monet-4

Qualitative Design

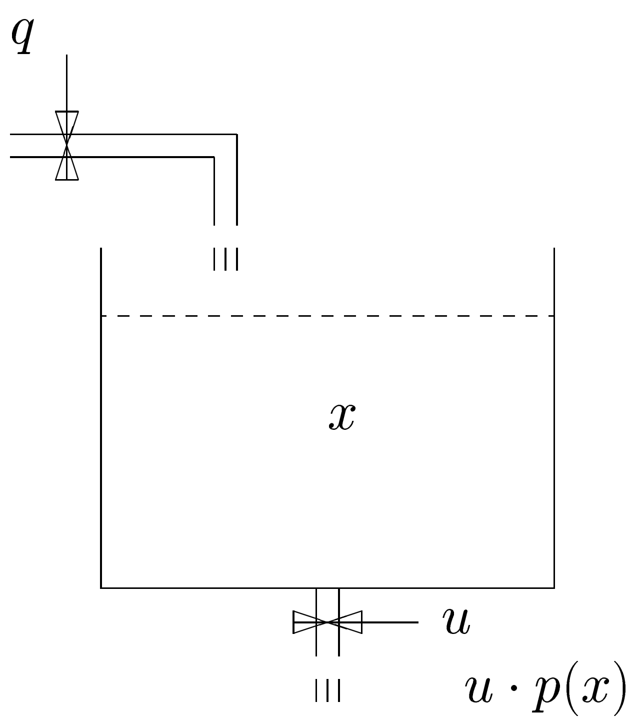
How does qualitative reasoning help in design?

1. Find qualitative properties of initial design.
2. Transform design to eliminate bad properties and add or ensure good ones.
3. Reach a design with the correct qualitative properties.
4. Construct a qualitative proof of correctness.
5. Accumulate the algebraic and numerical constraints required for the proof to hold.
6. Any remaining degrees of freedom may be used to optimize the design.

Heterogeneous ("fuzzy") Control

- Design classical controllers in their own operating regions.
- Overlapping operating regions defined by fuzzy set membership functions.
  - "appropriateness measures" for the controller
- The output of the heterogeneous controller is
  - the average output of the local controllers,
  - weighted by their appropriateness measures.
- Classical optimality within each region;
  Smooth transitions.
- Example: the water tank controller

Level Control of the Water Tank

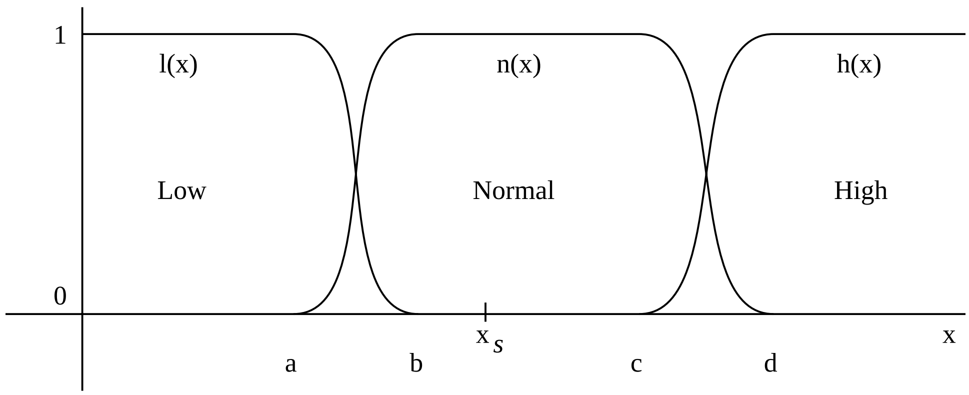


$$\dot{x} = f(x, u) = q - u \cdot p(x).$$

$$\begin{aligned} x &= \text{amount in tank} \\ q &= \text{inflow into tank} \\ u &= \text{drain area} \\ p(x) &= \text{influence of pressure at drain} \end{aligned}$$

A Heterogeneous Controller

The operating regions and their appropriateness measures:



The local control laws:

$$x \in Low \;\Rightarrow\; u_l(x) = 0$$
$$x \in Normal \;\Rightarrow\; u_n(x) = k(x - x_s) + u_s$$
$$x \in High \;\Rightarrow\; u_h(x) = u_{max}$$

The global control law:

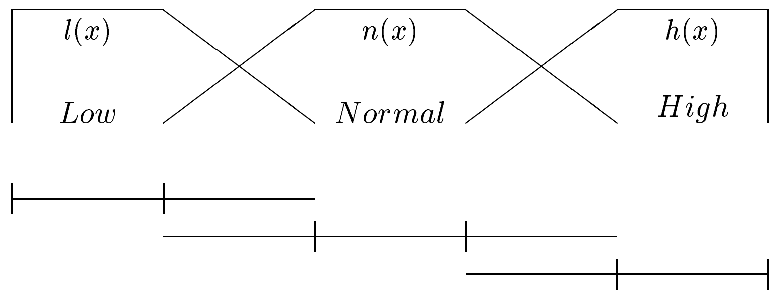
$$u(x) = l(x)u_l(x) + n(x)u_n(x) + h(x)u_h(x).$$

The discrete abstraction:

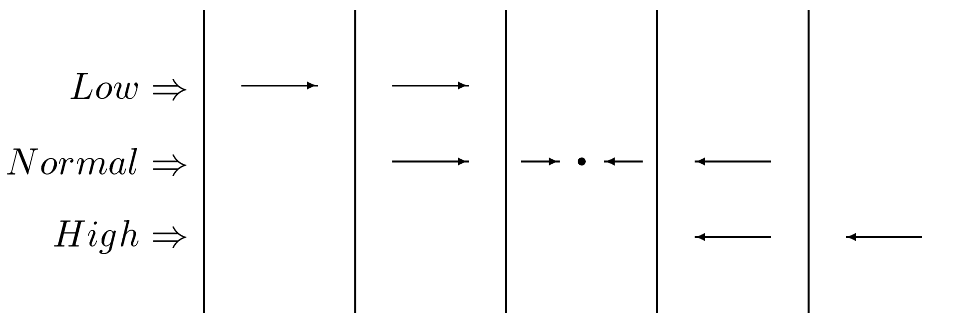
$$\boxed{\text{Low}} \longrightarrow \boxed{\boxed{\text{Normal}}} \longleftarrow \boxed{\text{High}}.$$

Qualitative Combination of Behaviors

- Overlapping operating regions for the local laws.



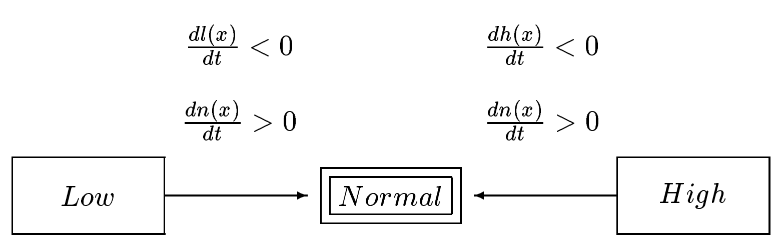
- Require qualitative agreement where laws overlap.



- Guarantee monotonic behavior in overlap regions.

$$
\begin{aligned}
Low &\Rightarrow q > 0 \\
Normal &\Rightarrow q_b < q < q_c \\
High &\Rightarrow q < u_{max} \cdot p(c)
\end{aligned}
$$

- Abstract the control law to a finite transition diagram.



Designing Qualitative Behaviors

- Simple but general qualitative models have reliable properties.
- The tank model: $\dot{x} = -f(x)$, where $f \in M^+$.
  - Converges to stable fixed point: $f(x) \longrightarrow 0$.
- The undamped spring: $\ddot{x} = -h(x)$, where $h \in M_0^+$.
  - Periodic oscillation.
- The damped spring: $\ddot{x} = -h(x) - f(\dot{x})$, where $f, h \in M_0^+$.
  - Converges to stable fixed point: $x \longrightarrow 0$.
  - Converges via spiral or nodal trajectory, depending on $sign(b^2 - 4c)$, where $b = f'(0)$ and $c = h'(0)$.

We can design qualitative behaviors by matching these models in local operating regions.

Design the Inverted Pendulum (applying torque at the pivot)

- The pendulum is: $\ddot{\theta} = -g \sin\theta - f(\dot{\theta})$.
  - $(0,0)$ is a stable attractor,
  - $(\pi, 0)$ is an unstable saddle.
- With controller: $\ddot{\theta} = -g \sin\theta - f(\dot{\theta}) + u(\theta, \dot{\theta})$.
- Design $u(\theta, \dot{\theta})$ so the model has the right qualitative behavior.
- Make $(0,0)$ a spiral repellor by adding a "negative friction" component $p(\dot{\theta})$.
- Make $(\pi, 0)$ a stable attractor by adding a "spring force" component $q(\theta - \pi)$.
- Constraints follow from requirement that net forces be monotonic functions of θ or $\dot{\theta}$.
- Another constraint follows from need to get from $nbd(0,0)$ to $nbd(\pi, 0)$.

The cart-pole version of the problem is a little more complicated.

Bottom: Attractor to Spiral Repellor

- Near $(\theta, \dot{\theta}) = (0, 0)$, the pendulum is qualitatively a damped spring.

  $\ddot{\theta} = -h(\theta) - f(\dot{\theta})$

  $\ddot{\theta} = -g \sin\theta - f(\dot{\theta})$

- The friction term $f(\dot{\theta})$ is responsible for the spiral in.
- We can get spiral out behavior if we have "negative friction" (pumping).
- Define the controller $u(\theta, \dot{\theta}) = p(\dot{\theta})$ such that

$$p(\dot{\theta}) - f(\dot{\theta}) \in M_0^+.$$

- The result is a "negatively damped spring":

$$\ddot{\theta} = -g \sin\theta - f(\dot{\theta}) + p(\dot{\theta})$$

  that will spiral away from $(0, 0)$.

Top: Saddle to Stable Attractor

- Near $(\theta, \dot{\theta}) = (\pi, 0)$, the pendulum has a saddle point.
- Changing variables to $\phi = \theta - \pi$ gives

$$\ddot{\phi} = g \sin \phi - f(\dot{\phi})$$

  with "negative spring force" making $(\pi, 0)$ unstable.
- Define the controller $u(\phi, \dot{\phi}) = -R(\phi)$ such that

$$R(\phi) - g \sin \phi \in M_0^+.$$

- The result is a damped spring:

$$\ddot{\phi} = -R(\phi) + g \sin \phi - f(\dot{\phi})$$

  which converges $\phi \longrightarrow 0$.

Can Pumping Reach from Bottom to Top?

- Let θ_1 be the *minimum* angle from which the Top controller can pick up the stopped pendulum.
- Let θ_0 be the maximum angle reached on the *preceding* cycle.
-
- In the end, the constraint is:

The Pendulum Controller

- **The Controllers**
  - **Bottom**: $\ddot{\theta} = -g \sin\theta - f(\dot{\theta}) + p(\dot{\theta})$
  - **Top**: $\ddot{\theta} = -R(\theta - \pi) + g\sin(\theta - \pi) - f(\dot{\theta})$
  - There is also a “Fast” region, to stop spinning.
- **The Constraints**
  - **Bottom**: $p(\dot{\theta}) - f(\dot{\theta}) \in M_0^+$.
  - **Top**: $R(\phi) - g\sin\phi \in M_0^+$.
  - **Transition**:
- **The Regions**:
  - **Bottom**: neighborhood around $(\theta, \dot{\theta}) = (0, 0)$.
  - **Top**: neighborhood around $(\theta, \dot{\theta}) = (\pi, 0)$.
  - **Fast**:
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Behavior of the Controller

References

- Benjamin J. Kuipers and Karl J. Åström. 1994. The composition and validation of heterogeneous control laws. *Automatica* **30**(2), February 1994.