Discrete–event modelling and diagnosis of quantised dynamical systems

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Abstract

The paper deals with the diagnosis of quantised continuous–variable systems whose state can be measured only by means of a quantiser. Hence, the on–line information used in the diagnosis is given by the sequences of input and output events. The diagnostic algorithm uses a representation of the quantised system by means of discrete–event models. Four different forms of such models will be explained and their usefulness for the solution of diagnostic tasks discussed. The paper shows that a *timed* discrete–event representation is necessary if the diagnostic task should be solved as quickly as possible under real–time constraints. The results are illustrated by diagnosing a batch process.

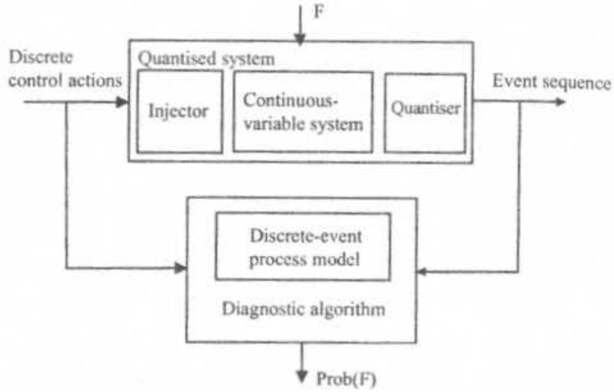

Fig. 1: Diagnosis of quantised systems

Introduction

Diagnosis of quantised systems. This paper is concerned with the diagnosis of dynamical systems with discrete inputs and outputs. As shown in Figure 1, the system under consideration is a continuous–variable continuous–time system, which can be described by some analytical model (set of differential equations). However, the system state x is accessible only through a quantiser, which generates an event whenever the state changes its qualitative value. The input assumes a sequence of discrete values v, which is transformed into a continuous input function $u(t)$ by the injector. Since the observations are based on the quantised signals, a qualitative model has to be used for the diagnosis. The system consisting of the continuous–variable system, the quantiser and the injector is called the *quantised system*.

Aim of the paper. This workshop paper should show how diagnostic methods can be elaborated for quantised continuous–variable dynamical systems. The development consists of two major steps.

- First, four different discrete–event representations of the quantised system are described.
- Second, diagnostic algorithms that use these models and the observed input and output sequences are given.

As the models distinguish concerning the information about the dynamical properties of the quantised systems, the diagnostic results differ concerning their precision. The severeness of these differences are shown by a numerical example.

Relevant literature. Results along this line of research have been obtained in two fields. The modelling problem for quantised systems has been investigated, for example, in (Lunze 1992), (Lunze 1994), (Lunze 1999), (Raisch, O'Young 1997) or (Stursberg, Kowalewski, Engell 1997). On the other hand, diagnosing quantised systems by means of a discrete–event representation has been investigated in (Lichtenberg, Steele 1996), (Lunze 1998), (Lunze, Schiller 1997), (Lunze, Schiller, 1999), (Sampath, Sengupta, Lafurtune, Sinnamohideen, Teneketzis 1995) or (Srinivasan, Jafari 1993). This paper uses the principle of consistency–based diagnosis (Hamscher, Console, and de Kleer 1992) which will be applied here to four different discrete–event representations.

Example: Diagnosis of a batch process

The class of diagnostic problems considered in this paper is illustrated by the batch process depicted in Figure 2. The dashed lines mark liquid levels, which are measured by sensors that indicate only if the level is higher or lower than its position. These sensors act as quantisers. The quantitative model is given in the Appendix.

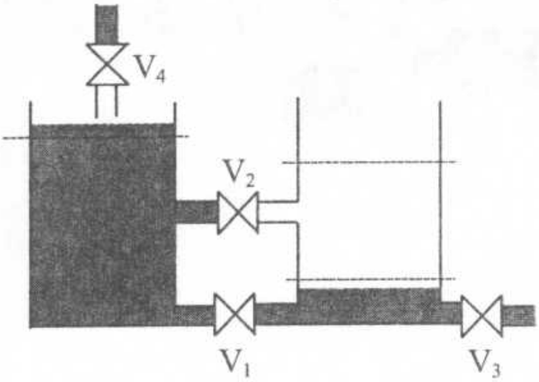

Fig. 2: Example of a batch process

The following operation from a batch process is considered. At $t = 0$ the liquid level in Tank 1 is "high" (i.e. higher than the dashed line) and the level in Tank 2 is "low" (i.e. lower than the lower dashed line). The aim is to bring the level in the right tank above the upper dashed line. To do this, the Valves V_1, V_2 and V_4 are opened and Valve V_3 closed.

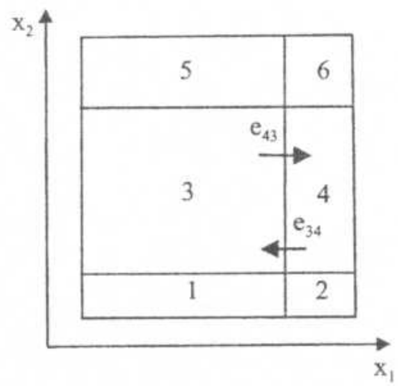

Fig. 3: Partition of the state space of the two tanks (x_1 = level of Tank 1; x_2 = level of Tank 2)

Since the only on-line information is obtained from the qualitative sensors, the behaviour of the system is considered in the partitioned state space depicted in Figure 3. The numbers $i = 1, 2, ..., 6$ in the figure refer to the enumeration of the state space partitions. If the trajectory of the system crosses one of the partition borders, an event is generated. Figure 3 shows as two examples the events e_{34} and e_{43}.

Four faults are considered where f_1, f_2 and f_4 denote the situation that the Valve V_1, V_2 or V_4 is not opened, respectively, and f_3 describes that Valve V_3 is not closed. f_0 symbolises the faultless system. Hence,

$$\mathcal{F} = \{f_0, f_1, f_2, f_3, f_4\}.$$

The diagnostic problem is to find the fault as quickly as possible after the control input, which opens the valves V_1, V_2 and V_4 and closes valve V_3, has been applied.

Quantised continuous–variable systems

Quantitative system description. This section explains important properties of the quantised system, which have to be taken into account when solving the modelling and diagnostic tasks. Continuous–variable systems

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), f), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0. \qquad (1)$$

are considered where $\boldsymbol{x} \in \mathbb{R}^n$ denotes the state and $\boldsymbol{u} \in \mathbb{R}^m$ the input vector. It is assumed that for any initial state $\boldsymbol{x}_0$ and input $\boldsymbol{u}(t)$ eqn. (1) has a unique solution, which is considered for the time interval $[0, T_h]$ and denoted by $\boldsymbol{x}_{[0,T_h]}$. Since the main ideas can be developed by considering a system without input (i.e. for $\boldsymbol{u}(t) = 0$), all further investigations are restricted to the autonomous system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), f), \quad \boldsymbol{x}(0) = \boldsymbol{x}_0. \qquad (2)$$

However, the modelling method and the diagnostic algorithm can be extended to system (1).

Quantisation of the state space. The *quantiser* maps the state space $\mathbb{R}^n$ onto a finite set

$$\mathcal{N}_x = \{0, 1, 2, ..., N\}$$

of qualitative values and, thus, introduces a partition of $\mathbb{R}^n$ into $N + 1$ disjoint sets $\mathcal{Q}_x(i)$, where i denotes the "number" of the partition. $\mathcal{Q}_x(i)$ denotes the set of states $\boldsymbol{x} \in \mathbb{R}^n$ with the same qualitative value i and $\delta\mathcal{Q}_x(i)$ the hull of this set. In the example, the state variables x_1 and x_2 are quantised independently so that the sets $\mathcal{Q}_x(i)$ represent rectangular boxes as shown in Figure 3.

Temporal quantisation. Since only the quantised state information is assumed to be available for diagnosis, the quantised system seems to remain in a given qualitative state as long as its trajectory $\boldsymbol{x}(t)$ does not cross a border

$$\delta\mathcal{Q}_{xij} = \delta\mathcal{Q}_x(i) \cap \delta\mathcal{Q}_x(j) \qquad (3)$$

between two adjacent state space partitions. A change of the qualitative value of the state $\boldsymbol{x}$ is called an *event*. The quantiser does not only determine which event occurs but also at which time the event is generated. The event that the system moves from qualitative state j to qualitative state i is denoted by e_{ij}. $\mathcal{E}$ is the set of all events that may occur. Upper-case letters like E_k represent variables denoting the occurrence of the k-th event whereas lower-case letters like $e, \tilde{e} \in \mathcal{E}$ denote particular events. Hence, $E_5 = e_{34}$ signifies that the system (2) changes its qualitative state from 4 to 3 while generating the fifth event.

Discrete–event behaviour of the quantised system. The behaviour of the quantised system is described by a timed event sequence

$$\boldsymbol{E}_t(0...T_h) = (E_0, T_0;\ E_1, T_1;\ E_2, T_2;\ ...; E_H, T_H). \quad (4)$$

E_k denotes the name and T_k the occurrence time of the k-th event. H is the number of events that the quantised system generates in the time interval $[0, T_h]$. If only the sequence of events are considered but the occurrence times neglected, the behaviour of the quantised system is described by the untimed event sequence

$$\boldsymbol{E}(0...H) = (E_0,\ E_1,\ E_2,\ ...E_H). \quad (5)$$

Clearly, every continuous–variable behaviour $\boldsymbol{x}(t)$ of the system (2) is associated with a unique timed event sequence (4) and a unique untimed event sequence (5), which is abbreviated by

$$\boldsymbol{E}_t(0...T_h) = \text{Quant}_t(\boldsymbol{x}_{[0,T_h]})$$

and

$$\boldsymbol{E}(0...H) = \text{Quant}(\boldsymbol{x}_{[0,T_h]}).$$

The qualitative modelling problem

For diagnosis, a model has to be used which generates for every given initial event e_0 the event sequence $\boldsymbol{E}_t(0...T_h)$ or $\boldsymbol{E}(0...H)$ for all faults $f \in \mathcal{F}$. Such a model is available if eqn. (2) is combined with the quantiser. However, this model includes continuous–variable and discrete–event parts. For diagnosis, a more compact model has to be found. An inherent problem of this modelling task results from the fact that these event sequences are not unique (Lunze, Nixdorf, Schröder 1999). This fact has to be explained now.

Nondeterminism of the discrete–event behaviour. The nondeterminism of the discrete–event behaviour of the quantised system means that the quantised system may generate one of a set of different event sequences $\boldsymbol{E}_t$ or $\boldsymbol{E}$ and it is impossible to select the true sequence in advance. The reason for this is given by the fact that the initial state $\boldsymbol{x}_0$ of the system (2) is unknown. After the first event e_0 has been observed at time t_0, the state of the system is known to lie in the set $\delta\mathcal{Q}(e_0)$, which includes all those states $\boldsymbol{x}$ for which the system generates the event e_0. For notational convenience, t_0 is assumed to be zero. Depending on $\boldsymbol{x}_0$ the system may produce one event sequence of the sets

$$\begin{aligned} \mathcal{S}_t(e_0, f) &= \{\boldsymbol{E}_t = \text{Quant}_t(\boldsymbol{x}_{[0,T_h]}) \,|\, \text{Eqn. (2) holds} \\ &\qquad \text{for some } \boldsymbol{x}_0 \in \delta\mathcal{Q}_x(e_0)\}. \quad (6) \\ \mathcal{S}(e_0, f) &= \{\boldsymbol{E} = \text{Quant}(\boldsymbol{x}_{[0,T_h]}) \,|\, \text{Eqn. (2) holds} \\ &\qquad \text{for some } \boldsymbol{x}_0 \in \delta\mathcal{Q}_x(e_0)\}. \quad (7) \end{aligned}$$

For the example, the reason for the nondeterminism of the behaviour can be seen from Figure 4. If the event e_{42} is observed as initial event, the tank system may follow any trajectory of the set depicted in the figure. Hence, it may generate any of the event sequence

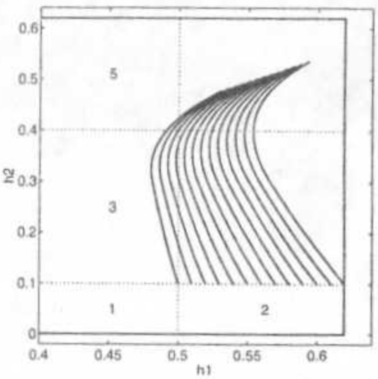

Fig. 4: Trajectories generated for fault f_1 and $e_0 = e_{42}$

$$\begin{aligned} \boldsymbol{E}_1 &= (e_{42}, e_{64}) \\ \boldsymbol{E}_2 &= (e_{42}, e_{34}, e_{43}, e_{64}) \\ \boldsymbol{E}_3 &= (e_{42}, e_{34}, e_{53}, e_{65}). \end{aligned} \quad (8)$$

Hence,

$$\mathcal{S}(e_{42}, f_1) = \{\boldsymbol{E}_1, \boldsymbol{E}_2, \boldsymbol{E}_3\} \quad (9)$$

holds.

Moreover, the temporal distance of the events may vary considerably, which yield to a huge set $\mathcal{S}_t(e_{42}, f_1)$ of timed event sequences.

Stochastic properties of the quantised system. A compact representation of the nondeterministic behaviour of the quantised system can be obtained by a statistical evaluation. It is assumed that the initial state $\boldsymbol{x}_0$ of the continuous–variable system (2) is uniformly distributed over the set $\delta\mathcal{Q}(e_0)$. Then the event sequence $\boldsymbol{E}_t$ is a random sequence with $\boldsymbol{E}_t \in \mathcal{S}_t(e_0, f)$. The probability that the event e has occurred before or at time t is denoted by

$$V_e(e, t, f) = \sum_k \text{Prob}\,(E_k = e,\ T_k \le t \,|\, F = f). \quad (10)$$

Since the initial event e_0 is assumed to be known, $V_e(e, 0, f)$ is known:

$$V_e(e, 0, f) \quad = \quad \begin{cases} 1 & \text{for } e = e_0 \\ 0 & \text{else} \end{cases} \quad \text{for all } f \in \mathcal{F}. (11)$$

The relation between the probabilities just defined and the event sequence $\boldsymbol{E}_t$ is obvious. An event e_{ij} occurs in at least one event sequence $\boldsymbol{E}_t \in \mathcal{S}_t(e_0, f)$ if and only if $V_e(e_{ij}, t, f) \not\equiv 0$.

Figure 5 shows the statistical properties of the quantised tank system. The strips depict the probability $V_e(e, t, f_1)$ in grey scale. The strips are shown only for the time interval in which $\frac{dV_e}{dt} > 0$ holds, because the event e may occur exactly in this time interval. The darker the strip is the more probable is the occurrence of the event until the corresponding time instant.

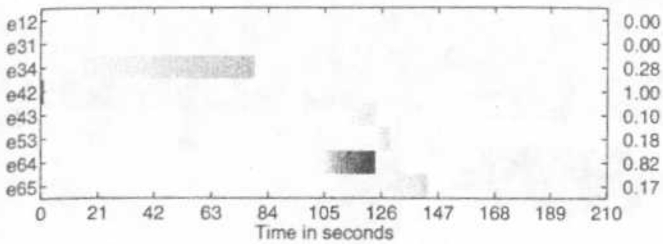

Fig. 5: Graphical representation of the statistical properties of the tank system for fault f_1 and initial event e_{42}

Figure 5 has been obtained by exhaustive simulation for a large number of initial states $x_0 \in \delta Q(e_{42})$. Such an exhaustive simulation should not be used in the diagnosis. Therefore, models have to be set up which represent the set of event sequences of the quantised system in a concise form.

Modelling aim. Since the behaviour is nondeterministic, a nondeterministic model has to be used. Such a model does not generate a unique event sequence $\boldsymbol{E}_t$, but a set $\mathcal{M}_t(e_0, f)$ of event sequences. The modelling aim is to find a representation of the quantised system such that the relation

$$\boxed{\mathcal{M}_t(e_0, f) \supseteq \mathcal{S}_t(e_0, f)} \qquad (12)$$

holds for all e_0, f and T_h. If the untimed sequences are considered, the modelling aim reads as

$$\mathcal{M}(e_0, f) \supseteq \mathcal{S}(e_0, f). \qquad (13)$$

According to eqn. (12) the model should generate all event sequences that the quantised system may generate over the same time horizon for the same initial event and fault. It has been shown that any diagnostic algorithm can find all possible faults in a quantised system if and only if the modelling aim (12) (or (13)) is satisfied (cf. (Lunze 1998)).

Qualitative modelling of the quantised system

In this section, four different solutions to the modelling problem will be given. Starting with a simple (untimed) model, the four models include more and more information about the quantised system. In the next section, it will be shown, how the diagnostic result can be improved due to this increasing information included in the model. The first two models can be merely used to generate the untimed sequences $\boldsymbol{E}(0...H)$, whereas the third and fourth model can be used to determine timed event sequences $\boldsymbol{E}_t(0...T_h)$.

Nondeterministic automata or Petri nets

The nondeterministic automaton $N(\mathcal{E}, R, e_0)$ with state transition relation $R(f) \subseteq \mathcal{E} \times \mathcal{E}$ and and initial state e_0 can be used as model of the quantised system if $R(f)$ includes all event pairs that the quantised system may generate subject to the fault f. In the automaton graph these pairs correspond to directed edges as shown in Figure 6. $R(f)$ can be found for a given quantised system by determining for every event e all possible successor events (Lunze 1994), (Raisch, O'Young 1997), (Förstner, Lunze 1999). Instead of the nondeterministic automaton, Petri nets can also be used as model of the quantised system (Lunze 1992).

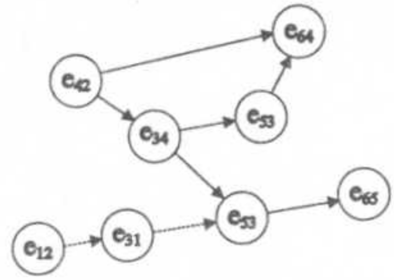

Fig. 6: Automaton graph representing the quantised tank system for fault f_1

The behaviour of the nondeterministic automaton can be interpreted as movement along the edges in its automaton graph. For the fault f_1 and initial event e_{42} the nondeterministic automaton generates the set $\mathcal{M}(e_{42}, f_1)$, which includes exactly the three event sequences $\boldsymbol{E}_1$, $\boldsymbol{E}_2$ and $\boldsymbol{E}_3$ given in eqn. (9). That is, for this example the modelling aim (13) can be satisfied with equality sign ($\mathcal{S}(e_{42}, f_1) = \mathcal{M}(e_{42}, f_1)$).

Stochastic automata

Compared with the nondeterministic automaton, the stochastic automaton $S(\mathcal{E}, P, e_0)$ generates additional information because its dynamical properties are described by the state transition probability P:

$$P(e, \tilde{e}, f) = \text{Prob}\,(E_1 = e \mid E_0 = \tilde{e}, f). \qquad (14)$$

In order to find P for a given quantised system, the right–hand side of this equation has to be determined for all possible event pairs $(e, \tilde{e})$, which is possible by means of the quantitative model (2) and the quantiser (Lunze 1994). These probability values are additional labels of the edges in the automaton graph.

For the tank example, it can be seen from Figure 4 that about 28% of all trajectories that generate the event e_{42} yield e_{34} as succeeding event and 72% of these trajectories have the successor event e_{64}. Hence, $P(e_{34}, e_{42}, f_1) = 0{,}28$.

The set $\mathcal{M}(e_0, f)$ of trajectories generated by the stochastic automaton again includes all paths within the automaton graph and, in addition to that, an evaluation of the probability of its appearance.

Timed automata

A first step towards a timed description can be made by using time intervals $[t_{min}, t_{max}]$ as additional labels of the state transitions of a nondeterministic or a stochastic automaton. t_{min} and t_{max} denote the minimum or maximum temporal distance between the events $\tilde{e}$ and e of the considered event pair, which can be determined for a given quantised system (Stursberg, Kowalewski, Engell 1997). The behaviour of the model is then described by all paths in the automaton graph and, in addition to that, by the cumulative time interval, which can be obtained by combining the time intervals of the individual state transitions according to the rules of interval arithmetic.

Semi–Markov processes

A further improvement is possible if the model describes with which probability the quantised system generates the event e if it has generated the event $\tilde{e}$ at τ time units before. Note that the probability depends now on the sojourn time $\tau = T_{k+1} - T_k$:

$$F_{e\tilde{e}}(\tau, f) = \text{Prob}\,(E_{k+1} = e, T_{k+1} \leq t + \tau \mid E_k = \tilde{e}, T_k = t, f) \quad \text{for } e \neq \tilde{e}. \quad (15)$$

A semi–Markov process $M_{\mathcal{T}}(\mathcal{E}, \mathcal{F}, F, e_0)$ with the state set $\mathcal{E}$ and the state transition probability distribution F generates the set $\mathcal{M}_t(e_0, f)$ of timed event sequences for given initial event e_0 and fault f:

$$\mathcal{M}_t(e_0, f) = \{(E_0, 0;\, E_1, T_1;\, ...;\, E_H, T_H) \mid \frac{d}{dt} F_{E_{k+1}E_k}(T_{k+1} - T_k, f) > 0, \; (k = 0, 1, ..., H-1)\}.$$

How to set up the models

All the models explained so far can be automatically generated from the quantitative description (2) and the given quantiser. For example, the probability distribution F of the semi–Markov model is given by

Timed Abstraction:

$$F_{e\tilde{e}}(\tau, f) = \begin{cases} \text{Prob}\,(E_1 = e, T_1 \leq \tau \mid E_0 = \tilde{e}, T_0 = 0, F = f) & \text{for } e \neq \tilde{e} \\ -\sum_{\bar{e} \neq e} F_{\bar{e}e}(\tau, f) & \text{else.} \end{cases} \quad (16)$$

On the right–hand side of the first equation, a pair $(\tilde{e}, e)$ of succeeding events is considered and the probability of its occurrence determined by means of eqn. (2) for given f. It has been proved in (Lunze 1999) that a semi–Markov process with the probability distribution F described by eqn. (16) satisfies the modelling aim (12). Similar relations between the given quantised system and the other three qualitative models have been elaborated in the references cited above.

The statistical evaluation necessary to determine the right–hand side of eqn. (16) is done by numerical simulation. Note that in these simulations only pairs of successive events have to be considered. The semi–Markov process includes the information obtained by the investigations of these event pairs and uses this information to generate arbitrarily long event sequences.

Process diagnosis by means of the qualitative models

The diagnostic problem

The diagnostic problem can be stated now as follows:

Given: Model of the quantised system
Observation $\boldsymbol{E}(0...H)$ or $\boldsymbol{E}_t(0...T_h)$
Find: $p_M(f, T_h)$, which describes whether fault f has occurred

The form of the diagnostic result $p_M(f, T_h)$ depends on the model used and will be explained later. However, for all models, p_M depends on the time horizon T_h of the observed data and $p_M(f, T_h) = 0$ signifies that the fault f is known not to occur.

The diagnostic problem will be solved in this section by applying the idea of *consistency–based diagnosis* to the four models proposed in the preceeding section. For a given observed event sequence it is tested for which candidate fault(s) $f \in \mathcal{F}$ this sequence is consistent with the qualitative model. Since also a model of the faultless system is used, the faulty behaviour can be diagnosed even if the fault set does not include the current fault.

Diagnosis by means of the semi–Markov model

As the semi–Markov model is the most general one among the four models, the diagnostic algorithm is explained for this model first. Assume that the event sequence $\boldsymbol{E}_t(0...T_h)$ has been observed until time T_h. The aim is to determine the probability that the quantised system with this fault f has generated the given timed event sequence:

$$p_M(f, T_h) = \text{Prob}\,(F \mid \boldsymbol{E}_t(0..T_h))$$

For the description of the algorithm it is assumed that *before* the time instant T_h the events $E_0, ..., E_H$ have occurred at the time points $T_0, ...T_H$ (Fig. 7). The last event, which has occurred at time $T_H < T_h$ is denoted by $\tilde{e}$ $(E_H = \tilde{e})$.

The algorithm uses the probability of the model to remain in the state $\tilde{e}$:

$$F_{\tilde{e}}(\tau, f) = \text{Prob}\,(T_1 > \tau \mid E_0 = \tilde{e}, T_0 = 0) = 1 - \sum_{\bar{e} \in \mathcal{E}} F_{\bar{e}\tilde{e}}(\tau, f). \quad (17)$$

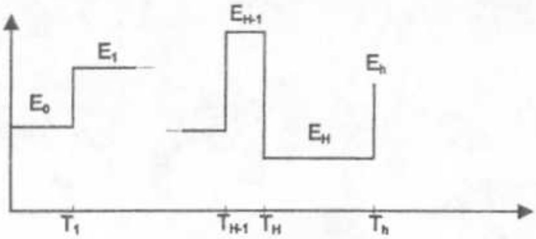

Fig. 7: Observed event sequence

Diagnostic Algorithm

Start with the initial fault probability

$$p_M(f,0) = \frac{1}{n_F}, \tag{18}$$

where n_F denotes the number of faults considered. For increasing time horizon T_h do the following:

1. Determine the auxiliary function

$$p_a(f,T_h) = \begin{cases} (1 - F_{\bar{e}}(T_h - T_H, f))\, p_M(f,T_H) \\ \quad \text{for } T_{H+1} > T_h \\ F_{\bar{e}\tilde{e}}(T_h - T_H, f))\, p_M(f,T_H) \\ \quad \text{for } T_{H+1} = T_h \end{cases} \tag{19}$$

   for all $f \in \mathcal{F}$. The first line concerns the case that at time T_h no new event occurs whereas the second line applies if the event $\tilde{e}$ occurs at time T_h.

2. Determine the diagnostic result at time T_h by

$$p_M(f,T_h) = \frac{p_a(f,T_h)}{\sum_f p_a(f,T_h)} \tag{20}$$

(for a proof cf. (Lunze 1998)). The proof uses results from probability theory to solve a decision problem for a semi–Markov process. In a real–time application, eqns. (19) and (20) are used for continuously increasing time horizon T_h.

Diagnosis by means of the timed automaton

If a timed automaton is used as qualitative model of the quantised system, the diagnostic result is an assertion saying whether a given fault f can be the cause of the observed event sequence or not, but no probabilistic information about the occurrence of the different faults is obtained. The Diagnostic Algorithm can be used with

$$F_{\bar{e}\tilde{e}}(\tau,f) = \begin{cases} 1 & \text{for } t_{min} \le \tau \le t_{max} \\ 0 & \text{else,} \end{cases}$$

which can be obtained from the timed automaton. Then, eqns. (18) - (20) yield a nonvanishing value of $p_M(f,T_h)$ if the observed event sequence can occur subject to fault f.

Diagnosis by means of the stochastic automaton

If the stochastic automaton is used, only the untimed event sequence $\boldsymbol{E}$ can be processed. The diagnostic result $p_M(f,T_h)$ gives the probability with which the quantised system can generate the observed event sequence for fault f. The Diagnostic Algorithm is used with $P(e,\tilde{e},f)$ replacing $F_{e\tilde{e}}(\tau,f)$. p_a and p_M are updated only after a new event $\tilde{e}$ has occurred:

$$p_a(f,T_H) = P(\bar{e},\tilde{e},f)\, p_M(f,T_{H-1}) \tag{21}$$

$$p_M(f,T_H) = \frac{p_a(f,T_H)}{\sum_f p_a(f,T_H)}. \tag{22}$$

Diagnosis by means of the nondeterministic automaton

If the nondeterministic automaton is used, $p_M(f,T_h)$ says only whether the observed untimed event sequence $\boldsymbol{E}(0...H)$ may occur for fault f or not.

Comparison of the diagnostic results

The diagnostic results obtained by the different models are compared now for the batch process example. First, the faulty behaviour of the tank system is analysed in order to evaluate the "difficulty" of the diagnostic task. Figure 8 compares the event sequences that the tank system generates for the different faults with initial event e_{42}.

- As the event sequence $(e_{42}, e_{34}, e_{53}, e_{65})$ may be generated by the faultless system as well as by the system with the faults f_1, f_2 or f_3, these three faults can only be discriminated if the temporal distances of the events are taken into account.
- Fault f_4 can be identified due to the fact that only the event pair (e_{42}, e_{34}) is generated. This, however, necessitates temporal information in the sense that the algorithm has to wait "long enough" before it outputs the fault f_4.
- For the faults f_1 and f_2 the quantised system may generate one of the three event sequences (9). Both faults can only be distinguished by using temporal information.

In the following diagrams the time $t = 0$ denotes the initial time when the tank system starts its movement in some initial state $\boldsymbol{x}_0$. The first event may occur later ($T_0 \ge 0$). In any case, the diagnostic algorithm starts at T_0.

First, consider the tank system for fault f_1 with (unknown!) initial state $\boldsymbol{x}_0 = (0.5 \quad 0)'$. The discrete–event behaviour is described by the untimed sequence $\boldsymbol{E}(0...3) = (e_{12}, e_{31}, e_{53}, e_{65})$ or the timed event sequence depicted in the upper part of Figure 9, where the dashes show at which time instances T_k the events E_k occur. The diagnosis by means of the four different models yield the following results:

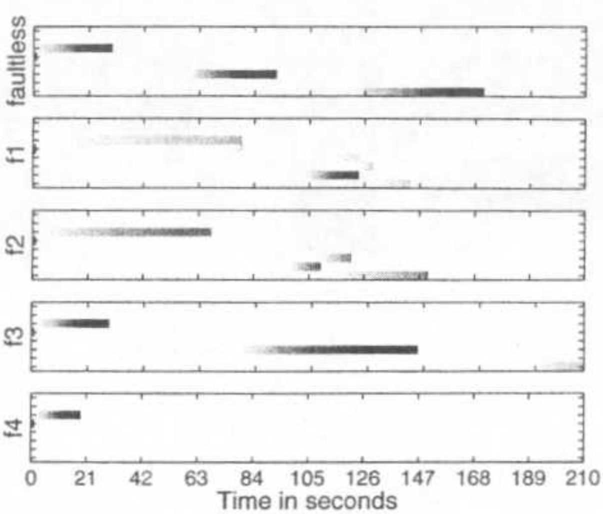

Fig. 8: Discrete–event behaviour of the batch process for different faults

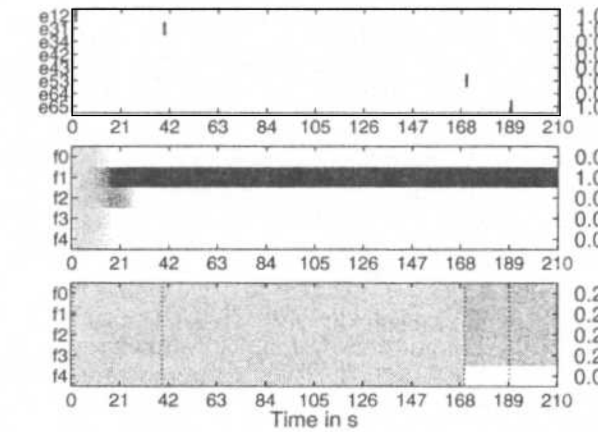

Fig. 9: Behaviour of the process subject to fault f_1 and diagnostic results

- As the observed untimed event sequence can occur for the faultless system and for the faults f_1, f_2 and f_3, the diagnostic algorithm using the untimed automata cannot identify the fault. Only the fault f_4 can be excluded after the fourth event has occurred (lowest part of Fig. 9).
- The middle part of Fig. 9 shows the diagnostic result, where the probability $p_M(f, T_h)$ is depicted in grey scale. Obviously, the fault f_1 is uniquely detected after about 30sec. That is, $p_M(f_1, T_h) = 1$ holds for $T_h > 30$, which is also indicated at the right margin of the figure. Note that the diagnosis is finished before the second event occurs.

The fault diagnosis takes more time if the event e_{42} is generated first. Figure 10 shows that for the (unknown) initial state $\boldsymbol{x}_0$ used now the first event occurs at time $T_0 = 32$. It is not before this time that the diagnostic algorithm is started. After the second event

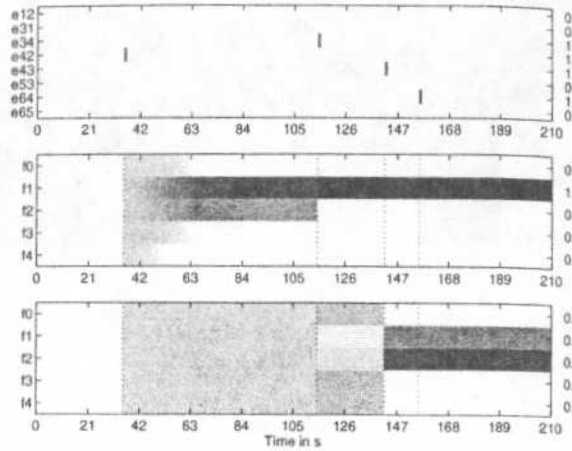

Fig. 10: Fault diagnosis for fault f_1 with e_{42} being the initial event

has occurred, the fault f_1 is uniquely determined. Thee lowest part of the figure shows that the diagnostic task cannot be solved by the untimed models.

If the timed automaton (without probabilistic evaluation of the state transitions) is used, the results are similar to the results obtained with the semi–Markov model. However, as the fault probabilities cannot be determined, all stripes are black instead of grey.

If the (untimed) nondeterministic automaton is used, the results are similar to those obtained for the (untimed) stochastic automaton, where again the stripes are black instead of grey. This comparison shows that an important information for diagnosis lies in the temporal distance between succeeding events. The probabilistic evaluation made here provides only additional information to distinguish the degree in which possible faults occur.

Conclusions

The paper has shown that quantised continuous–variable systems can be diagnosed by means of discrete–event representations of the quantised system. Four different models have been described, which include different information and, hence, necessitate different depth of knowledge about the quantised system. A diagnostic algorithm has been given, which can be used (with some modifications) for all four models. The results have been compared by means of a numerical example.

For the simplicity of presentation, only an autonomous system (2) has been used here. However, as the cited literature shows, all modelling and diagnostic steps can be generalised to the system (1) which takes into account the input $\boldsymbol{u}(t)$.

References

D. Förstner; J. Lunze, 1999, Qualitative modelling of a power stage for diagnosis, *Workshop on Qualitative Reasoning*.

W. Hamscher, L. Console, J. de Kleer, (Eds.), 1992, *Readings in Model-based Diagnosis*, Morgan Kaufman.

G. Lichtenberg, A. Steele, 1996, "An approach to fault diagnosis using parallel qualitative observers", *Workshop on Discrete Event Systems*, Edinburgh, pp. 290–295.

J. Lunze, 1992, "A Petri-net approach to qualitative modeling of continuous dynamical systems", *Systems Analysis, Modelling, Simulation* **9**: 88–111.

J. Lunze, 1994, "Qualitative modelling of linear dynamical systems with quantized state measurements", *automatica* **30**: 417–431.

J. Lunze, 1998, "Process diagnosis by means of a timed discrete-event representation", *IEEE Trans. on Systems, Man and Cybernetics* (submitted for publication).

J. Lunze, 1999, "A timed discrete-event abstraction of continuous-variable systems", *Intern. J. Control* (accepted for publication).

J. Lunze, B. Nixdorf, B., J. Schröder, 1999, "On the nondeterminism of discrete-event representations of continuous-variable systems," *automatica* **35**: 395–406.

J. Lunze, F. Schiller, 1997, "Qualitative Prozeßdiagnose auf wahrscheinlichkeitstheoretischer Grundlage", *Automatisierungstechnik* **45**: 351–359.

J. Lunze, F. Schiller, 1999, "An example of fault diagnosis by means of probabilistic logic reasoning", *Control Engineering Practice* **7**: 271–278.

J. Lunze; T. Serbesow, 1991, "Logikbasierte Prozessdiagnose unter Berücksichtigung der Prozessdynamik", *Messen, Steuern, Regeln* **34**: 163–165 und 253–257.

J. Raisch, S. O'Young, 1997, "A totally ordered set of discrete abstractions for a given hybrid or continuous system", In: P. Antsaklis, W. Kohn, A. Nerode, S. Sastry, (Eds.): *Hybrid Systems IV*, Lecture Notes in Computer Science, vol. 1273, pp. 342–360, Berlin: Springer-Verlag.

M. Sampath, R. Sengupta, S. Lafurtune, K. Sinnamohideen, D. Teneketzis, 1995, "Diagnosability of discrete event systems", *IEEE Trans.* **AC–40**: 1555-1575.

V.S. Srinivasan, M.A. Jafari, 1993, "Fault detection/monitoring using timed Petri nets", *IEEE Trans.* **SMC–23**.

O. Stursberg; S. Kowalewski; S. Engell, 1997, "Generating timed discrete models", *2-nd MATHMOD*, Vienna 1997, pp. 203–207.

Appendix: Quantitative model of the tank system

The coupled tanks can be described by the differential equations

$$\begin{aligned}
\dot{h}_1 &= \frac{1}{A_1}\left(\dot{Q}_4 - \dot{Q}_1 - \dot{Q}_2\right)\\
\dot{h}_2 &= \frac{1}{A_2}\left(\dot{Q}_1 + \dot{Q}_2 - \dot{Q}_3\right)\\
\dot{Q}_1 &= S_v\,\mathrm{sgn}(h_1 - h_2)\sqrt{2g|h_1 - h_2|}\\
\dot{Q}_2 &= \begin{cases} S_v\,\mathrm{sgn}(h_1 - h_2)\sqrt{2g|h_1 - h_2|} & \text{if } h_1, h_2 > h_v\\ S_v\sqrt{2g|h_1 - h_v|} & \text{if } h_1 > h_v,\ h_2 \le h_v\\ S_v\sqrt{2g|h_2 - h_v|} & \text{if } h_2 > h_v,\ h_1 \le h_v\\ 0 & \text{if } h_1, h_2 \le h_v \end{cases}\\
\dot{Q}_3 &= S_v\sqrt{2g|h_2|}\\
\dot{Q}_4 &= \dot{Q}_{40}.
\end{aligned}$$

h_1 is the liquid level in the left tank (Tank 1) and h_2 the level in the right tank (Tank 2). $\dot{Q}_i$ denotes the flow through the Valve V_i.

If V_i is closed, $\dot{Q}_i = 0$ holds instead of the given equation.

The system is considered for the following parameters:

$A_1, A_2 = 0,0154\mathrm{m}^2$	Cross-section of the cylindric tanks
$h_v = 0,3\mathrm{m}$	Height of the upper pipe
$S_v = 0,00002\mathrm{m}^2$	Cross-section of the valves
$\dot{Q}_{40} = 6\frac{\mathrm{l}}{\mathrm{min}}$	Flow through Valve V_4 (if opened)
$g = 9,81\frac{\mathrm{m}}{\mathrm{s}^2}$	Gravity constant