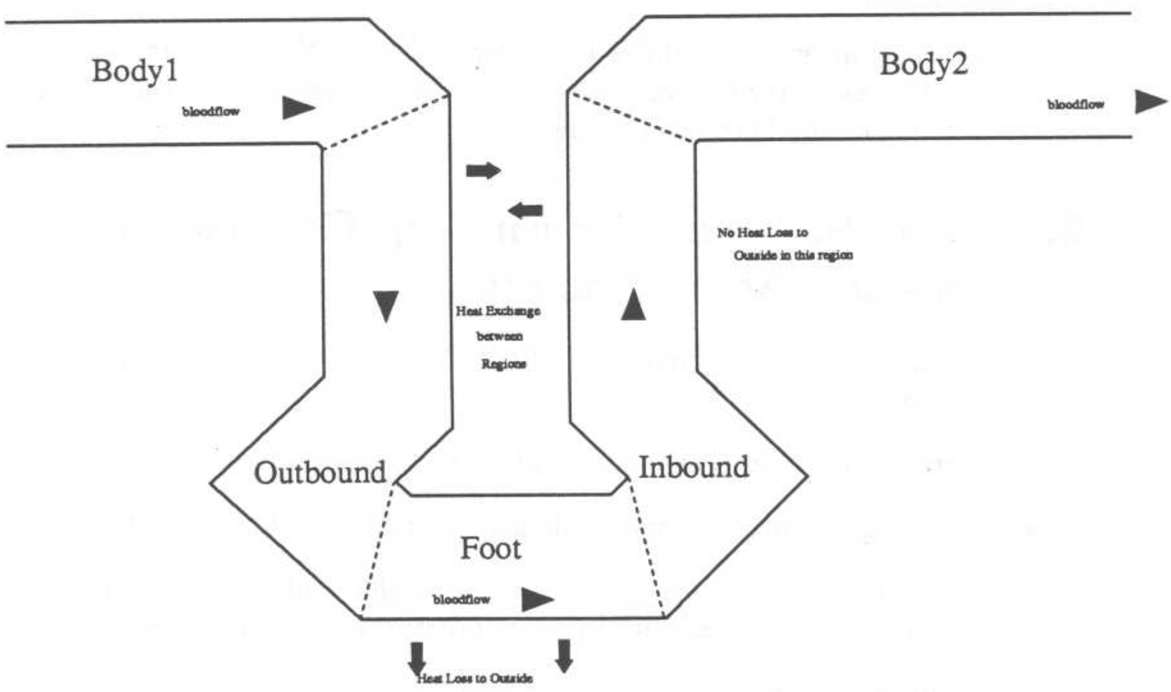


Figure 2.1: Qualitatively Distinct Regions of the Bird's Foot Example

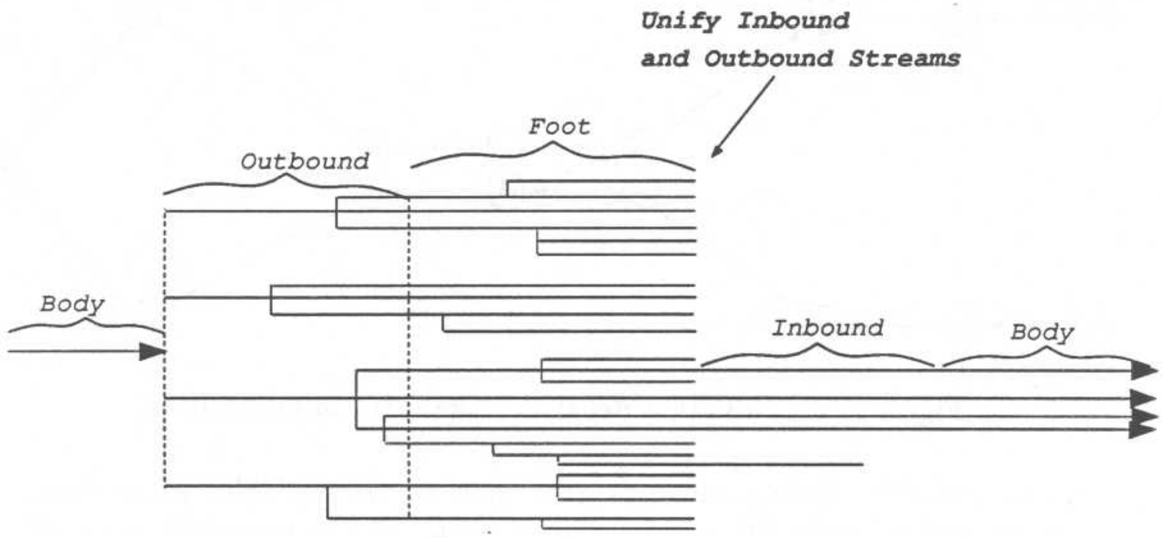


Figure 2.2: The Growth and Pruning of the Behavior Tree

Once the relative positions of TEMP and FACING-TEMP are chosen, the values and directions for the other variables are found using the CFILTER algorithm and the constraints of the model.

These are the initial states in the region. Each state spawns a tree of behaviors. These behaviors reach the end of the OUTBOUND region and continue into the FOOT region. At the end of the FOOT region, an attempt is made to continue each behavior into the INBOUND region. At this point, the continuation into the INBOUND region must unify with the behavior that was tracked in the OUTBOUND region. Figure 2.2 is a sketch of the growth and pruning of the tree of behaviors.

2.2.1 Spatial Unification

The behavior of a steady state mechanism is the succession of qualitative states encountered moving along a linear, *although not necessarily straight*, path through the mechanism. If the path twists back on itself, the behavior will visit the same physical place in multiple states. These states will not, generally, be adjacent in the behavior.

Any pair of such states gives two views of the same place. These two

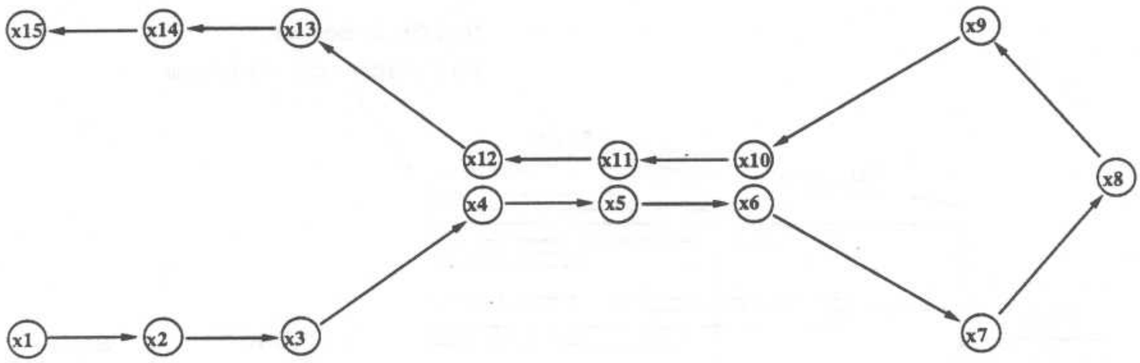


Figure 2.3: The Path a Behavior Takes May Intersect Itself

views are of the same parameters, they must be reconciliable. If the views are not reconciliable, the states are inconsistent, and no consistent behavior can contain both states.

Unifying Two Views of the Quantity At the point where the blood flows from FOOT into OUTBOUND, FACING-TEMP is instantiated to several qmags. Some of the initializations are impossible. Although they are locally consistent, they not consistent with any real behavior. For instance, the initialization where the stream returns to the body hotter than when it left is not physically possible.

All possible continuations of the assumption that TEMP $<$ FACING-TEMP at the top of OUTBOUND lead to contradictions. Eventually, all of the possible behaviors that are continuations of this initialization are pruned. Then, this initialization is itself pruned, and the system concludes that no possible steady state behavior returns the blood to the body warmer than when it left.

This is the style of reasoning used to find the behaviors—and to exclude all of the impossible behaviors. All of the possible local behaviors are instantiated and explored. Those which are not globally consistent lead to contradictions when multiple views of a region are unified. Contradictory behaviors are pruned, leaving the qualitatively distinct behaviors.

Chapter 3

Reasoning about the Steady State

3.1 Steps in the Reasoning

The two major examples in this paper are of counter-current exchange. In both cases, a single liquid stream, on its way out, is meeting with itself on its way back in. In each mechanism, a hot, outbound segment of a stream exchanges with the cold, inbound segment.

There are two viewpoints of such a counter-current exchange. From the hot side, both segments are warming; from the cold, both are cooling.

The hot, outbound liquid is chilling as it moves forward. From its viewpoint, the cold liquid is also chilling—that is, the farther the outbound liquid gets from its origin, the colder is the inbound liquid which it faces. Contrawise, the cold inbound liquid is warming as it goes forward, and from its viewpoint, the hot, outbound liquid is also warming.

These two distinct views result from having two different view of the *forward* direction—one towards one end of the mechanism, one towards the other. But these are two views are of the same system, and the views must be reconciliable. After all, from an *outside* viewpoint, the mechanism is at steady state—the temperature at any single point in the mechanism is constant over time and there is no warming nor cooling trend.

3.1.1 Generating the Correct Behaviors

This work's simulation and analysis of steady-state mechanisms depends crucially upon generating these multiple views, and on reconciling them.

Section 2.1.1 states several desirable conclusions about the behavior of the Bird's Foot. These conclusions will be reached by

- generating all of the qualitatively distinct, consistent steady-state profiles of the mechanism; and
- showing that all the behaviors are consistent with those conclusions.

For each behavior generated, two different views of the leg will be generated, one each for the INBOUND and OUTBOUND views.

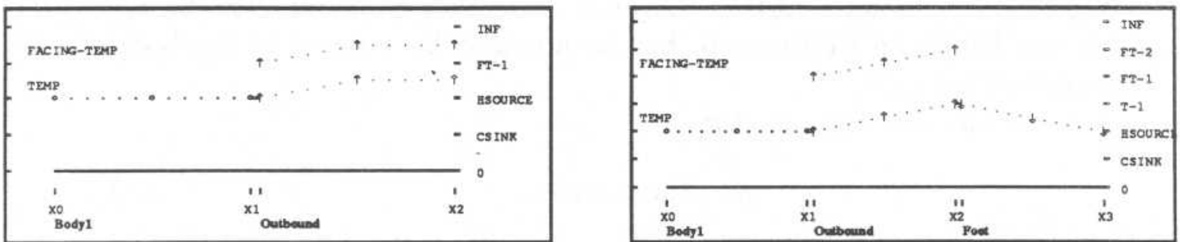
3.1.2 Initialization

The simulation begins in the BODY1 region, with the TEMP of the blood the body temperature. The blood remains at the body temperature as it passes into OUTBOUND. The temperature of the inbound blood, FACING-TEMP is at some temperature, unknown as yet.

Because FACING-TEMP is unknown, it could be at any place in the qualitative temperature space. This gives seven possible qualitative magnitudes:

- at freezing,
- between freezing and the outside temperature,
- at the outside temperature,
- between the outside temperature and the body temperature,
- at the body temperature,
- between the body temperature and infinity, and
- at infinity.

Each of these values for FACING-TEMP leads to a possible continuation of the behavior. The behavior branches seven ways, and continuations are generated.



(A) Continuation of Inconsistant Behavior into Outbound (B) Continuation of Inconsistant Behavior into Foot

Figure 3.1: A Continuation of an Inconsistent Behavior in OUTBOUND

3.1.3 Continuation and Transition

Each behavior is continued, moving outward along the leg. Two of the initializations have no continuations. Neither of the initializations where FACING-TEMP was at infinity and increasing, or at freezing and decreasing, have continuations. They are pruned immediately.

Eventually, all behaviors are either pruned, or they reach the region transition at the end of OUTBOUND and continue into FOOT. Again, each behavior is continued in the normal way, until it is pruned or reaches the end of the FOOT region.

3.1.4 Using Spatial Unification to Prune Behaviors

The blood is cooled in FOOT and enters the INBOUND region. The behaviors must now be unified spatially. Section 2.2.1 introduced the unification and its mathematical description. We will now consider how it applies to the Bird's Foot.

The point where the blood leaves the foot and enters the inbound region is the first place where the behavior returns to a place it has visited earlier in the behavior. This is the first place where the unification is performed, and it is where a large number of behaviors are pruned.

Up until now, the simulation has pursued some impossible behaviors that were locally consistent. For instance, Figure 3.1-A showed one instantiation of FACING-TEMP where the blood's final return temperature was hotter than the body temperature. This behavior was locally consistent - it had a consistent continuation, where both stream were even hotter nearer to the foot. But the simulation will find that this behavior leads to a contradiction.

Figure 3.1-B shows a further continuation of that behavior. The blood, which was heated in OUTBOUND, has been cooled back down to the body temperature in FOOT.

The original assumption, that

$$\text{HSOURCE} < \text{FT-1} \tag{3.1}$$

led, in this continuation, to the intermediate conclusion that

$$\text{HSOURCE} < \text{T-1} < \text{FT-1} < \text{FT-2} \tag{3.2}$$

The continuation through the foot set the blood's temperature leaving the foot at HSOURCE. As the blood enters INBOUND the TEMP of the blood must unify with the FACING-TEMP of the last state of the OUTBOUND region. This would mean that

$$\text{HSOURCE} = \text{FT-2}$$

But this would contradict Eq 3.2. Therefore, this continuation is inconsistent. It is pruned. Likewise, the other continuations through the foot, where the blood is cooled below HSOURCE or where it is cooled to between HSOURCE and T-1, also fail to unify with the value for FT-2 in Equation 3.2. These behaviors are also pruned. Eventually, all of the continuations that had the assumption of Equation 3.1 at their beginnings are pruned. This means that there are *no* consistent behaviors where the blood returns to the body hotter than HSOURCE.

Spatial unification does not have to occur after *every* region transition. Unification occurs only when transitioning into a region which interacts with a region through which the behavior has already passed. In the Bird's Foot example, this happens only at the transition from FOOT to INBOUND.

Chapter 4

Results

4.1 Results of the Bird's Foot Example

Hot crossover occurs when the hot stream is cooled to the maximum temperature of the cold stream. *Cold crossover* occurs when the cold stream is heated to the minimum temperature of the hot stream. Crossover occurs only in counter-current exchange, never in co-current mechanisms. There are five qualitatively distinct steady-state configurations of the Bird's Foot. Figure 4.1 shows them. They differ in the way that that crossover occurs.

C1 is the configuration where the least amount of heat is recovered. The warming in the INBOUND region never rewarms the blood to the temperature at which it entered FOOT. The returning blood at the hip is cooler than the OUTBOUND temperature at the ankle. No crossover occurs.

In C2, the returning blood recovers just as much heat in the INBOUND region as it lost in FOOT. Hot crossover occurs at the bottom edge of OUTBOUND, and cold crossover occurs at the top edge of INBOUND.

In C3, the returning blood recovers still more heat. Both crossovers now occur in the interior of their regions. Cold crossover is still above hot crossover.

In C4, both crossovers occur at the same place in the leg.

In C5, hot crossover occurs above cold crossover.

Figure 4.2 shows the a screen-image of the five steady state behaviors of the Bird's Foot mechanism generated by spatial reasoning in QSIM.

QSIM deduces these five behaviors. QSIM initially assumed all possible

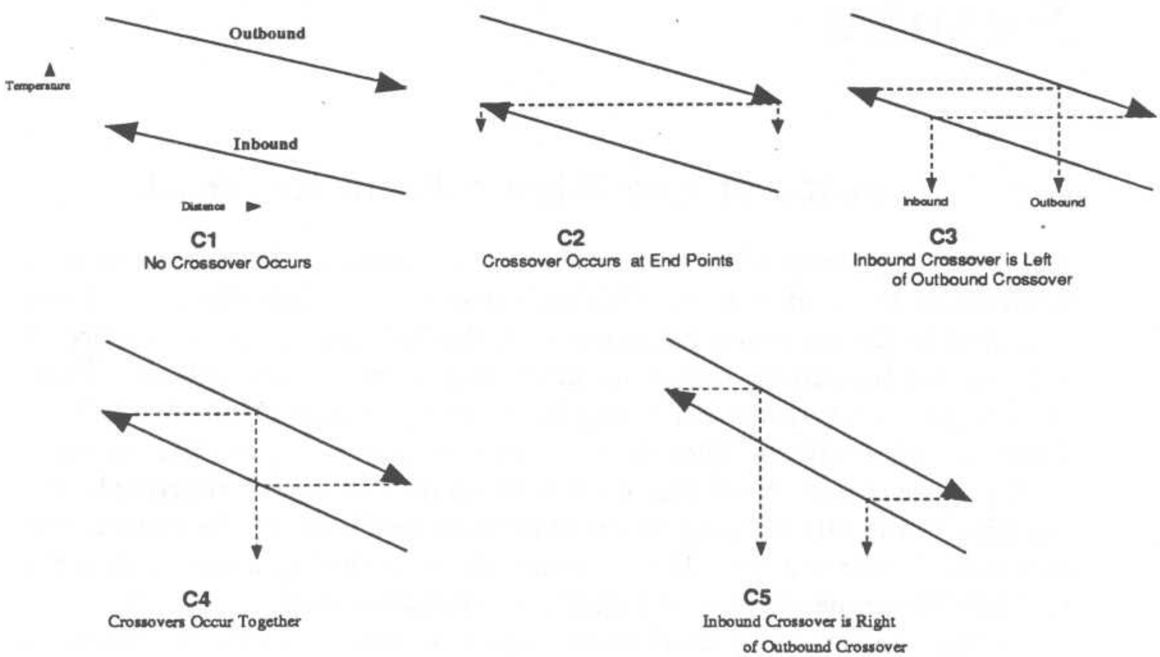


Figure 4.1: Five Qualitatively Distinct Behaviors of the Bird's Foot

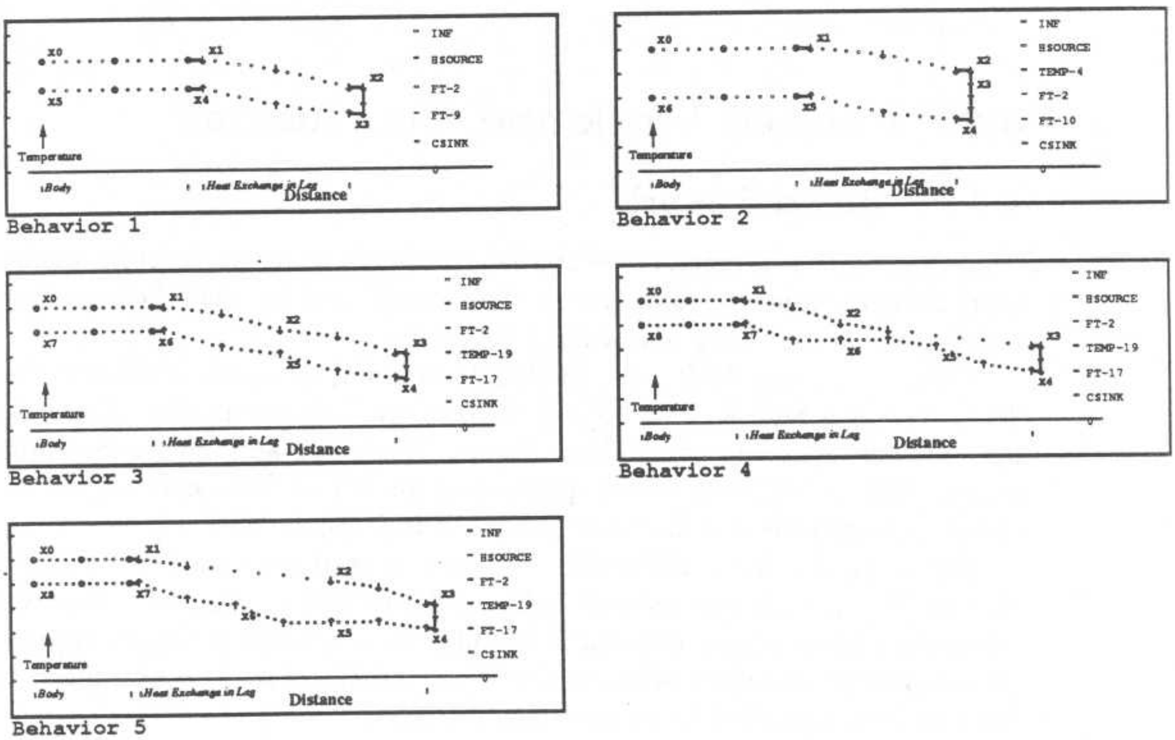


Figure 4.2: QSIM Generated Plots of the Bird's Foot Example

behaviors as outcomes, and pruned the inconsistent ones. These five behaviors, then, represent all possible consistent behaviors. In section 2.1.1 we listed four points that we felt a spatial reasoning program should be able to conclude about this mechanism. We can use QSIM's simulations to answer the four points.

- In no behavior does the blood return warmer than it left.
- Nowhere in any behavior is the blood ever chilled below the outside temperature.
- Nowhere in any behavior is the blood in the INBOUND region warmer than the blood immediately opposite in the OUTBOUND region.
- The mechanism can exhibit crossover—crossover occurs in four distinct ways. But there is one possible behavior in which crossover does not

occur.

4.2 Related Work and Conclusions

4.2.1 Related Work

Preceding work has represented spatial relationships as single point connections between adjacent components. DeKleer [2] and Forbus [4] demonstrate techniques for reasoning about fluid flowpaths.

Other work has dealt with qualitative reasoning about counter-current flow. Oyeleye and Kramer [9][10] develop qualitative models of counter-current flow, focusing on methods for resolving sign ambiguities at confluences. [12] and [13] hybridize qualitative models of heat exchangers with order-of-magnitude and LaPlace transform models, respectively.

Hayes [6] discusses difficulties in doing spatial reasoning with liquids. Forbus [5] contrasts understanding liquid flows from a static viewpoint with viewpoints of molecular collections of fluid. This work provides an approach comparable to the view taken in the Wire and Garden Hose examples; but has not been extended to counter-current flow.

4.2.2 Conclusions

This present work successfully models two moderately complex examples. This success shows that qualitative spatial reasoning is feasible. It further shows that such reasoning is about as difficult as is temporal qualitative reasoning. The two are closely related; major portions of the new techniques necessary for this effort (including the shared quantity spaces, the multiple region transitions, prohibited transitions) and the new data structures necessary for this effort (including especially the MECH as structure to impose order on the diverse QDEs) have immediate application back to temporal reasoning.

Qualitative reasoning is no longer limited to lumped parameter models. Mechanisms that can only be described with distributed parameters can be recast in ways that existing qualitative reasoning tools can handle. This has been demonstrated in mechanisms (counter-current exchangers) which loop back on themselves—where loops and recycle have long presented particularly difficult problems for qualitative reasoning.

The extended QSIM program finds all of the the possible distinct qualitative behaviors. It rules out many inconsistent predictions by assuming them and following them to a contradiction. This approach is essentially an all-paths search, with good pruning keeping the search space manageable.

Mechanisms of moderate size are fully explored in manageable amounts of computer time. The Flamingo Leg has ten constraints over nine parameters, in five regions. It builds about 5000 states to simulate ten behaviors of about 17 states each. It takes about 40 minutes on a Symbolics 3640 to run.

Mechanisms of moderate size are tractable to this program. This attests to both the power of the QSIM constraint filtering and the power of spatial unification as a way of representing spatial knowledge.

The representation are kept tractable by detecting contradictions in the generated behaviors. Detecting the contradiction involves representing the mechanism in multiple viewpoints, and spatially unifying these views. The spatial unification procedure is the most complex part of the reasoning. The theoretical analysis of the unification showed many possible problems that the examples did not present; however, the procedural ability do deal with these problems was kept in the code. This points to a larger set of flow problems to which the current program could be profitably applied.

Bibliography

[1] D. Bobrow. *Qualitative Reasoning about Physical Systems.* MIT Press, Cambridge, Mass, 1985.

[2] J. de Kleer and J.S. Brown. A qualitative physics based on confluences. *Artificial Intelligence*, 24:7–84, December 1984.

[3] A. Farquhar and B. Kuipers. The QSIM manual. 1987. Available from University of Texas AI Lab.

[4] K. Forbus. Qualitative process theory. *Artificial Intelligence*, 24:85–168, December 1984.

[5] K. Forbus and J. Collins. Reasoning about fluids via molecular collections. *AAAI-87*, 1987.

[6] P.J. Hayes. Naive physics I: Ontology for liquids. In Hobbs and Moore, editors, *Formal Theories of the Commonsense World*, pages 71–106. Ablex, 1985.

[7] B. Kuipers. Commonsense reasoning about causality: Deriving behavior from structure. *Artificial Intelligence*, 24:169–204, December 1984.

[8] B. Kuipers. Qualitative simulation. *Artificial Intelligence*, 29:289–338, December 1986.

[9] O. O. Oyeleye and M. A. Kramer. Qualitative simulation of chemical process systems: Steady state analysis. *Chemical Engineering Science*, March 1987.

[10] O. O. Oyeleye and M. A. Kramer. The use of causal information in determining steady-state qualitative process behavior. *Chemical Engineering Science*, 1987. (submitted).

[11] Guy L. Jr. Steele. *Common LISP, The Language*. Digital Press, 1984.

[12] D. Throop. Getting rules to talk to models. In J.S Kowalik, editor, *Proceedings of the Second Workshop on Coupling Symbolic and Numerical Computation*. North Holland, 1987.

[13] D. Throop. LaPipe: A program for reasoning about time varying phenomena. In *The NASA Ames Symposium on Knowledge Based Systems*. NASA Ames Research Center, 1987.

[14] D. Throop. Spatial unification: Qualitative spatial reasoning about steady state mechanisms. Technical Report AI89-95, Artificial Intelligence Laboratory, The University of Texas at Austin, February 1989.

[15] Thomas W. Weber. *An Introduction to Process Dynamics and Control*. John Wiley & Sons, 1973.