A Qualitative System Identification Method

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Abstract

A method which can qualitatively identify a dynamical system according to qualitative descriptions about the behavior of the observed system is proposed. The system identification problem is well known especially in control theory. It involves the identification of the causal structure of systems according to observed behabvior.

We introduce a Qualitative System Identification Method based on Qualitative Reasoning. The proposed method determines the most likely constraints that can satisfy a given observation set.

Our approach takes an analytical approach to the modeling process which generates a mathematical model of an observed dynamical system according to its behavior.

AI topic: qualitative system identification , identification of causal structure

Domain area: mathematical modeling

Language/Tool: CommonLisp / SparcStation

1 Introduction

The estimation of the behavior of a dynamical system, namely simulation, requires the causal structure of the system to described as differential equations and by determing their numerical solutions. System identification is the inverse problem of such simulation process. It is the process of identifying a causal structure between the variables that characterize the observed dynamical system, from observations about the behavior of the variables. This paper discusses a qualitative system identification method based on qualitative reasoning.

The method proceeds in two steps, given the situation in which a time series of characteristic variables is observed.

1. Estimation of Qualitative States described by Qualitative Values and Qualitative Derivatives.
2. Identification of causal structure as Qualitative Differential Equations.

Chapters 2 through 4 derive the basic mathematical relations. In order to estimate possible qualitative states, *State Estimation Rules* are derived in chapter 5. A method to estimate the qualitative differential equations is derived in chapters 6 through 7. An experimental result is presented in chapter 8.

2 Quantization of Value

The qualitative description of a numeric value is called its *Qualitative Value*. A qualitative Value is a symbolic value which consists of several landmarks and intervals in Euclid Space. The proposed method employs several symbols in order to develop our method in Qualitative Value Space in which the only landmark is "ZERO".

landmark	0	$\Longrightarrow$	[0]
interval	$(-\infty, 0)$	$\Longrightarrow$	[−]
interval	$(0, +\infty)$	$\Longrightarrow$	[+]
interval	$[0, +\infty)$	$\Longrightarrow$	[0+]
interval	$(-\infty, 0]$	$\Longrightarrow$	[−0]
interval	$(-\infty, +\infty)$	$\Longrightarrow$	[+0−]
interval	$(-\infty, 0)\ (0, +\infty)$	$\Longrightarrow$	[+−]

We define the basic operations of these Qualitative Values in Table.1.

Here, "$\subset$" describes *subset*. Therefore, the following relations exist between the Qualitative Values defined above.

Table 1: Addition and Multiplication of Qualitative Values

(A) *Additive Operations*

$+$	[+]	[0]	[−]	[0+]	[−0]	[+−]	[+0−]
[+]	[+]	[+]	[+0−]	[+]	[+0−]	[+0−]	[+0−]
[0]		[0]	[−]	[0+]	[−0]	[+−]	[+0−]
[−]			[−]	[+0−]	[−]	[+0−]	[+0−]
[0+]				[0+]	[+0−]	[+0−]	[+0−]
[−0]					[−0]	[+0−]	[+0−]
[+−]						[+0−]	[+0−]
[+0−]							[+0−]

(B) *Multiplication*

$\times$	[+]	[0]	[−]	[0+]	[−0]	[+−]	[+0−]
[+]	[+]	[0]	[−]	[0+]	[−0]	[+−]	[+0−]
[0]		[0]	[0]	[0]	[0]	[0]	[0]
[−]			[+]	[−0]	[0+]	[+−]	[+0−]
[0+]				[0+]	[−0]	[+0−]	[+0−]
[−0]					[0+]	[+0−]	[+0−]
[+−]						[+−]	[+0−]
[+0−]							[+0−]

$$
\begin{array}{lll}
[Q] \subset [+] & \Rightarrow [Q] = [+] \\
[Q] \subset [0] & \Rightarrow [Q] = [0] \\
[Q] \subset [-] & \Rightarrow [Q] = [-] \\
[Q] \subset [0+] & \Rightarrow [Q] = [0] \text{ or } [Q] = [+] \\
[Q] \subset [-0] & \Rightarrow [Q] = [0] \text{ or } [Q] = [-] \\
[Q] \subset [+-] & \Rightarrow [Q] = [+] \text{ or } [Q] = [-] \\
[Q] \subset [+0-] & \Rightarrow [Q] = [+] \text{ or } [Q] = [0] \text{ or } [Q] = [-]
\end{array}
$$

A qualitative description about a derivative value is called a *Qualitative Derivative*. Its meaning is as follows.

$$
\begin{array}{ll}
\partial Q = [+] & \Longrightarrow Q \text{ is increasing.} \\
\partial Q = [0] & \Longrightarrow Q \text{ is steady.} \\
\partial Q = [-] & \Longrightarrow Q \text{ is decreasing.}
\end{array}
$$

3 Quantization of Time

In this paper, characteristic values are assumed to have continuous behavior. Therefore, they must satisfy the following conditions[Nishida 89].

1. Once a Qualitative Value enters into an *interval*, it must stay there for a while.
2. A Qualitative Value goes through a *landmark* when it transfers from one interval to another.
3. Qualitative value passes through a *landmark* in an instant.

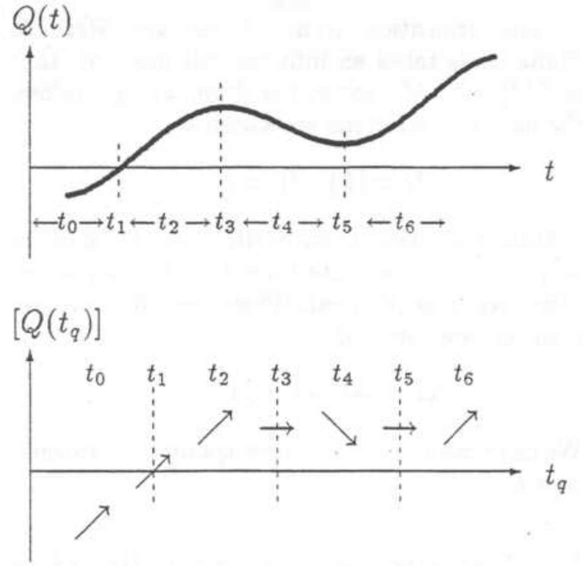
Value states can, accordingly, be divided into two types, namely, "*Static State*" and "*Momentary State*". Therefore, time is quantized. Corresponding with these two state types, two types of quantized time are defined, namely, "*Time Interval*" and "*Moment*". On the quantized time axis, *Time Intervals* and *Moments* appear alternately. *Figure*.1 shows the continuous function $Q(t)$ and its qualitative description $[Q(t_q)]$ on the quantized time axis t_q.

4 Basic Mathematical Relations

4.1 Assumption of Continuous Behavior

As assumed in the previous chapter, the Qualitative Value $[Q(t)]$ can not transit by jumping over any landmark nor any interval from quantized time t to t'. Therefore, in the Qualitative Value Space, which has arbitrary n landmarks and $n+1$ intervals,

Figure 1: Continuous function $Q(t)$ and its qualitative interpretation $[Q(t_q)]$ on *Quantized Time Axis* t_q



landmark	L_i	,	$i = 1, 2, \ldots, n$
interval	$(-\infty, L_1)$		
interval	(L_j, L_{j+1})	,	$j = 1, 2, \ldots, n-1$
interval	$(L_n, +\infty)$		

Qualitative Value $[Q(t)]$ must satisfy the following relations.

- Prohibition of jumping over landmark

$$\{([Q(t)] - L_i) \cdot ([Q(t')] - L_i) \subset [0, +\infty)\} \quad i = 1 \cdots n$$

- Prohibition of jumping over interval

$$\prod_{i=1}^{n} \prod_{j \neq i}^{n} \{([Q(t)] - L_i) + ([Q(t')] - L_j)\} \neq [0]$$

In the Qualitative Value Space that has only landmark "ZERO", the next relation must be satisfied. (In this case "Prohibition of jumping over interval" need not be considered.)

$$[Q(t)] \cdot [Q(t')] \subset [0+] \tag{1}$$

4.2 Rules to Distinguish Time Interval or Moment

Assuming a differentiable function $Q(t)$ defined in Euclid Space, $Q(t)$ satisfies the next relation.

$$Q(t') = Q(t) + \Delta t \cdot \left. \overline{\frac{dQ(\xi)}{dt}} \right|_{\xi = t \to t'}$$
$$\Delta t = t' - t$$

Quantizing this relation into the Qualitative Value Space, the following relations are derived.

$$[Q(t')] \subset [Q(t)] + \overline{\partial Q(t \to t')} \cdot [\Delta t] \tag{2}$$

$$[Q(t)] \subset [Q(t')] - \overline{\partial Q(t \to t')} \cdot [\Delta t] \tag{3}$$

Here, $[\Delta t]$ is a qualitative description of the temporal length taken by qualitative state transition.

Now assume three continuous quantized time $t_{prev} \to t_{now} \to t_{next}$. "Moment" is a point on a quantized time axis and has no length. Therefore, the following relations must be satisfied.

⟨⟨ **When** t_{now} **is a** *Moment* ⟩⟩

- Quantized time passes from t_{now} to t_{next} in an infinitesimal moment. Therefore, according to formula (2), $[Q(t)]$ satisfies the next relation.

$$[Q(t_{next})] \subset [Q(t_{now})] + \overline{\partial Q(t_{now} \to t_{next})} \cdot \varepsilon \tag{4}$$

- Quantized time goes back from t_{now} to t_{prev} in an infinitesimal moment. Therefore, according to formula (3), $[Q(t)]$ satisfies the next relation.

$$[Q(t_{prev})] \subset [Q(t_{now})] - \overline{\partial Q(t_{prev} \to t_{now})} \cdot \varepsilon \tag{5}$$

When formulae (4) and (5) are satisfied, t_{now} is judged to be a "Moment".

⟨⟨ **When** t_{now} **is a** *Time Interval* ⟩⟩

- Quantized time passes from t_{prev} to t_{now} in an infinitesimal moment. Therefore, according to formula (2), $[Q(t)]$ satisfies the next relation.

$$[Q(t_{now})] \subset [Q(t_{prev})] + \overline{\partial Q(t_{prev} \to t_{now})} \cdot \varepsilon \tag{6}$$

- Quantized time goes back from t_{next} to t_{now} in an infinitesimal moment. Therefore, according to formula (3), $[Q(t)]$ satisfies the next relation.

$$[Q(t_{now})] \subset [Q(t_{next})] - \overline{\partial Q(t_{now} \to t_{next})} \cdot \varepsilon \tag{7}$$

Here, ε describes an infinitesimal moment. Additive operation of a Qualitative Value with another Qualitative Value multiplied by ε is effective only when the Qualitative Value is on a landmark. For example, the following relations are satisfied.

$$[+] + [-] \cdot \varepsilon = [+]$$
$$[0] + [-] \cdot \varepsilon = [-]$$

When formula (6) and (7) are satisfied, t_{now} is judged to be a "Time Interval". When all of these four formulae are satisfied, t_{now} is judged to be either "Moment" or "Time Interval".

Here, in each formula, the term $\overline{\partial Q(t \rightarrow t')}$ is unknown. Therefore, this term is assumed to be as follows.

$$\overline{\partial Q(t \rightarrow t')} \subset [+0-]$$

4.3 Qualitative Integration Rule

Qualitative analysis of the behavior of a variable $Q(t)$ is performed using the following relation[de Kleer 84].

$$[Q(t')] \subset [Q(t)] + \partial Q(t) \qquad (8)$$

This relation is called the "*Qualitative Integration Rule*". This rule was derived from the following *mean value theorem* defined in Euclid Space.

$$\exists \xi \in (t, t') \quad \forall t, t'$$
$$Q(t') = Q(t) + (t' - t) \cdot \dot{Q}(\xi) \quad (9)$$

However, formula (8) is derived from formula (9) only under the condition that $Q(t_i)$ is a *strictly monotone function* at $t_i \in (t, t')$,namely, $\partial Q(t_i)$ does not change. Therefore, formula (8) can not describe the following transition.

$$[Q(t)] = [0], \ \ \partial Q(t) = [0]$$
$$\text{When } \partial^2 Q(t) = [+]$$
$$\Downarrow$$
$$[Q(t')] = [+], \ \ \partial Q(t') = [+]$$

We derived a new "*Qualitative Integration Rule*" using the next relation defined in Euclid Space.

$$Q(t') = Q(t) + \int_t^{t'} \dot{Q}(\xi) d\xi \qquad (10)$$

Quantizing formula (10) into Qualitative Value Space, the next relation is derived.

$$[Q(t')] \subset [Q(t)] + \partial Q(t) \cdot \delta t + \partial Q(t') \cdot \delta t' \quad (11)$$

Here, δt and $\delta t'$ are qualitative descriptions of the temporal length of each quantized time t and t'. Therefore, defining $[\Delta t]$ as a temporal length needed for state transition, the next relation is satisfied.

$$[\Delta t] \stackrel{\text{def}}{=} [t' - t] = \delta t + \delta t'$$

State transition from *Momentary State* to *Static State* takes an infinitesimal moment, that is, $[\Delta t] = \varepsilon$. Moreover, t is *Moment*. Therefore, the next two relations are satisfied.

$$\delta t = [0], \ \ \delta t' = \varepsilon$$

State transition from *Static State* to *Momentary State* takes a finite time, that is, $[\Delta t] = [+]$. Moreover, t' is *Moment*. Therefore, the next two relations are satisfied.

$$\delta t = [+], \ \ \delta t' = [0]$$

We call formula (11) the new *Qualitative Integration Rule*.

5 Estimation of Qualitative States

Estimation of possible qualitative states is performed by estimating the possible Qualitative Derivatives $\partial Q(t)$ according to the observed time series of Qualitative Values $[Q(t)]$. We defined the following two rules in order to estimate possible Qualitative States, asuming three continuous quantized time $t_{prev} \rightarrow t_{now} \rightarrow t_{next}$. These rules were derived using the basic mathematical relations derived in the previous chapter.

5.1 State Estimation Rules

⟨⟨ **State Estimation Rule.1** ⟩⟩

When t_{now} is a *Moment*. The Qualitative State, $([Q(t_{now})], \partial Q(t_{now}))$ is estimated in the following manner.

Given the sequence t_{prev}(*Time Interval*) $\rightarrow$ t_{now}(*Moment*) $\rightarrow$ t_{next}(*Time Interval*), the following relations are derived using the *Qualitative Integration Rule* (11).

$$[Q(t_{now})] \subset [Q(t_{prev})] + \partial Q(t_{prev}) \qquad (12)$$

$$[Q(t_{next})] \subset [Q(t_{now})] + \partial Q(t_{next}) \cdot \varepsilon \qquad (13)$$

Substituting the observed values $[Q(t_{prev})]$, $[Q(t_{now})]$, $[Q(t_{next})]$ into these formulae, yields $\{\partial Q(t_{prev}), \partial Q(t_{next})\}$. In addition, $\partial Q(t)$ must satisfy the *Assumption of Continuous Behavior* (1).

$$\partial Q(t_{prev}) \cdot \partial Q(t_{now}) \subset [0+]$$
$$\partial Q(t_{now}) \cdot \partial Q(t_{next}) \subset [0+] \quad (14)$$

Moreover, considering t_{now} is a *Moment*, according to formulae (4) (5).

$$\partial Q(t_{next}) \subset \partial Q(t_{now}) + [+0-] \cdot \varepsilon$$
$$\partial Q(t_{prev}) \subset \partial Q(t_{now}) - [+0-] \cdot \varepsilon \quad (15)$$

and $\partial Q(t_{now})$ which satisfies these formulae (14) (15) can be derived. Now, we have the relations possible when t_{now} is a *Moment* , as listed in Table.2. We call these relations the *State Estimation Rule.1*.

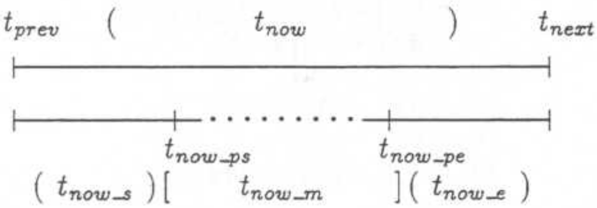
Table 2: State Estimation Rule.1

$[Q(t_{prev})]$	$[Q(t_{now})]$	$[Q(t_{next})]$	$\partial Q(t_{now})$
[+]	[0]	[+]	[0]
[+]	[0]	[0]	[0]
[+]	[0]	[−]	[−0]
[0]	[0]	[+]	[0]
[0]	[0]	[0]	[0]
[0]	[0]	[−]	[0]
[−]	[0]	[+]	[0+]
[−]	[0]	[0]	[0]
[−]	[0]	[−]	[0]
[+]	[+]	[+]	[+0−]
[−]	[−]	[−]	[+0−]

⟨⟨ **State Estimation Rule.2** ⟩⟩

When t_{now} is a *Time Interval*, we can not assume $\partial Q(t)$ to be constant. Therefore, we assume the interval t_{now} consists of several sub-intervals as shown in Figure.2.

Figure 2: Division of t_{now} into Sub-Intervals



Here, t_{now_s} is defined as the interval immediately after t_{prev}, t_{now_e} is defined as the interval immediately before t_{next}. In these sub-intervals, the qualitative states,

$$([Q(t_{now\_s})], \partial Q(t_{now\_s}))$$
$$([Q(t_{now\_e})], \partial Q(t_{now\_e}))$$

are estimated using the following relations derived from formula (11).

$$[Q(t_{now\_s})] \subset [Q(t_{prev})] + \partial Q(t_{now\_s}) \cdot \varepsilon \quad (16)$$

$$[Q(t_{next})] \subset [Q(t_{now\_e})] + \partial Q(t_{now\_e}) \quad (17)$$

Where $[Q(t_{now\_s})] = [Q(t_{now\_e})] = [Q(t_{now})]$. t_{now_m} is defined as the interval that includes *Moments* t_{now_ps} and t_{now_pe}. That is,

$$Interval \;\; t_{now\_m} : [t_{now\_ps}, t_{now\_pe}]$$

However, we can not say how many sub-intervals t_{now_m} is divided into.

We discuss how to estimate the qualitative states,

$$([Q(t_{now\_m\_j})], \partial Q(t_{now\_m\_j})),$$
$$j = 1, 2, \ldots\ldots$$

According to the *mean value theorem* (9), we can define a state which must be in the interval t_{now_m}. Such states can be estimated by assuming the *strict monotony* of function $Q(t)$ in the interval t_{now_m}. Assuming strict monotony of $Q(t)$ in t_{now_m}, $\partial Q(t_{now\_m})$ can be estimated using the next relation derived from the *Assumption of Continuous Behavior* (1).

$$\partial Q(t_{now\_s}) \cdot \partial Q(t_{now\_m}) \subset [0+]$$
$$\partial Q(t_{now\_m}) \cdot \partial Q(t_{now\_e}) \subset [0+]$$

However, care is needed here. When an observed system has several variables $\{Q_i : i = 1, \ldots\}$, we can not assume that $\{\partial Q_i(t_{now\_m}) : i = 1, \ldots\}$ appear at the same quantized time (defining ξ_i for each variables using formula (9), we cannot assume all $\{\xi_i : i = 1, \ldots\}$ appear at the same time). Therefore, considering the quantized time $t_{now\_m\_i}$ at which each variable $\partial Q_i(t_{now\_m})$ is assured to appear given monotony, we can state

$$\partial Q_j(t_{now\_m\_i}) \subset [+0-] \;\;,\;\; j \neq i$$

For example, consider the situation in which the next time series was observed.

	t_{prev}	t_{now}	t_{next}
$[X_1]$:	[0]	[+]	[0]
$[X_2]$:	[0]	[+]	[0]

Possible Qualitative States are estimated as follows.

Quantized Time	*Possible States* $([X_1], \partial X_1, [X_2], \partial X_2)$
t_{now_s}	([+], [+], [+], [+])
$t_{now\_m\_1}$	([+], [0], [+], [+0−])
$t_{now\_m\_2}$	([+], [+0−], [+], [0])
t_{now_e}	([+], [−], [+], [−])

We now have the relations for which the situation t_{now} is a *Time Interval.* These relations are given as Table.3. We call these relations *State Estimation Rule.2.*

5.2 Example of State Estimation

This section introduces an example of applying the *State Estimation Rules* to a simple case. Consider a dynamical system which has two characteristic variables, namely, X_1 and X_2. We assume that a time series of Qualitative Values $[X_1]$ and $[X_2]$ are observed as follows.

$[X_1(t)]$:[−] [0] [+] [+] [+] [0] [−] [−] [−] [0] [0]
$[X_2(t)]$:[+] [+] [+] [0] [−] [−] [−] [0] [+] [0] [0]

Quantized Time →

Applying *State Estimation Rule.1 and Rule.2,* the possible qualitative states listed in Table.4 can be estimated.

These Qualitative States should satisfy some kinds of *Constraints.* We discuss how to estimate the *Constraints* in the next chapter.

6 Identification of Constraints

6.1 Constraint Identification Method

In Qualitative Reasoning, we describe the causal structure between characteristic variables as *Constraints.* Qualitative Simulation[Kuipers 86] is performed by finding possible state transitions between several Qualitative States which satisfy the *Constraints.* Such Constraints describe a temporal causal structure of the dynamical system and are in fact, *Qualitative Differential Equations.* In this chapter, we discusse how to estimate *Qualitative Differential Equations* by finding causal relations between characteristic variables and their derivatives, namely,

$$\{[Q_i], \partial Q_i\} \quad , \quad i = 1, 2, \ldots$$

In order to find these relations, we use the following Polynomial.

$$[0] \subset \sum_{m=0}^{order} [C_{k_1,\ldots,k_n,l_1,\ldots,l_n}] \prod_{i=1}^{n} [Q_i]^{k_i} \prod_{j=1}^{n} \partial Q_j{}^{l_j} \quad (18)$$

$$m = k_1 + \cdots + k_n + l_1 + \cdots + l_n$$

Here, $\sum$ means an additive operation between Qualitative Values and $\prod$ means qualitative multiplication. $[C_{k_1,\ldots,k_n,l_1,\ldots,l_n}]$ are the coefficients of the terms which forming the Polynomial. "*order*" means the order of the Polynomial. Therefore, when $order = 1$, formula (18) is a *Linear Differential Equation.* By substituting Qualitative States

$$([Q_1], \partial Q_1, \ldots, [Q_n], \partial Q_n)$$

estimated by the *State Estimation Rules* into formula (18), the estimation of *Constraints* is performed by searching for possible sets of the coefficients $[C_{k_1,\ldots,k_n,l_1,\ldots,l_n}]$ satisfying the relation "$\subset$".

In our method, we begin searching the coefficients with $order = 1$. When possible coefficients can not be found, *order* is incremented.

6.2 Example of Identification of Constraints

Using the example discussed in the previous chapter, we estimated the Constraints. According to this example, the following 18 Constraints are estimated.

$$[0] \subset [X_1] + \partial X_1 + [X_2] + \partial X_2$$
$$[0] \subset [X_1] + \partial X_1 + [X_2] - \partial X_2$$
$$[0] \subset [X_1] + \partial X_1 + \partial X_2$$
$$[0] \subset [X_1] + \partial X_1 - \partial X_2$$
$$[0] \subset [X_1] + \partial X_1 - [X_2] + \partial X_2$$
$$[0] \subset [X_1] + \partial X_1 - [X_2]$$
$$[0] \subset [X_1] + \partial X_1 - [X_2] - \partial X_2$$
$$[0] \subset [X_1] + [X_2] + \partial X_2$$
$$[0] \subset [X_1] - \partial X_1 + [X_2] + \partial X_2$$
$$[0] \subset [X_1] - \partial X_1 + [X_2]$$
$$[0] \subset [X_1] - \partial X_1 + [X_2] - \partial X_2$$
$$[0] \subset [X_1] - \partial X_1 - \partial X_2$$
$$[0] \subset [X_1] - \partial X_1 - [X_2] - \partial X_2$$
$$[0] \subset \partial X_1 + [X_2] + \partial X_2$$
$$[0] \subset \partial X_1 + \partial X_2$$
$$[0] \subset \partial X_1 - [X_2] + \partial X_2$$
$$[0] \subset \partial X_1 - [X_2]$$
$$[0] \subset \partial X_1 - [X_2] - \partial X_2$$

In this case, the possible coefficients are found with polynomial of $order = 1$. Therefore, these Constraints are Qualitative Linear Differential Equations.

Moreover,

$$\begin{aligned} H_c &= -\sum_{i=1}^{k} p_c(s_i)\log p_c(s_i) = -k \cdot p_k \log p_k \\ &= \log k \end{aligned} \qquad (22)$$

According to formulae (21) (22), *Entropy reduction* is rewritten as follows.

$$H_r = 1.0 - \frac{H_c}{H_{max}} = 1.0 - \frac{\log k}{2n\log 3} \qquad (23)$$

According to formula (23), when the number of n variables is fixed, H_r depends only on the number of states k. Moreover, smaller numbers of k indicate *improved forcasting power.*

According to these relations, we chose the Simultaneous Constraints which are satisfied by the smallest number of states, as appropriate models.

7.3 Example of Appropriate Simultaneous Differensial Equations

Using the example introduced in chapter 5, we estimated the most appropriate Simultaneous Differential Equations. For the data given, the following 6 models are estimated.

$\langle\langle$ *Model No.1* $\rangle\rangle$ $[0] \subset [X_1] + \partial X_1 + [X_2] + \partial X_2$
$[0] \subset \partial X_1 - [X_2]$

$\langle\langle$ *Model No.2* $\rangle\rangle$ $[0] \subset [X_1] + \partial X_1 + \partial X_2$
$[0] \subset \partial X_1 - [X_2]$

$\langle\langle$ *Model No.3* $\rangle\rangle$ $[0] \subset [X_1] + [X_2] + \partial X_2$
$[0] \subset \partial X_1 + \partial X_2$

$\langle\langle$ *Model No.4* $\rangle\rangle$ $[0] \subset [X_1] + [X_2] + \partial X_2$
$[0] \subset \partial X_1 - [X_2]$

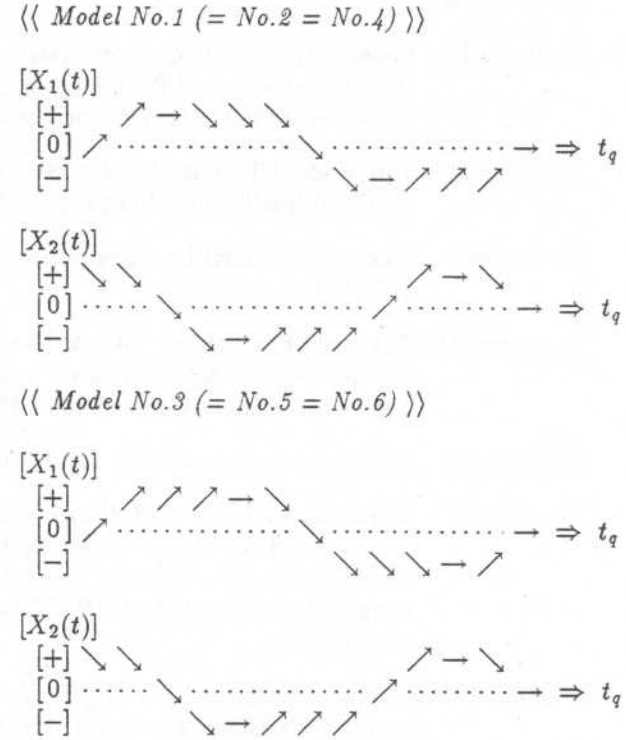
$\langle\langle$ *Model No.5* $\rangle\rangle$ $[0] \subset [X_1] - \partial X_1 + [X_2] + \partial X_2$
$[0] \subset \partial X_1 + \partial X_2$

$\langle\langle$ *Model No.6* $\rangle\rangle$ $[0] \subset [X_1] - \partial X_1 + [X_2]$
$[0] \subset \partial X_1 + \partial X_2$

8 Experimental Results of Qualitative Simulation

We can now show that the observed behavior can be reproduced by Qualitative Simulation [Kuipers 86] using the above mathematical models. Figure.3 shows qualitative simulation results from Model No.1 ~ Model No.6. Here, Models No.1, No.2 and No.4 are qualitatively equivalent. Moreover, Models No.3, No.5 and No.6 are also qulaitatively equivalent. This figure shows that the observed behavior is reproduced by each model.

Figure 3: Results of Qualitative Simulation



9 Conclusion

We have introduced a method that qualitatively models dynamical systems using the qualitative descriptions about the behavior of observed systems. Moreover, we showed experimental results in this paper. We built an experimental system on a SparkStation using the CommonLisp language. Inputs of the experimental system are the name of characteristic variables and the time series of the variables. The appropriate models are calculated automatically.

We discussed a development of our method in Qualitative Value Space in which the only landmark is "ZERO". However, Basic Mathematical Relations derived in Chapter.4 are not depend on the Qualitative Value Space. Therefore, the proposed method can be extended to cases where the Qualitative Value Space includes more than one landmark value , by (1)defining new symbols for the Qualitative Value Space , (2)defining the basic operations between these symbols and (3)deriving new *State Estimation Rules* from the Basic Mathematical Relations.

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Table 3: State Estimation Rule.2

$[Q(t_{prev})]$	$[Q(t_{now})]$	$[Q(t_{next})]$	$\partial Q(t_{now\_s})$	$\partial Q(t_{now\_m})^{(*)}$	$\partial Q(t_{now\_e})$
[+]	[+]	[+]	[+0−]	[+0−]	[+0−]
[+]	[+]	[0]	[+0−]	[−0]	[−]
[0]	[+]	[+]	[+]	[0+]	[+0−]
[0]	[+]	[0]	[+]	[0]	[−]
[0]	[−]	[0]	[−]	[0]	[+]
[0]	[−]	[−]	[−]	[−0]	[+0−]
[−]	[−]	[0]	[+0−]	[0+]	[+]
[−]	[−]	[−]	[+0−]	[+0−]	[+0−]

(*) : $\partial Q(t_{now\_m})$ assuming *strict monotony* of $Q(t)$ in t_{now_m}

Table 4: Example of Applying State Estimation Rules

Observed Time Series $[X_1][X_2]$	*Applied Rule*	*Qualitative State* $([X_1], \partial X_1, [X_2], \partial X_2)$
[−][+]		
[0][+]	*Rule.1* →	([0], [0+], [+], [+0−])
[+][+]	*Rule.2* →	([+], [+], [+], [+0−]) ([+], [0+], [+], [+0−]) ([+], [+0−], [+], [−0]) ([+], [+0−], [+], [−])
[+][0]	*Rule.1* →	([+], [+0−], [0], [−0])
[+][−]	*Rule.2* →	([+], [+0−], [−], [−]) ([+], [−0], [−], [+0−]) ([+], [+0−], [−], [−0]) ([+], [−], [−], [+0−])
[0][−]	*Rule.1* →	([0], [−0], [−], [+0−])
[−][−]	*Rule.2* →	([−], [−], [−], [+0−]) ([−], [−0], [−], [+0−]) ([−], [+0−], [−], [0+]) ([−], [+0−], [−], [+])
[−][0]	*Rule.1* →	([−], [+0−], [0], [0+])
[−][+]	*Rule.2* →	([−], [+0−], [+], [+]) ([−], [0+], [+], [+0−]) ([−], [+0−], [+], [0]) ([−], [+], [+], [−])
[0][0]	*Rule.1* →	([0], [0], [0], [0])
[0][0]		

7 Estimation of Appropriate Models

When an observed system has n characteristic variables, the system can be modeled by Simultaneous Differential Equations consisting n Constraints. In this section, we discuss a method to find the combinations of Constraints that have the strongest *forecasting power*[Cellier 91] as appropriate Simultaneous Differential Equations.

7.1 Exclusion of Inconsistency

The Qualitative States estimated in chapter 5 satisfy each Constraint estimated in chapter 6. However, there are the cases in which Qualitative States can not satisfy the "Simultaneous" Constraints. These cases occur due to the uncertainties of Qualitative Values in estimated Qualitative States (*ex:* $[0+], [-0], [+0-]$). For example, consider the next state (State A) which consists of 4 variables.

$$\text{State } A \quad : \quad ([X_1], \partial X_1, [X_2], \partial X_2) \subset ([+], [-0], [-], [0+])$$

State A satisfies each of the next Constraints.

$$[0] \subset \partial X_1 - [X_2]$$
$$[0] \subset \partial X_1 - \partial X_2$$

However, Qualitative States that satisfy these Simultaneous Constraints, that is,

$$\begin{array}{lll} ([+],[+],[+],[+]) & , & ([+],[0],[0],[0]) \quad , \\ ([+],[-],[-],[-]) & , & ([0],[+],[+],[+]) \quad , \\ ([0],[0],[0],[0]) & , & ([0],[-],[-],[-]) \quad , \\ ([-],[+],[+],[+]) & , & ([-],[0],[0],[0]) \quad , \\ ([-],[-],[-],[-]) & & \end{array}$$

do not include any states which satisfy State A, that is,

$$\begin{array}{lll} ([+],[-],[-],[0]) & , & ([+],[-],[-],[+]) \quad , \\ ([+],[0],[-],[0]) & , & ([+],[0],[-],[+]) \end{array}$$

This means that State A does not satisfy the Simultaneous Constraints. We call such cases "*Inconsistency*". Such Simultaneous Constraints should be excluded as candidates for appropriate models.

7.2 Searching Combinations of Constraints with Strongest Forecasting Power

Cellier[Cellier 91] introduced "*Entropy Reduction*" as a measure of the *forecasting power* of the Constraints. In our case, "*Entropy Reduction*" is defined as follows. Considering probability $p_c(s)$ of state s which satisfies the given Constraint Set c, the next relation is satisfied.

$$\sum_{\forall s} p_c(s) = 1.0$$

Entropy is defined as,

$$H_c \stackrel{\text{def}}{=} -\sum_{\forall s} p_c(s) \log p_c(s)$$

Entropy H_c takes maximum value,

$$H_{no\_constraint} = H_{max}$$

when all of the states occur with the same probability. This occurs when no Constraint is given. Moreover, when state occurence is deterministic,

$$H_{deterministic} = 0$$

Using these values, *Entropy Reduction* H_r is defined as follows.

$$H_r \stackrel{\text{def}}{=} 1.0 - \frac{H_c}{H_{max}} \tag{19}$$

H_r is a real number in the range between 0.0 and 1.0, and higher values usually indicate *improved forecasting power*.

Considering a dynamical system which has n characteristic variables, Qualitative States s are described as follows.

$$(\{[Q_i], \partial Q_i\}) \quad , \quad i = 1, \ldots, n$$

In the Qualitative Value Space with 3 values (only landmark "ZERO"), there exists 3^{2n} states. Now we assume k states satisfy Constraint c, and their probability $p_c(s)$ are the same, namely, p_k.

$$p_c(s_1) = p_c(s_2) =, \cdots, = p_c(s_k) \stackrel{\text{def}}{=} p_k$$

Therefore,

$$\sum_{\forall s} p_c(s) = k \cdot p_k = 1.0 \tag{20}$$

We rewrite p_k as p_{max} , in the case where $k = 3^{2n}$. According to formula (20),

$$p_{max} = 1/3^{2n}$$

Therefore,

$$H_{max} = -\sum_{\forall s} p_{max} \log p_{max} = 2n \log 3 \tag{21}$$