Unit A1.2 Qualitative Modeling

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Overview

- Ontologies for qualitative modeling
- Quantities and values
- Qualitative mathematics
- Reasoning with qualitative mathematics

Design Space for Qualitative Physics

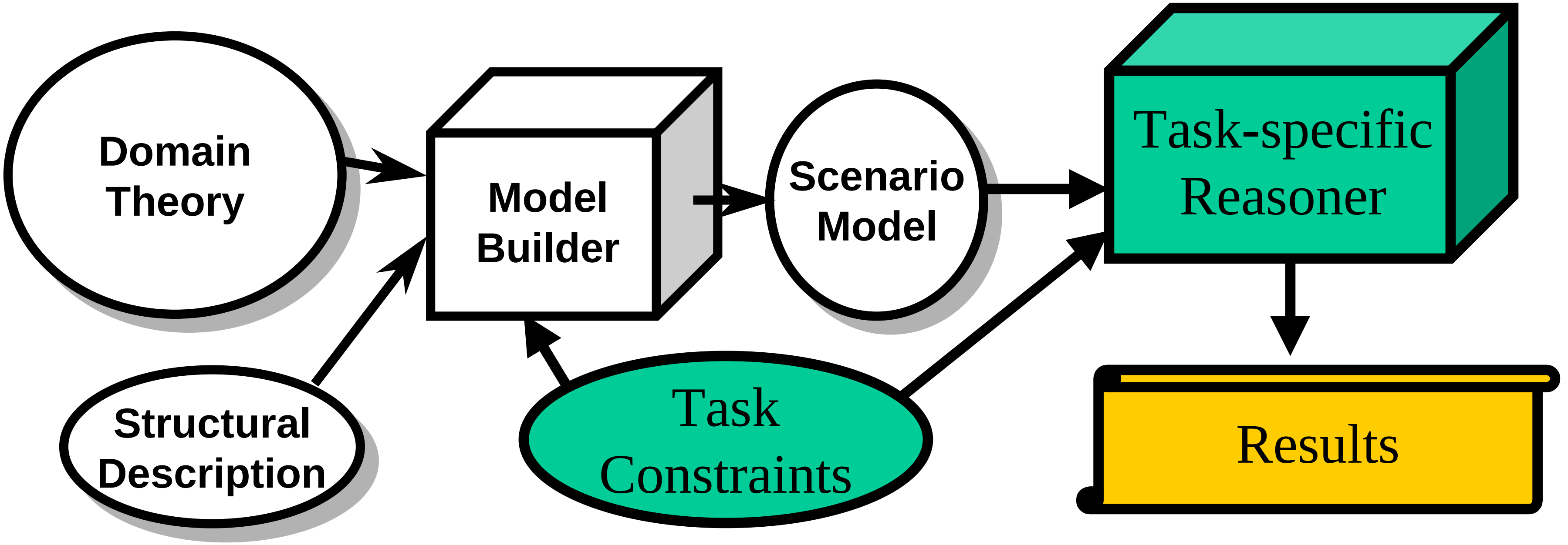
- Factors that make up a qualitative physics
  - Ontology
  - Mathematics
  - Causality
- Some parts of the design space have been well explored
- Other parts haven’t

Goal: Create *Domain Theories*

- Domain theory is a knowledge base that
  - can be used for multiple tasks
  - supports modeling of a wide variety of systems and/or phenomena
  - supports automatic formulation of models for specific situations.
- Examples of Domain theory enterprises
  - Engineering thermodynamics (Northwestern)
  - Botany (Porter’s group, U Texas)
  - Chemical engineering (Penn)
  - Electro-mechanical systems (Stanford KSL)

Organizing Domain Theories

- Domain theory = collection of general knowledge about some area that can be used to model a wide variety of systems for multiple tasks.
- Scenario model = a model of a particular situation, built for a particular purpose, out of fragments from the domain model.



Ontology

- The study of what things there are
- Ontology provides organization
  - Applicability
    - When is a qualitative relationship valid? Accurate? Appropriate?
  - Causality
    - Which factors can be changed, in order to bring about desired effects or avoid undesirable outcomes?

How Ontology addresses Applicability

- Figure out what kinds of things you are dealing with.
- Associate models with those kinds of things
- Build models for complex phenomena by putting together models for their parts

Ontology 0: Math modeling

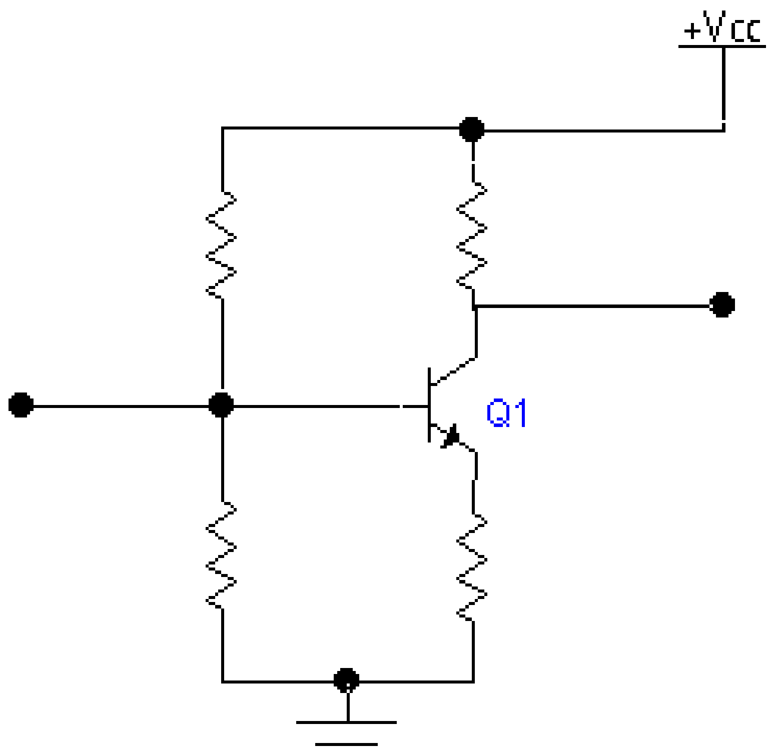
- Just start with a set of equations and quantities
- Many mathematical analyses do this
- QSIM does, too. QDE’s instead of ODE’s
- Advantage: Simplicity
- Drawback: Modeling is completely manual labor, often ad hoc.

Ontology 1: Components

- Model the world as a collection of components connected together
  - Electronic circuits
  - Fluid/Hydraulic machinery
  - etc -- see System Dynamics
- Model connections via links between properties
  - Different kinds of paths
  - Nodes connect more than two devices

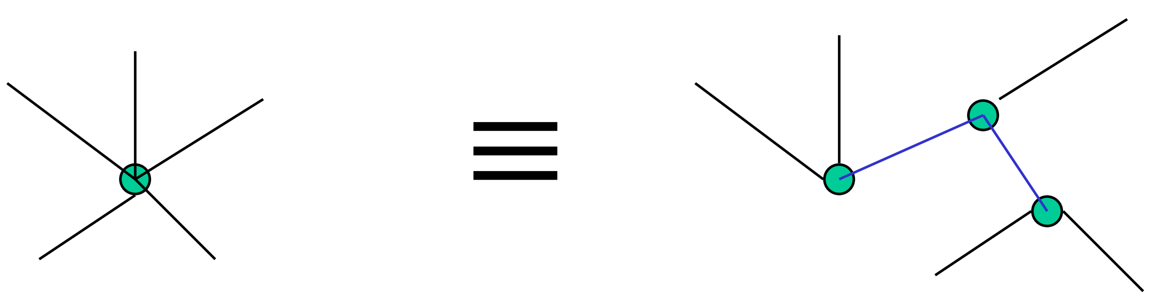
Classic case: Electronics

- Components include resistors, capacitors, transistors, etc.
- Each component has *terminals*, which are connec



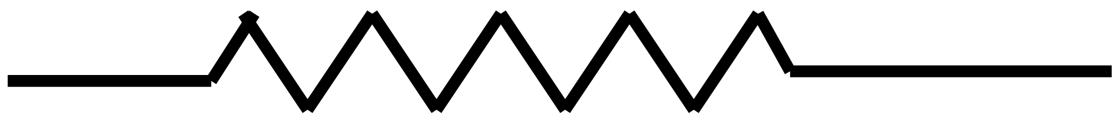
Nodes in electronics

- 2-terminal node = wire
- 3-terminal node = junction
- Can build any size node out of 2 & 3 terminal nodes
  - theorem of *circuit theory* in electronics.



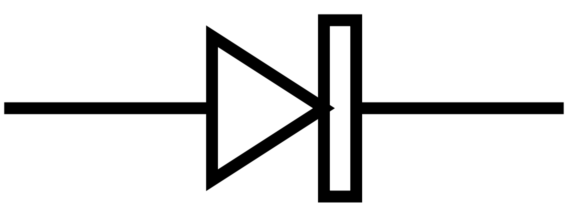
Component Laws

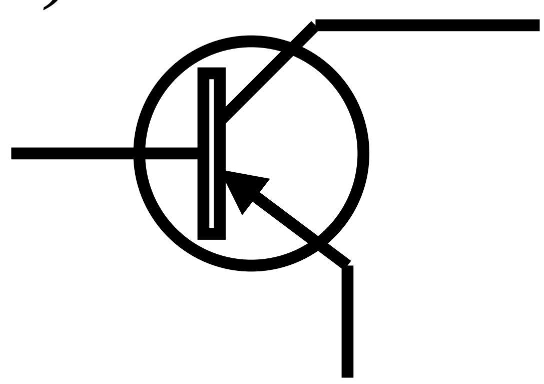
- Associate qualitative or quantitative laws with each type of component
- Example: Resistor
  - Quantitative version: V = IR
  - Qualitative version: [V] = [I] + [R]



States in components

- Some components require multiple models, according to *state* of the component
- Example: diode
  - Only lets current flow in one direction
  - *Conducting* versus *Blocked* according to polarity of voltage across it
- Example: Transistors can have several states (*cutoff*, *linear*, *saturated*, *etc.)*





Building circuits

- Instantiate models for parts
- Instantiate nodes to connect them together
- And then you have (almost) a model for the circuit, via the combination of the models for its parts

Other laws needed to complete models

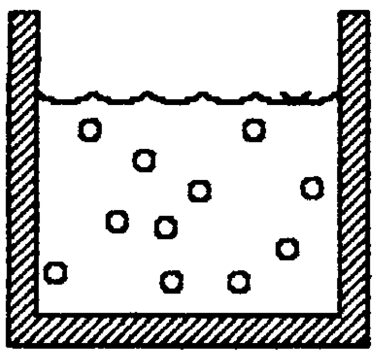
- Kirkoff's Current Law
  - Sum of currents entering and leaving a node is zero
    - i.e., no charge accumulates at nodes
  - Local, tractable computation
    - Example: **`0 = [i1] + [i2] + [i3]`**
- Kirkoff's Voltage Law
  - Sum of voltages around any path in a circuit is zero
  - In straightforward form, not local. Requires finding all paths through the circuit
  - Heuristic: Do computation based on exhaustive combination of triples of nodes.

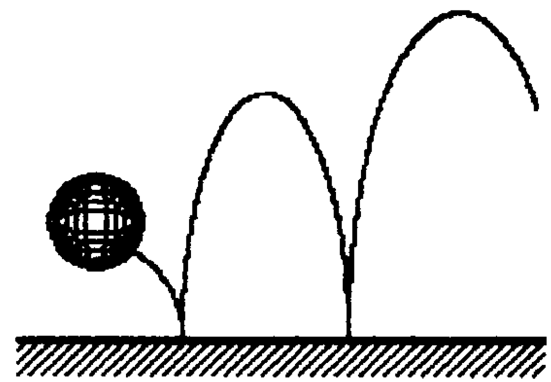
Component ontology is appropriate when…

- Other properties of “stuff” flowing can be ignored
- No significant “stuff” stored at nodes
  - Otherwise KCL invalid
- All interactions can be limited to fixed set of connections between parts

Component ontology often inappropriate

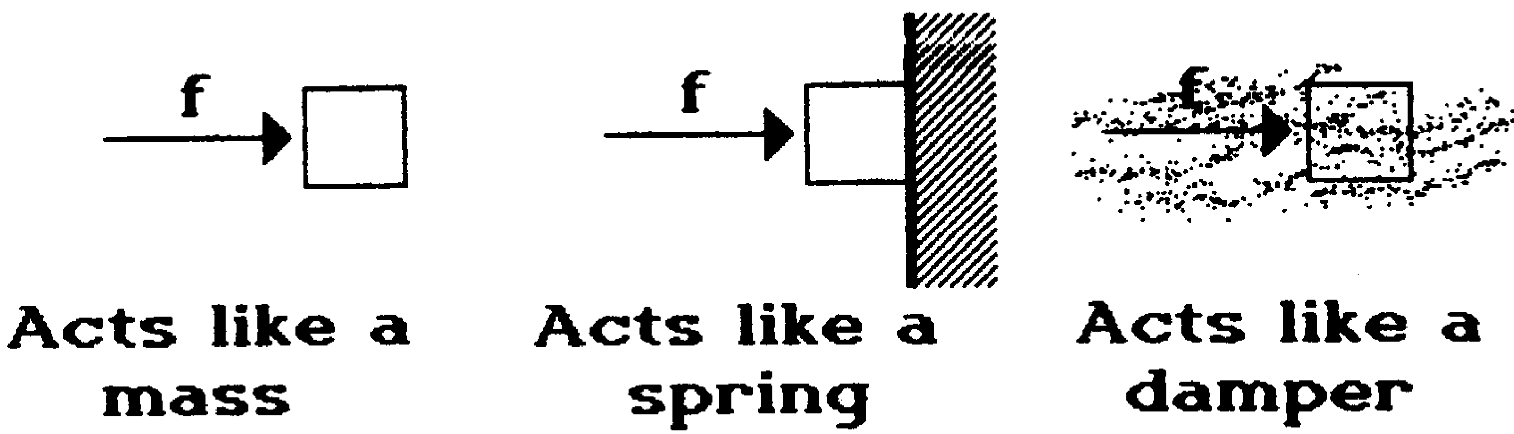
- Motion: Momentum flows??
- Real fluids accumulate





Components avoid interesting modeling problems

- Step of deciding what components to use lies outside the theory
- How should one model a mass?

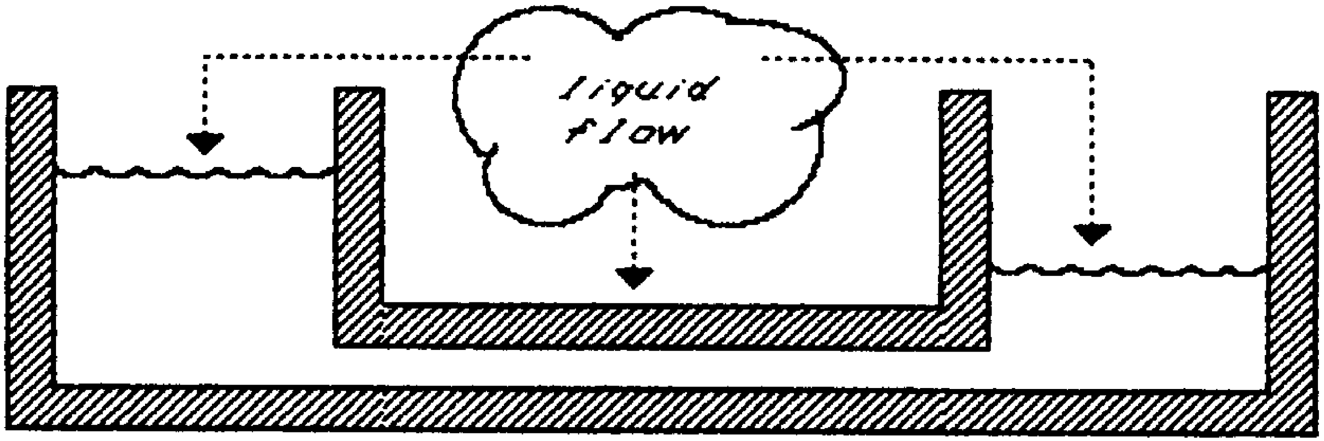


Ontology 2: Physical Processes

- All changes in world due to physical processes
- Processes act on collections of objects related appropriately.
- Equations associated with appropriate objects, relationships, and processes

Example: Fluids

- Entities include containers, fluid paths, heat paths.
- Relationships include connectivity, alignment of paths
- Processes include fluid flow, heat flow, boiling, condensation.



How processes help in modeling

- Mapping from structural description to domain concepts is part of the domain theory
- Given high-level structural description, system figures out what processes are appropriate.

The Number Zoo

Status Abstraction

Signs

Intervals

Fixed
Finite
Algebras

Ordinals

Fuzzy
Logic

Floating Point

Order of
Magnitude

Reals

Infinitesimals

Issues in representing numbers

- Resolution
  - Fine versus coarse? (i.e., how many distinctions can be made?)
  - Fixed versus variable? (i.e., can the number of distinctions made be varied to meet different needs?)
- Composability
  - Compare (i.e., How much information is available about relative magnitudes?)
  - Propagate (i.e., given some values, how can other values be computed?)
  - Combine (i.e., What kinds of relationships can be expressed between values?)
- Graceful Extension
  - If higher resolution information is needed, can it be added without invalidating old conclusions?
- Relevance
  - Which tasks is this notion of value suitable for?
  - Which tasks are unsuitable for a given notion of value?

What do we do with equations?

- Solve by plugging in values
  - When done to a system of equations, this is often referred to as *propagation*
  - $x+y=7$; if $x=3$, then we conclude $y=4$.
- Substitute one equation into another
  - $x+y=7$; $x-y=-1$; then we conclude $x=3$; $y=4$.

Signs

- The first representation used in QR
- The weakest that can support continuity
  - if **`[A]= -`** then it must be **`[A]= 0`** before **`[A]= +`**
- Can describe derivatives
  - **[∂A]=+** ≡ "increasing"
  - **[∂A]=0** ≡ "steady"
  - **[∂A]=-** ≡ "decreasing"

Confluences

- Equations on sign values
- Example: `[x]+[y] = [z]`
- Can solve via propagation
  - If `[x]=+` and `[z]=-` then `[y]=-`
  - If `[x]=+` and `[z]=+` then no information about `[y]`

Confluences and Algebra

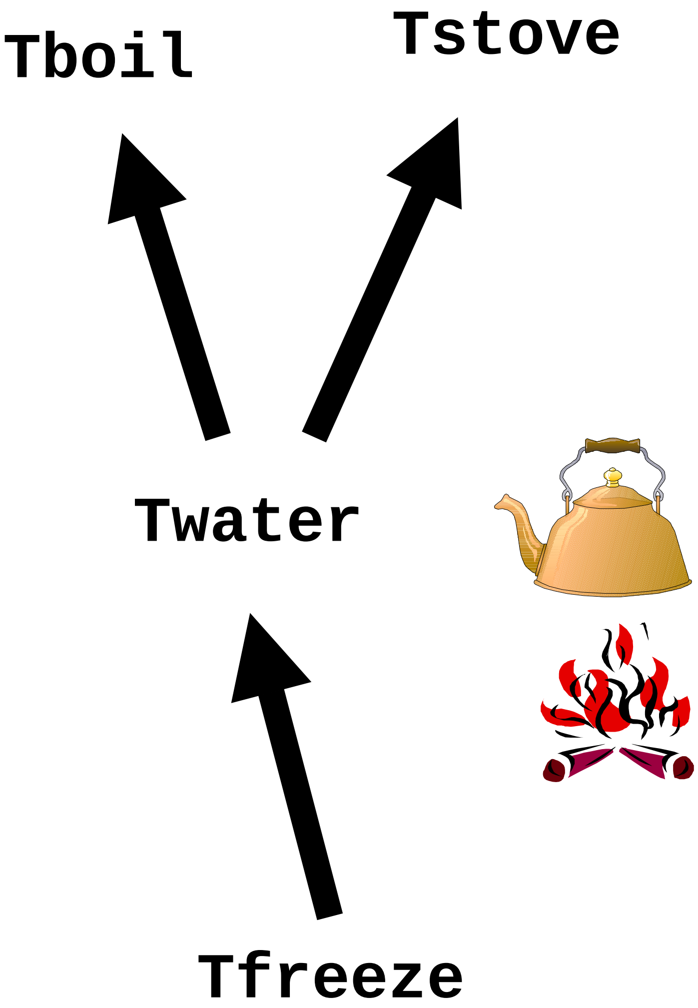
- Algebraic structure of signs very different than the reals or even integers
- Different laws of algebra apply
- Example: Can't substitute equals for equals
  - **`[X] = +, [Y] = +`**
  - **`[X] - [X] = 0`**
  - **`[X] - [Y] = 0`** ? Nope
    - (Suppose X = 1 and Y = 2)

Ordinals

- Describe value via relationships with other values
  A > B; A < C; A< D
- Allows partial information
  in the above, don't know relation between C and D
- Like signs, supports continuity and derivatives

Quantity Space

- Value defined in terms of ordinal relationships with other quantities
- Contents dynamically inferred based on distinctions imposed by rest of model
- Can be a partial order
- *Limit points* are values where processes change activation
- Specialization: *Value space* is totally ordered quantity space.



Landmark values

- Behavior-dependent values taken on at specific times
- Limit point $\Rightarrow$ Landmark
  - "The boiling point of water"
- $\neg$ [Landmark $\Rightarrow$ Limit point]
  - "The height the ball bounced after it hit the floor the third time."
- Landmarks enable finer-grained behavior descriptions

Monotonic Functions

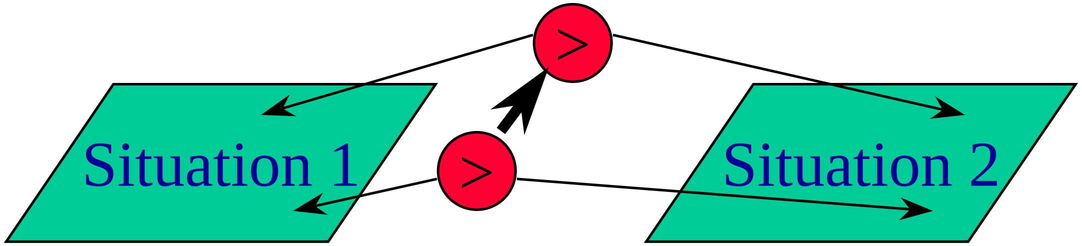
- Express direction of dependency without details
- Example: **`M+(pressure(w),level(w))`** says that **`pressure(w)`** is an increasing monotonic function of **`level(w)`**
  - When **`level(w)`** goes up, **`pressure(w)`** goes up.
  - When **`level(w)`** goes down, **`pressure(w)`** goes down.
  - If **`level(w)`** is steady, **pressure(w)** is steady.

Monotonic Functions (cont)

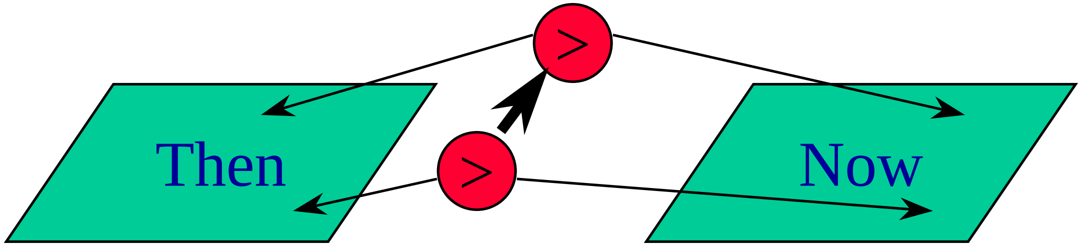
- Example:
  `M-(resistance(pipe),area(pipe))`
  - As **`area(pipe)`** goes up, **`resistance(pipe)`** goes down.
  - As **`area(pipe)`** goes down, **`resistance(pipe)`** goes up.
- Form of underlying function only minimally constrained
  - Might be linear
  - Might be nonlinear

What do we mean by “goes down”?

- Version 1: Comparative analysis



- Version 2: Changes over time



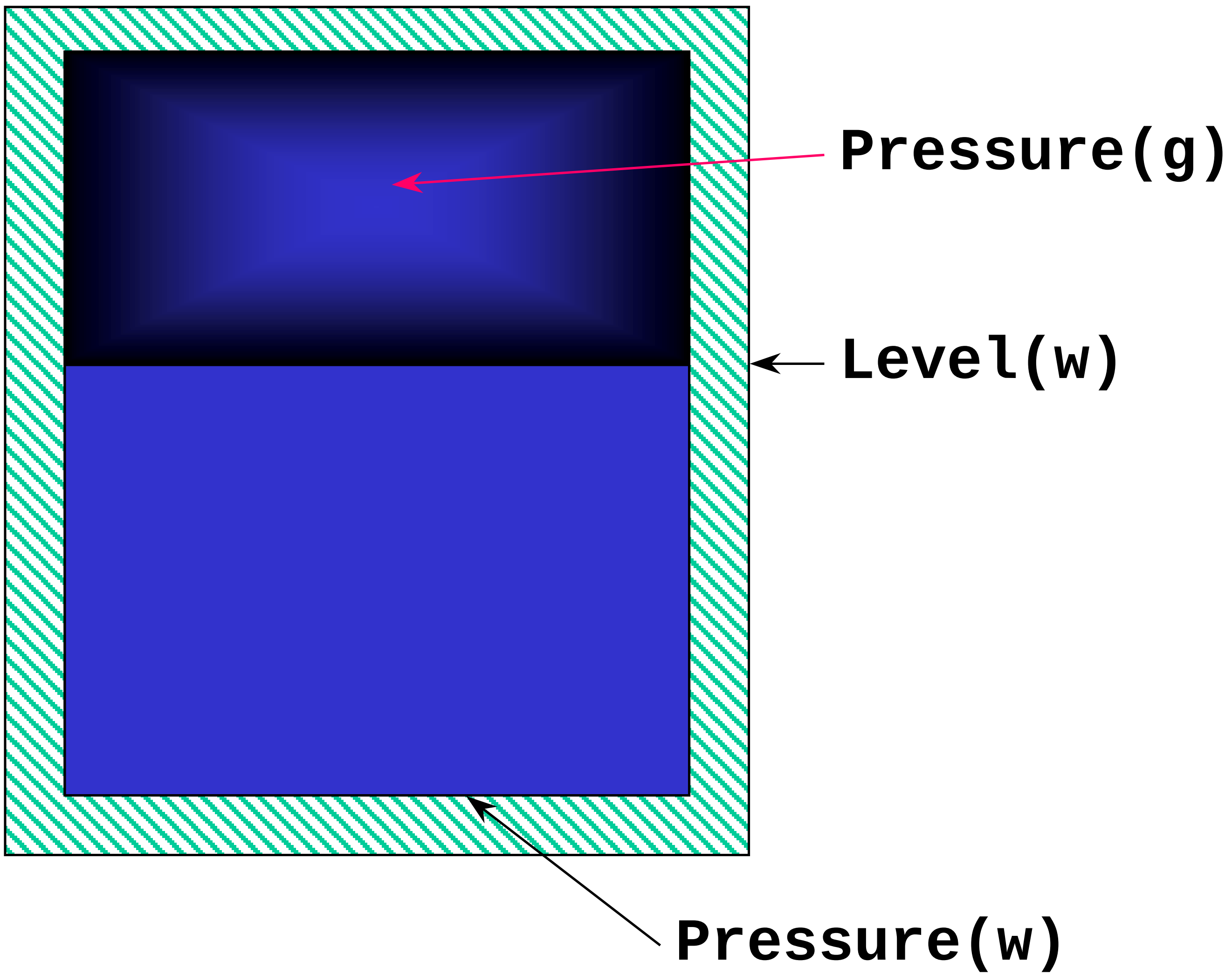
Qualitative proportionalities

- Examples
  - **`(qprop (temperature ?o) (heat ?o))`**
  - **`(qprop- (acceleration ?o) (mass ?o))`**
- Semantics of **`(qprop A B)`**
  - **$\exists$`f` s.t. `A = f(…, B,…)` $\wedge$ `f` is increasing monotonic in `B`**
  - For **`qprop-`**, decreasing monotonic
  - **`B`** is a causal antecedent of **`A`**
- Implications
  - Weakest causal connection that can propagate sign information
  - Partial information about dependency requires closed world assumption for reasoning

Qualitative proportionalities capture aspects of intuitive mental models

- “The more air there is, the more it weighs and the greater its pressure”
  - **`(qprop (weight ?air-mass) (n-molecules ?air-mass))`**
  - **`(qprop (pressure ?air-mass) (n-molecules ?air-mass))`**
- “As the air temperature goes up, the relative humidity goes down.”
  - **`(qprop- (relative-humidity ?air-mass) (temperature ?air-mass))`**
- Source: *Weather: An Explore Your World ™ Handbook.* Discovery Press

(qprop+ (pressure w) (pressure g))



Composability

- Can express partial theories about relationships between parameters
- Can add new qualitative proportionalities to increase precision

Cost of Composability

- Explicit *closed-world assumptions* required to use compositional primitives
- Requires understanding when you are likely to get new information
- Requires inference mechanisms that make CWA's and detect when they are violated

Causal Interpretation

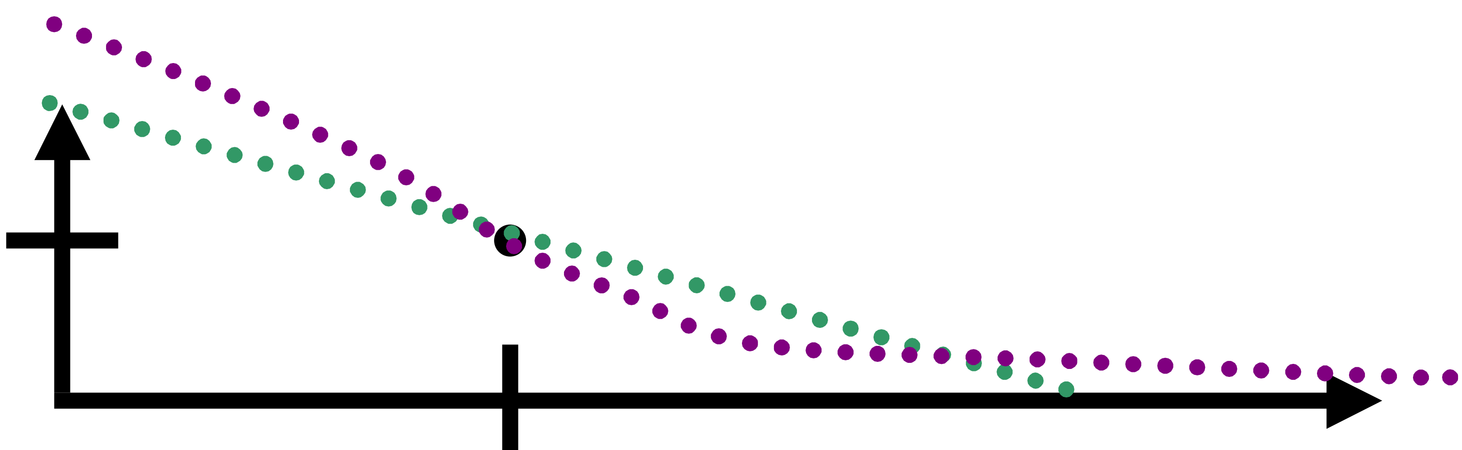
- **`(qprop+ A B)`** means that changes in B cause changes in A
- But not the reverse.
- Can never have both **`(qprop+ A B)`** and **`(qprop+ B A)`** true at the same time.

Resolving Ambiguity

- Suppose
  - `(Qprop A B)`
  - `(Qprop- A C)`
  - **`B`** & **`C`** are increasing.
  - What does **`A`** do?
- Without more information, one can't tell.

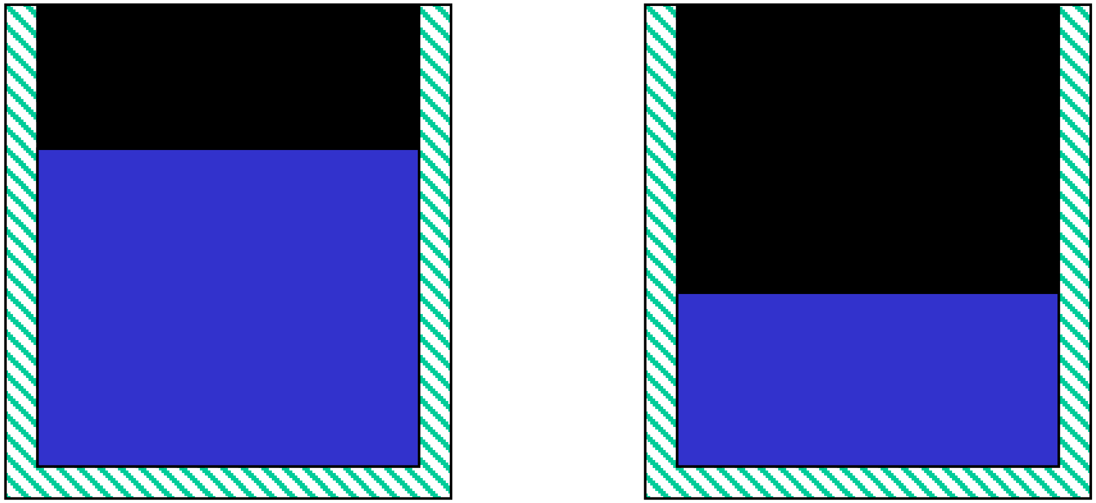
Correspondences

- Example:
  - `(correspondence ((force spring) 0)`
    `((position spring) 0)`
  - `(qprop- (force spring)`
    `(position spring))`
- Pins down a point in the implicit function for the qualitative proportionalities constraining a quantity.
- Enables propagation of ordinal information across qualitative proportionalities.



Explicit Functions

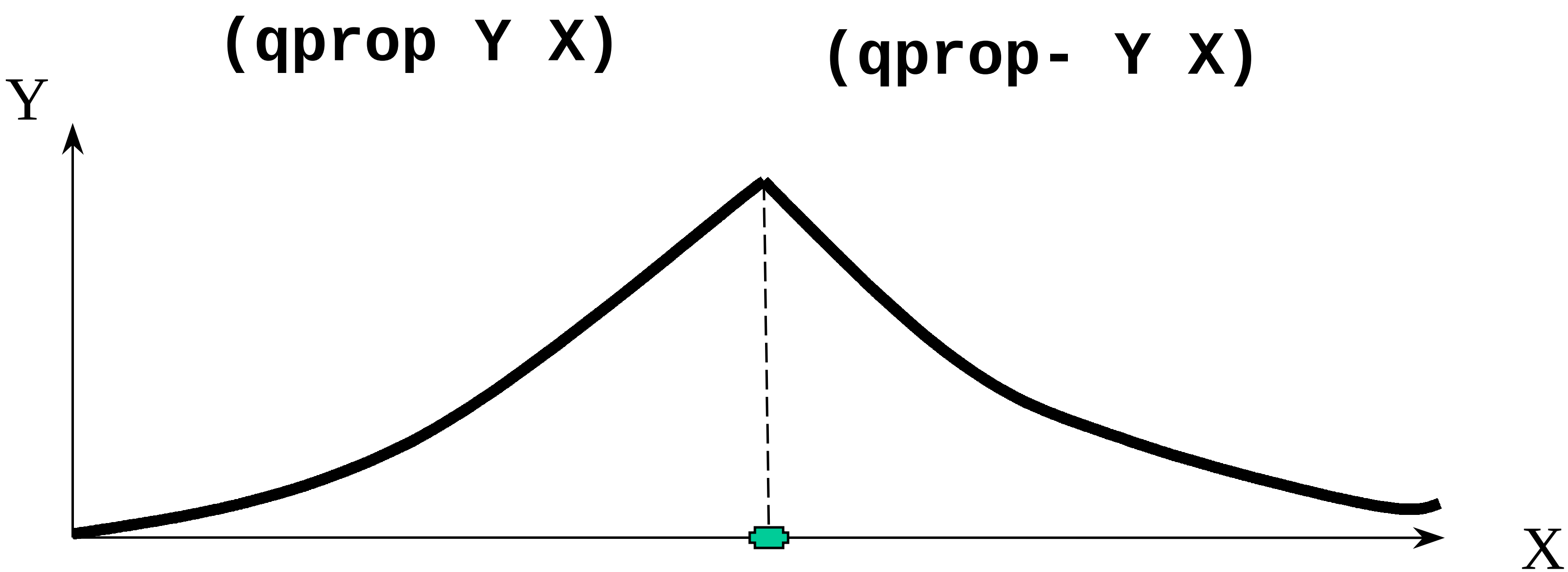
- Allow propagation of ordinal information across different individuals



Same shape, same size, same height ⇒
Higher level implies higher pressure

Representing non-monotonic functions

- Decompose complex function into monotonic regions
- Define subregions via limit points



Direct Influences

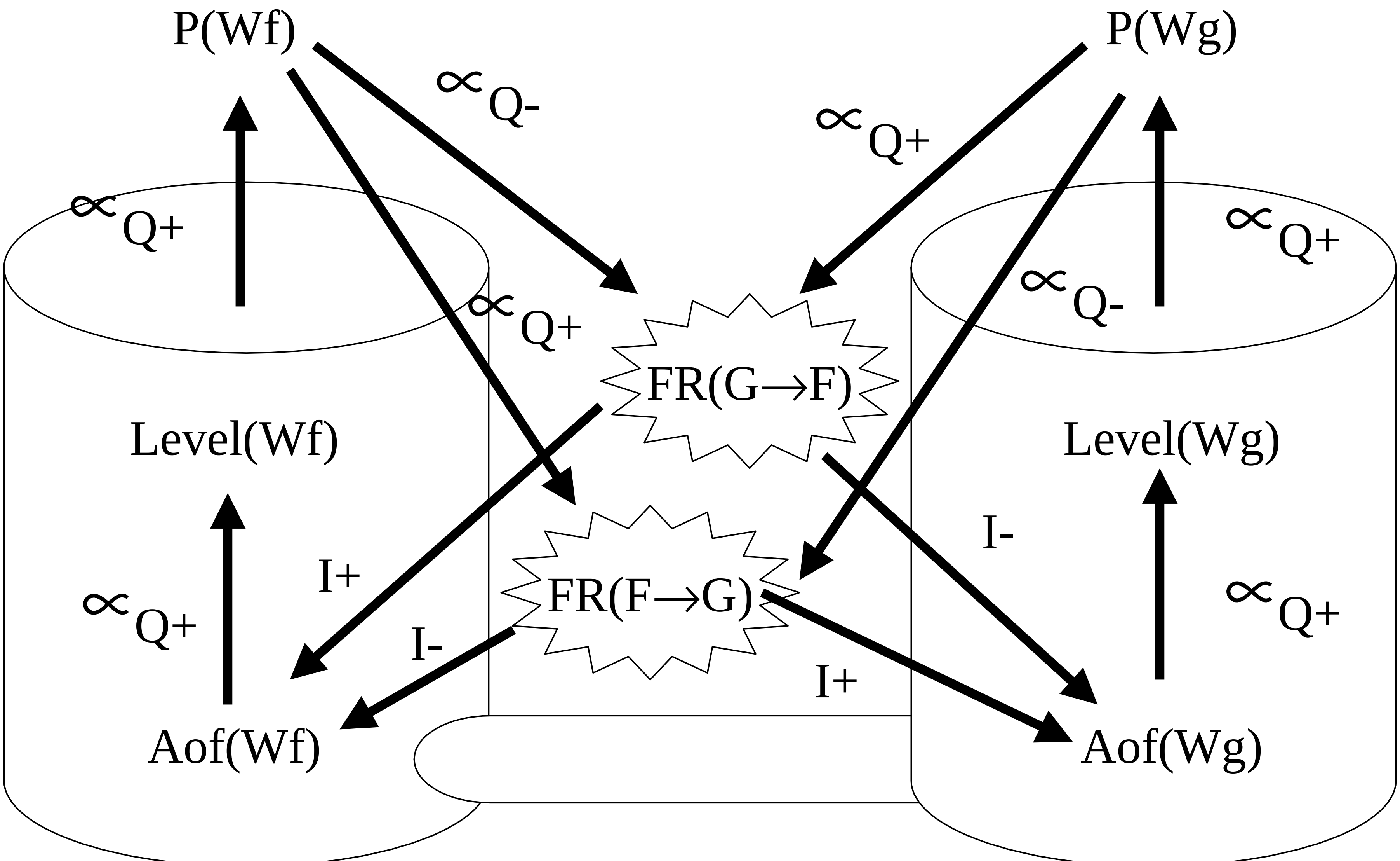
- Provide partial information about derivatives
  - Direct influences + qualitative proportionalities = a qualitative mathematics for ordinary differential equations
- Examples

```
I+(AmountOf(w),FlowRate(inflow))
I-(AmountOf(w),FlowRate(outflow))
```

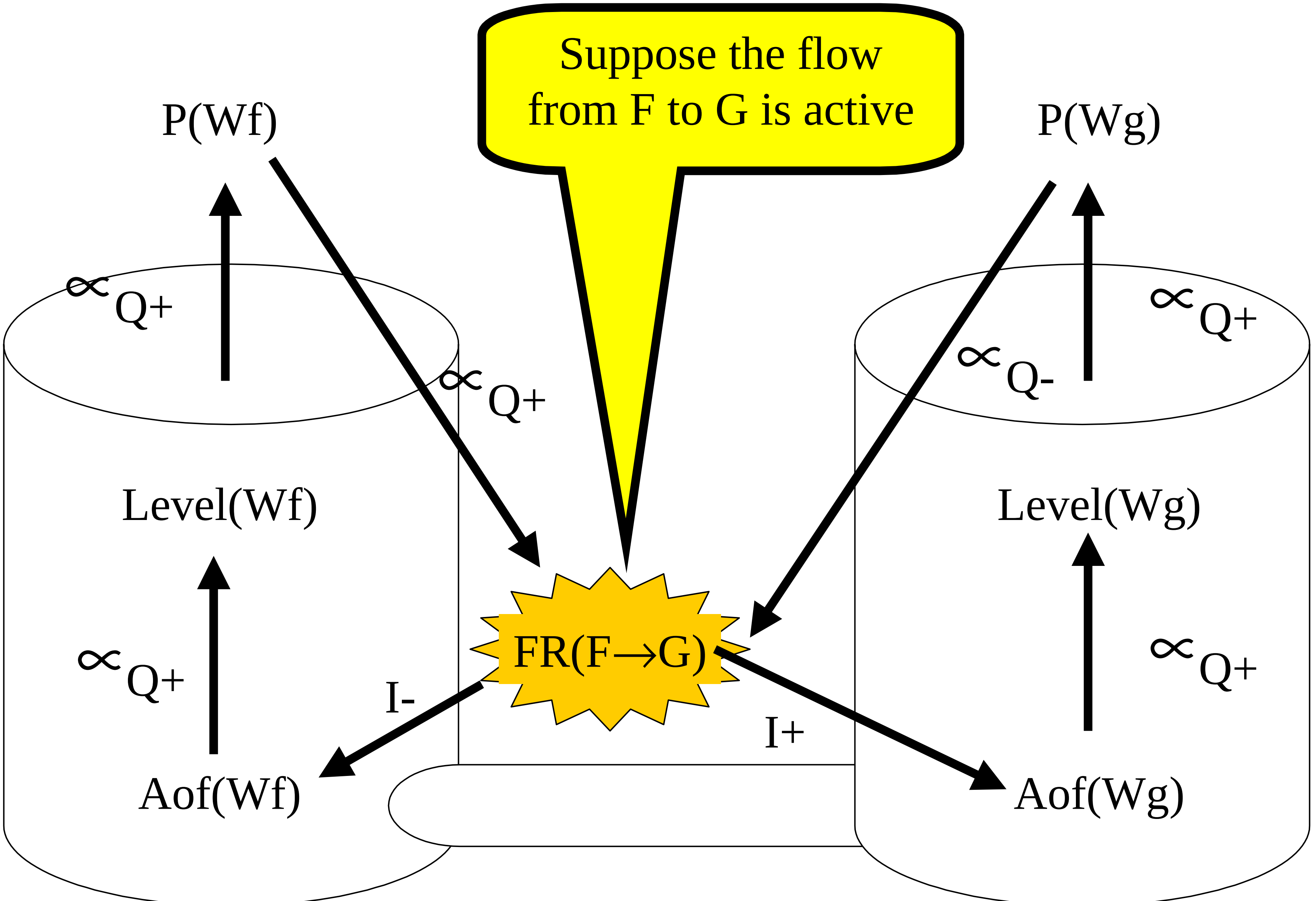
Semantics of direct influences

- **`I+(A,b)≡ D[A]=…+b+…`**
- **`I-(A,b)≡ D[A]=…-b+…`**
- Direct influences combine via addition
  - Information about relative rates can disambiguate
  - Abstract nature of qprop $\Rightarrow$ no loss of generality in expressing qualitative ODE's
- Direct influences only occur in physical processes (*sole mechanism assumption*)
- Closed-world assumption needed to determine change

Example of influences



Example of influences



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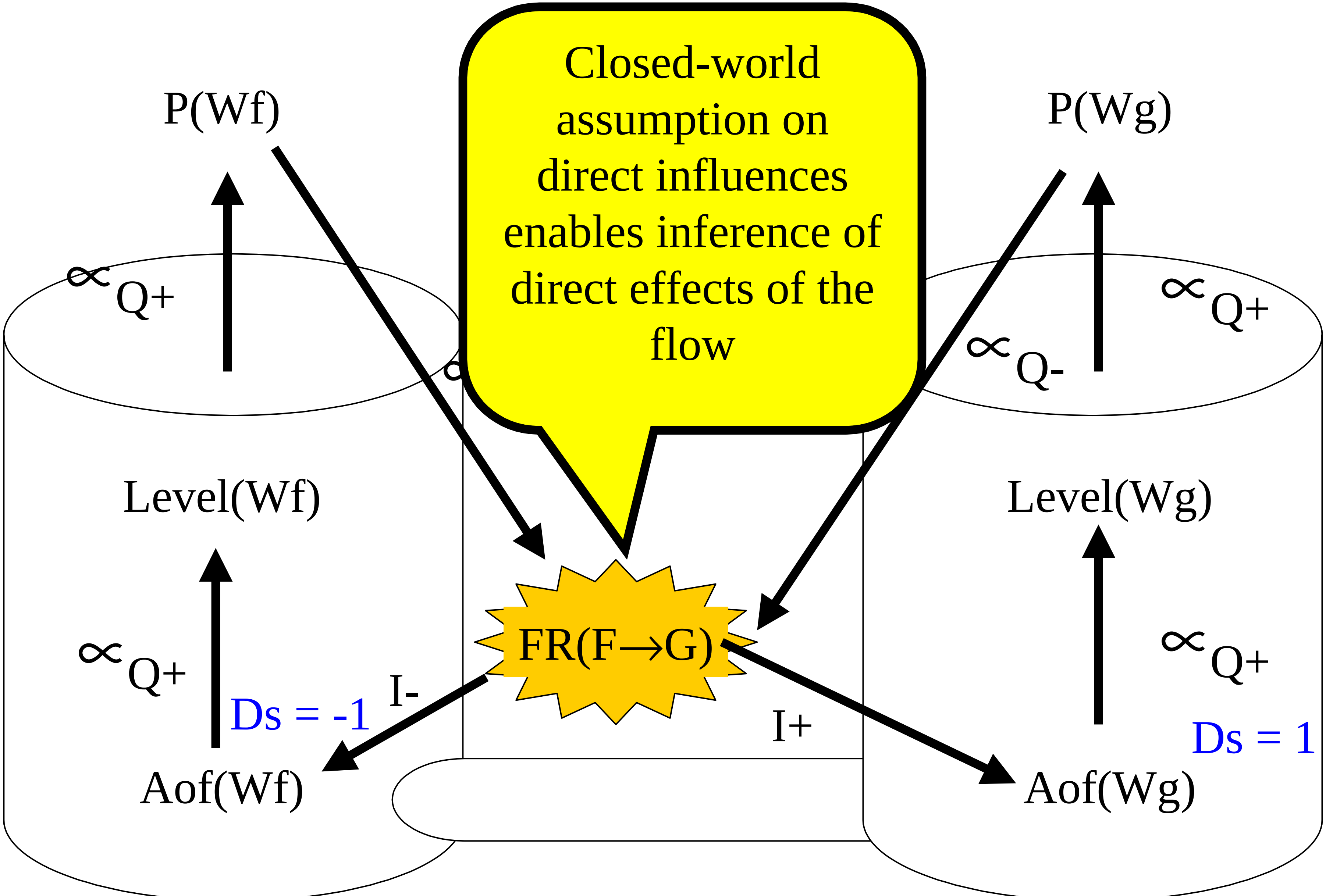

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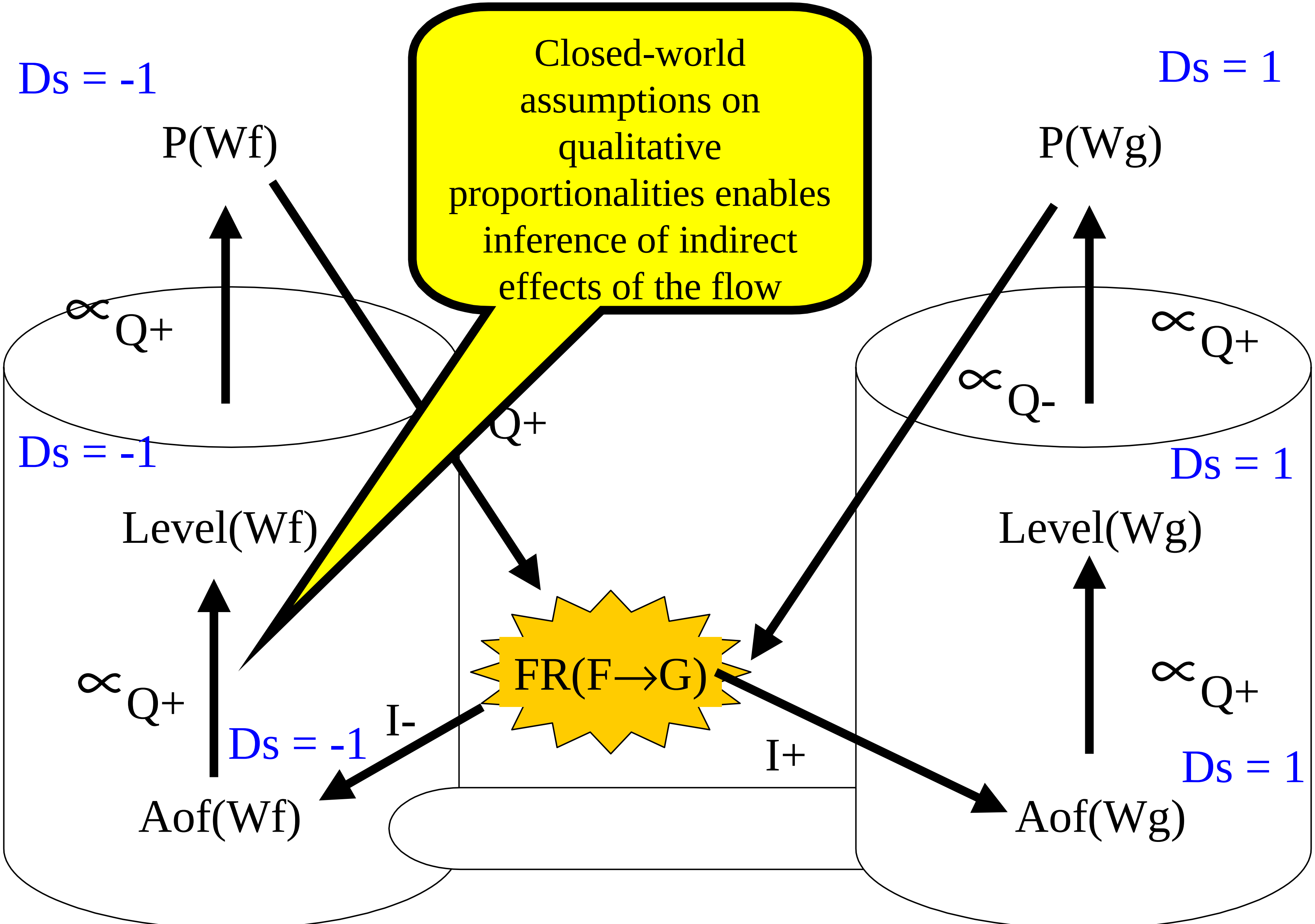

Example of influences