ASSEMBLING A DEVICE

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Abstract

A challenge for qualitative physics is to deduce the overall interactions of a device using only its structural decomposition and models of behavior for generic components. A qualitative physics based on confluences [1] attempts to achieve this goal. The aim is to work with a model (i.e., a set of confluences) while providing a physical interpretation. In fact, only a limited use of confluences has been made in previous work. A powerful use of such a model requires first developing a qualitative calculus. This means understanding the power and the limits of reasoning with confluences. We focus here on the physical interpretation of the calculus we have developed. We show that it is possible to extend the use of confluences and to infer global laws specific to the device. This means that it is possible to reassemble the device.

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Introduction

A general framework, "a Qualitative Physics based on confluences" has been proposed by de Kleer and Brown [1]. A model of a device is a set of confluences. Each confluence qualitatively describes local physical laws. From a physical point of view, this framework tries to capture the intuitive notion of perturbations propagating through a device.

However in previous work only a limited use of confluences has been made. We show here that an extensive use of confluences can be made. We start by a simple motivating example. It shows that a qualitative calculus has to be developed. In fact, qualitative calculus is reduced to a single rule, we call the Qualitative Gauss rule. The Qualitative Gauss rule is fundamental, for it is the link between the model, taken from a formal point of view, and the intended intuition. Applying the Gauss rule means combining the perturbations propagating through the device. In fact the Gauss rule infers the mutual influences between all the physical quantities involved in the whole device. Thus by connecting the different parts of the device, it reassembles it.

Is the sum of two pipes a pipe?

Consider a very simple example, a qualitative model for two connected pipes. For each pipe, there is a confluence describing the link between the sign of the pressure at the different ends of the pipe and the flow Q. The confluence (1) resp (2) for pipe 1, and pipe 2 are the following:

- (1) $\left[dP_{A} \right] \left[dP_{B} \right] \left[dQ \right] \cong 0^{1}$
- (2) $[dP_a] [dP_c] [dQ] \approx 0$

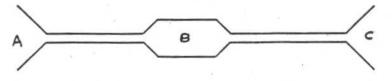


Figure 1: Two connected pipes.

This model describes separately the different parts of the physical device. It is obvious that the two connected pipes behave like a single pipe. A system performing qualitative reasoning should be able to make such a deduction. On this example this can be done in a very simple way by adding the two confluences (1) and (2).

$$(1)+(2) \quad \left[\mathrm{d}\mathbf{P}_{\mathbf{A}}\right] - \left[\mathrm{d}\mathbf{P}_{\mathbf{B}}\right] - \left[\mathrm{d}\mathbf{Q}\right] + \left[\mathrm{d}\mathbf{P}_{\mathbf{B}}\right] - \left[\mathrm{d}\mathbf{P}_{\mathbf{C}}\right] - \left[\mathrm{d}\mathbf{Q}\right] \cong 0$$

Eliminating $[dP_s]$ in (1)+(2), ² reduces the confluence for the two pipes into a confluence (3) expressing that the two connected pipes behave just like a single pipe.

(3) $\left[dP_{A} \right] - \left[dP_{c} \right] - \left[dQ \right] \approx 0$

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^{1 [}x] is the sign of the quantity x

² and using the fact that q + q = q for any qualitative quantity q (cf. prop. 2 Annex).

Here combining confluences produces another confluence which corresponds to a physical law, describing a global constraint that must satisfy the two connected pipes. Is it generally correct to combine confluences? What does this kind of combination rule mean for the interaction between separate elements of the device? To what extent applying such a rule leads to a description for all the interactions of the different parts of the device?

The Quantity Space, Basic Remarks

The basic quantity space considered in Qualitative Physics is the set $\{0, +, -\}$. In order to handle addition and product of signs it is necessary to add to the set S the element ?. A physical parameter takes its qualitative value in S-{?}. If we assume that $[dP_A] = -$ and $[dP_B] = -$ the following relation is meaningless for [dQ]:

 $\left[\mathrm{d}\mathbf{Q}\right] \cong \left[\mathrm{d}\mathbf{P}_{A}\right] - \left[\mathrm{d}\mathbf{P}_{B}\right] = ?$

The only information in this case is that the confluence is solved.

Generally, for a physical quantity x, [dx] is not necessarily ambiguous if a confluence is reduced to $[dx] \cong ?$. Its qualitative value may be deduced from another confluence.

The relation \cong must not be interpreted as the classical notion of equality of elements of a set. It is necessary to give a clear definition for the relation \cong , called qualitative equality:

$$a = b$$

or
$$a \cong b \quad iff \quad a = ?$$

or
$$b = ?$$

The relation \cong as defined above offers a clear semantic. It extends the notion of equality, for if a equals b, then a \cong b. Obviously working with confluences is not just like working within a vector space.

Qualitative Gauss rule

Theorem : Qualitative Gauss rule.

A variable can be eliminated between two confluences, provided that no other variable is eliminated at the same time.

This Qualitative Gauss rule expresses when it is correct to combine confluences. Combining confluences is fundamental to deduce links between different parts of a device, i.e. discovering the links between the qualitative values of physical quantities not involved directly in a same confluence. For example, the only variable eliminated when adding

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the confluences for the two connected pipes is $[dP_B]$. Thus the Qualitative Gauss rule shows that combining the two confluences is correct. It establishes a link between Q, P_A and P_c . So combining local confluences leads to a more global one. In other words, different parts of the device are fitted together.

It is also possible to deduce a confluence which gives the relation between the pressure at point $B_{,P_{B}}$, and the input pressures P_{A} and P_{C}

(1)-(2)
$$\left[dP_{A} \right] - \left[dP_{B} \right] + \left[dP_{c} \right] \approx 0$$
.

These two confluences are relations between internal ($[dP_B]$, [dQ]) and external variables ($[dP_A]$, $[dP_c]$).

From local to a global path.

Applying the Gauss rule makes physical sense. A way to understand a confluence is to consider that it describes a local path linking the physical variables at the boundary of a component and its internal variables. For example the confluence for a pipe links the pressures at its ends and the flow through the pipe. Combining confluences consists in connecting two paths and considering the entire path. Having a complete picture of a device is considered here as linking internal physical variables to boundary physical parameters. The links obtained directly show how the internal variables react to an external perturbation influencing the device.

It is not required to assign some particular values to the inputs. But once values are assigned to inputs, deducing the values of internal variables is straightforward. There is no need of any indirect proof, as in RAA [1]. Let's focus again on the basic example above and assume for instance that the inputs are $[dP_A] = +$ and $[dP_C] = 0$. It is obvious using the two inferred confluences that $[dP_B] = +$ and [dQ] = +. Even in this case RAA requires to make assumptions.

The pressure regulator revisited

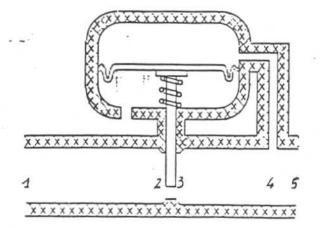


Figure 2: Pressure regulator.

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Reconsider the well known pressure regulator.

The model used here is slightly different from the model used by de Kleer and Brown [1]. P_i stands for the pressure at point I, Q is the flow (considered as uniform w.l.o.g.), A is the area under the valve. It is assumed that the valve is neither completely open or closed, and that the fluid flows from point 1 to point 5 (Q>0).

$\left[dP_{1}\right] - \left[dP_{2}\right] - \left[dQ\right] \cong 0$	Path (1,2)
$\left[dP_{2}\right] - \left[dP_{3}\right] - \left[dQ\right] + \left[dA\right] \cong 0$	Path (2,3)
$\left[\mathrm{d}\mathrm{P}_{3}\right]-\left[\mathrm{d}\mathrm{P}_{4}\right]-\left[\mathrm{d}\mathrm{Q}\right]\cong0$	Path (3,4)
$\left[dP_{4}\right] - \left[dP_{5}\right] - \left[dQ\right] \cong 0$	Path (4,5)
$[dP_4] + [dA] \cong 0$	(Valve)

The physical parameters at the boundary of the pressure regulator are P_1 and P_5 ³. The internal parameters are P_2 , P_3 , P_4 , Q, A. The aim here is to find the direct influences of the physical inputs P_1 , and P_5 , on the internal variables. The basic idea is to follow the different paths that start at the boundary and that lead to the area where the particular internal variable is involved.

For example there are three paths that lead to point 2: path $\{(1,2)\}$, path $\{(2,3) + (3,4) + (4,5)\}$ and path $\{(2,3) + valve + (4,5)\}$. Combining the input perturbations through the different paths leads in five steps to:

 $[dP_2] \cong [dP_1] + [dP_3].$

 $\begin{bmatrix} dP_3 \end{bmatrix} - \begin{bmatrix} dP_5 \end{bmatrix} - \begin{bmatrix} dQ \end{bmatrix} \cong 0 \\ dP_2 \end{bmatrix} - \begin{bmatrix} dP_3 \end{bmatrix} - \begin{bmatrix} dP_4 \end{bmatrix} - \begin{bmatrix} dQ \end{bmatrix} \cong 0 \\ Paths \{(3,4) + (4,5)\} \\ Paths \{(2,3) + (valve)\} \\ Paths \{(2,3) + (valve) + (4,5)\} \\ Paths \{(2,3) + (valve) + (4,5)\} \\ Paths from 2 to 5 through 3 and the valve. \\ All the paths from 1 and 5 to 2.$

In the same way relations are obtained for the other variables:

[dP4]	$\simeq - [dP_1] + [dP_5].$
[dA]	$\simeq - [dP_1] - [dP_5].$
[dQ]	$ \stackrel{\cong}{=} \left[dP_1 \right] + \left[dP_5 \right]. $ $ \stackrel{\cong}{=} \left[dP_1 \right] - \left[dP_5 \right]. $ $ \stackrel{\cong}{=} \left[dP_1 \right] - \left[dP_5 \right]. $

This set of confluences is relevant as far as simulation is concerned. It may be interesting to focus on another set of variables, for instance, in order to make postdiction. For example, consider the set $\{[dQ], [dA]\}$. Following the different paths leads to:

$$\begin{bmatrix} dP_1 \end{bmatrix} \cong \begin{bmatrix} dQ \end{bmatrix} - \begin{bmatrix} dA \end{bmatrix}. \\ \begin{bmatrix} dP_2 \end{bmatrix} \cong \begin{bmatrix} dQ \end{bmatrix} - \begin{bmatrix} dA \end{bmatrix}. \\ \begin{bmatrix} dP_3 \end{bmatrix} \cong \begin{bmatrix} dQ \end{bmatrix} - \begin{bmatrix} dA \end{bmatrix}. \\ \begin{bmatrix} dP_4 \end{bmatrix} \cong - \begin{bmatrix} dA \end{bmatrix}.$$

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³ P₅ is not considered as an input in the model of the pressure regulator given in [1]. Considering point 5 as passive is equivalent to the assumption $[dP_5] = 0$

$[dP_s] \cong - [dQ] - [dA].$

Combining all the local paths.

From now on, solving confluences for given inputs is straightforward. For instance, assume that:

$$\{[dP_1] = +, [dP_5] = 0\}$$
 or $\{[dP_1] = +, [dP_5] = +\}$.

In both cases deducing the qualitative values of dP₂ dP₄ and dA is obvious:

$\left[dP_{2} \right] \cong \left[dP_{1} \right] + \left[dP_{5} \right]$	leads to:	$[dP_2] = + .$
$[dP_{4}] \cong [dP_{1}] + [dP_{2}]$	leads to:	$[dP_4] = +$.
$[dA] \simeq - [dP_1] - [dP_5]$	leads to:	[dA] = -.

What does this mean? These confluences have been previously deduced by combining the influences through all the paths linking P_1 and P_5 and respectively P_2 , P_4 , and A These results can be expressed 'a la Forbus' [2]: 'All the perturbations influence positively the pressures at point 2 and 4 and negatively the area under the valve.'

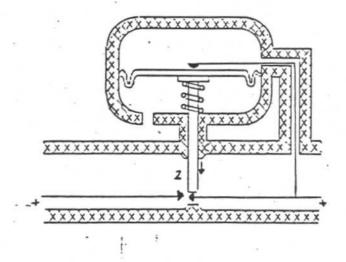


Figure 3: Pressure at point 2 increases.

What about the pressure P₃ and the flow Q?

In the first case $[dP_3] = +$ and [dQ] = +.

But the qualitative values of $[dP_3]$ and [dQ] are not determined in the second case. The reason is that, considering the two paths and $\{(3,4) + (4,5)\}$ and $\{(2,3) + \text{valve} + (4,5)\}$ leads to the two confluences : $\begin{bmatrix} dP_3 \end{bmatrix} - \begin{bmatrix} dP_3 \end{bmatrix} - \begin{bmatrix} dQ \end{bmatrix} \cong 0$ $\begin{bmatrix} dP_2 \end{bmatrix} - \begin{bmatrix} dP_3 \end{bmatrix} - \begin{bmatrix} dP_3 \end{bmatrix} - \begin{bmatrix} dQ \end{bmatrix} \cong 0$

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Thus the influence of $[dP_3]$ on $[dP_3]$ is positive in the first confluence, but negative in the second one. This means that the influences of $[dP_3]$ on $[dP_3]$ through the two paths are opposite.

The qualitative Gauss rule and causal heuristics.

Quasi-static assumption.

It has been pointed out [1] that physical laws often used in qualitative physics are equilibrium laws. A perturbation is considered as being immediately propagated through the device. This is the quasi-static assumption : 'the device goes through an infinity of infinitely close equilibrium states'. Using the Qualitative Gauss rule assumes that all the perturbations are propagated through the different paths at the same time. In particular, it is not necessary here to consider 'mythical time'.

Concurrent inputs are needed.

It is essential to consider that a device may have several inputs. All the communicating points with outside world must be considered as inputs. The differentiation between "active" and "passive" communicating points (i.e., inputs and outputs) is strongly related to the intended function of the device. A model that includes such a separation violates the "No Function in Structure Principle".

This idea corresponds to the good old mathematical 'adage': "The solution of a differential system is determined by the conditions at its boundaries."

The causal heuristics cannot deal with concurrent inputs.

Let's assume that $[dP_1] = [dP_3] = +$. The first causal heuristic applied to $[dP_1]$ infers that [dQ] = +. Causally propagating perturbations from 1 through the device retrieves $[dP_3] = +$. Thus the input value $[dP_3]$ is considered as a causal consequence of the pressure perturbation at point 1. One could attempt to invert the causal heuristics in order to take into account the perturbation at point 5 as an initial one. Thus we could deduce that the only value of [dQ] which could cause the increasing of pressure at point 5 is still [dQ] = +. Finally, some modified causal heuristics would probably assign the unique value + to $[dQ]_1$. Thus [dQ] = - would not be considered as a causal value. It is nevertheless obvious that this behavior is possible if $[dP_3]$ is the most important perturbation.

Perturbations can be propagated without assigning values to ambiguous variables.

Still assuming that $[dP_1] = [dP_s] = +$, point 3 is used when combining paths leading to point 2 in order to deduce $[dP_2] = +$. But the pressure at point 3 remains ambiguous. Thus it has been possible to propagate an input perturbation through an ambiguous variable with no assigned value. This is not possible using the causal heuristics. Causal reasoning is able to assign a value to a variable V only if values have already been assigned to all the previous variables involved in a causal path leading to V. Thus con-

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sidering the path {(2,3) + (3,4)}, it is impossible to deduce $[dP_2] = +$ without assigning a value to $[dP_3]$.

No particular values are assigned to ambiguous variables when propagating perturbations through all the possible paths. Only non ambiguous variables are computed and proved to be non ambiguous.

Choosing the main path.

When using the Gauss rule, no particular value is assigned to an ambiguous variable. A variable is ambiguous when at least two opposite paths lead to it from inputs. Choosing a value for such a variable means considering a path as more important than the other ones. Contrarily causal heuristics assign values to ambiguous variables. In fact applying the causal heuristics means **arbitrarily choosing** a main path. This choice is not based on physical considerations. We believe that it is possible to deduce the main path from intrinsic properties of the device.

Interpreting multiple solutions.

Multiple solutions correspond to multiple paths with opposite influences. Let's again consider the case: $\{[dP_1] = +, [dP_5] = +\}$. Propagating the computed values in the initial confluences leads to only one unsolved confluence:

$[dP_3] - [dQ] \cong +$

The solutions of this confluence can be are computed by a brutal branch and bound algorithm. We cannot think of any other method, even causal ones.

[dQ]	+	0	-	-	١
[dP,]	-	0.	+	+	+

Table 1: Possible values of $[dP_3]$ and [dQ].

Nevertheless, it is clear that only one behavior at a time is possible. The elegant way to handle multiple solutions is to select the main path. Trying to find out what is the main phenomena, can be achieved by performing order of magnitude reasoning. This requires focusing on the orders of magnitude of key parameters. It is not possible here to give a complete description of what this involves, only the general idea is exposed. Taking into account orders of magnitude can be done as shown in [9] using Non Standard Analysis. 'Non standard confluences' can be used, in the same way as classical ones.

Let's focus first on the relative orders of magnitude of the inputs. The five initial confluences can be kept. Let's assume that dP_s is negligible compared to dP_1 . Every pressure (except at point 5) has the same order of magnitude as dP_1 , and the flow is essentially influenced by dP_1 . Thus the flow and the pressure at point 3 increase.

Conversely, if dP_1 is negligible compared to dP_s , the flow Q decreases (but the pressure at point 3 remains ambiguous). Thus the formal solution of initial confluences [dQ] = 0 can be considered as an intermediate case, i.e. dP_s and dP_1 are of the same order of magnitude, by a kind of rule of continuity.

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Three possibilities remain for dP_3 when dP_1 is negligible compared to dP_5 . Focusing now on the tightness of the spring, k, leads to replace the confluence for the valve by a Non Standard confluence

$\left[\left[dP_{4}\right]\right] + k\left[\left[dA\right]\right] \cong 0.$

If k is infinitely large, the pressure at point 3 is of the same order of magnitude as the pressure at point 5. But if k is infinitely small, the pressure at point 3 decreases.

So, when the spring is very hard, the pressure at point 3 increases. The main perturbation path is 3 - 4 - 5. Conversely, if the spring is lax, the pressure at point 3 decreases. The influence of perturbation through the valve dominates all the other ones. There is a 'hole of pressure' at point 3.

What can the Qualitative Gauss Rule compute ?

As shown in the pressure regulator, the Qualitative Gauss rule allows to compute dependencies between 'internal' variables and variables considered as inputs. For given values of these inputs the qualitative values of the variables, which are completely determined, can be computed by applying the two very simple propagation rules :

Rule1: When the qualitative value of a variable is computed, replace in all confluences the variable by its value.

Rule2: When there is only one variable in a confluence, deduce the qualitative value of this variable.

These rules are the two basic rules usually used in order to solve confluences. Solving confluences is a very difficult problem (it can be proved that it is a N-P complete problem). But the two propagation rules above are much more simple (polynomial). Thus reducing initial confluences to a set of confluences where the search consists in applying only the propagation rules is fundamental. Such a set is called a relevant set.

Finding all the variables which are completely determined by special values of some inputs is obvious once a relevant set is found. Practically, finding first a relevant set provides good efficiency. The reason is that the most important computational work is done once and for all. This means 'compiling' the device, while computing solutions of the confluences for particular inputs means 'interpreting' it.

In the same time, this purely symbolic computation assembles the device. This means for the pressure regulator that the paths (i,j), with i < j, (except for j = 3), behave like a pipe.

Soundness of the Gauss rule.

Does the Qualitative Gauss rule always provide a relevant set? This is a kind of soundness problem. A first result is that, applying the Gauss Rule freely on a set of confluences provides a relevant set. This result is a consequence of the fundamental theorem (cf. Annex) which asserts that, when 'interpreted' for special values of inputs, the Gauss rule infers the qualitative values of all the non ambiguous variables.

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How to obtain a relevant set of confluences?

Practically, applying the Gauss rule freely leads to combinatorial explosion. Thus it must be controlled. It seems that the confluences that must be kept are provided by considering only chain paths, i.e. confluences which have been considered in the physical interpretation above. Thus controlling the Gauss rule can be done by mean of physical interpretation. For example, it seems that the following set of 14 confluences is relevant for the pressure regulator.

$[dP_1] -$	$[dP_{2}]$	-	[Ob]	$\simeq 0$
[dP,] -	[dP.]	_	dol	$\simeq 0$
	[dP,]			
	[dP.]			≅0
	[dP ₅]			
$[dP_3] -$	$[dP_4]$	-	[dQ]	≅ 0
[dP,] -	$[dP_{i}]$	-	[dQ]	≅0
[dP.] -	$[dP_{j}]$	-	[dQ]	≅0
	$[dP_{j}]$		[dP.]	
[dP] -	$[dP_{j}]$	+		≃ 0
$[dP_1] -$	[dP₄]	+	[dP ₃]] ≅ 0
	[dP₄]		[dP₃_	
	[dP₄]			≧ 0
$[dP_4] +$	$[d\Lambda]$	≅()	

Good models.

Unexpectedly, as a consequence of the fundamental theorem, it is possible to state whether a set of confluences is 'good'. From a physical point of view, it is clear that a device must remain steady if there are steady inputs. This is a stability property. This is the case for the model of the pressure regulator.

Three kinds of pitfalls may occur if the stability condition is not satisfied :

- 1. The model is not correct.
- 2. The model is correct, but the device is not well designed. No given input can produce a predictable answer at any point of the device.
- 3. The model is correct, and describe a well designed device, but the quantity space $\{0, +; -\}$ itself is too weak.

It can be proved using the fundamental theorem that, under reasonable assumptions about the kind of used confluences, the stability condition above holds if and only if the qualitative Gauss rule can deduce a non empty relevant set of confluences.

Power and limits of confluences.

Developing a qualitative calculus has allowed to extend the use of confluences. Practically, it consists in 'compiling' the device.

Previous theoretical results show the limits of confluences. For instance, interpreting multiple solutions requires an extended model, based on orders of magnitude.

Our goal was to find the power and limits of confluences. This is a step towards modeling complex devices.

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Annex : qualitative calculus.

P1. Quasi-transitivity of the qualitative equality.

Let a, b, c belonging to S, $b \neq ?$, such that $a \cong b$ and $b \cong c$ Then: $a \cong c$

The relation ' \cong ' is not transitive, but transitivity can be applied if the middle-term is "defined".

P2. Compatibility of addition and qualitative equality.

Let a, b, c belonging to S such that $a + b \cong c$ Then $a \cong c - b$

"- b" means (-) * b .

٢r

P3. Idempotency of addition. Let a belonging to S. Then a = a + a

TH1: Qualitative Gauss Rule

Let I, J, K, L be four mutually disjoint sets not containing 0.

Let x₀ be an element of S different from '?'.

Let x, for i belonging to I, J, K or L, a, for i belonging to K or L, and α and β be elements of S.

Assume that:

$$x_{0} + \sum_{i=1}^{\infty} x_{i} - \sum_{i=1}^{\infty} x_{i} + \sum_{k \in \mathcal{K}} a_{k} x_{k} \cong \alpha \quad (1)$$

$$- x_{0} + \sum_{i=1}^{\infty} x_{i} - \sum_{i=1}^{\infty} x_{i} + \sum_{k \in \mathcal{L}} a_{i} x_{i} \cong \beta \quad (2)$$

Then:

$$\sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i + \sum_{k \in RUL} a_k x_k \cong \alpha + \beta \quad (3)$$

Proof:

From (1) and (2) by applying P2,

$$\begin{array}{rcl} \mathbf{x_0} &\cong& -\sum\limits_{i=1}^{\infty} \mathbf{x_i} &+ \sum\limits_{i=1}^{\infty} \mathbf{x_i} &- \sum\limits_{k=L}^{\infty} \mathbf{a_k} \mathbf{x_k} &+ \alpha & (1') \\ \mathbf{x_0} &\cong& \sum\limits_{i=1}^{\infty} \mathbf{x_i} &- \sum\limits_{i=1}^{\infty} \mathbf{x_i} &+ \sum\limits_{k=L}^{\infty} \mathbf{a_i} \mathbf{x_i} &- \beta & (2') \end{array}$$

Applying P1 to (1') and (2') and x₀ leads to:

$$\sum_{i \in I} x_i - \sum_{i \in J} x_i + \sum_{i \in L} a_i x_i - \beta \cong - \sum_{i \in I} x_i + \sum_{i \in J} x_i - \sum_{k \in K} a_k x_k + \alpha \quad (3')$$

The Gauss rule is proved by applying P2, and then P3 to this qualitative equality. \Box Remark: Physical quantities can be eliminated using the qualitative Gauss rule because they always belong to $\{+, 0, -\}$.

TH.2: Fundamental theorem of qualitative calculus.

Let $A X \cong B$ be a qualitative linear system, where A is a squared matrix (n,n).

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The qualitative Gauss rule recursively applied to this system leads in a finite number of steps to the equation $X_1 \cong a$ where a is the qualitative value of X_1 if X_1 is uniquely determined.

Notation : A, X, and B are respectively a matrix and vectors of elements of S- $\{?\}$. Product of matrices and vectors is defined in a natural way. The relation \cong can be extended to vectors (two vectors of elements of S are qualitatively equal if and only if their components are qualitatively equal). The vector A X may have some components equal to ?.

When considering a device, X is the vector of its physical quantities. The system A X \cong B is obtained by adding to the set of confluences of the device the confluences $X_1 \cong B_2$, where B_1 are the values assigned to the inputs X_1 .

The proof of this theorem is too long to be given here. It is based on a theory of qualitative determinant and on a characterization of squared matrices with determinant + or -.

References

[1] J. de Kleer, J.S. Brown, "A Qualitative Physics Based on Confluences", Artificial Intelligence, Vol 24, 1984.

[2] K.D. Forbus, "Qualitative Process Theory", Artificial Intelligence Vol 24, 1984.

[3] J. de Kleer, "How circuits work", Artificial Intelligence Vol 24, 1984.

[4] B.C. Williams, "Qualitative Analysis of MOS circuits", Artificial Intelligence Vol 24, 1984.

[5] R. Davis, "Diagnostic Reasoning Based on Structure and Behavior", Artificial Intelligence Vol 24, 1984.

[6] M. R. Genesereth, "The Use of Design Descriptions in Automated Diagnosis", Artificial Intelligence Vol 24, 1984.

[7] H. G. Barrow, "VERIFY: A Program for Proving Correctness of Digital Hardware Designs", Artificial Intelligence Vol 24, 1984.

[8] B. Kuipers "The Limits of Qualitative Simulation", Proceeding IJCAI-85, 1985.

[9] O. Raiman, Order of Magnitude Reasoning, Proceeding of the fifth national conference on Artificial Intelligence, AAAI-1986.

[10] Y. Iwasaki, H. A. Simon, "Causality in Device Behavior", Artificial Intelligence Vol 29, 1986.

[11] J. de Kleer, J. S. Brown, "Theories of Causal Ordering", Artificial Intelligence Vol 29, 1986.

[12] Y. Iwasaki, H. A. Simon, "Theories of Causal Ordering: Reply to de Kleer and Brown", Artificial Intelligence Vol 29, 1986.

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