Qualitative Reasoning of a Temporally Hierarchical System Based on Infinitesimal Analysis

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Abstract

In ordinary qualitative reasoning(QR), qualitative behavior of the dynamical systems is predicted by assignment of qualitative values such as {+,0,−} into model variables based on proper transition rules. Unfortunately, due to ambiguities in these ordinarily introduced qualitative values,their arithmetic and transition rules cause predictions to be redundant,sometimes even inaccurate. In this paper,we present a new method of qualitative reasoning which, besides using hyperreal numbers,takes into account their ε-**H** ranking in describing both qualitative values and qualitative derivatives of variables and also employs a convergence filter to investigate the infinitesimal asymptotic behavior of qualitative variables. We applied this qualitative reasoning method to envision a temporally hierarchical complex system.The result shows that this method provides a more detailed and natural qualitative solution than previous methods like Kuipers's time abstraction in envisioning temporally hierarchical complex system.

1 Introduction

Limitations of ordinary qualitative reasoning(QR) using the {+,0,−} semantics have been discussed in many works [1]-[7],[11]-[17]. One of the major limitations is that scale information such as the relative magnitude of quantities or their temporal derivatives are not included so that further ambiguity arises in determining state transition among all the possible adjacent values of a current qualitative state. To compensate this too abstract qualitative representation, several researchers introduced the concept of order of magnitude(O(M)) proposed by Raiman [11] and the hyperreal numbers to extend the definition of qualitative value[2][16]. Davis's CHEPACHET[2] and Weld's HR-QSIM[16], though quite different in their motivations and aims, can be considered as examples of this extention from ordinary QR.

We believe, however, that the true virtue of introducing this kind of scale information lies in envisioning a complex system in which several scale hierarchies are involved. Introducing this scale information can approximately isolate interactions of subparts having different magnitudes or time-scales, which, in ordinary QR, must be taken into account together in equal levels. This means that by using scale information we can introduce some kind of hierarchization in envisionment to prune insignificant transitions. Though infinitesimal analysis is not explicitly used, Kuipers's "abstraction by time-scale" method [6] can be considered to lie along this line. He treats the complex system as composed of different time-scales. On account of the "extra-mathematical" nature of his method, however,this intuitively appealing method has several problems when its range of application is extended.

In this paper we developed a new method to deal with temporal hierarchization by introducing infinitesimal analysis [5] to realize what Kuipers was trying to do in a more formal and natural way. But in doing so, simple introduction of hyperreal number like in [2],[16] has proved to be insufficient. For this reason we developed a new QR scheme which can handle infinitesimal qualitative behavior in a right way. Our QR method features the following two newly introduced concepts: First,infinitesimal/infinite numbers are ranked (ε-**H** ranking) to be able to evaluate their relative magnitudes. Second a convergence filter is introduced in order to investigate more precisely how variables converge towards their equilibrium.

The paper is organized as follows:in Section 2, we discuss Kuipers's time-scale abstraction and its problems. Section 3 shows extensions of the qualitative values and transition rules for our new scheme. Section 4 presents our temporal hierarchization algorithm for envisionment. Section 5 gives an example which illustrates envisionment by our temporal hierarchization applied to the same problem in [6]. Finally,in Section 6 we discuss about related work.

We assume that the reader is familiar with standard theories of envisionment, as in [1],[7], and those with qualitative hyperreals proposed by Davis [2] and Weld [16]. In this paper, $[x]$ denotes the qualitative value of a variable x. ∂x and $\partial^2 x$ stand for the qualitative derivative of x, and the second order of derivative of x, respectively. And x_i denotes the variable in a state i, for example, x_1 is the variable in a state 1. We also use the symbols,ε,N,and $\mathbf{H}$, which represent infinitesimals, finite numbers, and infinite numbers respectively, in such a way that, when distance between a variable x and a landmark x_0 is infinitesimal: $[x-x_0]=\varepsilon$, finite:$[x-x_0]=N$, or infinite:$[x-x_0]=\mathbf{H}$.

2 Kuipers's Hierarchization and its Problems

Kuipers [6] deals with a complex system such as a collection of interacting equilibrium submechanisms. He gives as an example the body fluid regulation and describes the whole system in terms of two mechanisms, water and sodium balances, that operate at different response time.The basic principle of this time-scale abstraction is "A faster mechanism reaches its equilibrium instantaneously and, during this process, a slower mechanism can be treated as being constant". His approach consists of the following steps:1)Decompose a whole system into faster system and slower system and model each one separately, but with some sharing variables. 2)Faster to slower: envision first the faster system. Viewed from a faster system, slower variables are treated as constants (relative constancy of slower variables). 3)Slower to faster: from the point of view of the slower system, the faster system instantaneously reaches its equilibrium with its environment composed of the slower system. Hence faster variables move quasi-statically along with the equilibrium conditions determined by slower variables when the slower system changes (binding faster variables as a function of slower variables). 3)The behavior of the whole system is temporally joined in cascade from faster to slower (continuation).

The essence of his approach is decomposition of a model from the temporal point of view.This hierarchization reflects our naive reasoning in envisioning behaviors of such a complicated system; however, this approach has several problems when extended to be applied to more general cases: 1) Since his continuation is based on "cascade shift of attention", which handles mechanism by switching between submodels, he assumes that all the submodels are stable and reach their equilibrium in predetermined subsequent order. If the faster mechanism does not converge (for example,oscillates with negligible magnitude), we cannot use this kind of decomposition. 2) Kuipers decomposes the structure of a system completely, so that conjunction after such decomposition may lose some important information about the interaction between faster variables and slower variables which is involved in the original system. 3) When faster variables converge to equilibrium values, their derivatives must approach the same infinitesimal order of magnitude as those of the slower variable derivatives. His abstraction assumes that even in that stage faster variables change more quickly.

The problems are mainly caused by the too strong nature of the decomposition of a system. Since the difference in velocities between variables can be described using the O(M) of those derivatives,it is expected that the introduction of the O(M) for the qualitative derivatives gives us a more sophisticated and natural way to use the information of difference of the variable changing rate without any decomposition of the structure.

3 QR with Ranked Hyperreals

3.1 Extensions of Qualitative Values

We extend a qualitative hyperreal representation described by Weld [16] and Davis[2] in order to describe the infinitesimal/infinite behaviors of qualitative variables more exactly. As Davis discusses in [2], one of the problems of envisionment using qualitative hyperreals is that once a parameter and its derivative both become infinitesimal or both become infinite, it becomes impossible to say anything about their relative sizes. To solve this problem, we further divide infinitesimal and infinite interval into $\varepsilon, \varepsilon^2, \varepsilon^3, \cdots$ and $\mathbf{H}, \mathbf{H}^2, \mathbf{H}^3, \cdots$, respectively, when these high-order numbers are required to be evaluated in the course of envisionment, for example, when the convergence speeds of variables are required to be evaluated.

Note that in evaluating the interval's length we take its maximal width. Thus, whereas ε means ε-neighborhood interval around some landmark, it is simultaneously representing its interval's length so that ε can be treated also as a number in infinitesimal calculations in envisionment. Hence we can write $\varepsilon > \varepsilon^2 > \varepsilon^3 > \cdots$. On the other hand, $\mathbf{H}$ also means the interval greater than the hyperreal infinite number $\mathbf{H}$ where the latter is thought as a hyperreal number. Hence we can write $\mathbf{H} < \mathbf{H}^2 < \mathbf{H}^3 < \cdots$. We call this division of infinitesimal/infinite interval "ε-$\mathbf{H}$ ranking."

This ranking is illustrated in fig.1 and fig.2. Let l_0 be a certain landmark of a parameter. Fig.1 shows the former relationship between ε^i and ε^{i+1} for any integer i. In this figure, ellipse denotes the neighbor of l_0 whose radius is given by ε, N, and $\mathbf{H}$. Region inside the ellipse whose radius is equal to ε, and N represents the infinitesimal neighborhood of l_0, and the "finite-distance" neighborhood of l_0, respectively. The outside of this "finite-distance" neighborhood shows the infinite neighborhood of l_0.

If we zoom up the infinitesimal neighborhood, then we can define the similar structure in the ellipse by introducing power of ε. Based on this characterization, relationship between ε^2 and ε can be defined as being similar to that between ε and N.

In the similar way, in general, relationship between

ε^{i+1} and ε^i can be defined as being similar to that between ε and N. Also, we can define relation between $\mathbf{H}^{i+1}$ and $\mathbf{H}^i$ for any integer i, as shown in fig.2.

This "ε-**H** ranking." is formulated as:

Definition 1 (Qualitative value with ε-H ranking) *Let $l_0 < l_1 < l_2 < ..l_i(=0).. < l_n(< \mathbf{H})$ be the landmark values of a parameter x. Define the hyperreal qualitative value of x as:*

$$[x] = \begin{cases} \mathbf{H}[\mathbf{H^2},\cdots], & if\ x - \mathbf{H}[\mathbf{H}^2..] \le \mathbf{H}(\mathbf{H}^2..) \\ N(=\langle l_n, \mathbf{H}\rangle), & if\ x - l_n > \varepsilon\ and\ x < \mathbf{H} \\ l_n + \varepsilon[\varepsilon^2,\cdots], & if\ 0 < x - l_n \le \varepsilon(\varepsilon^2,..) \\ l_n, & if\ x = l_n \\ l_n - \varepsilon[\varepsilon^2,\cdots], & if\ 0 < l_n - x \le \varepsilon(\varepsilon^2,..) \\ \langle l_{n-1}, l_n\rangle, & if\ x - l_{n-1} > \varepsilon\ and \\ & l_n - x > \varepsilon \\ \cdots, & \\ l_0 + \varepsilon[\varepsilon^2,\cdots], & if\ 0 < x - l_0 \le \varepsilon[\varepsilon^2,..] \\ l_0, & if\ x = l_0 \\ l_0 - \varepsilon[\varepsilon^2,\cdots], & if\ 0 < l_0 - x \le \varepsilon[\varepsilon^2,..] \\ -N(=\langle -\mathbf{H}, l_0\rangle), & if\ l_0 - x > \varepsilon\ and \\ & x > -\mathbf{H} \\ -\mathbf{H},[-\mathbf{H^2},\cdots], & if\ \ -\mathbf{H}[\mathbf{H}^2..] - x \\ & \quad < -\mathbf{H}[\mathbf{H}^2..] \end{cases}$$

where

denotes the possible alternation and $\langle p1, p2\rangle$ is equivalent to the difference between open interval (p1,p2) and the two halos as in Weld's HR-QSIM. The quantity space of a variable x,$QS(x)$ is defined as:

$$\begin{aligned} QS(x) \quad = \quad & \{(\cdots,\mathbf{H^2}),\mathbf{H},N, \\ & l_n+\varepsilon(\varepsilon^2,\cdots), l_n, l_n-\varepsilon(\varepsilon^2,\cdots), \\ & \langle l_{n-1},l_n\rangle,\cdots,\langle l_0,l_1\rangle, \\ & l_0+\varepsilon(\varepsilon^2,\cdots), l_0, l_0-\varepsilon(\varepsilon^2,\cdots), \\ & -N,-\mathbf{H},(-\mathbf{H^2},\cdots).\} \end{aligned}$$

□

Definition 2 (O(M) and Sign of Qualitative Value) *Define the order of magnitude (O(M)) of a variable x as:*

$$abs(x) = \begin{cases} \mathbf{H}^i, & where\ x\ is\ \mathbf{H}^i\text{-}infinite \\ N(>0), & where\ x\ is\ finite \\ \varepsilon^i, & where\ x\ is\ \varepsilon^i\text{-}infinitesimal \\ 0, & where\ x = 0 \end{cases}$$

where i is integer. □

Using D2 with the sign of a variable x $sign(x)$, we can define the qualitative derivative(QD) of a variable. We also introduce the qualitative second-order derivatives of variables to "tame intractable branching" [8] where it is appropriate to evaluate.

Definition 3 (Qualitative Derivative) *For $i = 1, 2$, define the qualitative derivative of a variable x as:*

$$\partial^i x = sign(\frac{d^i x}{dt^i}) * abs(\frac{d^i x}{dt^i}).$$

□

Definition 4 (Qualitative Representation) *Define the qualitative representation of a variable state x, $Q_sR(x)$ as:*

$$Q_sR(x) = (\ [x], \partial x, \partial^2 x\ ).$$

□

Consider an example where qualitative differential equation(QDE) is $\partial x = -\varepsilon * [x]$ and its initial condition is $[x] = \varepsilon$. From QDE, we obtain $\partial^2 x = -\varepsilon * \partial x$. Hence the initial state: $Q_sR(x_1) = (+\varepsilon, -\varepsilon^2, +\varepsilon^3)$ is obtained through constraint propagation. Since ε^2 and ε^3 appear in ∂x , $QS(x) = \{\mathbf{H}, N, \varepsilon, \varepsilon^2, \varepsilon^3, 0, -\varepsilon^3, -\varepsilon^2, -\varepsilon, -N, -\mathbf{H}\}$ and the next candidate is the transition from $[x] = \varepsilon$ to $[x] = \varepsilon^2$. The next state $Q_sR(x_2) = (+\varepsilon^2, -\varepsilon^3, +\varepsilon^4)$ is derived from the above constraints.

Definition 5 (Qualitative Arithmetic) *Arithmetic between qualitative infinitesimal values is based on the O(M) reasoning proposed by Raiman[11]. We extend this arithmetic as follows: For any integers m and n such that $m < n$,*

$$\begin{aligned} (Addition) \quad & \varepsilon^m + \varepsilon^n \simeq \varepsilon^m \\ (Substitution) \quad & \varepsilon^m - \varepsilon^n \simeq \varepsilon^m \\ (Multiplication) \quad & \varepsilon^m * \varepsilon = \varepsilon^{m+1} \\ (Division) \quad & \varepsilon^{-m} = \mathbf{H}^m \\ (Comparison) \quad & (\varepsilon/\varepsilon) < (1/\varepsilon) < (1/\varepsilon^2) < \cdots \\ & < (1/\varepsilon^m) < .. \end{aligned}$$

where $A \simeq B$ means $abs(A) = abs(B)$. □

Note that ε/ε is finite. Intuitively, this relation is derived by an inequality $\varepsilon^2/\varepsilon(=\varepsilon) < \varepsilon/\varepsilon < 1/\varepsilon(=\mathbf{H})$. The proof can be found in Keisler[5].

3.2 Transition with ε-H ranking

Besides extensions of qualitative values,we extend definition of state interval, transition between qualitative values, and persistent and arrival time. The ε-**H** ranking adds many interesting characteristics to QR with hyperreals. But for limitations of space, this presentation is restricted to the extensions of transition and persistent and arrival time from Weld's HR-QSIM[16]. For more detail, see [15].

Definition 6 (Transition of Qualitative Values) *Qualitative values can have transitions only between two adjacent states, or inside a ε-neighborhood interval ($\varepsilon^i \leftrightarrow \varepsilon^{i+1}$) or inside a infinite interval ($\mathbf{H}^i \leftrightarrow \mathbf{H}^{i+1}$). The possible transitions are as follows:*

Table 1: time-distance table

		distance			
		0	ε	N	**H**
	0	0	**H**	**H**	**H**
∂x	ε	0	N	**H**	$\mathbf{H}^2$
	N	0	ε	N	**H**
	H	0	ε^2	ε	N

$$(.. \leftrightarrow \mathbf{H}^2 \leftrightarrow)\mathbf{H} \leftrightarrow finite \leftrightarrow \varepsilon(\leftrightarrow \varepsilon^2 \leftrightarrow ..) \leftrightarrow point$$

The steps which determine the transition of qualitative values are derived by envisionment using Welds' HR-QSIM method [16] except for inside ε *or* **H** *interval.* □

Definition 7 (Arrival time and persistent time) *For any variable x and any integer i, $I_s(x_i)$ be the O(M) of the interval's length of the state i, and $I_a(x_{i+1})$ be the minimum O(M) of the distance of x between the present state(i) and the next one(i+1). Persistent time($t_s(x_i)$) and arrival time($t_a(x_{i+1})$) are represented as the following equations:*

$$t_s(x_i) = I_s(x_i)/abs(\partial x_i)$$
$$t_a(x_{i+1}) = I_a(x_{i+1})/abs(\partial x_i)$$

Their values are derived by qualitative calculus with ε-**H** *ranking as shown in Section 3.1 and a time-distance table as shown in Table 1.* □

For example, when $I_a(x) = \varepsilon$ and $abs(\partial x) = \varepsilon^3$, $t_a(x) = (\varepsilon/\varepsilon^3) = \varepsilon^{-2} = \mathbf{H}^2$

Note that our ε-**H** ranking improves Weld's temporal filter[16]. For example, we can deal with ε-ordering rule[1] more concretely, which Weld a little bit trickily includes in "Temporal Continuity Rule". Consider an example where a variable x is 0, and the order of its derivative is ε^m (m:a certain integer). The next deduced transition is $[x] = \varepsilon^m$. Since $I_a(x)$ is the minimum O(M) of the distance of x between 0 and ε^m, for any integer i, $Ia(x) < Ia(\varepsilon^{i+m}) < Ia(\varepsilon^m)$. Hence we obtain that $t_a(x) = (I_a(x)/abs(\partial x)) < (I_a(\varepsilon^{(i+m}})/abs(\partial x)) = (\varepsilon^{i+m}/\varepsilon^m) = \varepsilon^i$. Consequently,$t_a(x) < \varepsilon^i$, that is, arrival time from x=0 to ε^m is less than ε^i. Since i is arbitrary, this means that a variable in a state changes faster than any other variables in an infinitesimal interval.

Then, how we can deal with the reverse case, that is, the transition of x from ε to 0 (x converges at 0)? We can classify the convergence into two types: x passes 0 after converging to 0 within a finite or an infinitesimal interval and x converges at 0 asymptotically. In the next section, we discuss about this case.

3.3 Convergence

When one variable converges monotonically, it would happen that envisionment is repeated infinitely. For example,consider the case mentioned in Section 3.1. Envisionment generates the following infinite sequence: $(x, \partial x, \partial^2 x) = (+\varepsilon, -\varepsilon^2, +\varepsilon^3), (+\varepsilon^2, -\varepsilon^3, +\varepsilon^4), (+\varepsilon^3, -\varepsilon^4, +\varepsilon^5)...$. However,in this case,it is obvious that $(x, \partial x, \partial^2 x)$ converges at (0,0,0). Our formalism can examine more precisely how a variable converge, (for the above example,$\partial x, \partial^2 x$ decreases as time passes),but it cannot judge the convergence. Hence,in addition to the extensions of QR discussed above, we must provide a rule which judges whether and how the convergence of variables occurs (In this section, for simplicity, we only deal with "monotonic convergence". However, our definitions can be easily generalized in order to deal with damped oscillation. Generalization of a convergence filter is discussed in [15].)

Definition 8 (Convergence Filter) *Let x_0 be the nearest landmark. Also let $I_s(x_i)$ denote the length of an interval(i:integer). If*

$$\exists i, \quad I_s(x_i) = \varepsilon$$

$\forall k$ such that $k > i$, $\exists m$

$$([x_k - x_0] = -\varepsilon^m, \partial x_k > 0) \vee ([x_k - x_0] = +\varepsilon^m, \partial x_k < 0)$$

$$and\ if \quad I_s(x_{k+1}) \leq \varepsilon * I_s(x_k)$$

then we shall say that **x converges at $\mathbf{x_0}$**.

And the arrival time t_a is defined as:

$$t_a = \sum_{j=i+1}^{\mathbf{H}} t_a(x_j).$$

□

This arrival time is calculated by the ordinary methods for infinite sum in nonstandard analysis[5] and it has several important features. Unfortunately, for limitation of space, we cannot give a detailed discussion about the arrival time here. In this paper, we only present two characteristics without their proof: **if $abs(t_a) \leq N$ then x passes x_0 and if $abs(t_a) \geq \mathbf{H}$ then x converges at x_0 asymptotically.** Precise discussion is given in [15].

The above definition is clear, but insufficient to determine the convergence with finite steps.It does not reduce the infiniteness of the determining process. Note that we should judge the convergence in finite time: when we observe that some elements of the sequence of a variable x approach at a point x_0, we determine that x converges at x_0. This reasoning process is an example of "persistence" in nonmonotonic reasoning[9],[10]. According to our commonsense reasoning, we provide a convergence filter rule as follows:

Definition 9 (Convergence Filter Rule) *If the states satisfy the condition D8 (convergent condition) until ε^4 occurs, then check the next transition. And if the condition D8 is also satisfied, the system judges* *that convergence has occurred. Arrival time is calculated as* $t_a = \sum_{j=i+1}^{\mathbf{H}} t_a(x_j)$ *where* i *is a certain integer such that* $I_s(x_i) = \varepsilon^3$. *If* $abs(t_a) < N$ *then* x *passes* x_0, *else if* $abs(t_a)) \geq \mathbf{H}$ *then* x *converges at* x_0 *asymptotically.* □

Consider the example mentioned above. $I_s(x) = \varepsilon, \varepsilon^2, \varepsilon^3$ and the candidate for the next state is $(x, \partial x, \partial^2 x) = (\varepsilon^4, -\varepsilon^5, \varepsilon^6)$. Since the former states and the candidate satisfy the conditions D8, we check the next transition. The next is $(\varepsilon^5, -\varepsilon^6, \varepsilon^7)$ and the condition is also satisfied. So we determine that x (and ∂x) converges at 0. And the arrival time is derived as follows: $t_a = (\varepsilon/\varepsilon^2) + (\varepsilon^2/\varepsilon^3) + ... = \sum_{j=2}^{\mathbf{H}} (\varepsilon^{i-1}/\varepsilon^i) = (1/\varepsilon) * \mathbf{H} \simeq \mathbf{H}^2 > \mathbf{H}$. Hence a variable x converges at 0 asymptotically.

4 QD Restriction

Time-scale abstraction is considered to be based on the two properties of the O(M) of qualitative derivatives. First, relative constancy of slower variables means that the O(M) of the derivatives of slower variables and their change are much less than the derivatives of faster variables. Second, binding of faster variables means that $O(M)$ of the derivatives of faster variables is much larger than slower ones. These temporal ontologies of variables can be more sophisticatedly represented by specifying range of derivatives; we call this hierarchy of qualitative derivatives QD restriction.

Definition 10 (QD Restriction) *Quantity space of qualitative derivatives of faster variables(f) or slower ones(s) should be represented as follows:*

$$\begin{aligned} QS(\partial f) &= \{.., +\mathbf{H}^2, +\mathbf{H}, +N, \\ &\quad +\varepsilon, +\varepsilon^2, ., 0, ., -\varepsilon^2, -\varepsilon, \\ &\quad -N, -\mathbf{H}, -\mathbf{H}^2, ..\} \\ QS(\partial s) &= \{+\varepsilon, +\varepsilon^2, ., 0, ., -\varepsilon^2, -\varepsilon\} \\ QS(\partial^2 s) &= \{+\varepsilon, +\varepsilon^2, ., 0, ., -\varepsilon^2, -\varepsilon\}. \end{aligned}$$

Direction of transition of qualitative derivative is determined by the signs of the second order derivatives of the qualitatives values and precedence of transition is determined by the arrival time. □

Note that the second order derivatives of the slower variables are also restricted. If their order of magnitude is N(finite), the derivatives transit into a finite interval. This contradicts the above definition. For example, consider $\partial x = \varepsilon$ and $\partial^2 x = N$. In order for ∂x to stay at ε,the order of persistent time for $\partial^2 x = N$ is lower than $(N - \varepsilon)/N = N/N$,that is,its order is equal to or lower than ε. If we consider that the third order derivatives meet the above requirements, $\partial^3 x$ will be $\mathbf{H}$. Hence,this fact contradicts the definition of slower variables: slower variables change much slower than faster variables in a finite interval. Therefore the second order derivatives are also required to be restricted.

One may say that QD restriction can be naturally embedded in QDE as shown in Davis[2],such that $\partial x = -\varepsilon^i * [x]$ where x is a slower variable and i is integer. However, this embedding is not sufficient. When x is $\mathbf{H}^i$, ∂x is $-$N. If the O(M) of the QD of a faster variable is N, then we cannot differentiate between a faster variable and slower one. Hence even in the above case, QD restriction is also needed.

5 QUASAR

We develop a program QUASAR (QUAlitative reasoning using time-Scale information Analysis by epsilon-eta ranking and Restriction of qualitative derivatives) which implements QR with ranked hyperreals, the convergence filter and QD restriction. QUASAR consists of two parts: setting part, and transition analyzer. Setting part calculates the constraints of QD from given QDE and then QD restriction is set up. Finally, it derives the initial states from an initial condition. Transition analyzer envisions a transition from a certain state i. In this section, first we show algorithm for transition analysis, and then illustrate how QUASAR works.

5.1 Algorithm for Transition

In this algorithm, we use a operator ":=" for substitution, for example, "$x := 3$" means that 3 is substituted for x. For simplicity, we assume that there is no branching before the state i.

Algorithm

Let i, j, k, l, m, n and p be integer. $x_i(m)$ stands for a variable of the system($1 \leq m \leq n$, n:the total number of the variables in QDE) in a state i. And also let $first_i$,min_i and $final_i$ denote the set of first candidates, minimum candidates, and final candidates respectively as defined below. Before transition analysis, all the sets are {}(empty).

1. Apply the transition rules to each variable:$x_i(j)$, and generate the list of the nearest qualitative adjacent value of each variable considered as the candidates of the next state transition: $first_{i+1} = \{(1 : \hat{x}_{i+1}(1)), ..., (n : \hat{x}_{i+1}(n))\}$.

2. Calculate each $t_a(\hat{x}_{i+1}(k))(k : integer, 1 \leq k \leq n)$. Compare the O(M) of $t_a(\hat{x}_{i+1}(k))$, and choose the set of candidates whose O(M) of the arrival time is minimum:(minimum candidates: $min_{i+1} = \{(j : \hat{x}_{i+1}(j))| \ \forall l, abs(\hat{x}_{i+1}(j)) \leq abs(\hat{x}_{i+1}(l))\}$).

3. Choose a variable, say $x_{i+1}(m)$, from min_{i+1}: $min_{i+1} := min_{i+1} - \{(m : x_{i+1}(m))\}$. Substitute $x_{i+1}(m)$ for $x_i(m)$ in the state i and apply the constraint propagation. If the constraints are satisfied, add this variable to the set of the final candidates($final_{i+1} := final_{i+1} \cup \{(m : x_{i+1}(m))\}$).

4. If $min_{i+1} \neq \{\}$,go to 3). If $min_{i+1} = \{\} \wedge final_{i+1} \neq \{\}$, go to 5). If $min_{i+1} = \{\} \wedge final_{i+1} = \{\}$, quit as failure.

5. Choose a variable, say $x_{i+1}(p)$ from $final_{i+1}$.($final_{i+1} := final_{i+1} - \{p : x_{i+1}(p)\}$). Apply the convergence filter rule to $x_{i+1}(p)$:

   (a) If the past four sequences of $x(p)$ satisfy the convergent condition, then check the next transition.

   (b) If the convergent condition is also satisfied, then determine that $x(p)$ converges at a point $x_0(x_{i+1}(p) := x_0)$.

   (c) If $x(p)$ converges,calculate the arrival time(t_a). If $abs(t_a) \leq N$ then $x(p)$ passes at the point, else $x(p)$ converge at the point asymptotically.

   (d) If $x(p)$ passes, store this state as the next state else store this state as the final state.

6. If $final_{i+1} \neq \{\}$ then go to 5).
   If $final_{i+1} = \{\}$ then quit as succeeded.

5.2 An Example

Let us consider a model of body fluid regulation. Body fluid system is regulated chiefly by amount of water(w),sodium(n) and concentration of sodium($c = n/w$) which is almost equal to osmotic pressure. It is known that the amount of body fluid change(water intake and excretion) is regulated by sensing osmotic pressure deviation, and this regulation takes place with response time 10 minutes, whereas change of the amount of sodium is regulated by the amount of water with response time more than one hour. This system can be modeled as follows:

$$\begin{aligned} \partial w &= [c - c_0] \\ \partial n &= \varepsilon * [w_0 - w] \\ c &= \frac{n}{w} \end{aligned}$$

where w is a faster variable,n is a slower variable. And n_0,w_0,$c_0 (= n_0/w_0)$ are quantities of sodium, water, and concentration of sodium at steady state, respectively. We consider the case when osmotic pressure is hypertonic($c > c_0$), and both water and salt are overloading($w > w_0, n > n_0$) to show how QUASAR works.

5.2.1 Setting Part

From the qualitative derivative equations, we calculate the constraints about derivatives and second order derivatives of faster and slower variables as follows:

$$\begin{aligned} \partial c &= \frac{\partial n - c\partial w}{w} \\ \partial^2 w &= \partial c \\ \partial^2 n &= \varepsilon * (-\partial w) \\ \partial^2 c &= \partial^2 n - 2\partial n \partial w - \partial^2 w + (\partial w)^2. \end{aligned}$$

Using these formulae, QD restriction are used to obtain into the quantity space of qualitative derivatives:

$$\begin{aligned} QS(\partial w) = QS(\partial^2 w) &= \{\cdots, +\mathbf{H}^2, +\mathbf{H}, +N, \\ &\quad +\varepsilon, +\varepsilon^2, \cdots, 0, \cdots, -\varepsilon^2, -\varepsilon, \\ &\quad -N, -\mathbf{H}, -\mathbf{H}^2, \cdots\}, \\ QS(\partial n) = QS(\partial^2 n) &= \{+\varepsilon, +\varepsilon^2, \cdots, 0, \cdots, -\varepsilon^2, -\varepsilon\}, \\ QS(\partial c) = QS(\partial^2 c) &= \{\cdots, +\mathbf{H}^2, +\mathbf{H}, +N, \\ &\quad +\varepsilon, +\varepsilon^2, \cdots, 0, \cdots, -\varepsilon^2, -\varepsilon, \\ &\quad -N, -\mathbf{H}, -\mathbf{H}^2, \cdots\}. \end{aligned}$$

After the above settings, the given initial conditions are propagated:Statel is an interval,where $Q_s R((w - w_0)_1) = (+N, N, -N)$, $Q_s R((n - n_0)_1) = (+N, -\varepsilon, -\varepsilon)$, $Q_s R((c - c_0)_1) = (+N, -N, -N)$. From this initial state,the transition process begins.

5.2.2 Transition

Using the algorithm mentioned in 4.3, we can derive the results of qualitative analysis as shown in Table 2. Here, we give one example: transition from state4 to state5 to illustrate how the results are obtained.

(Transition from state4 to state5)

1. The candidates are $[w - w_0] : +N \rightarrow \mathbf{H}$, $[n - n_0] : +N \rightarrow +\varepsilon$, $[c - c_0] : +\varepsilon^3 \rightarrow \varepsilon^4$:thus $first_5 = \{(w - w_0)_5 : H), ((n - n_0)_5 : +\varepsilon), ((c - c_0)_5 : +\varepsilon^4\}$.

2. Calculate each arrival time: $w : (\mathbf{H} - N)/N \simeq \mathbf{H}$, $n : N/\varepsilon \simeq \mathbf{H}$, $c : (\varepsilon^3 - \varepsilon^4)/\varepsilon \simeq \varepsilon^2$, c are the final candidates: $min_5 = \{((c - c_0)_5 : +\varepsilon^4)\}$.

3. Choose a variable $(c - c_0)_5$ (then $min_5 := \{\}$). Perform constraint propagation and the solution: $Q_s R((w - w_0)_5) = (N, +\varepsilon^4, -\varepsilon)$, $Q_s R((n - n_0)_5) = (N, -\varepsilon, -\varepsilon^5)$, $Q_s R((c - c_0)_5) = (+\varepsilon^4, -\varepsilon, +\varepsilon)$ are derived. Add it to the final candidates: $final_5 = \{(c - c_0)_5\}$.

4. $min_5 = \{\}$ and $final_5 = \{(c - c_0)_5\}$, so go to 5).

5. Choose $(c - c_0)_5$ (then $final_5 = \{\}$). It satisfy the convergent condition: that is to say, for a variable c and for integer $i = 2, 3,$ *and* 4, $Ia((c - c_0)_2) =$

ε, $Ia((c-c_0)_{i+1}) \leq \varepsilon * Ia((c-c_0)_i)$, $[c-c_0] = +\varepsilon^{i-1}$, and $\partial c < 0$. Check the next transition. the next state is $([c-c_0], \partial c, \partial^2 c) = (+\varepsilon^5, -\varepsilon, +\varepsilon)$, and the arrival time is ε^3. So we determines that $[c-c_0]$ converges at 0 and their arrival time is $t_a = \sum_{j=2}^{\mathbf{H}} \varepsilon^j \simeq \varepsilon^2$. Since $t_a < N$, c passes c_0: the state is stored as the state 5.

6. $final_5 = \{\}$, so quit as succeeded.

5.2.3 The Results

The results of qualitative analysis show that three kinds of the processes are involved. First, the amount of water(w) increases fast and the concentration of sodium(c) converges at the point(c_0). Second, when c reaches c_0, w stops increasing. Third, c passes c_0, and w begins to decrease slowly. The variable c remains to be in the neighborhood of c_0 only to give the driving force to adjust remained water imbalance after osmotic pressure is regulated. w and n decreases and,as infinite time passes, w, n and c reach their equilibrium. First and second process correspond to faster mechanisms in Kuipers's time-scale abstraction, and third process to slower mechanisms. But our results explain the interaction between faster variables(w) and slower ones(n) more clearly: while w changes quickly when $c - c_0 > \varepsilon$, w changes slowly with n when $c \simeq c_0$. Those behaviors clearly agrees with the body fluid regulation: if the osmotic pressure changes a little, this change is compensated mainly by renal function - slower mechanism. In Kuipers's time-scale abstraction, however, if "$c \simeq c_0$", one constraint:"$c = n/w = c_0$" should be given for qualitative simulation; interconnection must be always given from outside in order to simulate only the slower mechanism. Our method can cope with that case correctly.

6 Discussion

We combine qualitative representation based on Weld's qualitative hyperreals with envisionment based on Davis' CHEPACHET[2] and introduce ε-**H**ranking and the convergence filter with QD restriction specially for application of our method to a temporal hierarchical system. ε-**H** ranking and the convergence filter give more precise information of qualitative variables to ordinary QR. Providing some important knowledge of real numbers and the O(M) of QD makes the envisionment more accurate and reduces the ambiguities in qualitative values. Also it can represent interaction between faster and slower variables more naturally than "extra-mathematical" hierarchization, especially when derivatives of faster variables converge on the O(M) of the derivatives of slower one and extends his approach. Hence,our approach realizes Kuipers time-scale abstraction in a more mathematical way and extends his approach.

The limitation of this work is that this work will be computationally expensive when faster and slower variables are not well-defined. In other words, since QD restriction may not be applied in that case, so transition of qualitative derivatives is not restricted as in definition 10. QUASAR cannot detect whether a given model support QD restriction or not. To implement these automated detection is our future work.

7 Related Works

Little previous attention has been devoted to time-scale abstraction,except for Kuipers work [6]. In this section,we consider AI work related to temporal hierarchization.

Weld [16] extends qualitative values to qualitative hyperreal numbers,and develops a program that considers a role of one parameter in a system by comparing a normal system behavior with the exaggerated system behavior. He discusses about Kuipers approach and describes that his exaggeration can represent time-scale abstraction implicitly,whereas he does not discuss the methodology in detail. One may say that our QD restriction can be regarded as exaggeration of slower variables:we use the nonstandard analysis, and also introduce time-scale into the quantity space of derivatives. However,QD restriction is different from exaggeration. As shown in Section 5, the derivative of faster variables reaches the same order of those of slower variables. And in those states the derivative of slower variables is not exaggerated any more. Hence the whole behavior can be interpreted as combination of exaggerated behavior and not-exaggerated one. Original exaggeration method needs the continuation analysis to deal with time-scale abstraction,which generates the problems discussed in Section 2. So our approach includes exaggeration about time-scale and solve the problems of the continuation analysis of exaggeration.

Davis [2] combines order of magnitude reasoning with envisionment of qualitative differential equations. He divides the non-standard real line into seven disjoint intervals: −LARGE (infinite numbers), −MEDIUM (finite numbers), −SMALL (infinitesimals), ZERO, SMALL, MEDIUM, LARGE. He introduces variance of parameter,which is equal to our $I_s(x)$, and time duration, which is equal to persistent time. He illustrates quickly settling control parameter and observes that this example is similar to those studied by Kuipers. Our approach is also similar to his work. However, he does not discuss the cases when derivatives of faster variables converge on the order of derivatives of slower one. In those cases, the interaction between faster and slower variables necessarily appears.So, we cannot envision both of them separately. Our QUASAR can cope with this problem and simulate those cases much finer.

Iwasaki [4] discusses about the mixture of slower system and faster system in a viewpoint from causal ordering. She regards a mixed structure M as combination of equilibrium equations $Static(M)$ which represent a very short-term equilibrium description, and dynamic equations $Dynamic(M)$ which represent slower mechanisms. Her approach also uses pre-decomposition of the model and deals with both systems independently. Like the approaches mentioned above,she does not discuss the interaction between faster and slower variables and the problems about the continuation analysis.

Finally,note that our framework can deal with hierarchization of variables' magnitude, such as a system which includes a subsystem of infinitesimal sustained oscillation. Detailed treatment of this kind of system is our future work.

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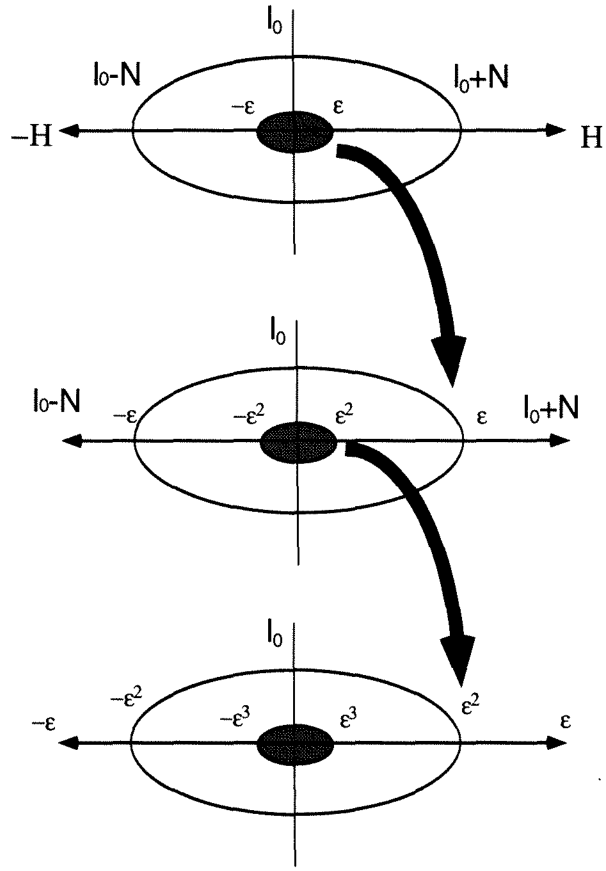

Figure 1: Relationship between the powers of ε

Table 2: Transition of an Example

state	1	2	3	4	(5)	6	7	8	9	10	(11)
$[w-w_0]$	N	N	N	N	N	N	N	N	ε	ε^2	0
$[n-n_0]$	N	N	N	N	N	N	N	N	ε	ε^2	0
$[c-c_0]$	N	ε	ε^2	ε^3	0	-ε	$-\varepsilon$	-ε	-ε^2	-ε^3	0
∂w	N	ε	ε^2	ε^3	0	-ε	-ε	-ε	-ε^2	-ε^3	0
∂n	-ε	-ε	-ε	-ε	-ε	-ε	-ε	-ε	-ε^2	-ε^3	0
∂c	-N	-ε	-ε	-ε	-ε	-ε	0	ε^2	ε^2	ε^3	0
$\partial^2 w$	-N	-ε	-ε	-ε	-ε	-ε	0	ε^2	ε^2	ε^3	0
$\partial^2 n$	-ε	-ε^2	-ε^3	-ε^4	0	ε^2	ε^2	ε^2	ε^3	ε^4	0
$\partial^2 c$	N	ε	ε	ε	ε	ε	ε^2	ε^2	-ε^2	-ε^3	0
t_a		N	N	ε	ε^2	N	N	N	**H**	**H**	$\mathbf{H}^2$

t_a for state4 $\rightarrow$ state5 $\quad t_a = \varepsilon^2 + \varepsilon^3 + \cdots = \varepsilon * (1/(1-\varepsilon)) = \varepsilon^2$
t_a for state10 $\rightarrow$ state11 $\quad t_a = 1/\varepsilon + 1/\varepsilon + ... = \mathbf{H}/\varepsilon = \mathbf{H}^2$

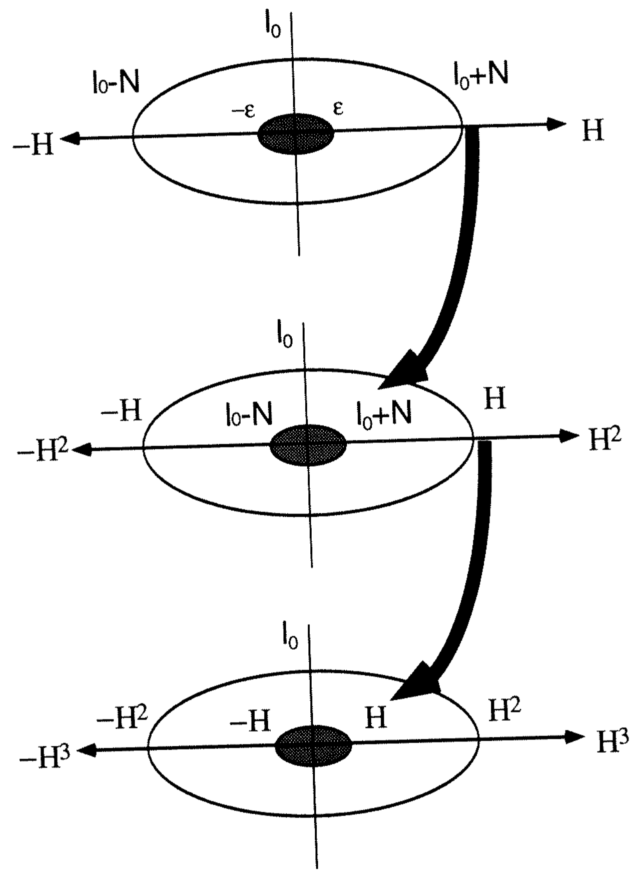

Figure 2: Relationship between the powers of **H**