Derivation of Exogenously-Driven Causality
Based on Assumptive Structural Equations

Takashi Washio, Ph.D.
Nuclear Reactor Laboratory
Massachusetts Institute of Technology
138 Albany Street, Cambridge, MA 02139
(617) 253-4211 / washio@mitvma.bitnet

Manuscript prepared for: The Third International Workshop on Qualitative Physics, Palo Alto, California (August 9-11, 1989)

Introduction

This research shows that the major knowledge of the exogenously-driven causality of physical systems can be derived from the knowledge of time-evolutional and functional causality which are generally assumed in our ontological hermeneutics of physics. A method is proposed to derive candidate exogenous variables, causal structures, and mythical causality of a system based on 'assumptive structural equations' which represent system-independent quantitative and causal assumptions that are, in our interpretation, assigned to each elementary physical law.

Method

Iwasaki and Simon proposed a causal ordering method based on structural equations [Iwasaki and Simon 86, Iwasaki 88]. Using their approach, each structural equation is formulated to represent a mechanism in a system. In the research reported here, we consider directions of influence among variables in each physical law which is more elementary than a mechanism. These directions are assumed system-independently in the physical interpretation. The knowledge of the assumed directions can be implemented comprehensively in each physical law's mathematical formula. The method for so doing is to separate variables in law ℓ into two groups, $X\ell$ and $Y\ell$, and arrange $X\ell$ on the right side of the equation and $Y\ell$ on the left side. $X\ell$ is a group of variables which unidirectionally influence the variables in the other group $Y\ell$ in the law ℓ. Equations formulated using this method are called 'assumptive structural equations'. For example, the physical laws of the electric heater shown in Fig.1 are represented by the following six assumptive structural equations:

$V/I = R$	(Ohm's Law)	(1)
$R = f(T_h)$	(Temperature Dependency of Resistance)	(2)
$\dot{H}_j = VI$	(Joule's Law)	(3)
$\dot{H}_c = K(T_h - T_a)$	(Heat Conduction Law)	(4)
$H_h/T_h = CM_h$	(Specific Heat Capacity Law)	(5)
$H_h = \int_0^t(\dot{H}_j - \dot{H}_c)dt$	(Time Integral)	(6)

V, I, R, H_h, T_h, and M_h:voltage, current, resistance, heat, temperature, and mass of heater, $\dot{H}_j$:Joule heat generation rate, $\dot{H}_c$:heat conduction rate, T_a:temperature of atmosphere, K:heat resistance, C:specific heat coefficient

The distinct time-evolutional causality from the time derivative of a variable to the variable itself is assumed in the Time Integral. The functional causality which is not explicitly related to time is assumed in the rest of the laws in our general interpretation. In Ohm's Law, voltage and current change their values mutually, but, resistance is interpreted not to be changed. Resistance can be changed only in the dependency law of temperature in an ordinary resistance device. Joule's Law stands for an irreversible process from voltage and current to heat generation rate. In the Heat Conduction Law, the heat resistance and the difference of temperature between two points change the heat conduction rate. But, temperature difference and heat conduction rate are interpreted not to change heat resistance. Heat conduction rate is generally interpreted to change the temperature difference through the Time Integral and Specific Heat Capacity Laws, but not through the Heat Conduction Law. In Specific Heat Capacity Law, heat and temperature change mutually, but do not affect the mass and the specific heat coefficient.

We now consider a procedure to derive candidate exogenous variables, causal structures, and the mythical causality of a system from a set of assumptive structural equations L representing the system. Let S be a set of all variables in L. When $Y\ell$ ($\ell \in L$) involves only a single element, the element and the equation ℓ are called a 'determined variable' and a 'determining equation' respectively. The procedure is as follows.

P1: Let the set of all determining equations in L be L1.
Let the set of all determined variables in S be S1.
Move variables belonging to S1 from $Y\ell$ to $X\ell$ in each equation ℓ in (L-L1).

P2: Let the set of all determining equations in (L-L1-...-L[i-1]) be Li.
Let the set of all determined variables in (S-S1-...-S[i-1]) be Si.
Move variables belonging to Si from $Y\ell$ to $X\ell$ in each equation ℓ in (L-L1-...-L[i-1]-Li). i=i+1.

P3: Repeat step P2, until no more determining equations are obtained.
The undetermined variables (S-S1-...-Sn) is the set of candidate exogenous variables S0, where n is the number of times that step P2 was applied.

A determining equation represents a law in which a variable on the left side is determined from the other variables on the right side. Thus, the determining equations derived in this procedure stand for the causal order of the system. The variables which are not determined are considered to be candidate exogenous variables.

We apply this method to the set of eq.(1) to (6) representing the electric heater. Initially, the following sets are defined:

$$L = \{(1),(2),(3),(4),(5),(6)\}; \quad S = \{V,I,R,H_h,T_h,M_h,\dot{H}_j,\dot{H}_c,T_a,K,C\} \tag{7}$$

At P1, the determining equations in L are eq.(2), (3), (4), and (6). Thus,

$$L1 = \{(2),(3),(4),(6)\}; \quad S1 = \{R,\dot{H}_j,\dot{H}_c,H_h\}; \quad (L-L1) = \{(1),(5)\}. \tag{8}$$

Among the variables in $Y\ell$ of equations in (L-L1), H_h at eq.(5) is included in S1. Hence, H_h is moved from Y5 to X5. Thus,

$$T_h = H_h/(CM_h) \tag{5'}$$

At P2, the new determining equation in (L-L1) is only eq.(5). Thus,

$$L_2 = \{(5)\}; \quad S_2 = \{T_h\}; \quad (L-L_1-L_2) = \{(1)\}. \tag{9}$$

Because T_h in S2 is not in Y1={V,I}, this procedure is terminated at P3, and

$$S0 = (S-S1-S2)=\{V,I,M_h,T_a,K,C\}. \tag{10}$$

M_h, T_a, K, and C in S0 do not appear on the left side of any equation in L. Hence, they are obviously exogenous in this system. The exogeneity of V and I is not still determined. If we suppose that V is exogenous, then V is moved from Y1 to X1, and the following new determining equation is obtained:

$$I = V/R \tag{1'}$$

Conversely, if I is taken as exogenous instead of V, V is determined. Hence,

$$V = IR \tag{1''}$$

These two interpretations of causal order are depicted in Fig.2. Fig.2(a) shows the case in which V is assumed to be exogenous. Intuitively speaking, this interpretation corresponds to the causal structure of an electric heater operated by a constant voltage power supply. Fig.2(b) is the case in which I is exogenous. This corresponds to the causal structure formed by a constant current power supply. Moreover, if we perturb any variables except the candidate exogenous variables, some of determining equations will be over-constrained, and a portion of the causal structure will be disrupted. Hence, the resultant causal interpretations stand for both of causal structures and feasible mythical causality under the system's normal operation.

Concluding Remarks

A method is proposed to derive the knowledge of the exogenously-driven causality of a system. The method is based on 'assumptive structural equations' which represent system-independent time-evolutional and functional causality which are assumed at each elementary physical law in our ontological hermeneutics. This method will provide highly beneficial knowledge for qualitative reasoning, simulation, and diagnosis.

Acknowledgments

The author expresses appreciation to Dr. Y. Iwasaki in Knowledge System Laboratory of Stanford University, Dr. H. Motoda of the Hitachi Advanced Research Laboratory, and Prof. M. Kitamura of Tohoku University for their useful comments. The author also thanks Dr. J.A. Bernard of the Nuclear Reactor Laboratory of the Massachusetts Institute of Technology for his encouragement. This research was supported by the Sandia National Laboratories.

References

[Iwasaki and Simon 86] Iwasaki, Y. and Simon, H.A., "Causality in Device Behavior," Artificial Intelligence, Vol.29, pp.33-61, 1986.

[Iwasaki 88] Iwasaki, Y., "Causal Ordering in a Mixed Structure," Proc. of the National Conference on Artificial Intelligence, pp.313, 1988.

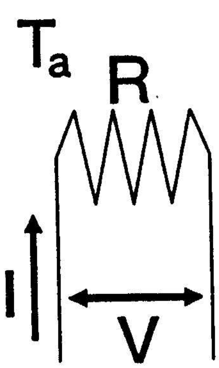
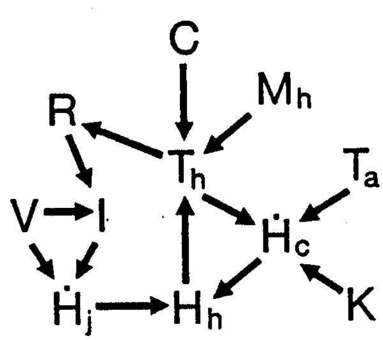
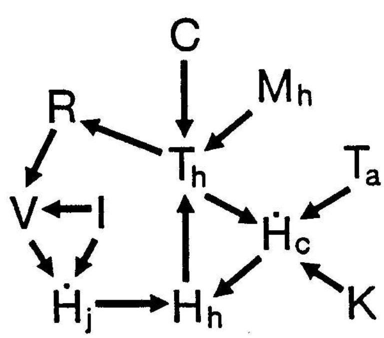


Fig.1 An electric heater.



(a) V is exogenous.



(b) I is exogenous.

Fig.2 Result of causal ordering.