Qualitative and Quantitative Mechanical Assembly Design

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Abstract

We describe a unified approach to computer-aided mechanical assembly design in which all design tasks are performed within a single computational paradigm supported by integrated design software. We have developed a prototype design environment for planar assemblies, called HIPAIR, that supports diverse design tasks. We organize the design tasks around the fundamental task of contact analysis, which we automate by configuration space computation. Configuration space is a complete, concise representation of rigid body interactions that contains the requisite qualitative and quantitative contact information for all design tasks. We describe practical algorithms for the key tasks of dynamical simulation and kinematic tolerancing. HIPAIR allows designers to perform computations that lie outside the scope of previous software and that defy manual analysis. It computes qualitative and quantitative functional changes, allowing designers to study assembly function under a range of operating conditions, to find and correct design flaws, and to evaluate the functional effects of part tolerances. HIPAIR has been tested on hundreds of pairs and on a dozen assemblies. It performs at interactive speed on assemblies of ten parts with tens of thousands of contacts.

Introduction

We describe a unified approach to computer-aided mechanical assembly design in which all design tasks are performed within a single computational paradigm supported by integrated design software. Mechanical assembly design is the task of devising an assembly of parts that performs a function reliably and economically. It is a ubiquitous activity that spans mechanical, electrical, and biomedical engineering. Designers need to devise, analyze, and compare competing design prototypes to produce optimal designs. Computer-aided design reduces design time and improves quality by allowing designers to substitute electronic prototypes for physical prototypes in diverse tasks.

Reasoning about part contacts plays a central role in mechanical assembly design. Contacts are the physical primitives that make assemblies out of collections of parts. Assemblies perform functions by transforming motions via part contacts. The shapes of the interacting parts impose constraints on their motions. For example, a door rotates about its hinges and meshed gears rotate in unison. Contact constraints largely determine the function of assemblies. Designers compute contact constraints to validate function and to measure performance. They correct design flaws by modifying part contacts. They choose part tolerances based on the variation in assembly function that they produce.

Contact analysis, also called kinematic analysis, determines the qualitative and quantitative relation between the function of an assembly and the shapes and motions of its parts. The primary qualitative information is the interacting parts, the touching features, and the configurations where these change. Design tasks require specialized qualitative information, such as jamming and failure modes for validation, impacts and sticking for simulation, and dependencies between part parameters and function variation for tolerancing. The primary quantitative information is the legal assembly configurations and the compliant motions in contact configurations. The specialized information quantifies the qualitative information with jamming configurations, impact forces, and function variations.

Contact analysis is difficult and time-consuming even for experienced designers due to the quantity and complexity of the contact constraints. Designers need to ensure that the intended contacts occur, to derive their constraints, and to guarantee that unintended contacts cannot occur. The difficulty is greatest in assemblies with multiple contacts, meaning that different parts or part features interact at various stages

*WWW site: http://www.cs.huji.ac.il/~josko/cad.html

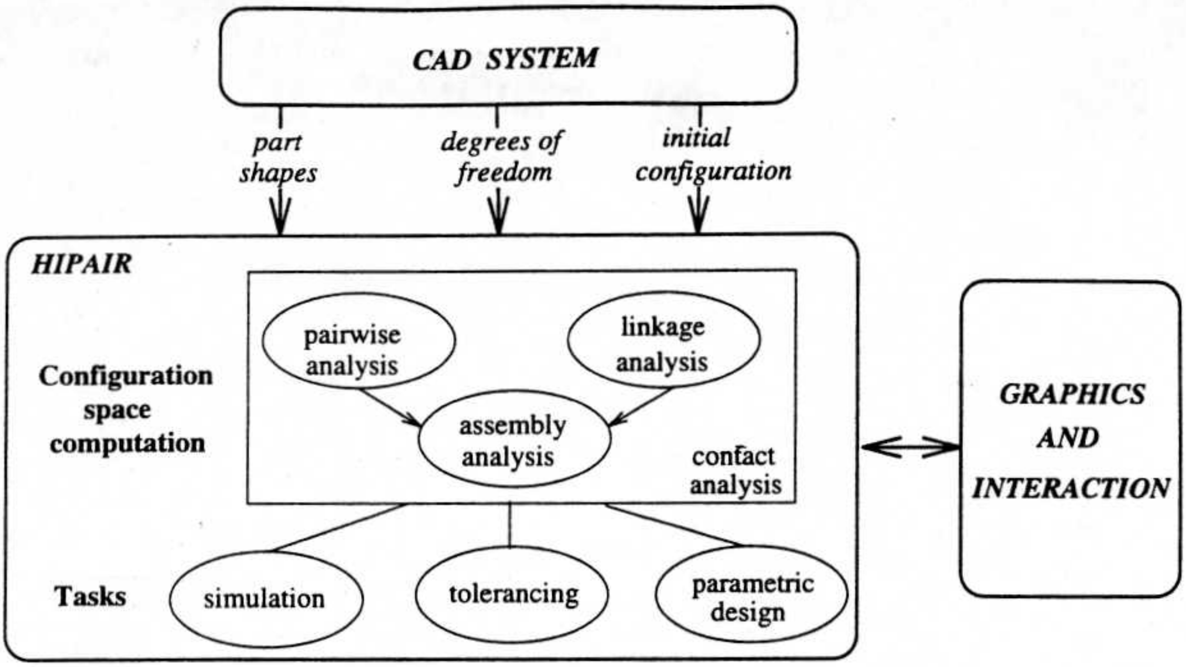


Figure 1: HIPAIR mechanical design environment.

of the work cycle. Manual analysis of these assemblies is error-prone and time-consuming at best and is often infeasible. Multiple contacts pervade modern mechanisms and account for 65% of the 2500 mechanisms in Artobolevsky's encyclopedia (Joskowicz and Sacks, 1991). The most common examples are gears, cams, clutches, and ratchets. Designers analyze multiple contacts when testing for part interference, jamming, cam under-cutting, and gear backlash. In kinematic tolerance analysis, designers study part variations that introduce multiple contacts into assemblies whose nominal function has permanent contacts, such as joint play in linkages.

Previous research in mechanical engineering, graphics, robotics, and artificial intelligence does not provide general algorithms for contact analysis. Mechanical simulation research (Haug, 1989; Schiehlen, 1990) focuses on efficient methods of solving the contact constraints of permanent contact assemblies, such as linkages and manipulators. Research in gear design (Litvin, 1994) and in cam design (Gonzales-Palacios and Angeles, 1993) addresses narrow classes of contacts. Assembly planning research (de Mello and Lee, 1991) focuses on the combinatorics of sequencing assembly steps for simple part shapes and motions. Graphics research in physically based modeling (Baraff, 1992; Cremer and Stewart, 1989; Mirtich and Canny, 1995) provides fast collision detection algorithms for polyhedral objects, but does not address the other aspects of contact analysis in mechanical design. Robotics research (Latombe, 1991) studies contact analysis in the context of robot motion planning. The planners use a configuration space representation for the possible contacts between the robot and the obstacles, and search this space for collision-free paths. Most research addresses a single polyhedral robot moving amidst fixed polyhedral obstacles. It does not provide practical algorithms for curved shapes or for multiple moving parts, which are the norm in mechanical assemblies. Qualitative reasoning research studies contact analysis for the task of producing symbolic, qualitative explanations of the workings of mechanisms. Faltings (Faltings, 1990; Faltings, 1992), Forbus et al. (Forbus *et al.*, 1991), and Joskowicz and Sacks (Joskowicz and Sacks, 1991) use the configuration space representation to derive qualitative descriptions of assembly contacts and kinematics. The programs handle assemblies of planar parts with one degree of freedom and multiple contacts. We build on this work, extending it to general planar assemblies and to tolerancing and dynamical simulation.

We have developed a unified approach to contact analysis and to computer-aided assembly design based on configuration space computation. In previous work, we have shown that configuration space is a complete, concise representation of rigid body interaction. We have demonstrated that performing contact analysis on an assembly is equivalent to computing its configuration space. The configuration space contains the requisite qualitative and quantitative information for design tasks involving contacts. It models permanent and multiple contacts uniformly.

In this paper, we survey our recent work on HIPAIR with an emphasis on the qualitative reasoning aspects. Figure 1 summarizes our approach. The core module automates contact analysis via configuration space computation. The task modules use the configuration spaces to support reasoning about contacts. We have implemented a prototype design environment for planar assemblies, called HIPAIR, based on this model. It automates dynamical simulation and provides novel support for validation, tolerancing, and parametric design. HIPAIR allows designers to perform computations that lie outside the scope of previous software and that defy manual analysis. It allows them to visualize assembly function under a range of operating conditions, to find and correct design flaws, and to evaluate the functional effects of part tolerances. We have tested HIPAIR on hundreds of pairs and on a dozen assemblies with up to ten moving parts. We can analyze assemblies with thousands of contacts in a few seconds on a workstation. The running times in the paper are for a Silicon Graphics Indigo 2 workstation with 64MB of main memory and a 250 Mhz processor. All the figures in the paper are direct HIPAIR output.

Configuration space

The configuration space of an assembly of parts is a parameter space whose points (tuples of parameter values) specify the spatial configurations (positions and orientations) of the parts. The parameters represent translations and rotations of parts with respect to a fixed global coordinate system. For example, a gear pair has a two-dimensional configuration space in which each gear configuration is specified by a rotation parameter. The configuration space dimension equals the total number of degrees of freedom of the parts in the assembly.

Configuration space partitions into free space where the parts do not touch and into blocked space where some parts overlap. The common boundary, called contact space, contains the configurations where some parts touch without overlap and the rest do not touch. Only free space and contact space are physically realizable. Free space represents the realizable motions of the parts and contact space represents the couplings between their motions induced by contacts.

We illustrate these concepts on a Geneva pair (Figure 2). The driver consists of a driving pin and a locking arc segment mounted on a cylindrical base (not shown). The wheel consists of four locking arc segments and four slots. The driver rotates around axis O_d and the wheel rotates around axis O_w. Each rotation of the driver causes a nonuniform, intermittent rotation of the wheel with four drive periods where the driver pin engages the wheel slots and with four dwell periods where the driver locking segment engages the wheel locking segments.

The configuration space of the Geneva pair is two-dimensional with coordinates the orientations θ and ω of the driver and the wheel. The shaded region is the

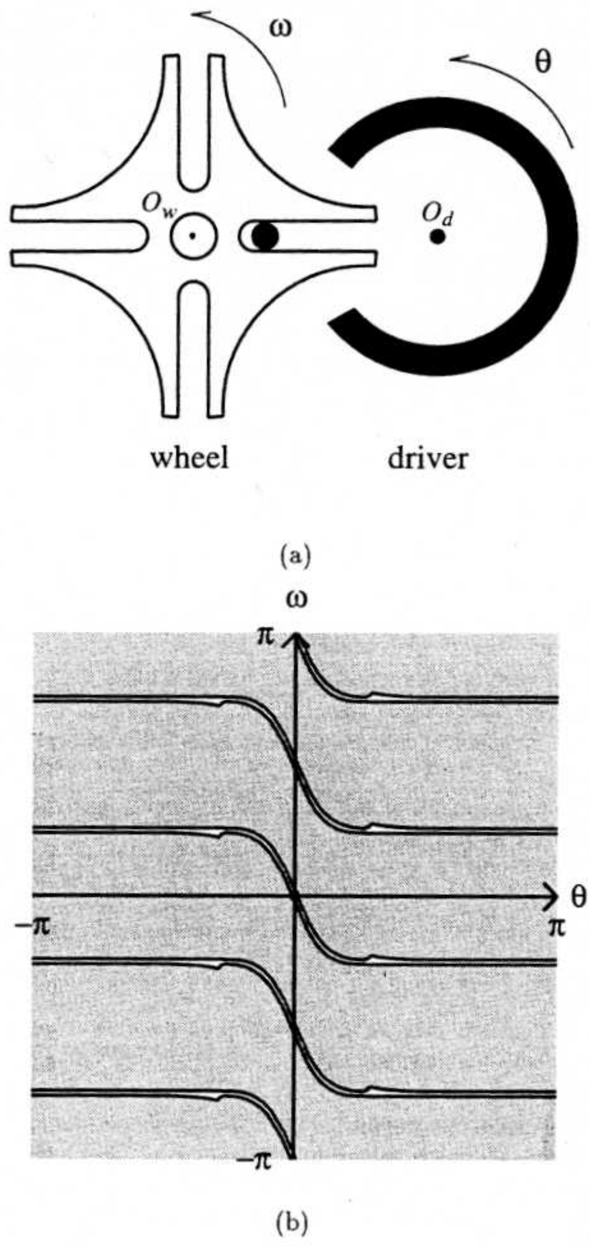


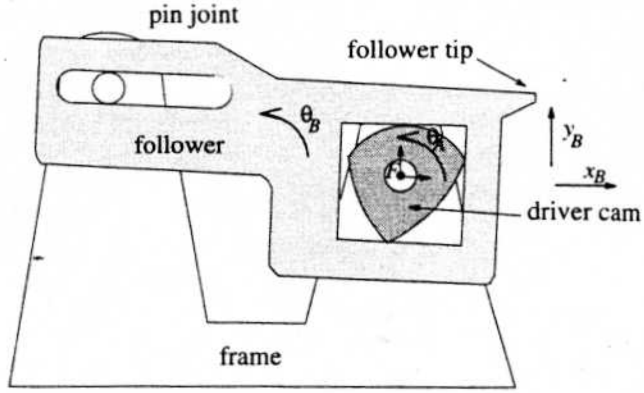
Figure 2: Geneva pair and its configuration space. The pair is displayed in configuration $\theta = 0$, $\omega = 0$, marked by the dot at the configuration space origin.

blocked space where the driver and the wheel overlap. The white region is the free space. It forms a single channel that wraps around the horizontal and vertical boundaries, since the configurations at $\pm\pi$ coincide. The width of the channel measures the potential backlash of the pair. The curves that bound the free and blocked regions, called contact curves, form the contact space. The functional forms of the contact curves encode the contact relations between the wheel and the driver. The horizontal segments represent contacts between the locking arc segments, which hold the wheel stationary. The diagonal segments represent contacts between the pin and the slots, which rotate the wheel. The ranges of the contact curves express the contact conditions; contact changes occur at curve endpoints.

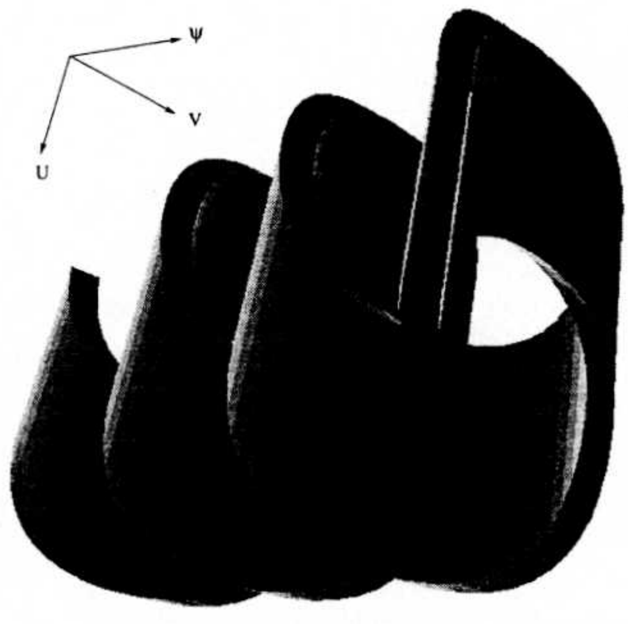
Configuration space encodes in a uniform geometric framework the quantitative and qualitative information for reasoning about part contact in all mechanical assemblies. It represents all the motion constraints induced by part contacts and the configurations where contacts change. It specifies the space of kinematic functions under all external forces, and thus constitutes global representation for contact analysis. The functions under specific forces are paths in configuration space that consist of contact and free segments separated by contact change configurations. For example, clockwise rotation of the driver produces a path that follows the contact curves on the bottom of the free space from right to left. The kinematic function consists of horizontal segments alternating with diagonal segments. The pin makes contact with the slot at the start of the diagonal segments and breaks contact at the end.

The configuration space of an assembly is compositional: it is determined by the configuration spaces of its pairs of parts (Joskowicz and Sacks, 1991). Although general configuration space computation is intractable in the worst case, it is manageable in practice because mechanical assemblies have characteristics that distinguish them from arbitrary collections of parts.

We have developed fast, robust configuration space computation programs for planar pairs with two degrees of freedom (Sacks and Joskowicz, 1995) and for general planar pairs (Sacks and Bajaj, 1997)(Figure 3. The planar algorithm is 100 times faster than the general algorithm (seconds versus milliseconds). The savings is important because 90% of assemblies are planar based on our survey of 2500 mechanisms (Joskowicz and Sacks, 1991) and on an informal survey of modern mechanisms, such as VCR's and photocopiers.



(a)



(b)

Figure 3: Movie camera film advance. The driver cam rotates about a shaft on the frame, while the enclosing follower is attached to the frame by a pin joint. As the cam rotates clockwise, the follower tip engages the film (not shown), pushes it down one frame, and retracts. (a) driver cam at configuration is $(0, 0, \theta_A)$ and follower at configuration is (x_B, y_B, θ_B). (b) configuration space.

Kinematic tolerance analysis

The goal of tolerance analysis is to compute the variation in the function of mechanical assemblies resulting from manufacturing variation in the shapes

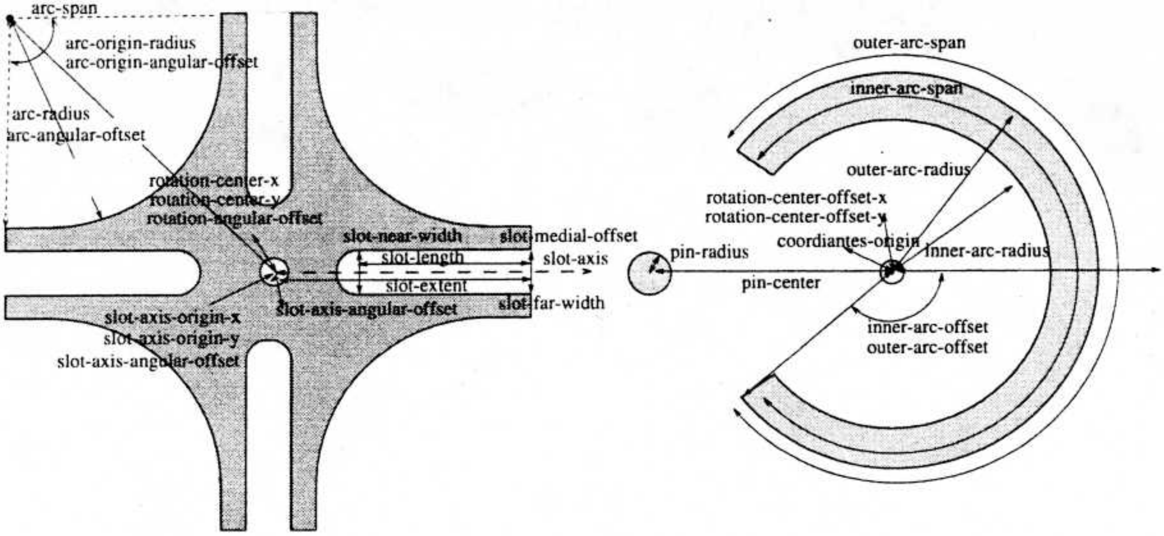


Figure 4: Parametric model of the Geneva pair.

and configurations of their parts. Kinematic tolerance analysis computes the qualitative and quantitative variations of the kinematic function determined by the series of contact constraints over the assembly work cycle. For example, a meshed pair of rotating gears undergoes a series of tooth contacts that impose a relation between the gear angular velocities. Ideal gears transmit rotation linearly, whereas real gears exhibit backlash and chatter because of axis misalignment and gear profile imperfections. Designers use kinematic tolerance analysis to guarantee correct assembly function and to reduce manufacturing cost. Worst-case analysis derives guaranteed upper and lower bounds on the variation, while statistical analysis derives probabilistic bounds. The analysis complements tolerancing for assembly, which verifies that the parts can be assembled despite shape variations.

Tolerance specifications define the allowable variation in the shape and configurations of the parts of an assembly. The most common are parametric and geometric specifications(Voelcker, 1993). Parametric specifications restrict the parameters of the assembly model to intervals of values. For example, a tolerance of $r = 1 \pm 0.1$ restricts the radius r of a disk to the interval $[0.9, 1.1]$. Geometric specifications restrict part features to zones around the nominal features, typically to fixed-width bands, called uniform profile tolerance zones, whose boundaries are the geometric inset and offset of the nominal features. For example, a uniform geometric profile tolerance of 0.1 on a disk of radius 1 constrains its surface to lie inside an annulus with outer radius 1.1 and inner radius 0.9. We discuss parametric tolerances because they are best suited to kinematic tolerance analysis. We analyze geometric tolerances by translating them into parametric tolerances or by a direct method (Joskowicz *et al.*, 1997).

Parametric kinematic tolerance analysis consists of contact analysis and sensitivity analysis steps. The contact analysis derives the functional relationship between the tolerance parameters and the assembly kinematic function. The sensitivity analysis determines the variation of the kinematic function over the allowable parameter values. Contact analysis has not been automated previously. It is difficult to perform manually because the contact constraints are numerous, are complicated, and vary during the work cycle. Multiple contacts occur in nominal designs with higher pairs, such as gears, cams, clutches, and ratchets. Part variations produce multiple contacts even in assemblies whose nominal designs involve only permanent contacts. Sensitivity analysis algorithms are well developed. The principal methods are linearization, statistics, and Monte Carlo simulation (Chase and Parkinson, 1991).

We have generalized the configuration space representation to model kinematic variation of toleranced parts and have developed a contact analysis algorithm for parametric planar assemblies with one degree of freedom per part (Joskowicz *et al.*, 1997; Sacks and Joskowicz, 1997b). We couple the contact analysis with sensitivity analysis to obtain a program that derives worst-case and statistical bounds on kinematic variation along with qualitative changes in kinematic function, such as jamming, under-cutting, and interference. The program is fast enough to be practical for complete functional models of complex assemblies and for parametric representations of geometric tolerances, such as offsets, which typically require many parameters. The extension to general planar assemblies is straightforward.

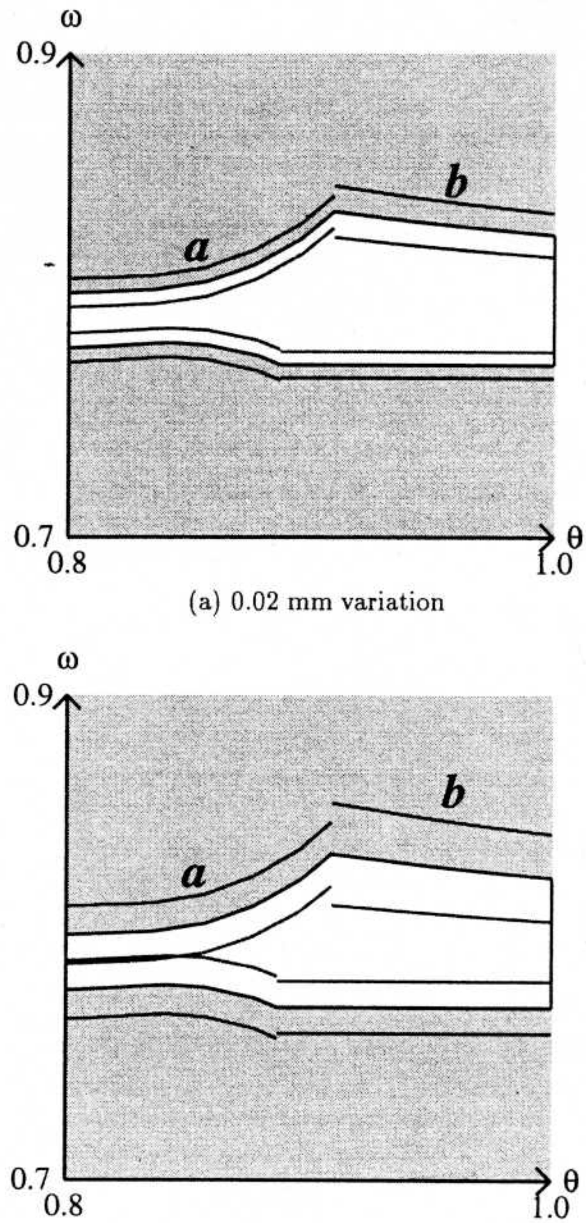
Worst-case analysis of pairs

We model kinematic variation by generalizing the configuration space representation to toleranced parts. The contact curves are parameterized by the tolerance parameters. As the parameters vary around their nominal values, the contact curves vary in a band around the nominal contact space, which we call the contact zone. Figure 4 shows a 26 parameter model of the Geneva pair and Figure 5 shows sample contact zones, each computed to 0.01% accuracy in 20 seconds. The contact zone defines the kinematic variation in each contact configuration: every pair that satisfies the part tolerances generates a contact space that lies in the contact zone.

Each contact curve generates a region in the contact zone that represents the kinematic variation in the corresponding part contact. The region boundaries encode the worst-case kinematic variation over the allowable parameter variations. They are smooth functions of the tolerance parameters and of the assembly configuration in each region. They are typically discontinuous at region boundaries because the contact curves depend on different parameters, as on the boundary between regions a and b in Figure 5. The variation at transition points is the maximum over the neighboring region endpoints. The contact zone also captures qualitative changes in kinematics, such as jamming, under-cutting, and interference. For example, the Geneva pair can jam when the contact zones of the upper and lower channels overlap, meaning that the channel closes for some allowable parts. The figure shows that this occurs when the variation equals 0.04 mm per parameter.

We compute the contact zone from the parametric model of the pair. The inputs are the part models, the nominal values and allowable ranges of the parameters, and an error bound. The outputs are closed-form expressions for the contact zone boundary. We first compute the nominal contact space with HIPAIR, obtaining a collection of contact curves of the form $y = f(x)$. We then derive parametric contact curves $y = f(x, \mathbf{p})$ by instantiating the contact table entries of the nominal curves with the symbolic tolerancing parameters $\mathbf{p}$ instead of with the nominal values. As $\mathbf{p}$ ranges over the allowable values, the parametric curves range over the contact zone. We compute closed-form expressions for the upper and lower boundaries of the contact zone by linearizing f around the nominal $\mathbf{p}$ values, making the standard tolerancing approximation that the kinematic variation is linear in the parameter variations.

Table 1 shows the results of the sensitivity analysis for the two regions shown in Figure 5. In region a where the driver pin touches the corner of the wheel slot, the two most important parameters are the wheel slot-axis-angular-offset and the driver pin-radius. The



(a) 0.02 mm variation

(b) 0.04 mm variation

Figure 5: Detail of the contact zone of the Geneva pair in the region where the driver locking segment disengages from the wheel locking segment and the driver pin engages the slot of the wheel. The center curves are the nominal contact space. The upper and lower curves bound the contact zone.

part	parameter	nom. val.	sensitivity %	
driver	pin-radius	4.5	**8**	0
	pin-center	56.5	7	0
	outer-arc-radius	46.0	0	12
	outer-arc-span	49.416	0	3
	outer-arc-offset	-2.4708	0	3
	inner-arc-radius	36.0	0	0
	inner-arc-span	4.9416	0	0
	inner-arc-offset	-2.4708	0	0
	rotation-center-off-x	80.0	7	11
	rotation-center-off-y	0.0	3	4
wheel	slot-axis-origin-x	0.0	7	0
	slot-axis-origin-y	0.0	3	0
	slot-axis-angular-off	0.0	**43**	0
	slot-extent	60.0	3	0
	slot-length	40.0	0	0
	slot-medial-offset	0.0	7	0
	slot-near-width	10.0	0	0
	slot-far-width	10.0	3	0
	arc-origin-radius	80.0	0	12
	arc-origin-angular-off	0.0	0	**28**
	arc-radius	46.683	0	**12**
	arc-angular-offset	0.0	0	0
	arc-span	1.5708	0	0
	rotation-center-x	0.0	7	11
	rotation-center-y	0.0	3	4
	rotation-angular-off	0.0	0	0

Table 1: Geneva pair nominal parameter values and relative sensitivities. Lengths are in millimeters, angles in radians.

former accounts for 40%–45% of the variation, while both account for 49%–52% of the variation. In region *b* where the driver locking arc touches the wheel locking arc, the two most important parameters are the wheel arc-origin-angular-offset and arc-radius. The former accounts for 25%–50% of the variation, while both account for 38%–59% of the variation. Statistical analysis shows that the average kinematic variation is much smaller than the worst-case bounds. We derive similar results for an 82-parameter model a camera shutter mechanism.

Assemblies and statistical tolerancing

The contact zone model of worst-case kinematic variation generalizes to assemblies. The assembly contact space is a semi-algebraic set in configuration space: a collection of points, curves, surfaces, and higher dimensional components. As the assembly tolerance parameters vary around their nominal values, the components vary in a contact zone around the nominal contact space. We compute the kinematic variation in a nominal operating mode, that is for specific external forces and initial conditions. This analysis is far easier than contact zone computation, yet suffices for assemblies with a moderate number of operating modes, which is the norm. We compute the nominal motion path by simulation or by measurement, split it into fixed contact segments, and perform sensitivity analysis on each segment. The computation is simple and fast because it involves a single curve instead of the entire nominal contact space.

We use the worst-case analysis to perform a statistical analysis. The inputs are the pairwise contact zones, the nominal motion path, and the joint distribution of the tolerance parameters. The outputs are the distributions of the kinematic variation in the contact zones and along the motion path. We compute the kinematic distributions by propagating the input distributions through the linearized contact functions.

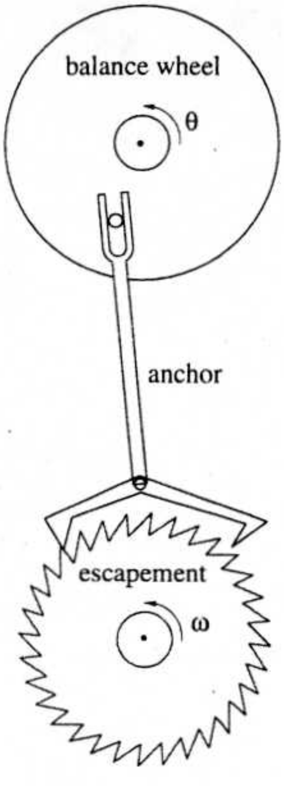
We have developed a comprehensive method of kinematic tolerance analysis based on our kinematic variation algorithms. The analysis is practical for complex models with many parts and parameters because the computation time is proportional to the product of the number of interacting pairs and the number of parameters per pair, both of which tend to be linear in the size of the model.

Dynamical simulation

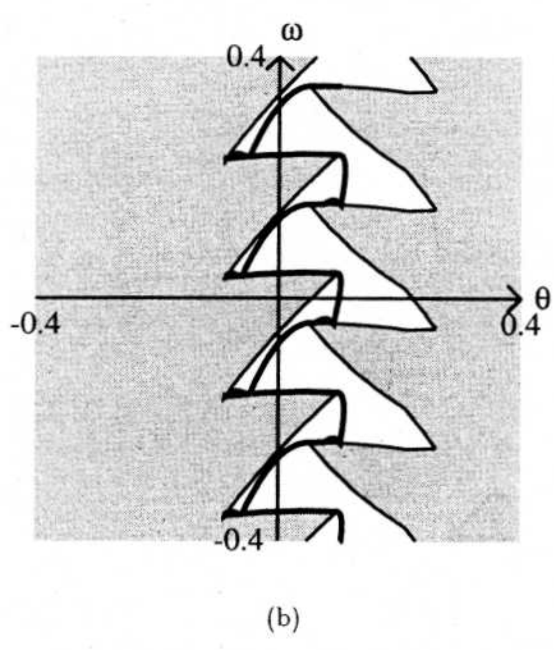
We have developed a dynamical simulator for planar assemblies based on configuration space computation (Sacks and Joskowicz, 1997a; Sacks and Joskowicz, 1996). Dynamical assembly simulation allows designers to visualize part motions, to validate assembly function, to optimize performance, and to compute loads for stress analysis. Simulation is an iterative process in which equations of motion for the parts of a system are integrated over time. Contact analysis plays a central role in simulation because it is necessary to determine the touching parts and the ensuing contact forces at each time step.

Mechanical systems simulators (Haug, 1989; Schiehlen, 1990) provide contact models for lower pairs and require the user to provide models for other contacts. This is appropriate for linkages and robot arms, but requires an excessive modeling effort for assemblies with multiple contacts and complex contact geometry, such as clock escapements, gear chains, and part feeders. General rigid-body simulators (Baraff, 1992; Cremer and Stewart, 1989; Mirtich and Canny, 1995) compute the dynamics of general polyhedral assemblies without user contact analysis by testing for part collisions at each time step, whose worst-case complexity is quadratic in the geometric complexity of the parts. The simulators speed up contact analysis with collision detection heuristics, such as spatial partitioning, which avoids comparisons between distant parts, and coherent computation, which predicts current contacts based on the past (Lin *et al.*, 1996).

These simulators have several potential drawbacks for mechanical assembly simulation. The collision detection heuristics are designed for loosely coupled systems where few part are close together at most times and where part velocities are small relative to inter-



(a)



(b)

Figure 6: (a) Escapement-type balance governor. (b) Anchor/escapement configuration space with correct motion path.

part distances, such as a moving object in a static world, pendulums, rolling balls, and rock slides. It is unclear how well the heuristics work in the mechanical domain where most parts interact, contact changes are common, and parts are driven fast. The algorithms approximate curved parts with polyhedra, which creates spurious discontinuities in the contact functions that distort the dynamics of high-speed systems and increases the running time when the parts interact often, as do many parts in mechanical systems.

Our simulator replaces collision detection and manual modeling with configuration space computation. The user inputs the part shapes, masses, moments of inertia, friction and restitution coefficients, external forces, and initial configurations. The simulator precomputes the configuration spaces of the interacting pairs in the assembly, producing a complete description of the pairwise contacts. At each simulation step, it computes the contact forces in the current state, combines them with the external forces, and predicts the next state by integrating the Newtonian equations of the parts. The configuration spaces provide the contact data (which parts touch and where) for contact force computation. The simulator tests for part collisions between steps, which create discontinuities in the contact forces and in the part velocities, by querying the configuration spaces for transitions from free to contact space. It terminates the step at the collision time, updates the state, and resumes simulation. The queries take linear time in the number of parts and are independent of their geometric complexity. The simulator never misses a contact transition, regardless of the integration step size, because it maintains an explicit model of the entire integration step in configuration space.

We have simulated planar assemblies such as the Geneva pair, a clock escapement governor (described next), a three-degree of freedom movie film advance pair, and a planar knee reconstructed from CT data. The computations are ten to twenty times faster than real-time at 0.1% accuracy.

We illustrate configuration space simulation on a simple but realistic scenario involving the design of an escapement-type balance governor of the type used in clocks and in other timing mechanisms (Figure 6a). It consists of a balance wheel, an anchor, and an escapement. Each part rotates around a fixed axis. The balance wheel is oscillated by a spiral spring with the pin vertical in the neutral position. The balance wheel pin engages the anchor fork and oscillates the anchor. The escapement rotates clockwise due to a constant torque imposed by a weight. The anchor pallets alternately engage and release the escapement, causing it to rotate by one tooth each anchor period. The impacts provide the energy that oscillates the balance wheel. This mechanism is extremely hard to analyze with mechanical systems simulators because the user needs to compute the contact sequences and the contact constraints for many part features.

The goal of the simulation is verify the qualitative function, the period, and the other dynamical properties. Suppose we want the escapement to turn two teeth per second. The moments of inertia are 1 Newton-centimeter^2 for the balance, 1 Newton-centimeter^2 for the anchor, and 5 Newton-centimeter^2 for the escapement. We pick a spring coefficient of π^2 to obtain a natural spring period of two cycles per second. We simulate the mechanism with a range of driving torques and find that it works with -20 Newton-centimeters, but fails with -30 Newton-centimeters. Figure 6b shows the anchor/escapement configuration space with the correct motion path. The failure path starts correctly, but fails to clear the bottom, left horizontal segment of the contact space when moving from left to right. Instead, it reverses direction and ends in the bottom, left corner.

Conclusion

We present a unified approach to computer-aided mechanical assembly design in which all design tasks are performed within a single computational paradigm supported by integrated design software. We organize design tasks around the fundamental task of contact analysis, which we automate with configuration space computation. Configuration space is a complete, concise representation of rigid body interactions that contains the requisite quantitative and qualitative information for design tasks involving contacts. We describe the HIPAIR program that supports planar assembly design. HIPAIR has been tested on hundreds of pairs and on a dozen assemblies with up to ten moving parts.

Configuration space provides a comprehensive understanding and a computational characterization of assembly contacts that systematizes diverse assembly design tasks. It uniformly models permanent and changing contacts. HIPAIR frees designers from contact analysis, which is often tedious, error-prone, or infeasible. In dynamical simulation, we replace collision detection with pairwise configuration space computation and querying, which is faster and more robust for mechanical assemblies. In kinematic tolerancing, we replace manual modeling and random parameter sampling with contact variation zones in which all parameter variations are considered. HIPAIR allows designers to perform computations that lie outside the scope of previous software and that defy manual analysis. They can visualize assembly function under a range of operating conditions, can find and correct design flaws, and can evaluate the functional effects of part tolerances.

The next step in our research is to extend HIPAIR to spatial assemblies. We need to define solid models of the parts, to derive contact constraints for spatial features, and to develop configuration space computation algorithms. The third step is impractical for general assemblies because of the high dimension of the configuration spaces. Even a single pair, which has six degrees of freedom, is probably impractical. Instead, we plan to develop specialized techniques by restricting the part geometry, motions, and interactions based on application constraints. The challenge is to obtain the data for specific tasks, especially the global data that only configuration space can provide, without computing entire configuration spaces.

We will address this challenge by dimension reduction and by selective computation. We can analyze parts with one degree of freedom, such as spatial gears and cams, by an extension of our planar algorithm. After deriving the spatial contact constraints, we can reuse the rest of the program. A similar approach applies to assemblies whose nominal motions are planar and whose off-plane motions do not cause contact changes. In general assemblies, we can construct individual configuration space regions that track the changing assembly configuration (Joskowicz and Taylor, 1996).

The next step in our mechanical assembly design research is to extend the task coverage and to improve the algorithms. Extensions to parametric tolerance analysis include geometric form tolerances, nonkinematic parameters, and objective functions. Extensions to parametric design include general planar pairs and assembly design. Both require better parameter space exploration strategies that are less dependent on the configuration space representation. New tasks include tolerance synthesis, configuration design, flexible parts, stress analysis, functional classification, and an online assembly database. Many of these tasks require better tools for configuration space visualization and interpretation.

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References

(Baraff, 1992) Baraff, David 1992. *Dynamic Simulation of Non-Penetrating Rigid Bodies.* Ph.D. Dissertation, Cornell University.

(Chase and Parkinson, 1991) Chase, K. W. and Parkinson, A. R. 1991. A survey of research in the application of tolerance analysis to the design of mechanical assemblies. *Research in Engineering Design* 3(1):23–37.

(Cremer and Stewart, 1989) Cremer, J. F. and Stewart, A. J. 1989. The architecture of *newton,* a general-purpose dynamics simulator. In *Proceedings of the 1989 IEEE International Conference on Robotics and Automation.* 1806–1811.

(de Mello and Lee, 1991) Mello, Luiz S. Homemde and Lee, Sukhan, editors 1991. *Computer-Aided Mechanical Assembly Planning.* Kluwer Academic Publishers.

(Faltings, 1990) Faltings, Boi 1990. Qualitative kinematics in mechanisms. *Artificial Intelligence* 44(1–2):89–120.

(Faltings, 1992) Faltings, Boi 1992. A symbolic approach to qulaitative kinematics. *Artificial Intelligence* 56(2-3).

(Forbus *et al.*, 1991) Forbus, K.; Nielsen, P.; and Faltings, B. 1991. Qualitative spatial reasoning: the clock project. *Artificial Intelligence* 51(1-3).

(Gonzales-Palacios and Angeles, 1993) Gonzales-Palacios, Max and Angeles, Jorge 1993. *Cam Synthesis.* Kluwer Academic Publishers, Dordrecht, Boston, London.

(Haug, 1989) Haug, Edward J. 1989. *Computer-Aided Kinematics and Dynamics of Mechanical Systems,* volume I: Basic Methods. Simon and Schuster.

(Joskowicz and Sacks, 1991) Joskowicz, Leo and Sacks, Elisha 1991. Computational kinematics. *Artificial Intelligence* 51:381–416.

(Joskowicz and Taylor, 1996) Joskowicz, L. and Taylor, R. H. 1996. Interference-free insertion of a solid body into a cavity: An algorithm and a medical application. *International Journal of Robotics Research* 15(3):211–229.

(Joskowicz *et al.*, 1997) Joskowicz, Leo; Sacks, Elisha; and Srinivasan, Vijay 1997. Kinematic tolerance analysis. *Computer-Aided Design* 29(2):147–157.

(Latombe, 1991) Latombe, Jean-Claude 1991. *Robot Motion Planning.* Kluwer Academic Publishers.

(Lin *et al.*, 1996) Lin, Ming C.; Manocha, Dinesh; Cohen, Jon; and Gottschalk, Stefan 1996. Collision detection: Algorithms and applications. In *Proceedings of the 2nd Workshop on Algorithmic Foundations of Robotics.*

(Litvin, 1994) Litvin, Faydor L. 1994. *Gear Geometry and Applied Theory.* Prentice Hall, New Jersey.

(Mirtich and Canny, 1995) Mirtich, Brian and Canny, John 1995. Impulse-based dynamic simulation. In Goldberg, K.; Halperin, D.; Latombe, J.C.; and Wilson, R., editors 1995, *The Algorithmic Foundations of Robotics.* A. K. Peters, Boston, MA.

(Sacks and Bajaj, 1997) Sacks, Elisha and Bajaj, Chandrajit 1997. Sliced configuration spaces for curved planar bodies. *International Journal of Robotics Research.* to appear.

(Sacks and Joskowicz, 1995) Sacks, Elisha and Joskowicz, Leo 1995. Computational kinematic analysis of higher pairs with multiple contacts. *Journal of Mechanical Design* 117:269–277.

(Sacks and Joskowicz, 1996) Sacks, Elisha and Joskowicz, Leo 1996. Dynamical simulation of planar assemblies with changing contacts using configuration spaces. Technical Report 96-046, Purdue University.

(Sacks and Joskowicz, 1997a) Sacks, Elisha and Joskowicz, Leo 1997a. Dynamical simulation of assemblies of planar, 1dof parts with changing contacts using configuration spaces. In *Proceedings of the 1997 IEEE International Conference on Robotics and Automation.* IEEE Computer Society Press.

(Sacks and Joskowicz, 1997b) Sacks, Elisha and Joskowicz, Leo 1997b. Parametric kinematic tolerance analysis of planar mechanisms. *Computer-Aided Design* 27(1). to appear.

(Schiehlen, 1990) Schiehlen, W. 1990. *Multibody systems handbook.* Springer-Verlag.

(Voelcker, 1993) Voelcker, Herbert 1993. A current perspective on tolerancing and metrology. *Manufacturing Review* 6(4).