A Qualitative Approach to Mechanical Constraint

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The goal of Qualitative Mechanics (QM) is to produce a common sense theory of mechanical analysis sufficient to describe the behavior of rigid body mechanisms. In order for machines to reason about manipulation of objects in the physical world we need to develop theories of mechanics sufficiently flexible to describe not only common mechanisms such as the gear trains, pistons, and ratchets, but also unusual devices such as mutilated gears, Wankel engines, and clock escapements. These descriptions may be used to predict the behavior of an unknown mechanism, determine the suitability of a given device for a task, diagnose mechanical failures, and critically analyze new mechanisms.

There is a great deal of interest in developing AI tools to assist in mechanics, both by mechanical engineers and AI researchers. Our approach is to build qualitative theories which are applicable to a wide range of real problems and construct programs to test these theories. This paper begins with a short definition of terms, then describes the computations of mechanical constraints on motion, and concludes with examples which have been implemented using these ideas.

One of our major investigations in the analysis of mechanisms has been a general notion of mechanical constraint.[1] Constrained motion is essential to mechanics; a *machine* is defined as "any device consisting of two or more resistant, relatively constrained parts which may serve to transmit and modify force and motion so as to do work." In space an object is assumed free to move in any direction unless it is specifically constrained. Reasoning in 1, 2, or 3 dimensions as well as ignoring translation or rotation only requires changing the set of directions considered.

We assume as input a place vocabulary which specifies the set of objects involved, the contact relations that hold between the objects, and the orientation of the contact surfaces. The results of the analysis are the directions in which an object is free to move. This result is an initial step in understanding the qualitative kinematics of motion. Combining dynamical information (as produced by Qualitative Process theory) to this result will provide a set of state transitions which describe the possible behaviors of the device over time.

To describe direction in space, without resorting to diagrams or mathematics, people typically use words such as right, left; up, down; and front, back relative to some frame of reference. For consistency we use this order and assign "+" to the first of each of these pairs and "−" to the second with "0" meaning center. This corresponds to the signs of the numerical values in a right handed cartesian coordinate system. Thus upper, left, back would be the 3-tuple [− + −] (corresponding to left, up, and back).

[1] Used here a constraint is a reaction force which absolutely prevents a body from moving a certain way.

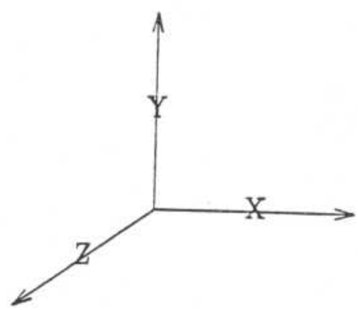

Figure 1: Right-handed coordinate system

Rotational directions are also the signs of the numerical values in a right handed Cartesian coordinate system. A counter-clockwise rotation will be "+" and a clockwise rotation will be "–" when looking along a positive axis toward the origin. The way we normally perceive the movement of the hands of a clock is (0 0 –), which is clockwise along the Z axis.

Surfaces are represented by the qualitative direction of the surface normal, and the direction to the center of rotation, if fixed. Places where the surface normal is not defined (corners) are represented by the adjacent surfaces.

Contact relations show which objects are in contact as well as the contact surface. Contact relations may be provided explicitly or by computations on the place vocabulary.

We begin by selecting a fixed component. This may be done heuristically to simplify analysis or may be specified by the user to facilitate analysis of kinematic inversions[2].

When an object comes in contact with a sufficiently constrained obstacle, the obstacle is capable of preventing linear motions of the object into the open half-plane defined by the contact surface. The obstacle will also prevent an object from rotating clockwise about any axis clockwise of the object's surface normal, as well as counter-clockwise about any axis counter-clockwise of the surface normal (in each plane). Rotation about an axis along the surface normal is not constrained.

The requirements for necessary and sufficient constraint of the obstacle is that exactly those directions which would be constrained on the object must already be constrained on the obstacle. For example the top block in a stack of blocks cannot move downward (relative to the Earth) because it has contact along a surface which has a downward surface normal and the block below it has its downward movement similarly constrained.

In figure 2 object B is in contact with object W. The surface normal of B is left ([– 0]). B will be unable to translate in any leftward direction ([+ 0], [+ +], [+ –]), rotate counterclockwise about any axis above (counterclockwise of left) the contact ([0 +], [+ +], [– +]), nor rotate clockwise about any axis above (clockwise of left) the contact ([0 –], [+ –], [– –]) as long as the W cannot move in any of these directions.

[2] *Inversion* is the making of different mechanisms by fixing different links in a kinematic chain.

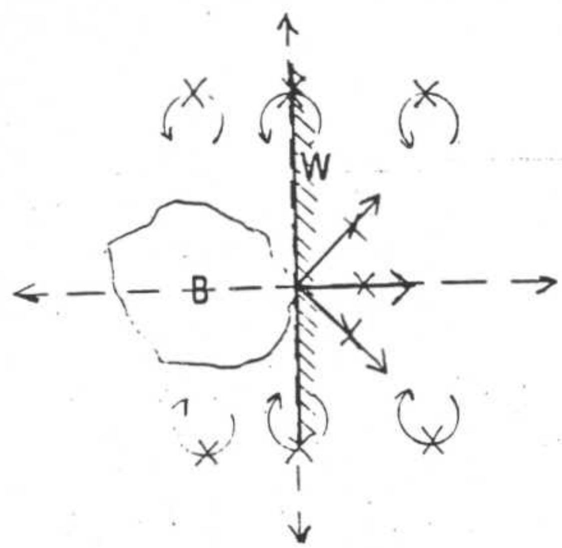

Figure 2: Constraints imposed by surface contact.

When one surface slides off another surface there will be an instant when contact between two convex corners may occur. In this case the surface tangent is not clearly defined at the point or line of contact. To know what motions can occur in this case, we must first determine from the place vocabulary the contacts which are possible between each adjacent surface and the corners.

The constraints imposed by contact between two convex corners are the intersection of those imposed by contact with adjacent surfaces, provided each half plane of the obstacle corresponding to the regions to be constrained is itself constrained.

Contact at a concave corner needs no special analysis since the constraints imposed are just the union of those imposed by each surface.

The propagation of constraint has been implemented in a program called *ALEX*. The recoil escapement shown in figure 3 will be used to demonstrate analysis of a single state. Here "*" indicates "+", "−", or "0". The direction of the surface normal **N** is (0 −), the direction to the center of rotation of the wheel **W** is (+ −), and the direction to the center of rotation of the pallet **P** is (+ +). In this type of escapement, when the pallet arm contacts the scape wheel, the wheel is not constrained by the pallet arm. As a result the pallet's continued swing drives the wheel backward, causing the entire clock mechanism to move backward (recoil), until the force of the clock drive is sufficient to overcome the inertia of the pallet.

```
Contact (Pallet, Wheel, [0 -] [+ +])
Contact (Wheel, Pallet, [0 +] [+ -])
Constraint (Pallet, [* *] [0])
Constraint (Wheel, [* *] [0])
-->
```

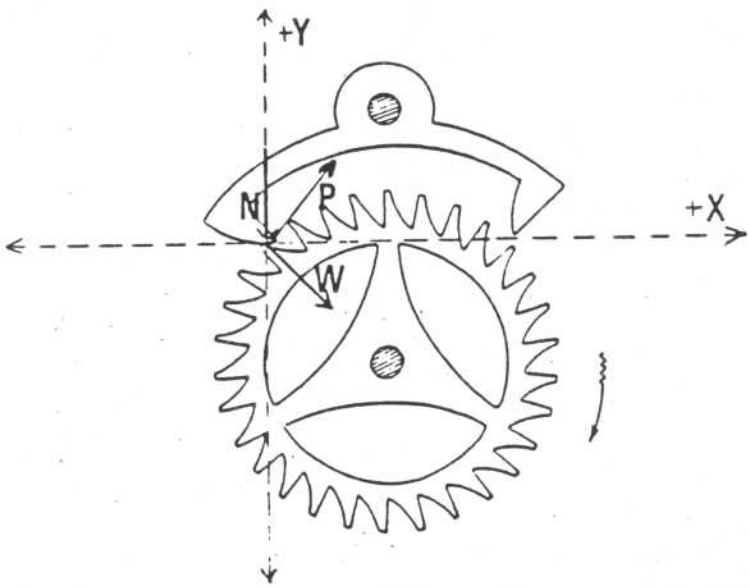

Figure 3: Recoil Escapement

```
Rotational-Freedom (Pallet, [+])
Rotational-Freedom (Pallet, [-])
Rotational-Freedom (Wheel, [+])
Rotational-Freedom (Wheel, [-])
```

The deadbeat escapement (figure 4) was an improvement over the recoil escapement because the contact surface of the pallet arms corresponded to a circle centered at the pallet arbor. This meant that the pallet arbor would fall along any surface normal of the contact surface. In the sample situation the pallet is free to move, but the wheel is stopped dead in the clockwise direction. These examples again demonstrate the need to consider partially constrained motions. If the pallet were considered completely constrained, both escapements would have the same behavior.

```
Contact (Pallet, Wheel, [- -] [+ +])
Contact (Wheel, Pallet, [+ +] [+ -])
Constraint (Pallet, [* *] [0])
Constraint (Wheel, [* *] [0])
-->
Rotational-Freedom (Pallet, [-])
Rotational-Freedom (Pallet, [+])
Rotational-Freedom (Wheel, [+])
```

We have presented one aspect of the kinematic analysis of a mechanism, the constraint

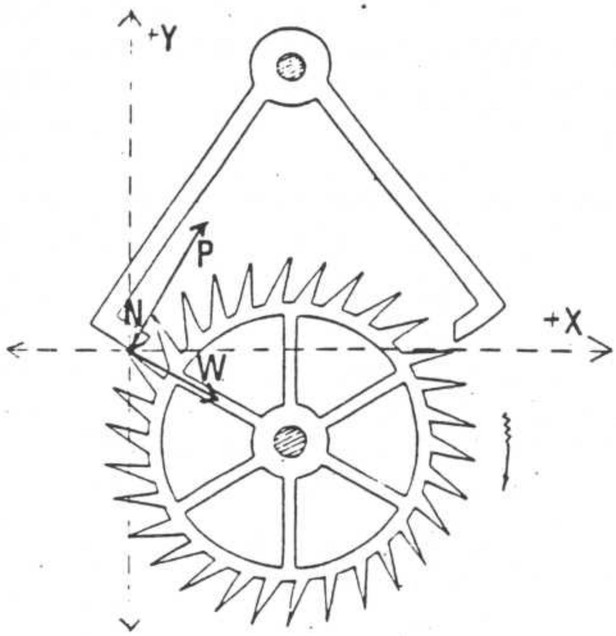

Figure 4: Deadbeat Escapement

of motion. Our approach is capable of describing a wide variety of behaviors of rigid body mechanisms.

The next phase of our research will be the transfer of motion. A constraint may be considered a transfer of sufficient force to exactly cancel the motion of a body, so constraint of motion is a special form of the more general transfer of motion. Moving bodies will transfer force along their surfaces by exactly the same means used to constrain motion, i.e. perpendicular to a line that is tangent to both surfaces at the point of contact. Friction will act parallel to the surface and in the opposite direction of the relative motion.