# A Qualitative System Theory: Introducing Global View into Qualitative Reasoning

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## Abstract

Many of the qualitative simulation algorithms so far proposed depend upon local propagation paradigm, and hence produce spurious states. Recent approaches use topological constraints in phase space diagrams for filtering the spurious states. We propose an alternative method of recognizing global properties in the graphically expressed qualitative model. Qualitative versions of such system-theoretic concepts as stability and observability are used to analyze the global properties of the system.

We first present structural conditions for recognizing qualitative stability or instability in the qualitative model.

Then, we generalize several qualitative properties such as *sign/ potential stability, sign observability, potential periodicity/constant, invariant sign patterns*, using the concept of inertia.

These qualitative properties with respect to interia can then be associated with the structure of graphically expressed dynamic systems. A principle of reasoning about the qualitative properties based on structural changes of graphs is also presented, and used for qualitative reasoning at the structural level.

## 1 Introduction

Qualitative physics [de Kleer and Brown 84] (or qualitative process theory [Forbus 84]) has been studied for predicting and explaining the behavior of the physical system by using symbolic computation. [de Kleer and Brown 84] use logical proof as the explanation for the physical behavior. They pointed out that the logical proof has undesirable features for making causal accouts, and proposed *mythical causality*. While their modeling is rather component oriented, [Forbus 84] developed process oriented modeling. [Kuipers 86] , however, starts from abstracting the mathematical model preserving qualitative information in the model. Both [Kuipers 86] , and [de Kleer and Bobrow 84] developed simulation algorithms on their qualitative models. Both of them introduce higher order derivatives to predict change precisely. The problems of these qualitative reasoning methods are:

1. Although these methods provide modeling perspectives, they are not yet ready for the automatic or interactive generation of the qualitative models of large-scale systems such as industrial processing plants.

2. They do not use global properties which may not come from the local propagation of the constraint or state.

[Falkenhainer and Forbus 88] focus on the problem of the modeling by considering the granularity of the model. We use a state-space model of the system which can be expressed by signed directed graph. It is pointed out that a state-space model is difficult to obtain, however, there are many system identification methods in control theory which may be used to to a qualitative level, not to a numerical level. It is also pointed out that linear system is too weak to express the real systems. Again, there are many method for approximating non-linear systems (piece-wise linear) which can be used in a qualitative level.

We focus on the second problem in this paper. Since the behavior is expressed as cause-effect sequences

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through time, most techniques try to develop the behavior in the time dimension. That is, qualitative simulation [de Kleer and Bobrow 84, Kuipers 86] and causal ordering[Iwasaki and Simon 86]. It has been pointed out that the constraints of qualitative equations may not be strong enough to generate a unique behavior [Kuipers 85]. Other constrains such as geometrical or topological constraints [Lee and Kuipers 88, Struss 88, Zhao 91] are used to filter out spurious behavior. We presented an alternative method for filtering out spurious behavior using global properties, which can be checked on the graphically expressed qualitative model [Ishida *et al.*, 81, Ishida 89, Ishida 92a, Ishida 92b]. [Trave and Dormoy 88] also proposed to use qualitative stability for qualitative reasoning.

The global properties of the system, such that an oscillation will converge on some point or not, can be discriminated to some extent by checking the sign structure of the graph.

Our method seems to be more suitable for symbolic computation than with the method using geometric conditions, however ours suffers from the limitation that the target system must be linear.

Another way of analyzing a qualitative model is to consider the model in a state-space dimension [Ishida *et al.*, 81, Ishida 89, Rose and Kramer 91]. That is, an investigation on how properties are affected by making structural changes.

Other reseach also uses structural constrains such as causal graph [Iwasaki and Simon 86, Dechter and Perl 91] and bond graph [Top and Akkermans 91]. But they are used for different purposes; causal ordering, generating qualitative models and so on.

Section 2 shows the qualitative model of dynamical systems. The qualitative model is a signed directed graph obtained by keeping the qualitative information of sign structures of a linear system. In section 3, global analyses such as stability analysis are made on the qualitative model. Section 4 first presents a generalization

of qualitative properties using the concept of intertia is also presented. Then, the structural sensitivity analysis is proposed which explores the structural change in the graph models and how the change affects the qualitative properties discussed in section 3.

# 2 Qualitative Model of a Dynamical System

## 2.1 The qualitative model

Qualitative theory of linear systems, which has been studied extensively in econometrics [Quirk 65, Quirk 68] and mathematical ecology [Jefferies 74], is used as an analysis tool for the qualitative model. In many systems such as chemical processing plants dynamical behavior is expressed or approximated by a linear differential equation:

(2-1) $dx/dt = Ax, A \in R^{n \times n}$.

We use the qualitative model expressing the signed matrix $A_s$. [2]

In the model, an arc is directed from node i to node j with the sign of $(A_s)_{ij}$.

The graphical expressions appear in ecosystems, econometrics, chemical reactions, control systems of processing plants and so forth.

Most of the results of qualitative system theory are obtained for the state-space expression of this linear system. Thus, in order to directly use this qualitative system theory, we transform the model into this state-space expression. All the interactions whose phase lag are $n > 1$ are divided into n sequential interactions of phase lag 1 by introducing n-1 dummy variables (nodes). On the other hand, variables (nodes) connected by the interaction of phase lag 0 are regarded as one variable (node). In the global analysis of section 3, we assume the systems under discussion are already normalized.

Non-linear systems must be first linearized in the following manner. Develop the system around the point of interest, then neglect the higher order non-linear terms. This linear approximation is only valid in the neighborhood of the point. The linearized system must be expressed in state-space expression for later analysis.

**Example 2.1**

The model for the pressure regulator is shown below.

$dXs/dt = -a \cdot Po$
$dQi/dt = b \cdot (DP - c \cdot Qi^2/Xs)$
$dPo/dt = e \cdot (Qo^2 - f \cdot Po)$
$DP = Pi - Po$
$Qi = Qo$

[2] Signed matrix $A_s$ of A is a triple value matrix defined as follows:
$(A_s)_{ij} = +, -, 0$ if $(A)_{ij} > 0, < 0, = 0$ respectively.

Xs : area available for the flow through the valve
Po : pressure at outlet
Pi : pressure at inlet
DP : pressure drop across the valve
Qi : inflow to the valve
Qo : outflow from the valve
a, b, c, e, f: appropriately chosen positive constants.

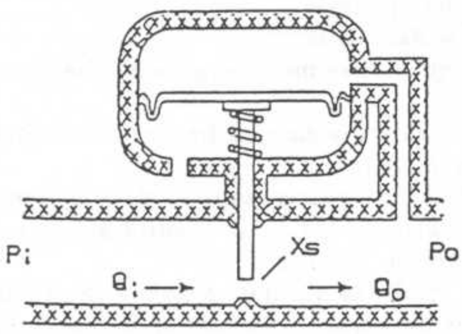


Fig. 1 Diagram of the pressure regulator

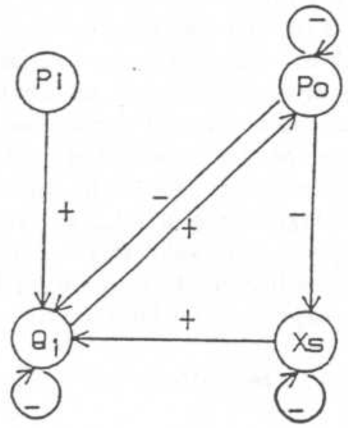


Fig. 2 Qualitative model of the pressure regulator

Fig. 1 shows a diagram of a pressure regulator.

Fig. 2 is the signed digraph expressing the qualitative model. Since the phase lag of all the arcs is normalized to 1, only the sign is indicated in the arc.

### 2.2 Causality and system theoretic concepts

In dynamical system theory, many concepts such as observability and stability have been studied. Since these concepts have intuitive explanatory power, they may be used as aids for causal account. There seems to be an important relation between the concept of observability and causality. In system theory, observability is defined as:

> "A system is said to be observable by an observer if it is possible to determine the initial state by observing the output signal from the observer during a finite time starting from the initial time."

We can use the observability [3] (or its dual concept of controlability) as a tool to check the potential causability. It is not against our intuition to say that the event dX = + (or −) can cause the event dY = + (or −) only when X is observable from Y.

## 3 Global Analysis

The main advantage of using state-space expression (2-1) of the qualitative model is that it allows many system theoretic analyses, especially the global analysis. This section presents several results which can be used as a tool for a global analysis on the qualitative model.

### 3.1 Qualitative stability analysis

Following a convention, we will call the qualitative property by modifying the adjective *sign (potential)* if the qualitative property requires that all (some of) instances of the model must satisfy the property. For example, a model is called *sign (potentially) stable* if all (some of) instances of the model are stable [4].

A property of a system is called qualitative if it is determined only by the sign structure of the qualitative model. In this section, we discuss the qualitative property of the qualitative model. Two kinds of qualitative stabilities, and qualitative observability are defined as follows.

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[3] The observability of the linear system can be checked by a matrix. Let $y = Cx, y \in R^{1\times m}, C \in R^{n\times m}$ be observed output of the linear system (2-1), then the observability from y can be known by testing whether or not the matrix $[C, CA, CA^2, \ldots, CA^{n-1}]$ have the full rank.

[4] The solution of the system will asymptotically converge on an equilibrium point. In the linear systems under discussion, all the eigen values of the system matrix have negative real parts.

**Definition 3.1.** (sign stability and potential stability)

A qualitative model $A_s$ is called sign (potential) stable if all (some of) instances of the model are stable [5].

**Definition 3.2.** (sign observability)

The qualitative model with the observer is said to be sign observable if all instances of the model are observable from the observer.

In the example 2.1, the graph indicates that the model can be decomposed into two strongly connected components [6] corresponding to the subsystem Pi and the subsystem consisting of Po, Q, Xs. Pi is observable from the subsystem of Po, Q, Xs and not in opposite way. Notice, however, that even if the model is decomposed into strongly connected components, the affecting subsystem may not be observable from the affected subsystem in such cases that the affecting subsystem has a constant mode or two effects canceling each other. (Obviously, the affected subsystem is not observable from the affecting subsystem.)

A necessary and sufficient condition for a qualitative model to be sign stable is obtained with the concept of sign observability.

**Theorem 3.3.**

A qualitative model is sign stable if and only if the qualitative model has the following properties.

(1) There is no positive loop [7]

and there exists at least one negative loop,

(2) there is no positive circuit of length two,

(3) there is no circuit of length greater than two, and

(4) by setting the subsystem of negative loops as observer, the rest of the

subsystem is sign observable from the observer.

The conditions (1)-(3) guarantee that the system does not have divergent mode [8]. In order to further guarantee that the system does has neither constant mode nor pure periodical modes we only have to put the condition that that the signals are always observable from the node having a negative loop. In other words, if the system has pure periodical modes then the signal may not be observable by the cancellation of the oscillations of different phase. Likewise, if the system has a constant mode the signal is not observable.

**Example 3.4.**

To demonstrate the power of the sign stability, let us consider the mass-spring system whose state-space expression is as below:

$dX/dt = V$

$dV/dt = -kX - fV$ where $k$ and $f$ are positive constants.

Fig. 3 shows the diagram of a mass-spring system with dashpot.

The qualitative model of the mass-spring system is shown in Fig. 4a. This system is known to be sign stable, since all the conditions of theorem 3.3 are satisfied. Thus, oscillation will always converge eventually. When $f = 0$ (when there is no loop at node V), however, the system is always in a pure periodical mode.

The necessary and sufficient conditions for a qualitative model to be potentially stable have not yet been obtained. We present some heuristics which will be used to identify the potentially stable sign structure.

**Theorem 3.5.**

A qualitative model is potentially stable if the subgraph of the the qualitative model is potentially stable.

As presented in section 4, one of the powerful heuristics used in the system theory is that

> "A property of a system is preserved after a change of the system if the property is *locally invariant* to the change."

Since the stability holds even with a small change of parameter, the qualitative model obtained by adding arcs to the sign stable qualitative model has a stable instance supposing the added arcs represent the interactions with small absolute values. This argument is also true when adding arcs to the stable instances of a potentially stable qualitative model.

We have obtained another sufficient condition for the potential stability.

**Theorem 3.6.**

A qualitative model of n nodes is potentially stable if the signed digraph has the negative circuit of length exactly k for every integer k = 1, 2, ...,n.

**Proof** (see [Ishida *et al.*, 81] )

As for the necessary condition, we obtained the following theorem.

**Theorem 3.7.**

[5] The solution of the system will asymptotically converge on an equilibrium point.

[6] Strongly connected component is such subgraph that for all the pairs of nodes in the subgraph there exists a path from both sides.

[7] A circuit is a closed path where the path is a graph connecting many nodes by arcs of the same direction sequentially. The sign of a circuit is a multiplication of all the signs of the arcs included in the circuit. The length of the circuit is the number of all the arcs included in the circuit. The circuit of length 1 is called a loop.

[8] For the system (2-1), divergent mode, pure periodical modes, and constant mode are realized when the matrix A has eigenvalues with positive real part, pure imaginaries, and 0 respectively.

If a qualitative model is potentially stable then the signed digraph has a set of negative circuits whose sum of length is equal to k for every integer k = 1 ... n .

**Proof** (see [Ishida *et al.*, 81])

**Example 3.8.**

Consider the qualitative stability of the pressure regulator example. Since the subsystem Pi is always constant, only the subsystem composed of Po, Q, Xs is analyzed. By theorem 3.3, this model is not sign stable because of the circuit of length 3. However, this model is potentially stable, since the graph has negative circuits of length 1, 2, and 3. Notice, however, these analyses are valid only in the neighborhood of the equilibrium point where the changes around the point are considered. In order to consider the neighborhood of a different point, we must use different models linearized on the other point.

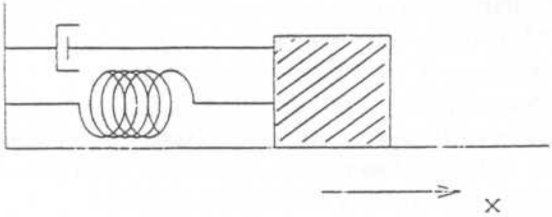



Fig. 3 Mass-spring system with a dashpot.

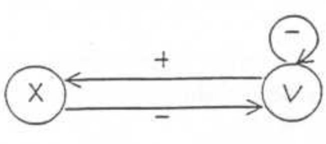



Fig. 4a
Qualitative model of
mass-spring system

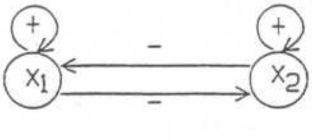



Fig. 4b
Qualitative model
having invariant sign patterns

Other than the conditions for sign stability and potential stability so far proposed, the following condition of sign instability can also be used as a tool to check the qualitative stability, since the potential stable class is the complementary set of the sign unstable class.

**Theorem 3.9**

The qualitative model obtained by making all the signs of arcs to some nodes opposite in the sign stable model is sign unstable.

**Proof** (see [Ishida *et al.*, 81])

## 3.2 Invariant sign pattern of a system

There is a class of qualitative model in which the initial sign pattern can be specified from the current sign pattern. In this section, we define the new concept of *invariant sign pattern*. Also, we discuss the relation between it and the class of qualitative stability.

**Definition 3.10.** (invariant sign pattern)

A sign pattern $x_s$ is called *invariant sign pattern* of a qualitative model if the model stays at the sign pattern $x_s$ all the time, once it attains the state.

It is easily checked whether or not a given qualitative model has *invariant sign patterns.*.

**Theorem 3.11.**

A strongly connected qualitative model has an invariant sign pattern if

(1) All the circuits have positive sign, and

(2) All the reconvergent fanout paths [9] between two nodes have the same sign.

**Proof** (see [Ishida 89])

The invariant sign pattern itself can be obtained from the sign structure of the qualitative model.

**Theorem 3.12.**

A sign pattern $x_s$ is an invariant sign pattern of a strongly connected qualitative model if it satisfies

(1) $(x_s)_i = +$ or $-$ for all$i = 1 \ldots n$, and

(2) $(x_s)_i = +(-)$ if there exists an arc $(x_k, x_i)$ such that $sgn(x_k, x_i)(x_s)_k = +(-)$ where$sgn(x_k, x_i)$ is the sign associated with the arc $(x_k, x_i)$.

**Proof**

Immediate from the sign equation.

**Theorem 3.13.**

If a strongly connected qualitative model has an invariant sign pattern $x_s$ then all the sign subpatterns converge on the invariant sign pattern. Sign subpattern is the sign pattern obtained by replacing some (but not all) of $+$ or $-$ with 0 in the original sign pattern.

Since the qualitative model is strongly connected, all the elements of the sign pattern vector will converge on

[9] Reconvergent fanout paths are such paths that share the initial and terminal nodes.

a non-zero pattern except the trivial all zero pattern. Further, they are not undetermined, for the qualitative model has two invariant sign patterns whose sign is opposite to each other. Thus, the primary sign pattern will converge on an invariant sign pattern which has the sign pattern as sign subpattern according to the dynamics of the system.

In connection with the qualitative stability discussed in the previous section, the next theorem holds.

**Theorem 3.14.**

If the sign equation $x_s = A_s x_s$ has a solution then the qualitative model of the sign structure $A_s$ is sign unstable.

If the sign equation has the solution $x_s$ then the solution of all the instances of the qualitative model with $A_s$ does not converge on 0.

**Theorem 3.15.** [10]

If there is a qualitative state assignment for the qualitative model such that the total effect on each node is not definite sign then the qualitative model potentially has an equilibrium point in the subspace specified by the assignment.

This assignment can be obtained by solving a sign equation of $0 = Ax_s$. This means that the matrix potentially have 0 eigenvalue.

**Example 3.16.**

A qualitative model shown in Fig. 4b has an invariant sign pattern $x_s = (+-)$ and hence $(-\ +)$ (If a qualitative model has a invariant sign pattern $x_s$, then $-x_s$ also.) For example, $(x_s)_1 = +$ is preserved for all the time, since feedback circuit from both $x_1$ itself and $x_2$ keep $x_1$ increasing. Similarly, sign patterns of $(x_s)_2$ are also preserved. Thus, the subpattern $(+\ 0)$ and $(0\ -)$ will fall into the sign pattern $(+\ -)$ by theorem 3.13. Further, it is also known by theorem 3.15 that this model has an equilibrium point in subspace $(+\ +)$ or $(--)$ in case the system has a constant mode.

As we have known that the qualitative model of the pressure regulator example is potentially stable, it does not have any invariant sign pattern. The model does not have a non-zero equilibrium point.

# 4 Structural Sensitivity Analysis

An important way of reasoning about dynamic systems is to investigate how qualitative properties are affected (or preserved) by applying a few structural changes on the model. In fact, the aim of this reseach is to extract this type of reasoning and establish a separate qualitative reasoning based on the analysis of structural changes.

Before formalizing this qualitative reasoning in terms of a system structure, we will list up the qualitative properties.

## 4.1 Qualitative Properties of a System

Other than these qualitative properties so far discussed, we will consider the following in this paper; *potentially periodic* (sign structures that may have periodic solutions.), *sign constant*(sign structures that have constant solutions) and *sign observable* (sign structures that are always observable by an observer).

We use the following notations to identify these properties by the inertia [11] .

$I(p,q,i)$: a class of qualitative models, all instances of which have the same inertia (p,q,i).

$P(p,q,i)$: a class of qualitative models, at least one instance of which has the inertia (p,q,i).

$I_c(-,-,i), i \geq 1$: a class of qualitative models, all instances of which have the inertia (-,-,i) where $-$ indicates they may change, but $i \geq 1$ includes at least one zero eigenvalue.

$P_p(p,q,i), i \geq 2$: a class of qualitative models, at least one instance of which has the inertia (p,q,i) where $i \geq 2$ includes at least two pure imaginary eigenvalue.

With this notation, *sign stable* and *potentially stable* qualitative models are written as $I(n,0,0)$ and $P(n,0,0)$ respectively. And *potentially periodic* and *sign constant* are written as $P_p(p,q,i), i \geq 2$ and $I_c(-,-,i), i \geq 1$.

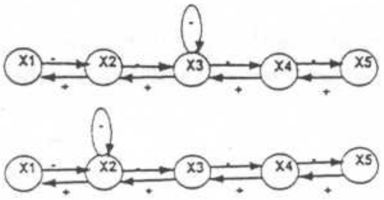
Several relationship between these properties are held by continuously moving eigenvalues on a complex plane. For example, it is known that if a qualitative model is both $P(p,q,0)$ and $P(p-k, q+k, 0)$ then it is $P(p-k, q, k)$ also.

Some of the sign structures whose qualitative properties are already known are listed in the appendix. The

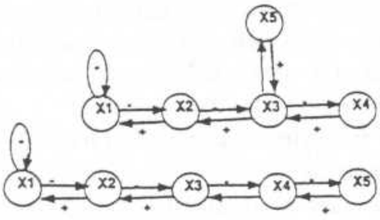
[10] This theorem can be generalized to the non-linear system dx/dt = f(x). That is, the subspace where an equilibrium point can potentially exist is specified by solving the possible sign pattern for 0 = f(x).

[11] Inertia of a matrix $A \in R^{n \times n}$ is defined as triple (p,q,i) of three integers where p is the number of eigenvalues of A with a positive real part, q with a negative real part, and i with a zero real part.

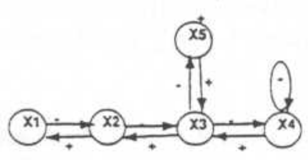
next example shows some of the sign structures which are already known.




(1)




(2)




(3)

Figs. 4 Several Qualitative Models Expressed by Graph

**Example 4.17**

Figs. 4 show the qualitative models of one negative loop. We call the negative circuit of length two a *chain*.

The model (1) is not *sign stable* by 1 in the appendix, since it is not *sign observable* from the element with a negative loop. It is *potentially periodic* $P_p(0,1,4)$ by 2-(a) in the appendix (it may have a periodic solution, but is not *sign non-constant* by 2-(b) in the appendix (it does not have a constant solution). The models (2), on the other hand, are not *potentially periodic* but they are *sign constant*. The models (3) are *sign stable* $I(0,5,0)$, since they are *sign observable* from the element with negative loop.

## 4.2 Qualitative Properties and Structural Changes

### 4.2.1 Structural Changes

Structural changes to the qualitative model are expressed well by the graphical expressions. Typical structural changes include; *system connection* (connecting two subsystems A,B by adding at least two arcs:arc from A to B and arc from B to A.), *system connection by a chain*( adding two arcs between nodes i and j so that $a_{ij}a_{ji} < 0$ ), *adding a specific subsytem* to the original system, *deleting a specific subsytem* from the original system and *changing the sign* of a few interactions.
We will introduce the concept of *open(closed)* property. A property of the system is called *open(closed)* if the property holds (does not hold) by changing the parameters (i.e. the value of the elements of the system matrix in a linear system) sufficiently small [12].

The property of an inertia $(p,q,0), p$ or $q \geq 1$ is open. That is, the matrix obtained by changing sufficiently small value of the elements will have the inertia $(p,q,0), p \geq 1$ or $q \geq 1$ if the original matrix has that inertia. [13] However, the property of an inertia $(p,q,i), i \geq 1$ is closed. The following principle plays an important role in structural analysis.

*Structural Preservation Principle*

If a qualitative property is *open* and *potential* then it is preserved after (1) system connection and (2) adding any arcs.

Since $P(p,q,0), p$ or $q \geq 1$ is *open* and *potential* properties, the following results directly come from the above principle.

- If a qualitative model belongs to $P(p,q,0), p$ or $q \geq 1$ then the model obtained by adding any arcs to this model is also $P(p,q,0), p$ or $q \geq 1$. This cannot be applied to the inertia preserving qualitative model $P(p,q,i)(i \geq 0)$. Because i may change even if changes to parameters are sufficiently small.

- Connection of the model $P(p_1,q_1,0)$ and $P(p_2,q_2,0)$ ) will result in the model $P(p_1+p_2, q_1+q_2, 0), p_1, q_1, p_2, q_2 \geq 1$.

[12] More formally said, a property is open(closed) if it is defined on an open(closed) set.

[13] This follows from the fact that the eigenvalues of the matrix A moves continuously on a complex plane when the value of the elements of the matrix A changes continuously, and that the half plane $Re > 0 (< 0)$ is an open set.

It should be noted, however, that even a *closed* property can be preserved by changing the structure in a certain manner. For example, models under conditions (1-3) of appendix 1 obtained by connecting two *potentially periodic* models, are *potentially periodic*.

*periodic.* Table 1: Qualitative Properties Preserved
* under conditions(1-3) in appendix 1
* * under conditions that all elements have negative loop and (2-3) in appendix 1

| Structural Change | Qualitative Properties Preserved |
|---|---|
| Addition of any arc | $P(p,q,0), p$ or $q \geq 1$ |
| System Connection | Potential Stability (Connection of $P(p_1,q_1,0)$ and $P(p_2,q_2,0)$ result in $P(p_1+p_2, q_1+q_2, 0)$, $p_1, q_1, p_2, q_2 \geq 1$) |
| System Connection by a *chain** | Potential Periodicity ($P_p(p,q,i), i \geq 2$), Sign Constant ($I_c(-,-,i), i \geq 1$), Not Sign Observable |
| Connection of one element with a negative loop, Addition of negative loop* | Sign Observability |
| Connection of the *chain* of length 2 by a *chain* | Potential Constant, Not Potential Constant |
| Connection of the *chain* of length 3 by a *chain* | Potential Periodicity |
| Making opposite the signs of the arcs of q elements** | I(0,n,0) is changed to I(p,q,0) |

Table 1 summarizes the results as to what qualitative properties are preserved (or how they are changed) by different structural changes. Proofs of these results without any citation are referred to in [Ishida 92a].

### 4.2.2 Qualitative Reasoning by Structural Analysis

Based on the *structural preservation principle* and other rules stating the relation between structural changes and qualitative properties, qualitative reasoning on the (graphically expressed) qualitative model can be made in the following two steps:

(1) Identify the nearest (in terms of sign structure) graph of a known qualitative property. (2) Reason about the qualitative property of the given graph by *extrapolating* the nearest graph whose qualitative property is already known.

Let us show how this structural level of qualitative reasoning operates with the above two steps on some examples.

**Example 4.18**

Let us first consider the qualitative model shown in Fig. 5. In the first step, the nearest known structure is a *sign stable* $I(0,5,0)$ subgraph that is obtained by deleting the arc corresponding to $a_{13}$, $a_{22}$, $a_{44}$ and $a_{55}$. In the second step, using the *structural preservation principle* the model shown in Fig. 1, is known to be *potentially stable* $P(0,5,0)$. In fact, we can compose a stable instance by assigning sufficiently small values to the elements corresponding to the deleted arcs.

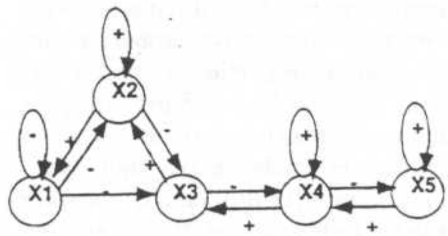



Fig. 5 Graphically expressed qualitative model

If we identify three subsystems consisting of $\{x_1\}, \{x_2, x_3\}, \{x_4, x_5\}$ as the nearest known structure, since they are $I(0,1,0)$, $I(2,0,0)$ and $I(2,0,0)$, then the total system is known as $P(4,1,0)$ by the connection rule stated in the previous section. The third subsystem consisting of $x_4, x_5$ is known to be $I(2,0,0)$ by the sign change rule stated in the last row in Table 1. It is also known that the model is *potentially periodic* $P_p(2,1,2)$ by the fact that the model is $P(0,5,0)$ and $P(4,1,0)$.

## 5 Conclusion

We have shown that such global properties as stability and observability can be investigated purely from the qualitative information of dynamical interaction.

So far we have discussed a global analysis of a linear system. As often done in system theory, the results of linear system can be used for non-linear systems in the following three manners:

(1) Non-linear systems can be approximated as linear systems in the neighborhood of the equilibrium point as in the example 2.1, and hence the results for linear systems hold there.

(2) The results of linear system $dx/dt = Ax$ holds for the non-linear system $dx/dt = A(t)x$ if the change of $A(t)$ is very slow.

(3) By *locally invariant* heuristics, some properties such as stability of the system $dx/dt = Ax + \epsilon \cdot F(x,t)$ do not change from that of $dx/dt = Ax$ if $\epsilon$ is sufficiently small.

We can use these approaches to the qualitative analysis for the non-linear system. That is, we divide the non-linear system into a set of linear systems each of which is an approximation of the non-linear system at some point and the neighborhood of the point. Summing up the results of these linear systems, the qualitative aspects of the non-linear systems are analyzed.

We have presented several qualitative properties of dynamic systems and discussed the relation between them and structural changes made on graphically expressed dynamic systems. Two discussions were (1) principles of how some qualitative properties are changed (or what qualitative properties are preserved) by structural changes. (2) Based on the principle, we presented a new qualitative reasoning about the qualitative properties which are made on the sign structure of the graph. The computer program *Q classifier* has been implemented which analyzes the given sign structure (or its nearest substructure) and outputs the qualitative properties. It successfully analyzes the system if the sign structure or its nearest sign strucuture are already associated with some qualitative properties. However, it consumes a large amount of time since it randomly generates the instance of the sign strucure and numerically analyzes the properties.

Compared with other methods which use phase space for the analysis of the global properties, our method seems more efficient since it directly operates on the system structure rather than generating trajectories. Of course, the limitation of our method is that it works only on linear systems or local areas near the equilibrium of non linear systems.

It may not be adequate to compare our method with many qualitative simulation methods since the tasks for them are quite different, such as analysis of the qualitative properties with our method and generation of step-by-step behavior for qualitative simulation. In fact, future research should be addressed to combine both approaches for more sophisticated reasoning. For example, our qualitative analysis can be used for filtering out supurious states generated by qualitative simulation and that qualitative simulation can be used to investigate qualitive properties.

## Appendix (Typical structures of qualitative properties)

1. *Sign Stable* $I(0, n, 0)$

   The qualitative model is sign stable [Jefferies and Klee 74] if and only if

   (1) All the loops have a non-positive sign, and at least one loop has a negative sign.

   (2) All the circuits of length 2 must have a non-positive sign.

   (3) There must be no circuit of lengths greater than 3.

   (4) It is sign observable from the elements with a negative loop [Ishida *et al.*, 81].

2. *Potentially Periodic and Constant* $P_p(p,q,i), i \geq 2, I_c(-,-,i), i \geq 1$

   (a) A qualitative model under conditions (1-3) above, may have a periodic solution if and only if it passes the color test.

   (b) If the graph of the qualitative model under conditions (1-3) above passes the matching test [Jefferies and Klee 74], then the model does not have a constant solution.

3. *Sign Observable*

   (a) The sign observable qualitative model under conditions (1-3) above is a graph that does not pass the color test [Jefferies and Klee 74] , but passes the matching test.

   (b) A qualitative model under conditions (2-3) above, that is chain structure with one negative loop at the end is sign observable from the element.

   (c) A qualitative model consisting of an element with a single loop and more than two subsystems of the same structure connected to the element, is not sign observable.

4. I(p,q,i)

   (a) If the graph of a qualitative model is a circuit of length n, then the qualitative model is I(p,q,i).

   (b) If the graph of a qualitative model has no loop and satisfies the conditions (2-3) above, then it is I(0,0,n).

# References


[de Kleer and Brown 84] A qualitative physics based on confluences. *Artificial Intelligence* , 24, 7-83, 1984.

[Forbus 84] Qualitative process theory. *Artificial Intelligence* 24, 85-168, 1984.

[de Kleer and Bobrow 84] Qualitative Reasoning with
Higher-Order Derivatives. *Proc. of AAAI 84* , 86-91, 1984.

[Kuipers 86] Qualitative simulation. *Artificial Intelligence*
29, 289-337, 1986.

[Falkenhainer and Forbus 88] Setting up large-scale qualitative model. *Proc. of AAAI 88*, 301-306, 1988.

[Struss 88] Global filters for qualitative behaviors *Proc. of AAAI 88*, 301-306, 1988.

[Lee and Kuipers 88] Non-intersection of trajectories in qualitative phase space: a global constraint for qualitative simulation. *Proc. of AAAI 88*, 286-290, 1988.

[Quirk 65] Qualitative economics and stability of equilibrium. *Rev. Economic Studies*, 32,311-326, 1965.

[Quirk 68] The correspondence principle, a macroeconomic
application. *Int. Econ. Rev.*, 9, 294-306, 1968.

[Jefferies 74] Qualitative stability and digraph in model ecosystem. *Ecology*, 55,1415-1419, 1974.

[Ishida *et al.*, 81] Some results on
the qualitative theory of matrix. Trans. of *SICE* , 17
49-55 , 1981.

[Ishida 89] Using Global Properties for Qualitative Reasoning: A Qualitative System Theory, *Proc. of IJCAI 89* , pp. 1174-1179 , 1989.

[Rose and Kramer 91] Qualitative Analysis of Causal Feedback, *Proc. of AAAI 91* , pp. 817-823, 1991.

[Ishida 92a] A Qualitative Analysis on Dynamical Systems: Sign Structure, Memories of Kyoto University, Vol. 54, No. 1, 1992.

[Ishida 92b] A Graphical Approach to Qualitative Reasoning on Dynamic Systems, Proc. of ECAI92.

[Trave and Dormoy 88] Qualitative Calculus and Applications, IMACS Trans. on Scientific Computing-88.

[Iwasaki and Simon 86] Causality in device behavior *Artificial Intelligence* 29, 3-32, 1986.

[Kuipers 85] The Limits of Qualitative Simulation. *IJCAI 85*, pp.128-136, 1985.

[Zhao 91] Extracting and Representing Qualitative Behaviors of Complex Systems in Phase Spaces, *Proc. of IJCAI 91* , pp. 1144-1149, 1991.

[Dechter and Perl 91] Directed Constraint Networks: A Relational Framework for Causal Modeling, *Proc. of IJCAI 91* , pp. 1164-1170 , 1991.

[Top and Akkermans 91] Computational and Physical Causality, *Proc. of IJCAI 91* , pp. 1171-1176 , 1991.

[Jefferies and Klee 74] When is a Matrix Sign Stable? *Can. J. Math*, 29-2,315-326, 1974.