TROUBLESHOOTING : WHEN MODELING IS THE TROUBLE

P. Dague

Mailing address: Paris IBM Scientific Center 36 Ave R. Poincare 75116 Paris France Telephone: 33 (1) 45 05 14 00 P. 27-40 Address: earn/ bitnet dague at frpoil1

P. Devès

Electronique Serge Dassault 55 Quai M. Dassault S' Cloud 92214 France

O. Raiman

Paris IBM Scientific Center 36 Ave R. Poincare Paris 75116 France

Abstract

When troubleshooting, finding components where changes in behavior occur only guides the search. In a misbehaving device, a correct component can indeed change its behavior. Distinguishing faulty components from correct ones requires modeling all possible correct behaviors of components. For analog circuits, there is a lack of numerical models and a lack of information on essential parameters. Therefore, modeling becomes quickly the real trouble. Under the assumption that "a defect leads to significant change in the behavior of a device", we show that Qualitative Reasoning is a solution. To exploit the above assumption, order of magnitude reasoning is used for modeling and defining a strategy. Experience with the expert system DEDALE has shown that this assumption applies to a wide class of cases, and that Qualitative Reasoning can be used for a real size application.

Submitted to AAAI-87 (also submitted to IJCAI-87)

Paper type: Full paper

Topics: Engineering Problem Solving Commonsense Reasoning Troubleshooting Qualitative Reasoning

i

Track: Engineering

Keywords: Troubleshooting, diagnosis, analog electronics, model-based reasoning, qualitative reasoning, expert system.

Introduction

The challenge of troubleshooting is to localize, in a misbehaving device, replaceable or modifiable faulty components¹.

A classical approach requires providing a set of dependency relations between failures and faults. The efficiency of such "shallow" reasoning relies on the description of all possible cases of failures. This knowledge is strongly dependent on a particular device and often not completely available. Troubleshooting another device having the same functioning principles leads to reconsider the knowledge base.

The model-based paradigm ([1],[4]) leads to a more general approach, since only models of correct behavior for generic components have to be given. An interesting feature of this approach is that basically there is no need either for any fault model or for a set of heuristically defined dependencies between failures and faults. The device specific knowledge is organized around a structural decomposition of the device. It is assumed that any correct behavior of a complex device can be predicted using its structure and the models of its components. Thus, a difference between the predicted behavior of a block (i.e. a set of connected components), presumed correct, and the corresponding observed behavior indicates that at least one defect is present in the block. The task of troubleshooting is then to identify those differences and to progressively refine their localization until focusing on a small faulty replaceable part.

But identifying differences between the presumed correct behavior and the available observations requires defining relevant models of behavior for generic components. Numerical models used in classical simulation algorithms to predict the behavior of a well functioning device, are not adequate for troubleshooting purpose, as soon as correct components are led to work out of their normal functioning limits. In such cases, there is a lack of numerical models. On the other hand, basic qualitative models [6] handle mainly signs of quantities and are not powerful enough to find inconsistencies between the predicted behavior and observations. Therefore in both cases the predictive procedure may fail to detect conflicts. Modeling becomes quickly the trouble.

To overcome this difficulty, an idea is to consider the expert's heuristic which consists in reasoning only about the main changes in the behavior of a device. This means making the fundamental assumption that a defect leads to significant changes in the behavior of a device. Under this assumption, this paper highlights the efficiency of a *qualitative model-based* approach for troubleshooting. Qualitative models of behavior are described by introducing *order of magnitude equations*. Only conflicts resulting from differences in order of magnitudes are considered and de-

A component is an elementary physical element having a well defined function.

tected by solving order of magnitude equations. Reducing the complexity of this qualitative reasoning is done by defining a strategy exploiting the consequences of the assumption for the functional decomposition of the device. The idea is to focus first on small blocks where *deviations*, i.e. significant changes with respect to what in this paper we shall call for short the "designed behavior"² are observed.

First, the difficulty of providing a general approach for troubleshooting analog circuits is explained. Secondly, the consequences of the fundamental assumption are underlined. A third part shows how qualitative reasoning is done in the expert system DEDALE³. Fourth, a simple example of troubleshooting is given. Fifth, the strategy of DEDALE to troubleshoot more complex circuits is explained.

1. Complexity of Troubleshooting in Analog Circuits

Troubleshooting analog circuits is a delicate operation which requires high level skills and know-how. Today, it is not as embedded in computer aided design and test as it is for troubleshooting digital electronics. In a digital circuit, correct or misbehaving, design defines in a way the possible correct behavior of components. In a misbehaving analog circuit, a component can behave in a radical way differently than in its designed behavior, while remaining correct. The information required to describe the possible correct behavior of components of a component is often too complex or not available. Knowing the designed behavior of components enables to answer the question "how does a circuit work?". This is already a difficult task [6], that requires deducing the function of the entire circuit from its structure and from models of designed behavior of its generic components. But finding "why a circuit does not work?" is even more complex, because knowing the designed behavior of components modeling. A device independent approach for troubleshooting analog circuits⁴ must then cope with the following complex problems:

- ² the behavior of a device when it fulfils the function for which it has been designed.
- ³ DEDALE is an expert system for troubleshooting analog hybrid circuits, that has been jointly developed by Electronique Serge Dassault and IBM.
- ⁴ The case of intermittent failures is not taken into account in this paper: it is assumed the faulty circuit is in a steady electric state, and observations are reproducible.

Lack of Numerical Models

Numerical models of behavior for each component are often useless. Time dependance and non linearity make such models complex to use, except for some simple components (e.g. Ohm's law for resistors).

Example: A model of a transistor in a normal state requires specification of a dozen of parameters. Today, only simulation of correct functioning of a circuit uses such models.

Multiple Correct Behaviors

The behavior of each analog electronic component depends, most of the time, on all the components which are connected to it. This interdependancy has not a well defined direction (no concepts of input/output for terminals of a component). A defect on a component may influence the functioning state of other ones. Thus, predicting the behavior of the different components becomes quickly very complex.

Example: A transistor which is basically an analog component may have many different behaviors such as current amplifier, switch, etc, according to its electronic environment. For instance, too high a value of resistor on the emitter of a transistor causes this transistor to change from its normal (current amplifier) functioning state to an open (no current) functioning state.

Lack of Measurements

The observation of analog electronic behavior involves providing the values of state variables, some of them are not accessible to measurement. The inability in an analog circuit to measure currents is the most crucial of these limitations, because current is an essential state variable, that appears frequently in behavior models of generic components.

2. Exploiting Significant Change in Behavior

Some basic difficulties are encountered when setting troubleshooting of analog circuits in a model-based approach. In order to explain these difficulties, and to set the debate in a well defined framework, the General Diagnosis Engine (De Kleer and Williams [2]) is taken here as a reference. The General Diagnostic Engine, consists essentially of three parts: a predictive procedure, a measurement strategy, and an ATMS (Assumption-based Truth Maintenance System). The predictive procedure makes behavioral predictions from observations and assumptions of good functioning by using models and structure; it also detects symptoms (i.e. inconsistencies between predictions or inconsistencies between predictions and observations). A symptom expresses that among a set of assumptions on the correctness of components, at least one is false. The ATMS manages these assumptions, and determines from symptoms minimal conflicts (a conflict is a set of components, at least one of which is faulty), and generates a complete set of minimal candidates

(a candidate is a set of components which are all assumed faulty, and explains, i.e. intersects, all the $conflicts)^{5}$. The diagnosis procedure is incremental and guided by the measurement strategy.

De Kleer and Williams emphasizes the role of ATMS in the GDE, assuming the existence of a predictive procedure and of a strategy. The adequacy of a GDE to real problems is highly linked to the efficiency of the predictive procedure and of the strategy.

As far as analog circuits troubleshooting is concerned, as shown above, the basic difficulty is that modeling is a trouble. The lack of accurate models of correct behavior for generic components prevents from detecting *conflicts*. Under the fundamental assumption, that a defect leads to significant change in the behavior of the circuit, modeling becomes possible. Exploiting this assumption makes relevant the use of qualitative models of behavior. Nevertheless, the existence of multiple models of correct behavior and the lack of measurements to select one of them imply that several behaviors for each component must be considered. Thus, prediction in its strong sense is limited. Instead of implementing a predictive procedure, the idea, here, is to use a problem solver that checks if a set of qualitative models for connected components and a set of observations lead to an inconsistency between orders of magnitude of parameters involved.

A qualitative "big crunch" [7] (a brute force approach) is often not desirable. In order to reduce the complexity of this checking process, it is more efficient to search for a *conflict* only once focused on a little set (compared to the size of the circuit) of components. Exploiting again the consequence of the main assumption for the *hierarchical decomposition (structural and functional)* of the circuit leads to the following heuristic: a defect on a component induces significant changes of behavior of higher level blocks containing it. The strategy of DEDALE is the following:

- First, if there is a deviation in the behavior of a high level function (significant change with respect to the designed behavior), focus the search for the defect inside this function. So, the rule deviation -> focusing replaces the implication symptom -> conflict. This is an heuristic rule because a deviation of a high level function does not imply necessarily a defect of one of its components. Typically this deviation can be induced by a defect in an other function, connected to it.
- Secondly, if there is no significant observable change in the behavior of a function, assume in a first step that there is no defect inside this function. So, the rule is: no deviation -> don't focus. Once again this is only an heuristic rule since a defect inside a high level function can change its behavior in an unobservable way.

⁵ Every superset of a conflict must be a conflict, and every superset of a candidate must be a candidate. Representing minimal conflicts and minimal candidates is then sufficient.

3. Qualitative Reasoning in DEDALE

3.1. Order of Magnitude Reasoning

Reasoning about significant changes in the behavior of a circuit means performing qualitative reasoning. Models handling only signs of quantities (using a Quantity Space⁶ reduced to $\{0, +, -\}$) fail, even in simple cases, to distinguish among radically different behaviors. In order to take into account significant changes, the qualitative value of a quantity that must be considered, is both its sign and its relative order of magnitude. To describe order of magnitude relations, three key operators \ll , \cong , \sim are defined, which represent the following intuitive concepts:

 $A \ll B$, stands for A is negligible in comparison to B

 $A \cong B$, stands for A is close to B, i.e. (A - B) is negligible in comparison to B

 $A \sim B$, stands for A has the same order of magnitude as B. The underlying idea is that if $A \sim B$, then $B \ll C$ implies $A \ll C$.

A formal system FOG [5] defines a set of rules that can be applied on these relations (see Appendix). The basic axioms are the following: \sim and \cong are both equivalence relations ⁷, \ll is a partial ordering which defines thresholds in the order of magnitude scale. Two key features of FOG are the interpretation of the formal system in Non Standard Analysis, and the structure of the Quantity Space induced by the properties of the three operators. The representation in Non Standard Analysis leads to an algebraic calculus on order of magnitude equations, and solving these equations leads to updating the relations that hold in the Quantity Space. Combining these two representations leads to an efficient implementation of the formal system.

⁶ The concept of Quantity Space is understood here as defined in Qualitative Physics.

⁷ The equivalence classes defined by \cong are sub-classes of the equivalence classes defined by \sim , because A \cong B implies A \sim B.

3.2. Library of Qualitative Models

Some simple components have only one correct behavior, easily described by a unique model: Ohm's law for resistors, Kirchoff'laws for nodes (remember that even a node is a component since it can be faulty). But, in general, generic components may have several possible correct behaviors, and different models are needed to describe all of them. A model for a component consists of a set of constraints linking the electrical parameters attached to this component⁸. Voltages are linked by numerical constraints and currents by qualitative constraints. These models are based on physical laws and expertise. This expertise is required to describe all the qualitative correct behaviors of complex components and to specify ranges for the numerical values of voltages corresponding to each behavior. Two different models of behavior of a component correspond to a significant change in terms of order of magnitude of at least one parameter. To reason about changes of behavior implies to add more constraints describing the evolution of a given parameter between two different models.

For a given circuit, what is known is its designed behavior. This means that for each component the model, M_N , corresponding to its designed behavior inside the circuit can be selected in the library of models. The values of parameters for this particular model, called nominal values, are available by simulation or by measurement on a correct circuit. They are noted: V^N , I^N ,... The relevant information to describe a possible behavior (model M_I) for a component are the constraints linking the values of parameters in M_I and the values of these parameters in M_N , i.e. each model M_I is described by its variation with respect to the model M_N taken as reference.

For instance, assume that the designed behavior of a transistor is its "normal" state. The model in the library of normal state for a transistor is the following:

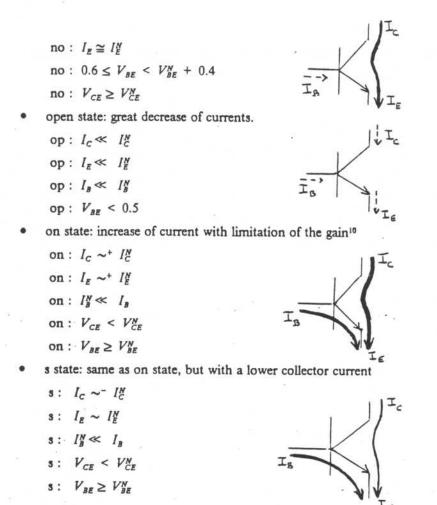


With this designed behavior, some correct behaviors for the transistor are:

• normal state: no significant change in currents, but some possible changes in voltage. no : $I_C \cong I_C^N$

* To simplify, here, only the main parameters, voltages and currents in direct mode, are considered.

⁹ V_{BE} stands for the base-emitter voltage, and V_{CE} stands for the collector-emitter voltage of a transistor. In those examples, voltages are expressed in volts and the numeric equality between voltages is to be understood at 10 %.



3.3. Assumptions and local consistency

Presuming that a component is correct requires attaching a qualitative model of correct behavior to this component. This implies selecting a model among the several models of correct behavior. This choice can be made only if relevant observations are available. Recall that the only observations available are measurements for voltages, not for currents intensity. Since there is not a one to one mapping between ranges for voltages and qualitative models of behavior, different models are generally consistent with the observations. Thus, selecting a model involves making an assumption. For example, consider the transistor T1 for which measurements indicate the following changes with respect to the nominal values:

$$V_{BE}^{N} = 0.74$$
 $V_{BE} = 1$
 $V_{CE}^{N} = 2$ $V_{CE} = 0.25$

¹⁰ A \sim^+ B stands for: A > B, A \sim B, and \neg (A \cong B); and A \sim^- B for: A < B, A \sim B, and \neg (A \cong B).

viii

Two models of correct behavior (on,s) for T1 are consistent with these observations. The two corresponding assumptions are noted T1(on) and T1(s).

Presuming now that a block B, i.e. a set of connected components, is correct implies attaching a model of correct behavior to each of its components. Thus, an assumption for a block B is a set a_g of elementary assumptions for each of its components. A(B) stands for the set of all potential assumptions a_g for the block B. The topology of a circuit allows to define a set of links L(B) between terminals of the components of the block B. A link stands for a connection between two terminals of two different components, and is viewed like a constraint¹¹. An assumption a_g is consistent if it satisfies all the constraints attached to L(B). FOG checks whether this set of constraints is satisfied or not. The set of assumptions a_g that satisfy L(B), is noted C(B)¹². If C(B) is empty, then the set of components in B is a conflict. This means that there is a defect in B. In all cases:

size[C(B)] < size[A(B)].

Minimal *conflicts* are searched by focusing first on minimal blocks composed of a node and the components connected to it. Such conflicts are minimal in the strong sense that no observation available could reduce their size¹³.

4. Example of Diagnosis

Consider a simple basic functional block of an analog circuit, called a voltage follower. The components of the voltage follower are: transistors T1 and T2, five resistors Ri and nodes Ni. The measurements for voltage are taken at the terminals of the different components. These values are available for the designed behavior (see Fig. 1) and for the circuit to troubleshoot (see Fig. 2).

- ¹² Notice that C(B) expresses local consistency strictly inside B, not at its "boundaries".
- ¹³ The reason is that unlike the models of other components the model of a node in itself is not very restrictive. To detect an inconsistency about Kirchoff's law for currents in a node needs to know the order of magnitude of each of these currents, so to have modeled the behavior of each component connected to that node.

¹¹ A link just expresses that a same electrical signal propagates on the two terminals. It implies voltage is the same on the two terminals, and currents are opposite.

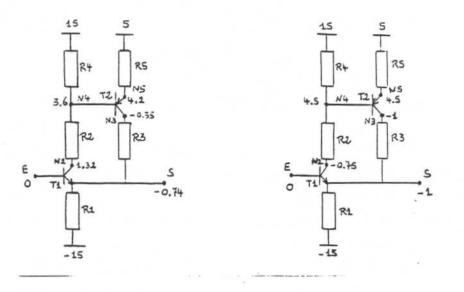


Fig. 1 Follower: designed behavior

Fig. 2 Follower: misbehaving.

For this example, a *deviation* is observed at the boundary of the voltage follower: the observed voltage on input E is equal to its nominal value, but the voltage on output S is appreciably less than its nominal value. This *deviation* leads to focus on the components of the follower. The assumptions of correct behavior for transistors, consistent with the measurements, are: T1(on) and T1(s) for T1; T2(op) for T2.

The sets of possible assumptions for the minimal blocks are:14

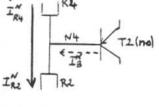
B2 = [N2, T1, R2]	$A(B2) = \{ \{ N2,T1(on),R2 \}, \{ N2,T1(s),R2 \} \},\$
B3 = [N3, T2, R3]	$A(B3) = \{ \{ N3, T2(op), R3 \} \}$
B4 = [N4, T2, R2, R4]	$A(B4) = \{ \{ N4, T2(op), R2, R4 \} \}$
B5 = [N5,T2,R5,R4],	$A(B5) = \{ \{ N5, T2(op), R5, R4 \} \}.$

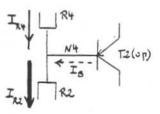
Let's examine the consistency of A(B4). The designed behavior implies:

$I_{R2}^N \cong$	$(-I_{R^4}^N)$	(1)
$I_B^N \ll$	INR2	(2)

The assumptions of correct behavior for components of B4 give:

 $T2(op): I_{g} \ll I_{g}^{N}$ (3) $R2 : I_{R2} \sim^{+} I_{R2}^{N}$ (4) $R4 : I_{R4} \cong I_{R4}^{N}$ (5) $N4 : (I_{R2} + I_{g}) \cong (-I_{R4})$ (6)





¹⁴ Assumptions of correct behavior for resistors and nodes, that correspond to a unique model, are noted by the name of the component. $(3) + (7) \rightarrow l_B \ll l_{R2}$ (9)

 $(6) + (9) \rightarrow I_{R2} \cong (-I_{R4})$ (10)

(8) + (10) -> contradiction by definition of \sim^+

Thus, C(B4) is empty. The same reasoning leads to:

C(B2) = { { N2,T1(on),R2 } }. C(B3) = { { N3,T2(op),R3 } }. C(B4) = { }. C(B5) = { }.

Since C(B4) and C(B5) are empty, at least one component among N4, T2, R2 and R4 is faulty and at least one component among N5, T2 and R5 is faulty. Thus, two minimal *conflicts* are identified:

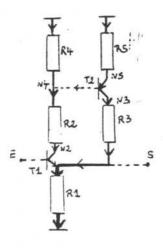
<N4,T2,R2,R4> and <N5,T2,R5>.

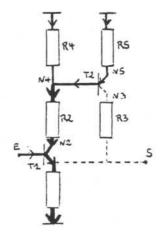
If the nodes are correct, then the three minimal candidates are:

[T2], [R2,R5] and [R4,R5]. This means that the set of defects of the circuit contains at least one of these three sets.

With the more restrictive assumption that there is a unique defect, it is certain that T2 is the faulty component, because T2 is the only component that may cause both *conflicts*. Moreover, the kind of electrical defect can then be discovered. Finding the behavior of the faulty component is obtained by suppressing its assumption of correct behavior. Then, instead of detecting an inconsistency, the qualitative reasoning above forces the behavior of the faulty component. Here, suppressing the assumption that T2 is correct (in particular suppressing equation (3)), forces already:

 $(6) + (8) \rightarrow \neg (l_B \ll l_{R2})$





designed behavior

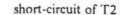


Fig.3 Main currents

xi

The complete reasoning for the other terminals of T2 leads to:

- $l_c \ll l_b$
- $I_E \cong I_B$.

This shows that the defect on T2 is a short-circuit between the base and the emitter. Unlike the situation in shallow reasoning, where all possible faults have to be described beforehand, no model of misbehavior are needed here, but even more, such models can be discovered. Finally the qualitative reasoning describes the main changes in the behavior of the circuit due to the defect (see Fig.3). Notice that T1 is correct, although its behavior has changed: T1(on) instead of T1(normal).

5.Strategy

The case of troubleshooting seen above does not require using a strategy, since there are few components involved. Troubleshooting circuits containing about a hundred of components (see Fig. 4) is more complex. Exploiting again the fundamental assumption that a defect leads to significant change in the behavior of the circuit leads to define three basic strategies.

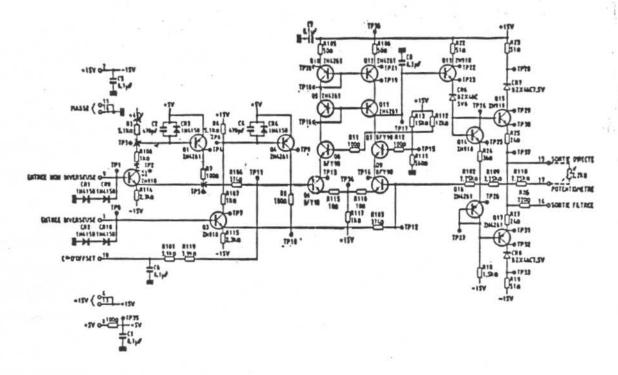


Fig.4 A typical circuit troubleshooted by DEDALE

5.1. Top-Down Strategy

The top-down strategy consists in focusing on functional hierarchical blocks where there are *deviations*. A *deviation* is a significant change between the observed and the designed behavior at the boundary of the block. This process is iterative, until reaching a basic function B, for which the sub-functions are components. It is then possible to apply models of behavior for these components ¹⁵. The search for small *conflicts* inside B leads to construct the set of locally consistent assumptions C(B).

5.2. Horizontal Strategy

If the set C(B) is not empty, that is no minimal *conflict* has been so far detected inside B, an other block B' must be considered. The horizontal strategy selects a hierarchical block B' among those included in the same higher level function B than B. A general heuristic for this choice is to focus first on a block B' connected to B (beginning the search for a link where a *deviation* was observed, that led to focus on B).

5.3. Bottom-Up Strategy

If C(B') too is not empty, then the set C(B U B') of assumptions consistent with block B and B' is constructed. C(B U B') is the set of assumptions that satisfy L(B) U L(B') U L(B,B'), where L(B,B') is the set of links, between B and B'. Thus, C(B U B') is included in C(B) \times C(B').

The strategy adopted to check inconsistencies between B and B' consists in studying first, for each link 1 belonging to L(B,B'), the set B_i of components of B and B' linked by 1 (this block does not appear in the hierarchical decomposition). The size of B_i does not exceed the maximal number of components connected to a same node, independently of the sizes of B and B'.

If some $C(B_i)$ is empty, a minimal conflict has been detected.

If no $C(B_i)$ is empty and if nevertheless $C(B \cup B')$ is empty, the only information is that $B \cup B'$ is a *conflict*, which is in general not minimal. But all possible observations on components of $B \cup B'$ are available to search minimal *conflicts* and it is important to notice that this case is rare. Indeed, if there is actually a defect in B for example, it means that the observations of measurable

¹⁵ For higher level functions, precise models of behavior, describing exhaustively all good functioning states, are not available. The only knowledge of the designed behavior simply allows to observe deviations. In particular, no assumption is made during the top-down process.

parameters on B and on all its components cannot permit to distinguish the behavior of the faulty block B from a possible correct behavior of B.

If all sub-fonctions B, B',... of a hierarchical function B have been connected in this way, there is a consistency at a higher level of the hierarchy.

The bottom-up strategy garantees to detect the smallest conflicts, with respect to the functional decomposition of the circuit.

Conclusion

The expert system DEDALE has been implemented in VM/PROLOG [8]. It has 4 components: (1) An object oriented language which allows to describe structurally and functionally a circuit; (2) A library of qualitative models for generic components; (3) A problem solver which performs order of magnitude reasoning: FOG; and (4) rules for strategy. Today, hypothetical reasoning is performed using backtracking. Future work involves implementing a more powerful algorithm using both a dependency-directed backtracking and an ATMS, as described in [3].

The expert system DEDALE is now experimented on real size applications in a factory environment to troubleshoot complex analog circuits, for which modeling is difficult. According to the first results, for about 75 % of investigated failures, there are significant change in the behavior of the circuit. In these cases, DEDALE is able to find the defects. This shows that this fundamental assumption can make troubleshooting possible when modeling is the trouble. For a real size application trying to overcome this difficulty by performing qualitative reasoning was a challenge. Experience with DEDALE indicates that reasoning about qualitative models was necessary to cope with incomplete information and is quite successful, when guided by a strategy exploiting this assumption.

The 25 % remaining cases of failures are not really due to faulty components, but rather to components that are led to work at the limits of their designed behavior. In such cases there are no significant deviations inside the circuit. Experience has shown that such failures are identified before trying a model-based approach. Specific heuristics can be added to DEDALE to try to handle these cases as well.

These results show both the efficiency and the scope of this approach.

References

- 1. Davis, R., Shrobe, H., Hamscher, W., Wieckert, K., Shirley, M. and Polit, S., Diagnosis based on description of structure and function, *Proceedings of the National Conference on Artificial Intelligence*, Pittsburg, PA (August, 1982) 137-142.
- 2. De Kleer, J., and Williams, B.C., Reasoning about multiple faults, *Proceedings of the National Conference on Artificial Intelligence*, Philadelphia, PA (August, 1986) 132-139.
- De Kleer, J., and Williams, B.C., Back to backtracking: controlling the ATMS, Proceedings of the National Conference on Artificial Intelligence, Philadelphia, PA (August, 1986) 910-917.
- 4. Brown, J.S., Burton, R. R. and de Kleer, J., Pedagogical, natural language and knowledge engineering techniques in SOPHIE I, II and III, D. Sleeman and J.S. Brown (Eds.), *Intelligent Tutoring Systems*, (Academic Press, New York, 1982) 227-282.
- 5. Raiman, O., Order of magnitude reasoning, Proceedings of the National Conference on Artificial Intelligence, Philadelphia, PA (August, 1986) 100-104.
- 6. De Kleer, J., How circuits work, Artificial Intelligence 24 (1984) 205-280.
- 7. Brown, A.L., Qualitative knowledge, causal reasoning, and the localization of failures, Artificial Intelligence Laboratory, TR-362, Cambridge, M.I.T. (1976).
- 8. VM/Programming in Logic, Program description and operations manual, IBM (1985).

Appendix

Here are some rules of FOG ([A] stands for the sign of Λ):

 $A \cong A$ $A \cong B \rightarrow B \cong A$ $A \cong B, B \cong C \rightarrow A \cong C$ $A \cong B, [C] = [A] \rightarrow (A + C) \cong (B + C)$ $A \sim B \rightarrow B \sim A$ $A \sim B, B \sim C \rightarrow A \sim C$ $A \sim B \rightarrow [A] = [B]$ $A \cong B \rightarrow A \sim B$ $A \ll B, B \ll C \rightarrow A \ll C$ $A \ll B, B \sim C \rightarrow A \ll C$ $A \cong B \rightarrow (A - B) \ll B$ $A \ll B \rightarrow (B + A) \cong B$ $A \ll B \rightarrow -A \ll B$ $A \sim B, [A] \neq 0 \rightarrow \neg (A \ll B)$ $A \sim^+ B \leftrightarrow A \sim B, \neg (A \cong B), [A - B] = +$ $A \sim B \leftrightarrow A \sim B, \neg (A \cong B), [A - B] = -$