monet-1

Models and Simulation
for
Monitoring and Control

- In **Monitoring**:
  - *"What if the system has Fault X?"*
  - *"Can we still achieve goal G?"*
  - *"Can we still prevent disaster D?"*
  - Predicted behavior does/does not match observations.

- In **Control** (design and validation):
  - *"What if we include Feature Y?"*
  - *"Can we possibly reach state S?"*
  - *"Will we necessarily reach state S?"*
  - *"Is behavior B possible?"*
  - Predicted behavior does/does not match design goals.

Knowledge Is Always Incomplete

- In **diagnosis**:

  By definition, device state is not known.

- In **design**:

  Need to test before design is complete.

Numerical simulation

- requires precise parameters and functions;
- requires assumptions (e.g., linearity) about functions;
- predicts *one* possible trajectory.

Qualitative simulation

- can express incomplete knowledge
  of parameters and functions,
- predicts a branching tree of all possible behaviors.

The Qualitative Representation

- **The Qualitative Structure Language**

  variables
  quantity spaces
  influences and constraints
  partially known functions
  bounds and envelopes

- **The Qualitative Behavior Language**

  qualitative value: landmark or interval
  direction of change
  qualitative state
  behavior tree (transition graph)

- **Where's the Power?**
  - Explicitly show all possible behaviors.
  - Each behavior is divided into monotone segments.

Intractable branching was a problem, but now abstraction and model decomposition keep branching under control.

The Key Guarantee Is Soundness

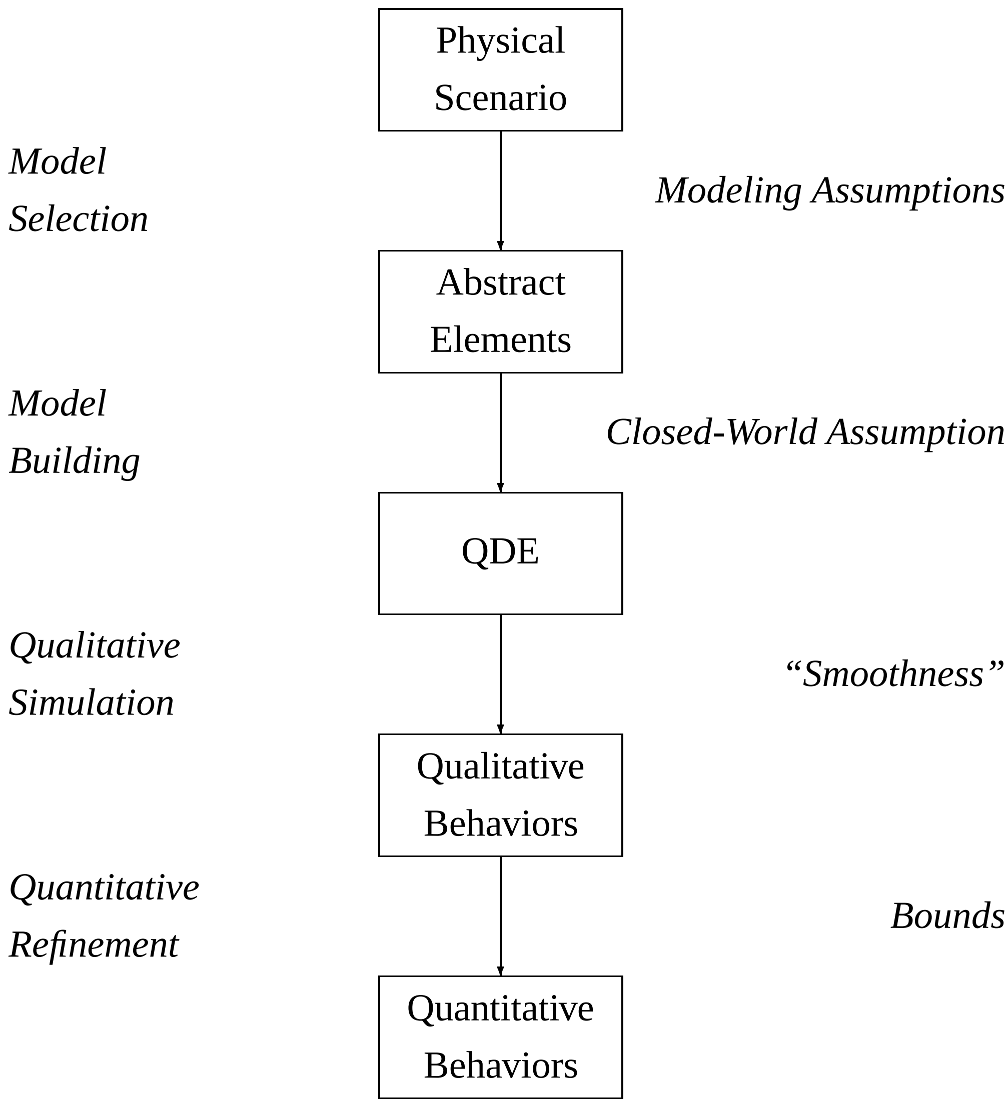
Given a qualitative model and initial state,
All possible behaviors are predicted.

1. generate all possibilities.
2. discard only those provably inconsistent.
3. explicitly note any assumptions.
4. all real behaviors must remain (modulo the assumptions).

This is **not** because soundness is the most important property for engineering or commonsense problem-solving. (It's not.)

- Traditional methods don't emphasize soundness, so we have complementary strengths.
- New methods can build on soundness, carefully maintained.

Model-Building and Simulation



Each stage will provide:

- explicit assumptions,
- explicit guarantees.

Qualitative Models

…express incomplete knowledge of mechanisms.

- **Quantity spaces**:

  abstract the real number line
  to a sequence of **landmark values**
  with qualitative significance.

$$\begin{array}{rcccccccc} amount & : & -\infty & \cdots & 0 & \cdots & AMAX & \cdots & +\infty \\ pressure & : & -\infty & \cdots & 0 & \cdots & PBURST & \cdots & +\infty \end{array}$$

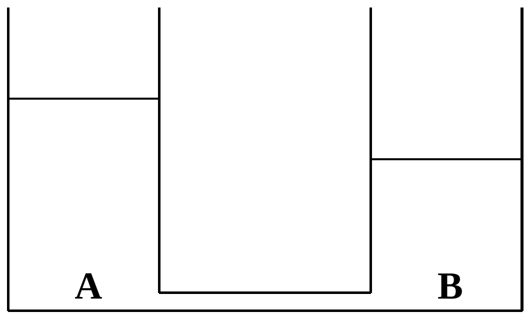
- **Monotonic function constraints**:

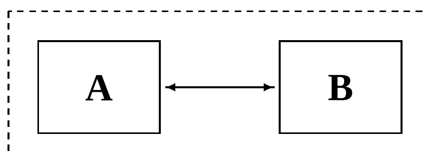
  abstract continuous functions
  preserving qualitative relationships.

$$\begin{aligned} pressure &= M^+(amount) \\ outflow &= M^+(pressure) \end{aligned}$$

A finite set of qualitative models covers an infinite set of linear and non-linear ordinary differential equations.

The U-Tube:
A Simple Two-Tank Model

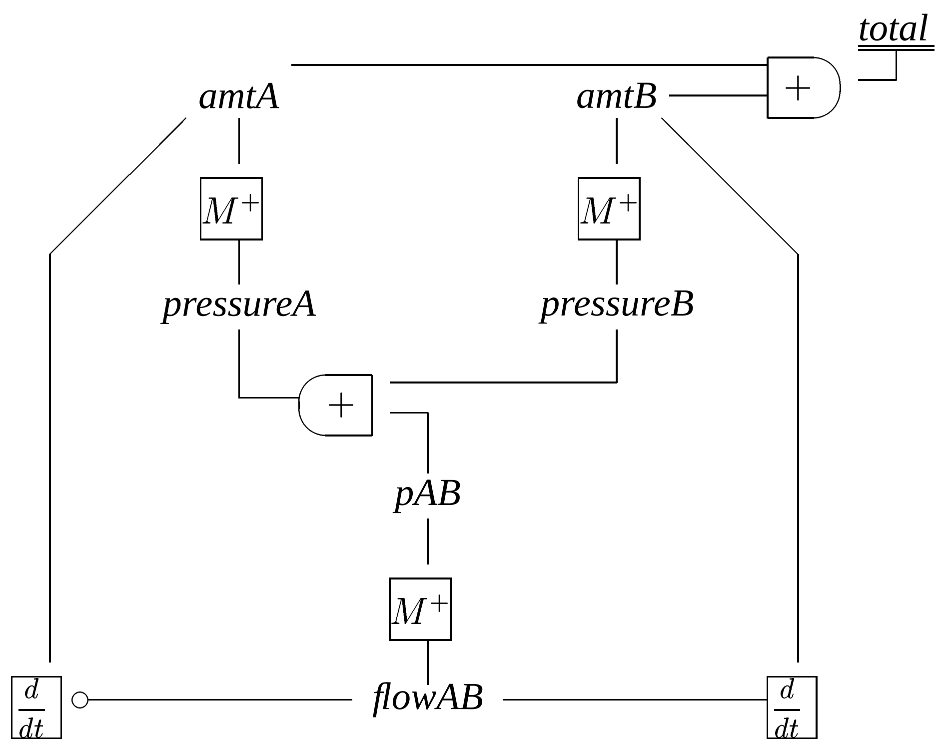




The Closed Two-Tank System

- classic simple equilibrium system
- generalizes naturally to more complex systems
- wide applicability to realistic systems

The Two-Tank Constraint Model



A Qualitative Differential Equation (QDE)

$$B' = f(g(A) - h(B)) \qquad f, g, h \in M^+$$
$$A + B = total \qquad constant(total)$$

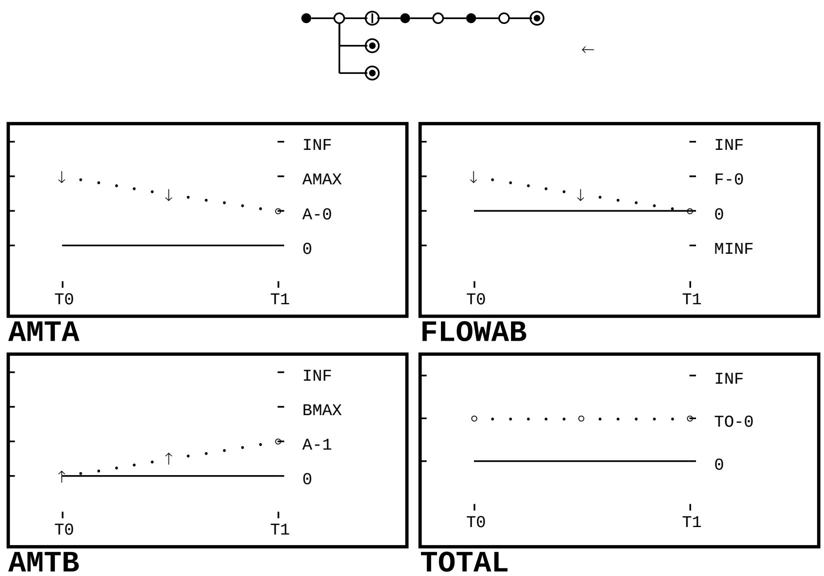
with underspecified functional constraints f, g, h.

The U-tube Model in QSIM

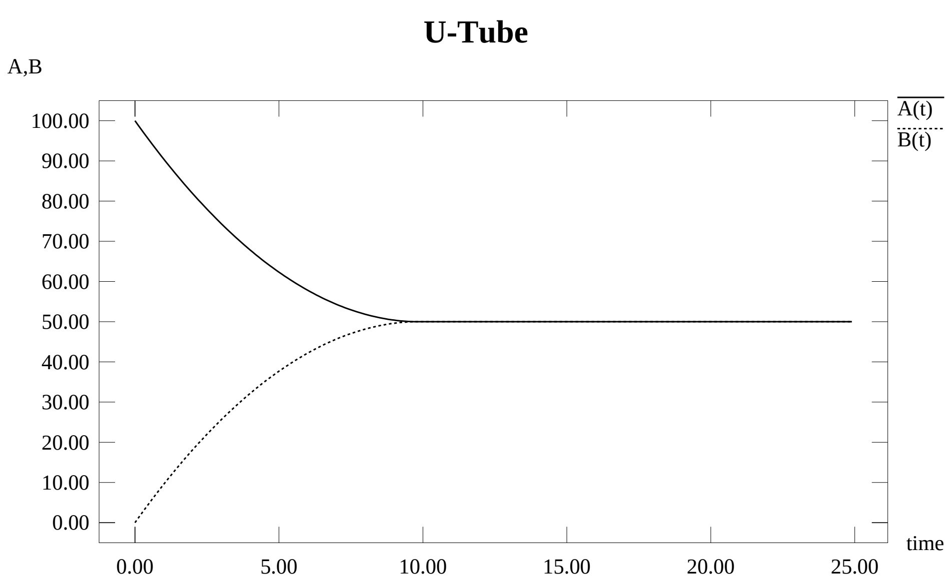
```
(define-QDE U-Tube
  (quantity-spaces
    (amtA       (     0 AMAX inf))
    (pressureA  (     0      inf))
    (amtB       (     0 BMAX inf))
    (pressureB  (     0      inf))
    (pAB        (minf 0      inf))
    (flowAB     (minf 0      inf))
    (mflowAB    (minf 0      inf))
    (total      (     0      inf)))
  (constraints
    ((M+ amtA pressureA)          (0 0) (inf inf))
    ((M+ amtB pressureB)          (0 0) (inf inf))
    ((add pAB pressureB pressureA))
    ((M+ pAB flowAB)              (minf minf) (0 0) (inf inf))
    ((minus flowAB mflowAB))
    ((d/dt amtB flowAB))
    ((d/dt amtA mflowAB))
    ((add amtA amtB total))
    ((constant total)))
  (transitions
    ((amtA (AMAX inc)) -> tank-A-overflow)
    ((amtB (BMAX inc)) -> tank-B-bursts)))
```

Qualitative and Quantitative Behaviors: Equilibrium

- **Describes a class of behaviors:**

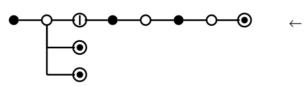


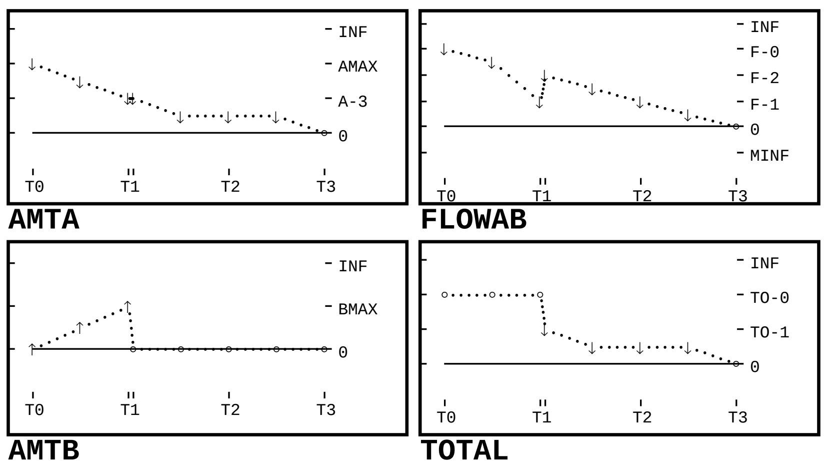
- **Describes an instance of that class:**



Qualitative and Quantitative Behaviors: Overflow and Burst

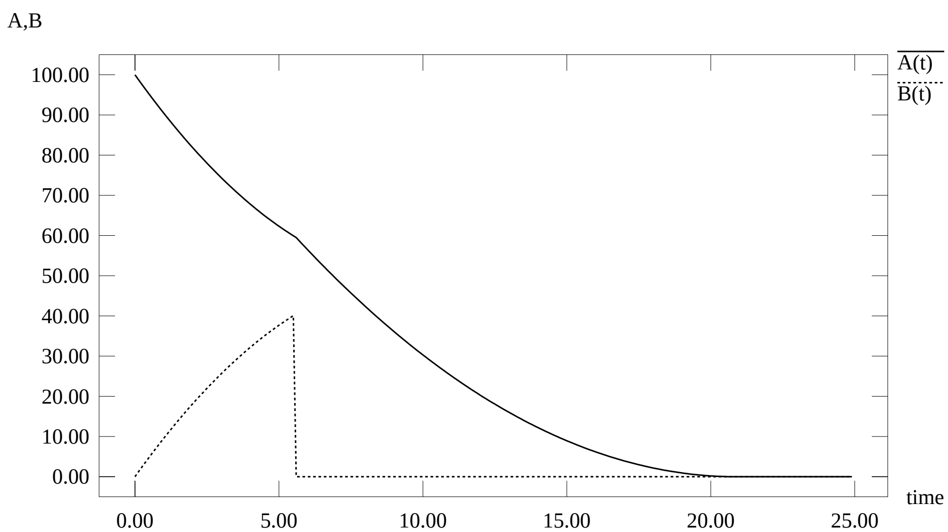
- **Describes a class of behaviors**:





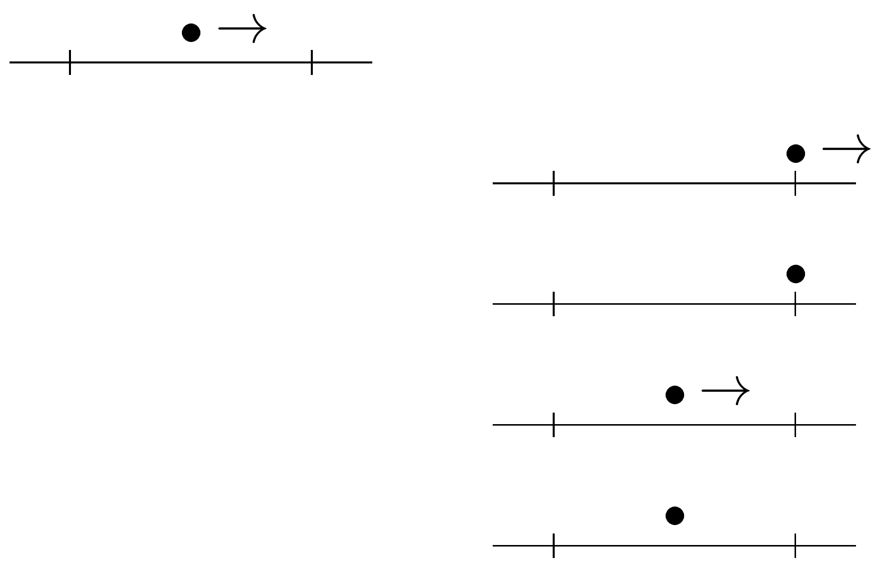
- **Describes an instance of that class**:

U-Tube

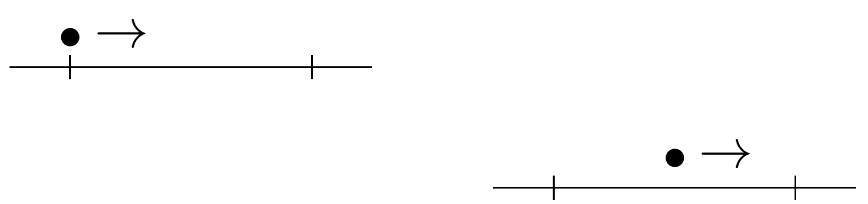


Qualitative Transitions

Moving toward a limit point



Moving from a landmark



The Transition Rules

Time Point to Interval

$P1$	$\langle l_j, std \rangle$	$\langle l_j, std \rangle$
$P2$	$\langle l_j, std \rangle$	$\langle (l_j, l_{j+1}), inc \rangle$
$P3$	$\langle l_j, std \rangle$	$\langle (l_{j-1}, l_j), dec \rangle$
$P4$	$\langle l_j, inc \rangle$	$\langle (l_j, l_{j+1}), inc \rangle$
$P5$	$\langle (l_j, l_{j+1}), inc \rangle$	$\langle (l_j, l_{j+1}), inc \rangle$
$P6$	$\langle l_j, dec \rangle$	$\langle (l_{j-1}, l_j), dec \rangle$
$P7$	$\langle (l_j, l_{j+1}), dec \rangle$	$\langle (l_j, l_{j+1}), dec \rangle$

Time Interval to Point

$I1$	$\langle l_j, std \rangle$	$\langle l_j, std \rangle$
$I2$	$\langle (l_j, l_{j+1}), inc \rangle$	$\langle l_{j+1}, std \rangle$
$I3$	$\langle (l_j, l_{j+1}), inc \rangle$	$\langle l_{j+1}, inc \rangle$
$I4$	$\langle (l_j, l_{j+1}), inc \rangle$	$\langle (l_j, l_{j+1}), inc \rangle$
$I5$	$\langle (l_j, l_{j+1}), dec \rangle$	$\langle l_j, std \rangle$
$I6$	$\langle (l_j, l_{j+1}), dec \rangle$	$\langle l_j, dec \rangle$
$I7$	$\langle (l_j, l_{j+1}), dec \rangle$	$\langle (l_j, l_{j+1}), dec \rangle$
$I8$	$\langle (l_j, l_{j+1}), inc \rangle$	$\langle l^*, std \rangle$
$I9$	$\langle (l_j, l_{j+1}), dec \rangle$	$\langle l^*, std \rangle$

(Intermediate Value and Mean Value Theorems)

The QSIM Algorithm

Efficient limit analysis by constraint filtering.

Group and filter at larger and larger scales:

- Propose transitions for each parameter.
  - Filter for consistency with current state.
- Form tuples at each constraint.
  - Filter for consistency with constraint.
  - Filter for consistency with corresponding values.
- Local consistency (Waltz) filtering on tuples.
  - Filter for pairwise consistency of tuples.
- Form possible successor states.
  - Filter for global consistency.

QSIM on the Ball

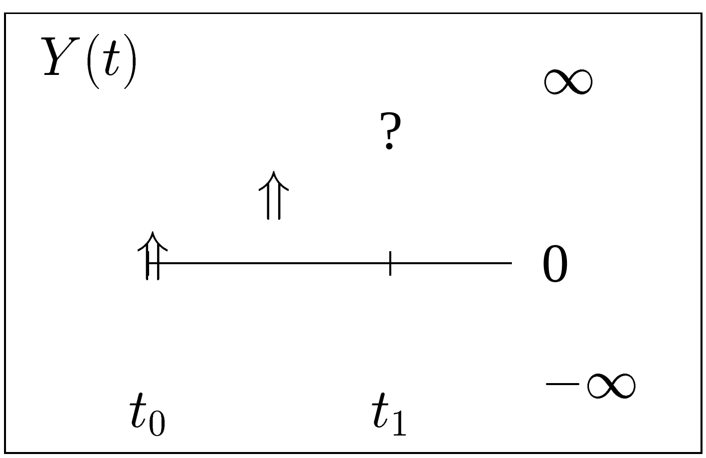
Structural Description:

$$DERIV(Y, V)$$
$$DERIV(V, A)$$
$$A(t) = g < 0$$

$$\frac{d^2}{dt^2}Y(t) = A(t) = g < 0$$

State: Rising toward the peak.

$$QS(A, t_0, t_1) = \langle g, std \rangle$$
$$QS(V, t_0, t_1) = \langle (0, \infty), dec \rangle$$
$$QS(Y, t_0, t_1) = \langle (0, \infty), inc \rangle$$



Find the Transitions

$$QS(A, t_0, t_1) \Rightarrow QS(A, t_1)$$

$$I1 \qquad \langle g, std\rangle \Rightarrow \langle g, std\rangle$$

$$QS(V, t_0, t_1) \Rightarrow QS(V, t_1)$$

$$\begin{array}{ll} I5 & \langle (0,\infty), dec\rangle \Rightarrow \langle 0, std\rangle \\ I6 & \langle (0,\infty), dec\rangle \Rightarrow \langle 0, dec\rangle \\ I7 & \langle (0,\infty), dec\rangle \Rightarrow \langle (0,\infty), dec\rangle \\ I9 & \langle (0,\infty), dec\rangle \Rightarrow \langle L^*, std\rangle \end{array}$$

$$QS(Y, t_0, t_1) \Rightarrow QS(Y, t_1)$$

$$\begin{array}{ll} I4 & \langle (0,\infty), inc\rangle \Rightarrow \langle (0,\infty), inc\rangle \\ I8 & \langle (0,\infty), inc\rangle \Rightarrow \langle L^*, std\rangle \end{array}$$

(The possibility that $Y(t_1) = \infty$ is excluded by methods we won't cover here.)

Filter the transition tuples:
Constraint Consistency
Waltz Consistency

$DERIV(Y,V)$		$DERIV(V,A)$	
$(I4, I5)$	c	$(I5, I1)$	c
$(I4, I6)$	c	$(I6, I1)$	
$(I4, I7)$		$(I7, I1)$	
$(I4, I9)$	w	$(I9, I1)$	c
$(I8, I5)$	w		
$(I8, I6)$			
$(I8, I7)$	c		
$(I8, I9)$	c		

$c =$ excluded by constraint filter.
$w =$ excluded by Waltz filter.

Global Interpretations

Y	V	A
$I4$	$I7$	$I1$
$I8$	$I6$	$I1$

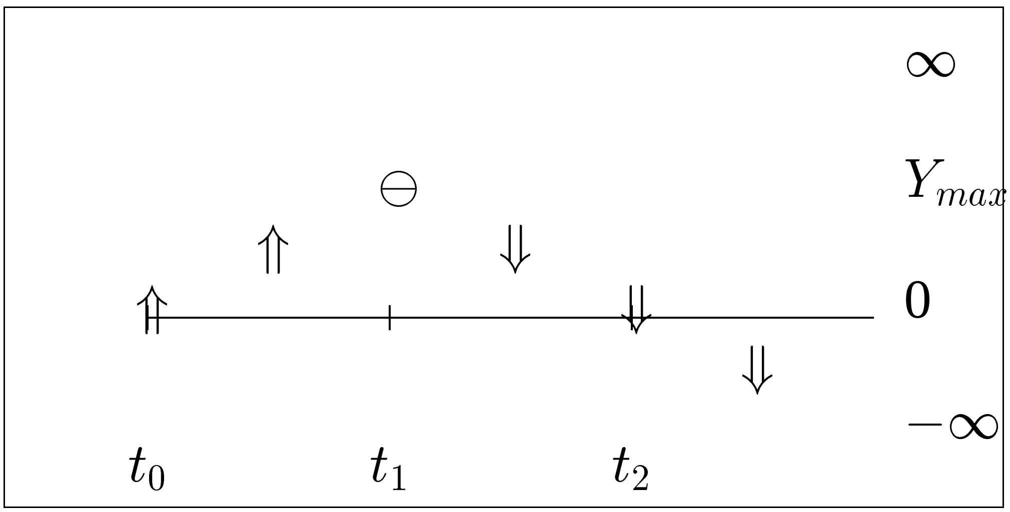
The “No Change” Filter excludes $(I4, I7, I1)$.

Define the Next State

$$QS(A, t_1) = \langle g, std \rangle$$
$$QS(V, t_1) = \langle 0, dec \rangle$$
$$QS(Y, t_1) = \langle Y_{max}, std \rangle.$$

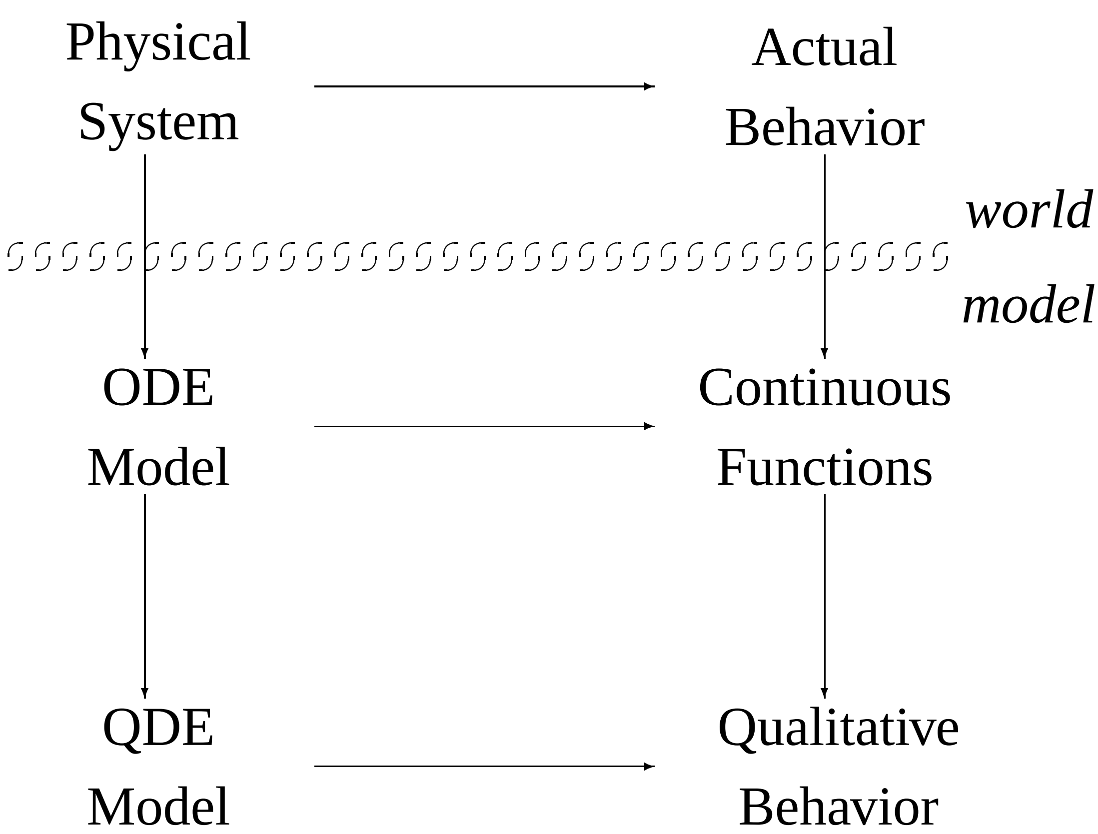
The new landmark $0 < Y_{max} < \infty$ has been discovered.

The Tree of Successor States Defines the Qualitative Behaviors



- The ball goes up . . .
- It stops, defining a new landmark value of Y
- . . . and falls back . . .

Qualitative Models are Abstractions of Ordinary Differential Equations



$$DiffEqs \vdash ODE, State(t_0) \rightarrow Beh_k$$

$$QSIM \vdash QDE, State(t_0) \rightarrow or(Beh_1, Beh_2, \ldots Beh_n)$$

Guaranteed coverage:

- **Theorem**: All real solutions are predicted.
- But not all impossible disjuncts Beh_i are filtered out.

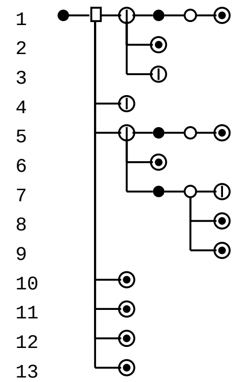
Progressive Filtering of the Behaviors

1. **Basic qualitative simulation**
   - Qualitative value transitions;
   - Constraint filtering to create states;
2. **State-based filters**
   - Higher-order derivative constraints;
   - Order of magnitude constraints;
3. **Behavior-based filtering**
   - Non-intersection of trajectories in qualitative phase space;
   - Energy conservation and dissipation;
   - Semi-quantitative constraints;
4. **Levels of abstraction**
   - Ignore/collapse certain descriptions;
   - Time-scale abstraction;

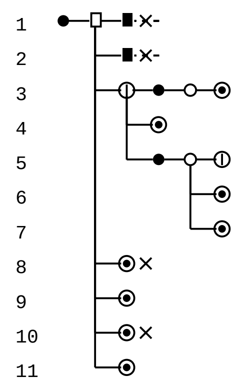
Each step is careful to preserve validity.

Prune the Behavior Tree

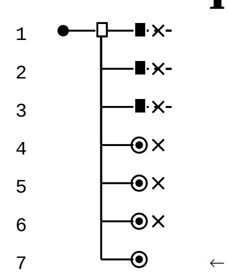
- QSIM gives 13 behaviors



- One filter prunes down to 7 behaviors

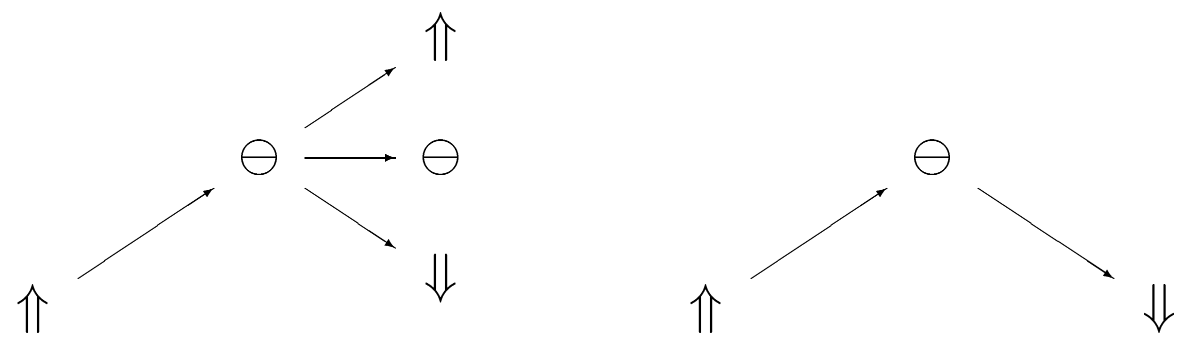


- Another prunes down to a unique behavior



Higher-Order Derivative Constraints

Problem: the highest-order derivative may be unconstrained, causing intractable "chatter."



Using Higher-Order Derivatives

- Chatter produces three-way branches at critical points ($f'(t) = 0$).
- We get a one-way branch if we know that $f''(t) < 0$.

Algebraically Derive The Curvature

- Higher-order derivatives can be derived algebraically from the constraint model (QDE).
- Higher-order terms just push the problem up a level.
- Derive **HOD constraints**, which apply only when $f'(t) = 0$.

Order of Magnitude Reasoning

Order of Magnitude relations

- $A \cong B$ — A is close to B.

$$A \cong A$$
$$A \cong B \rightarrow B \cong A$$
$$A \cong B, B \cong C \rightarrow A \cong C$$
$$A \cong B, [C] = [A] \rightarrow (A + C) \cong (B + C)$$

- $A \sim B$ — A has the same order of magnitude as B.

$$A \sim B \rightarrow B \sim A$$
$$A \sim B, B \sim C \rightarrow A \sim C$$
$$A \sim B \rightarrow [A] = [B]$$
$$A \cong B \rightarrow A \sim B$$

- $A \ll B$ — A is negligable compared to B.

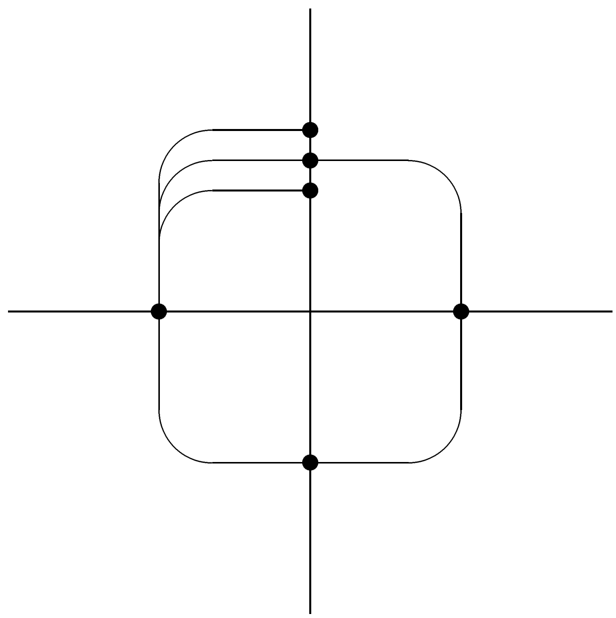
$$A \ll B, B \ll C \rightarrow A \ll C$$
$$A \ll B, B \sim C \rightarrow A \ll C$$
$$A \ll B \rightarrow -A \ll B$$

Propagate. Then filter inconsistent states.

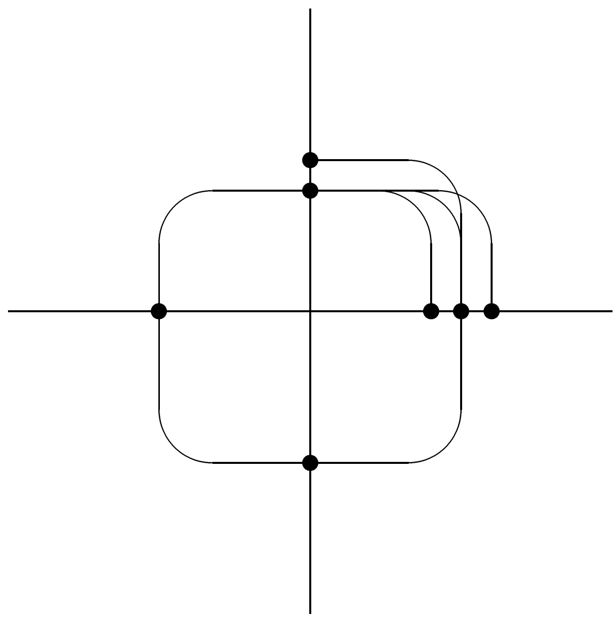
Qualitative Phase Portraits

Phase portraits provide global, two-dimensional constraints on possible behaviors:

- Lyapunov ("energy") constraints.



- Trajectories may not intersect.



Improved QSIM Filters

- Improved filters and analysis [Cem Say]
  - L'Hôpital filter
  - infinity filter
  - sign-equality assumption
  - relative duration
  - etc.
- Lyapunov analysis [Hofbaur]
- Discontinuous change [Biswas & Mosterman]

What Are The Unique Strengths of Qualitative Reasoning?

- Ability to express incomplete knowledge and reason effectively with it.
- Ability to predict the *entire* behavior, from beginning to end.
- Guarantee that all possibilities are covered, within explicitly enumerated modeling and simulation assumptions.
- Multiple successors to a state.

What Else Is Required for Practical Value in Applications?

- **Automated model-building**.

  Identification and representation of modeling assumptions.

- **Tractable qualitative simulation**.

  Everything else builds on the framework generated by qualitative simulation.

- **Semi-quantitative inference**.

  As uncertainty $\rightarrow 0$, we want
  prediction error $\rightarrow 0$ as well.

- **Complementary methods**:

  Monte Carlo simulation
  fuzzy representations and inference
  optimization methods
  probabilistic reasoning
  temporal logic

Times Have Changed

In the beginning, we argued against the position:

- Numerical methods are necessary and sufficient for reasoning about physical systems.

We showed that . . .

- QR alone can derive surprisingly strong conclusions.
- QR is an important, though implicit, part of most quantitative reasoning.

Now, we need to build **hybrid reasoning** systems.

- compositional model-building
- algebraic reasoning
- qualitative simulation
- semi-quantitative reasoning (bounds and envelopes)
- parameter estimation (Kalman filters)
- numerical simulation (Monte Carlo)

Research Problems

- Integrate QSIM with numerical simulation, algebraic manipulation, data handling package (MATLAB, LabView, etc.).
- Is there a QSIM Completeness Theorem?
  - Is there a set of filters that leaves only real behaviors?
  - Or is there a Gödel-like incompleteness theorem, saying that the properties of real dynamical systems are too rich to be captured by any symbolic theory?

More Information:
http://www.cs.utexas.edu/users/qr

Qualitative Reasoning

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