OBSERVATION OF UNMEASURABLE STATES BY MEANS OF QUALITATIVE MODELS

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Abstract:

In control engineering, diagnosis and process supervision are important tasks where qualitative models can be applied. In this paper, the problem of observing the internal states of a dynamical system is solved by means of a qualitative model that has the form of a nondeterministic automaton. In the qualitative observation problem, the current qualitative state of a system has to be inferred from measurement sequences of qualitative inputs and outputs. An observation algorithm is described and illustrated by applying the algorithm to a three-tank system. Due to the structure of the qualitative model used, the observation algorithm can be practically applied under real-time constraints.

1   Introduction

This paper is concerned with the reconstruction of unmeasurable signals of a dynamical system by means of a model and of the sequences of measurable input and output signals. This problem is called the *observation problem*. It has to be solved because there are a lot of physical variables that are unaccessible to measurement but have to be known if for the technological process under consideration control actions should be appropriately selected.

In the following, the observation problem is formulated and solved on a qualitative level of abstraction, which is reasonable for process supervision and diagnosis. The qualitative values of internal signals of the process have to be found from given qualitative input and output data. This problem is called *qualitative observation problem*. The paper shows, that a nondeterministic automaton as a qualitative model of the process is appropriate for solving this problem.

The literature on qualitative reasoning, which is surveyed in [10], [12], focuses mainly on the prediction of the qualitative behaviour of a given system by means of qualitative simulation. For a given qualitative initial state, the future movement of a physical system has to be determined. The observation problem dealt with in this paper refers to the "inverse" problem: For a given output sequence the initial state has to be found. Hence, this paper differs from the literature on qualitative reasoning not only with respect to the kind of model used but concerning the aim for which the qualitative model is set up.

The paper is organised as follows: In Section 2 and Section 3 the state space model of dynamical systems and signal quantisations are described. Qualitative signals can then be defined in accordance with these partitions. In Section 4, the qualitative observation problem is given and compared with the classical observation problem, which is well known in control theory. In Section 5 the qualitative model of the process is presented, which is used for the qualitative observation algorithm given in Section 6. The applicability of the observatoin algorithm is demonstrated in Section 7 for an interconnected tank system.

2   State space models of dynamical systems

The behaviour of dynamical systems is described by signals, i.e. by time-dependent variables. Dynamical systems transform input signals $u_i(t)$, $(i = 1, \ldots, m)$ given by the environment of the system into output signals $y_i(t)$, $(i = 1, \ldots, r)$ that can be observed by the environment. This fact is described by the block diagram in Figure 1, where the arrows denote signals and the block a dynamical system.
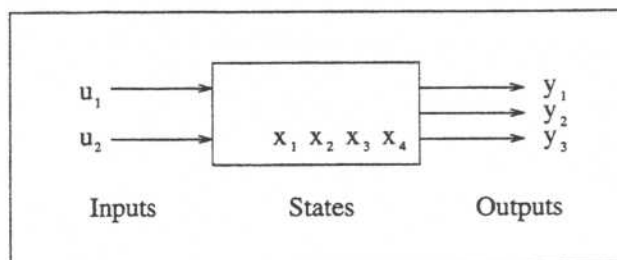

Figure 1: General dynamical system

For the description of the dynamical behaviour, the state $x = (x_1, x_2, \ldots, x_n)'$ of the system plays an im-

portant role. If all state variables $x_i(\bar{t})$, $(i = 1, \ldots, n)$ at time $\bar{t}$ as well as the input signals $u_i(t)$ in the time interval $\bar{t} \leq t \leq T$ are known, the output signals $y_i(t)$ can be uniquely predicted for the same time interval. Thus, all the information about the past and the possible future evolution of the system is stored at each instant of time in the vector $\boldsymbol{x}(\bar{t})$. In contrast to this, if only the output vector $\boldsymbol{y}(\bar{t}) = (y_1(\bar{t}), y_2(\bar{t}), \ldots, y_r(\bar{t}))'$ is known at time $\bar{t}$, the future behaviour cannot be uniquely determined.

For the sake of simplicity of presentation we focus on the class of linear multiple-input multiple-output (MIMO) discrete time systems although all results can be easily extended to nonlinear systems. The dynamics is modelled for discrete time points $t = kT$ which are enumerated by the integer variable k. The correlation between the state $\boldsymbol{x}(k) \in \mathbb{R}^n$ and input $\boldsymbol{u}(k) \in \mathbb{R}^m$ at time k and the successor state $\boldsymbol{x}(k+1)$ or the output $\boldsymbol{y}(k) \in \mathbb{R}^r$, respectively, is given by the *state space model*

$$\begin{align}
\boldsymbol{x}(k+1) &= \boldsymbol{A}\boldsymbol{x}(k) + \boldsymbol{B}\boldsymbol{u}(k), \tag{1}\\
\boldsymbol{y}(k) &= \boldsymbol{C}\boldsymbol{x}(k) + \boldsymbol{D}\boldsymbol{u}(k), \tag{2}\\
\boldsymbol{x}(0) &= \boldsymbol{x}_0. \tag{3}
\end{align}$$

where $\boldsymbol{x}_0$ denotes the initial state and $\boldsymbol{A} \in \mathbb{R}^{n\times n}$ $\boldsymbol{B} \in \mathbb{R}^{n\times m}$ $\boldsymbol{C} \in \mathbb{R}^{r\times n}$ and $\boldsymbol{D} \in \mathbb{R}^{r\times m}$ are matrices describing the system parameters. Discrete-time dynamical systems (1)–(3) map a given initial state $\boldsymbol{x}_0$ and an input sequence

$$U = (\boldsymbol{u}(0), \boldsymbol{u}(1), \ldots, \boldsymbol{u}(T)) \tag{4}$$

into unique state and output sequences

$$\begin{align}
X &= (\boldsymbol{x}(0), \boldsymbol{x}(1), \ldots, \boldsymbol{x}(T+1)) \tag{5}\\
Y &= (\boldsymbol{y}(0), \boldsymbol{y}(1), \ldots, \boldsymbol{y}(T)). \tag{6}
\end{align}$$

3 Quantisation of the signals

In process supervision the operator is interested in knowing the qualitative rather than the quantitative behaviour of the process. For example the operator wants to know whether the process works "stable" or whether the outputs are in the desired region. In order to represent such an assessment of the sequences (5) and (6), partitions $\mathcal{Q}_x$, $\mathcal{Q}_u$ and $\mathcal{Q}_y$ of the spaces of the state, input and output signals are introduced. A *partition* $\mathcal{Q}$ of some set $\mathcal{M}$ defines disjoint subsets $\mathcal{Q}_i$, which altogether cover the set $\mathcal{M}$:

$$\bigcup_i \mathcal{Q}(i) = \mathcal{M} \tag{7}$$

$$\mathcal{Q}(i) \cap \mathcal{Q}(j) = \emptyset \; \forall \; i \neq j. \tag{8}$$

For technical reasons, these subsets are often rectangular. Thus, the partition is given by *intervals* for each of the variables (Figure 2).
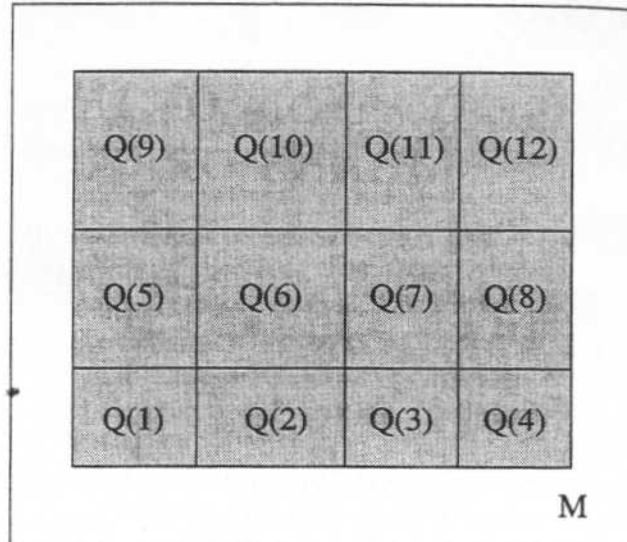

Figure 2: Partition of a two-dimensional space

The qualitative values $[\boldsymbol{u}(k)], [\boldsymbol{x}(k)], [\boldsymbol{y}(k)]$ of the input, state and output at time k are defined by

$$\begin{align}
[\boldsymbol{u}(k)] = v(k) &\iff \boldsymbol{u}(k) \in \mathcal{Q}_u(v(k)) \tag{9}\\
[\boldsymbol{x}(k)] = z(k) &\iff \boldsymbol{x}(k) \in \mathcal{Q}_x(z(k)) \tag{10}\\
[\boldsymbol{y}(k)] = w(k) &\iff \boldsymbol{y}(k) \in \mathcal{Q}_y(w(k)). \tag{11}
\end{align}$$

That is, these values are the numbers $v(k)$, $z(k)$ or $w(k)$, respectively, of the regions to which $\boldsymbol{u}(k)$, $\boldsymbol{x}(k)$ or $\boldsymbol{y}(k)$, respectively, belong at time k.

Eqns. (4)–(6) yield the sequence of qualitative inputs

$$[U] = ([\boldsymbol{u}(0)], [\boldsymbol{u}(1)], \ldots, [\boldsymbol{u}(T)]), \tag{12}$$

the sequence of qualitative states

$$[X] = ([\boldsymbol{x}(0)], [\boldsymbol{x}(1)], \ldots, [\boldsymbol{x}(T+1)]) \tag{13}$$

(also called *qualitative trajectory*) and the qualitative output sequence

$$[Y] = ([\boldsymbol{y}(0)], [\boldsymbol{y}(1)], \ldots, [\boldsymbol{y}(T)]). \tag{14}$$

4 Classical and qualitative observation problem

In control theory, the following problem has been intensively studied (cf. eqns. (4)–(6)):

Classical observation problem:
Given: Input sequence U, Output sequence Y.
Find: State sequence X.

That is, for given input and output sequences, the unknown initial state $\boldsymbol{x}_0$ and the whole state sequence X has to be reconstructed.
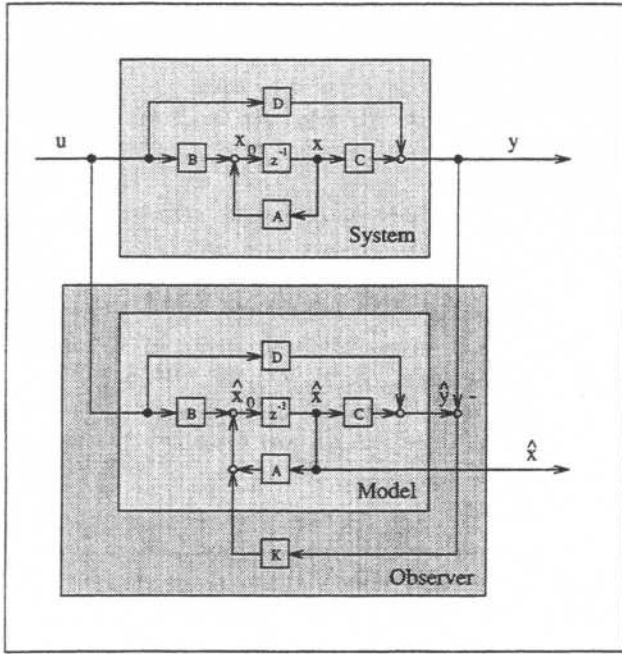
Figure 3: Classical Luenberger observer

Figure 4: Qualitative observer

This problem can be solved by means of the classical *Luenberger observer* depicted in Figure 3 [8]. As the main component, the observer includes a model (1)–(3) of the system whose state x has to be reconstructed. The feedback of the difference $y - \hat{y}$ between the predicted and the actual output is used to adjust the actual state of the model assuring that

$$\lim_{k \to \infty} ||x(k) - \hat{x}(k)|| = 0 \qquad (15)$$

holds. That is, at each time k the model state $\hat{x}(k)$ is an approximation of the system state $x(k)$.

We want to mention two important facts for the application of Luenberger observers. First, it is known that the observation problem can be solved if and only if the system is *observable* [1], which can be checked in terms of the matrices A, B, C and D. Second, it is known how to find a feedback matrix K (cf. Figure 3) such that the *convergence* (15) of the observer is ensured.

In process supervision, a similar observation problem has to be solved (cf. eqns. (12)–(14)):

Problem of qualitative state observation:

Given: Qualitative input sequence $[U]$,
Qualitative output sequence $[Y]$.
Find: Qualitative state sequence $[X]$.

This problem is similar to the classical observation problem. However, the important distinction concerns the qualitative nature of the known signals $[U]$ and $[Y]$ and of the state sequence $[X]$ to be reconstructed.
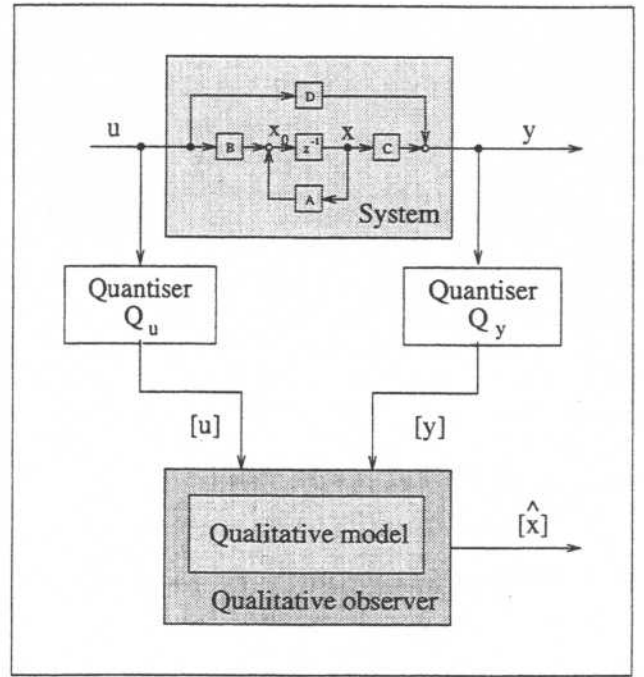
The remainder of this paper is devoted to the solution of this problem. A qualitative observer, which to a certain extend is similar to the Luenberger observer, will be explained (Figure 4). Like in the Luenberger observer, a model of the system (1)–(3) plays the key role in the observation algorithm. We will use a qualitative model of the form described in Section 5. Then the observation algorithm will be given in Section 6 and illustrated by an example in Section 7.

5 Nondeterministic automata as qualitative models

The modelling problem

This section surveys the results described in [9] and extends the model proposed there. For details, the reader is referred to this reference and to [10], [7]. This is an alternative approach compared with the methods in qualitative reasoning described, for example, in [2], [5].

For every given qualitative initial state $[x_0] = z(0)$ and input sequence $[U] = V$, the system (1)–(3) generates a set of qualitative states and qualitative outputs which can be described as follows. First, the set $\mathcal{X}(k|V, z(0))$ of (quantitative) states that the system can assume at time k is given by

$$\mathcal{X}(0|V, z(0)) = \mathcal{Q}_x(z(0)) \qquad (16)$$

$$\mathcal{X}(k+1|V, z(0)) = \left\{ \tilde{x} \,\middle|\, \begin{array}{l} \tilde{x} = Ax + Bu \\ u \in \mathcal{Q}_u(v(k)) \\ x \in \mathcal{X}(k|V, z(0)) \end{array} \right\} \qquad (17)$$

By using these sets, the sets of qualitative states and

outputs are given by

$$[\mathcal{X}(k|V, z(0))] = \{[\boldsymbol{x}] | \boldsymbol{x} \in \mathcal{X}(k|V, z(0))\} \quad (18)$$

$$[\mathcal{Y}(k|V, z(0))] = \left\{[\boldsymbol{y}] \left| \begin{array}{l} \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u} \\ \boldsymbol{x} \in \mathcal{X}(k|V, z(0)) \\ \boldsymbol{u} \in \mathcal{Q}_u(v(k)) \end{array} \right. \right\} (19)$$

(cf. eqns. (1)–(3)).

The aim of qualitative modelling is to find a concise representation of the relation among the qualitative values of input v, state z and output w. Due to the fact that $[\mathcal{X}(k|V, z(0))]$ is, in general, a set with more than one element unless the matrix $\boldsymbol{A}$ in eqn. (1) satisfies the restrictive condition given in [9], a nondeterministic automaton N is used as qualitative model of the system (1)–(3) with quantised signals.

Nondeterministic automata

The nondeterministic automaton

$$N(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L(z', w|z, v), z_0) \quad (20)$$

has at each time k some output $w(k) \in \mathcal{N}_y$, input $v(k) \in \mathcal{N}_v$ and state $z(k) \in \mathcal{N}_z$. The *behaviour relation*

$$L(z', w|z, v) : \mathcal{N}_z \times \mathcal{N}_v \times \mathcal{N}_z \times \mathcal{N}_w \longrightarrow \{0, 1\} \quad (21)$$

provides the information, whether a transition from state z to state z' while getting the input v and giving the output w is allowed ($L(z', w|z, v) = 1$) or not allowed ($L(z', w|z, v) = 0$). The behaviour relation $L(z', w|z, v)$ can be expressed for any combination of inputs $v \in \mathcal{N}_v$ and outputs $w \in \mathcal{N}_w$ as a Boolean transition matrix $\boldsymbol{L}(w|v) \in \{0, 1\}^{N \times N}$, where N is the number of automaton states. For each element of these matrices

$$\left(\boldsymbol{L}(w|v)\right)_{z'z} = L(z', w|z, v) \quad (22)$$

holds.

This notation is illustrated for the example where $\mathcal{N}_z = \{1, 2, 3\}$, $\mathcal{N}_v = \{1, 2\}$ and $\mathcal{N}_w = \{1, 2\}$ holds. The automaton is described by 4 matrices for any combination of the inputs $v \in \mathcal{N}_v$ with the outputs $w \in \mathcal{N}_w$:

$$\boldsymbol{L}(1|1) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix},$$

$$\boldsymbol{L}(1|2) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix},$$

$$\boldsymbol{L}(2|1) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad (23)$$

$$\boldsymbol{L}(2|2) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}.$$

Each of the columns of the transition matrix $\boldsymbol{L}(w|v)$ gives a set of successor states. The state $z(k) = 2$ in the example (23) has successor states $z(k + 1) = 2$ and $z(k + 1) = 3$ for the input $v(k) = 2$ while having the output $w(k) = 1$ (second column of the matrix $\boldsymbol{L}(1|2)$).

In Figure 5, an automaton graph is drawn for the nondeterministic automaton of example (23). The nodes represent the states of the automaton. All directed edges show state transitions where the corresponding inputs and outputs are given by the drawing style of the edge. All movements with $v = 2$ have dashed edges to distingush them from movements with $v = 1$ (normal lines). Also, any edge generating the output $w = 2$ is black, while the edges with output $w = 1$ are grey.
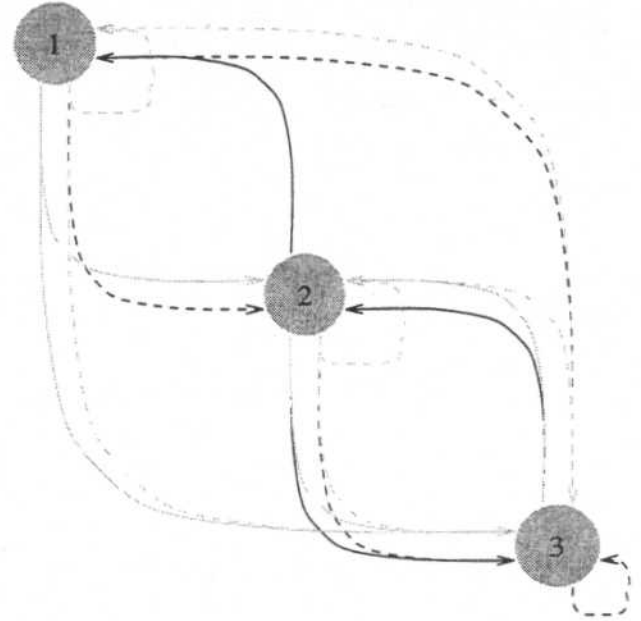

Figure 5: Automaton graph

Qualitative modelling

In order to use the automaton as qualitative model, one now identifies each automaton state with a qualitative state $z(k) = [\boldsymbol{x}(k)]$, each automaton input with a qualitative input $v(k) = [\boldsymbol{u}(k)]$ and each automaton output with a qualitative output $w(k) = [\boldsymbol{y}(k)]$.

To make the automaton (20) a qualitative model of the system (1)–(3), it is necessary that the automaton generate **all** qualitative trajectories $[\boldsymbol{X}]$ and output sequences $[\boldsymbol{Y}]$ that the system (1), (2) can generate. This can be ensured if the behaviour relation L generates the set of all qualitative successor states that the system can take from arbitrary states $\boldsymbol{x}$ belonging to one qualitative state $z = [\boldsymbol{x}]$ while getting an arbitrary input $\boldsymbol{u}$ belonging to one qualitative input $v = [\boldsymbol{u}]$ and having an output $\boldsymbol{y}$ that is given by (2) with qualitative value $w = [\boldsymbol{y}]$.

In extension of the results of [9] it can be proved that the automaton has to satisfy the relation

$$L(z', w|z, v) \geq L^*(z', w|z, v) \ \forall \ w, v, z, z' \qquad (24)$$

with

$$L^*(z', w|z, v) \ = \ \begin{cases} 1 & \text{if} & \begin{array}{l} \exists \ \boldsymbol{x}, \boldsymbol{u} \text{ with} \\ [\boldsymbol{x}] = z \, , [\boldsymbol{u}] = v \\ [\boldsymbol{Ax} + \boldsymbol{Bu}] = z' \\ [\boldsymbol{Cx} + \boldsymbol{Du}] = w \end{array} \\ 0 & \text{else} \end{cases} \quad (25)$$

Eqn. (25) can be easily extended to a set S of systems. A nondeterministic automaton is a qualitative model of a set S of systems of the form (1)–(3), if the behaviour relation L satisfies eqns. (25),(24) for all matrices $\boldsymbol{A} \in \mathcal{A}$, $\boldsymbol{B} \in \mathcal{B}$, $\boldsymbol{C} \in \mathcal{C}$, $\boldsymbol{D} \in \mathcal{D}$, where the sets $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ define the set S. That is, the qualitative model simultaneously describes the qualitative behaviour of all systems (1)–(3) of the set S. For making the automaton a qualitative model of a nonlinear system, the linear model used in eqn. (25) only has to be substituted by the nonlinear model.

Qualitative simulation

Since qualitative models are, in general, used for qualitative simulation, we mention in passing that our model can also be used for predicting the future qualitative behaviour of the system under consideration. In qualitative simulation, the qualitative initial state $z(0)$ and a sequence of qualitative inputs

$$V \ = \ (v(0), v(1), \ldots, v(T)) \qquad (26)$$

are known and one is interested in the set $\mathcal{W}$ of all possible output sequences

$$W \ = \ (w(0), w(1), \ldots, w(T)) \qquad (27)$$

that the system (1)—(3) may produce. The simulation task can be solved by means of the model (20),(24) because it can be proven that the set $\mathcal{Z}(V, z(0))$ of trajectories generated by the model includes the set $[\mathcal{Z}(V, z(0))]$ of qualitative trajectories of the system (1)—(3):

$$\mathcal{Z}(V, z(0)) \ \supseteq \ [\mathcal{X}(V, z(0))] \qquad (28)$$

(for details cf.[10]).

For better handling, a state transition relation and an output relation

$$F(z'|z, v) \ : \ \mathcal{N}_z \times \mathcal{N}_z \times \mathcal{N}_v \longrightarrow \{0, 1\} \quad (29)$$
$$G(w|z, v) \ : \ \mathcal{N}_w \times \mathcal{N}_z \times \mathcal{N}_v \longrightarrow \{0, 1\} \quad (30)$$

are introduced, which can be derived from the behaviour relation $L(z', w|z, v)$ with the following equivalences:

$$F(z'|z, v) = 1 \iff \exists \ w \ \text{with} \ L(z', w|z, v) = 1 \ (31)$$
$$G(w|z, v) = 1 \iff \exists \ z' \ \text{with} \ L(z', w|z, v) = 1 \ (32)$$

If an actual state $z(k)$ and the actual input $v(k)$ is known, the transition relation F gives a set of successor states $z(k+1)$ while the output relation G gives a set of outputs $w(k)$.

As an example, the automaton (23) is started with initial state $z(0) = 2$ and the input $v(0) = 2$. The set of successor states $z(1) \in \{2, 3\}$ as well as the set of outputs $w(1) \in \{1, 2\}$ can be obtained from the behaviour relation L.

The observation problem, which will be solved now, is a completely different problem, because the initial state $z(0) = [\boldsymbol{x}(0)]$ is unknown but a sequence $W = [\boldsymbol{Y}]$ of outputs is given.

6 A qualitative observation algorithm

Now we are able to solve the qualitative observation problem given in Section 4 by means of the qualitative model $N(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L(z', w|z, v), z_0)$. The problem is to find the set of all qualitative trajectories that are consistent with the model (1)–(3), the sequence of qualitative inputs V and the sequence W of measured qualitative outputs. This set is defined by

$$[\mathcal{X}(V, W)] = \qquad (33)$$
$$\left\{ ([\boldsymbol{x}(0)], [\boldsymbol{x}(1)], \ldots) \middle| \begin{array}{l} \exists \ \boldsymbol{x}(k), \boldsymbol{y}(k), \boldsymbol{u}(k) \\ \text{satisfying (1) and (2)} \\ [\boldsymbol{y}(k)] = w(k), \\ [\boldsymbol{u}(k)] = v(k), \\ [\boldsymbol{x}(0)] = z_0 \\ \forall \ k \in \{0, 1, 2, \ldots, T\} \end{array} \right\} .$$

The aim now is to find the set $[\mathcal{X}(V, W)]$ with the help of the qualitative model N.

Because of the nondeterministic behaviour of the qualitative model, there is a **set** of states that the automaton can assume at each time k. To represent this set in a convenient way, a binary state vector $z \in \{0, 1\}^N$ is introduced where

$$z_i(k) \ = \ \begin{cases} 1 & \iff \text{state } i \text{ at time } k \text{ possible} \\ 0 & \iff \text{state } i \text{ at time } k \text{ impossible} \end{cases} (34)$$

holds. Then, a set of possible qualitative initial states can be represented by a vector z_0.

We are now in a position to describe the observation algorithm by means of which the qualitative state $z(k)$ can be recursively determined. For a given set $z(k)$ of qualitative states at time k, known input $v(k)$ and measured output $w(k)$, the set of successor states $z(k+1)$ can be determined by means of the relevant matrix $\boldsymbol{L}(w(k)|(v(k))$ defined in eqn. (22) as follows:

$$z(k+1) \ = \ \boldsymbol{L}(w(k)|v(k)) \circ z(k) , \qquad (35)$$
$$z(0) \ = \ z_0 = (1, 1, \ldots, 1)' . \qquad (36)$$

The Boolean matrix multiplication operator $\circ$ is defined as usual: Addition is replaced by logical or and

multiplication is replaced by logical and:

$$z(k+1) \;=\; \begin{pmatrix} \bigvee_j \, (L(w(k)|v(k)))_{1j} \wedge z_j(k) \\ \vdots \\ \bigvee_j \, (L(w(k)|v(k)))_{Nj} \wedge z_j(k) \end{pmatrix} \quad (37)$$

If N is a qualitative model of the system (i.e. (24) holds), then eqn. (35) gives a set of qualitative trajectories $\mathcal{Z}(V, W)$ which is a superset of the desired one $[\mathcal{X}(V, W)]$:

$$\mathcal{Z}(V, W) \;\supseteq\; [\mathcal{X}(V, W)] . \quad (38)$$

Thus, eqn. (35) can recursively be applied to construct a superset of all qualitative trajectories the system can go through.

In summary, the following algorithm describes the solution to the qualitative observation problem:

Qualitative observation algorithm:

Given:

Qualitative input sequence $V = [U]$,
Qualitative output sequence $W = [Y]$,
Qualitative model $N(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L, z_0)$.

Start: $k = 0$
$z(0) = z_0 = (1, \ldots, 1)'$

Iterate: $(k = 0, 1, \ldots, T)$
$z(k+1) = L(w(k)|v(k)) \circ z(k)$

Result:

Set of qualitative trajectories $\mathcal{Z}(V, W)$ represented by $(z(0), z(1), \ldots, z(T+1))$

Example

For the simple example given in Section 5 we can see that the knowledge of the outputs really effects the set of successor states. Assume that the automaton (23) is a qualitative model of a given system (1)–(3). With the given sequences of qualitative inputs and outputs

$$\begin{aligned} V &= (1, 2, 1) \\ W &= (1, 2, 2) \end{aligned}$$

the observation algorithm yields the following sequence of sets of possible states

$$\mathcal{Z}(V, W) = (\{1, 2, 3\}, \{2, 3\}, \{3\}, \{2\}).$$

The observation algorithm starts by assuming that the system (1)–(3) has an arbitrary qualitative initial state: $z(0) \in \{1, 2, 3\}$. After two time steps the qualitative state of the system is uniquely determined $(z(2) = 3, z(3) = 2)$ by the observation algorithm. $\square$

Extension of the observation algorithm

Due to the existence of spurious solutions the set of reconstructed qualitative states is a superset of $[\mathcal{X}(V, W)]$ given in eqn. (33). An estimate of the probability that a reconstructed state can be really a qualitative state of the system (1)–(3) can be determined if a stochastic automaton S is used as qualitative model of (1)–(3) rather than the nondeterministic automaton N.

The automaton graph of the stochastic automaton

$$S(\mathcal{N}_z, \mathcal{N}_v, \mathcal{N}_w, L_s(z', w|z, v), p_0) , \quad (39)$$

has the same structure (edges) as that of N but additional weights for each edge [13]. The weights are given by the conditional probability

$$L_s(z', w|z, v) =$$
$$\mathrm{Prob} \left(\begin{array}{c} [x(k+1)] = z' \\ [y(k)] = w \end{array} \middle| \begin{array}{c} [x(k)] = z \\ [u(k)] = v \end{array} \right) \quad (40)$$

The matrix representation of the behaviour relation only changes slightly: the matrices are no longer Boolean, but their elements take values from the unit interval: $L_s(w|v) \in [0, 1]^{N \times N}$.

With the stochastic automaton, a discrete probability distribution $p(k)$ over the regions of the qualitative states is given at each instant k. Thus, the initial condition p_0 of the stochastic automaton is the probability distribution of x_0. If no information about the qualitative initial state is available, the automaton may take a uniform distribution over all qualitative states as initial state p_0. The Boolean multiplication in the observation algorithm (35) can be replaced by the usual matrix-vector multiplication:

$$\begin{aligned} p(k+1) &= L_s(w(k)|v(k)) \; p(k) , & (41) \\ p(0) &= p_0 = (1/N, \ldots, 1/N)' . & (42) \end{aligned}$$

With this extension of the qualitative model the observation algorithm remains, in principle, the same as given above. However, as additional information with every qualitative state a measure of the probability is determined with which the system (1)–(3) will really assume the qualitative state.

7 Process supervision of a tank system

Figure 6 shows an interconnected tank system to which the qualitative observation algorithm is applied [11]. The input u to the system is the inflow into Tank 1. The only output y is the outflow of Tank 3. The states x_1, x_2 and x_3 of the system are the levels of the liquid in the tanks. We want to focus our attention to the qualitative observation of the level x_2 of Tank 2.

A quantitative model (1)–(3) of the tank system has been derived from a continuous–time model with the
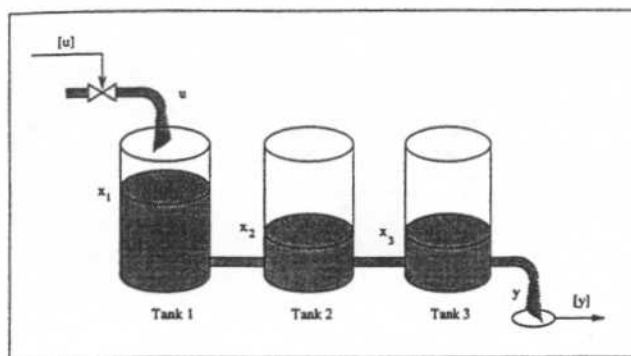
Figure 6: Experimental setup of the connected tank system

sampling time of 0.5 seconds. It has the parameter matrices:

$$A = \begin{pmatrix} 0.6737 & 0.2582 & 0.0608 \\ 0.2582 & 0.4763 & 0.2217 \\ 0.0608 & 0.2217 & 0.5042 \end{pmatrix} \quad (43)$$

$$B = \begin{pmatrix} 0.0813 \\ 0.0161 \\ 0.0024 \end{pmatrix} \quad (44)$$

$$C = (\; 0.000 \quad 0.000 \quad 1.6666 \;) \quad (45)$$

$$D = 0.000 \, . \quad (46)$$

The inputs can only assume one of two discrete values:

$[u]$	$=$	off	u	$=$	0
$[u]$	$=$	on	u	$=$	1

The qualitative measurement of the outflow can give one of 5 qualitative values from "tiny" to "huge", which represent a partition of the output space

$[y]$	$=$	tiny	y	$\in$	$[\,0\,,0.2\,]$
$[y]$	$=$	small	y	$\in$	$(\,0.2\,,0.4\,]$
$[y]$	$=$	normal	y	$\in$	$(\,0.4\,,0.6]$
$[y]$	$=$	large	y	$\in$	$(\,0.6\,,0.8\,]$
$[y]$	$=$	huge	y	$\in$	$(\,0.8\,,1.0\,]$

For the tank levels, quantisation into 3 qualitative values for each tank is done.

$[x_i]$	$=$	low	x_i	$\in$	$[\,0\,,0.2\,]$
$[x_i]$	$=$	med	x_i	$\in$	$(\,0.2\,,0.4\,]$
$[x_i]$	$=$	high	x_i	$\in$	$(\,0.4\,,0.6\,]$

Hence, the qualitative model has $3^3 = 27$ states. For the state space model (1)–(3), (43), (44), (45), (46) and the given partitions a stochastic automaton was generated with a modified cell mapping algorithm [4]. As the automaton has 372 edges, the behaviour relation is not given here.

The qualitative observation problem is to determine the qualitative level of Tank 2 from qualitative measurements of the inflow and the outflow. The number of possible combinations of qualitative inputs and

outputs is $2 * 5 = 10$. At each time step, one of these combinations is measured and the qualitative observer (41) is applied. An example of sequences $[U], [Y]$ is shown in Figure 7.
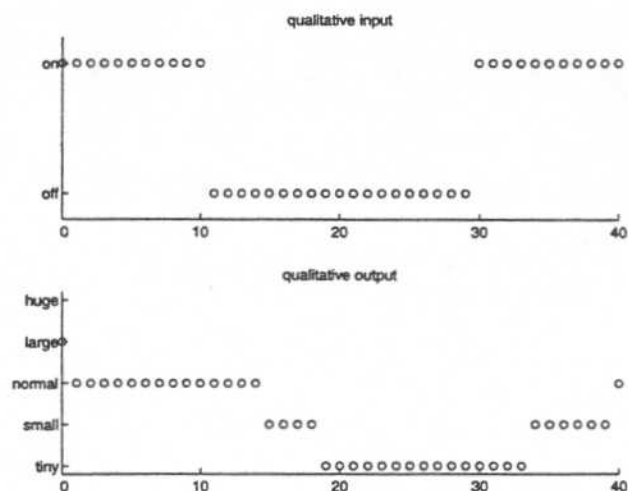

Figure 7: Qualitative input and output measurements

Note that there is a set $\mathcal{X}(V, W)$ of qualitative trajectories that will lead to the measured sequences $[U]$ and $[Y]$ which is represented in the lower part of Figure 8. For example, at time $k = 13, 18$ or 33 two different qualitative states are really possible. The qualitative observer yields the result shown in the upper part of Figure 8. Here the probabilities generated by the qualitative observer are drawn in grayscale: the darker the bar, the more likely is the qualitative state.
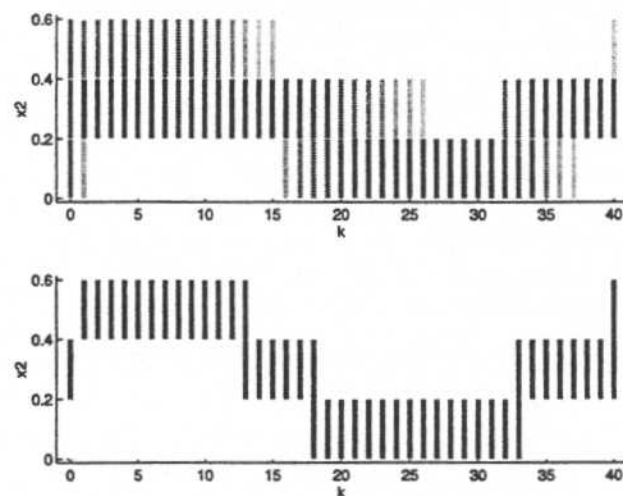

Figure 8: Reconstructed and real qualitative state trajectories for Tank 2

We can see that this estimation for the probability of each qualitative state given by the qualitative observer is useful for an operator. In our example, during the time steps $k = 3$–20, Tank 2 is surely not empty and it is unlikely that its level increases to an overflow.

During the time steps $k = 21$–30, the probability for an empty tank is growing, which causes the operator to open the valve at time $k = 30$ (cf. top of Figure 7). The result of his action is visible in an increasing probability for the "med"-level of Tank 2, which is qualitatively the correct dynamic behaviour of the system. Due to the simplicity of the observation algorithm, the observation problem can be solved in real time.

This numerical example shows that spurious solutions do not influence the solution of the observation problem as seriously as qualitative simulation. The reason for this is given by the fact that the additional qualitative output information is exploited.

Conclusions

The paper shows that qualitative models of dynamical systems can be used to reconstruct information about the internal states of the system from qualitative input/output sequences. The proof of the relation (38), which represents the basis of the observation algorithm, is given in [7].

The formulation of the approach here was restricted to linear systems and to precise qualitative measurements without noise. However, stochastic automata as qualitative models can also be used for nonlinear systems and noisy data. Nonlinearities only influence the behaviour relation of the automaton while noisy measurements can be handled by generating discrete probability distributions for the qualitative inputs and outputs.

For technical applicability, methods for the generation of qualitative models have to be found, that do not presuppose the knowledge of a precise quantitative model (1)–(3) or a set S of such models. An approach to this *qualitative identification* problem, which only refers to qualitative measurement data, has been proposed in [6].

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