Mathematical Aspects of Qualitative Reasoning

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Introduction

Qualitative reasoning has received more and more attention in the last few years. Several approaches have been proposed and implemented (/deKleer-Brown 84/, /Williams 84/, /Forbus 84/, /Kuipers 84/, /Raulefs 84/, /Voss 86/, etc.), and several attempts have been started to investigate the applicability of the suggested techniques to practical problems (/Chandrasekaran-Milne 85/). This is due to the fact that obviously a great deal of human reasoning about processes and the behavior of systems is done without using exact numerical information and formalisms even in cases where they are available.

Sometimes, the qualitative arguments are gained by abstraction from existing quantitative models. It is not the topic of this paper to question whether this relationship really suggests a useful understanding of the role of qualitative models. Here, it might be sufficient to point out that, in history as well as in the individual human lerning process, a qualitative understanding and description of a physical process in almost every case **preceeds** the development of formalisms which capture more details, explain more observations or allow calculations for specific cases (/Struss 87/).

Nevertheless, most of the examples given in the literature about qualitative reasoning construct qualitative descriptions of physical processes from given quantitative descriptions, mainly given as sets of equations or differential equations, and use the latter as a scale for determining the value and the correctness of the results obtained by the qualitative analysis. Using the mathematical description of a physical system as the "gold standard" (/Kuipers 86/) instead of using the observation of the physical system itself seems to be a natural and even necessary step. However, one of the aims of this paper is to uncover the problems which are raised by this step.

Going from an exact and unique description of a system in terms of real-valued variables and functions to a qualitative description comprising different distinguishable, but essentially similar instances of a described process or system obviously introduces some vagueness and ambiguity into the analysis and its results. Although the hope that these ambiguities simply express different possible developments according to varying parameters or initial conditions was recently abandonned (/Kuipers 85/) there is still no sufficient understanding why the existing approaches to qualitative reasoning may predict behaviors of a system which contradict the possible solutions to its "precise" description.

/Kuipers 86/ treats the example of a frictionless spring and detects that his qualitative simulation algorithm QSIM is incapable of deciding whether the amplitude of its oscillation is increasing, decreasing, or constant. He explains this observation with the local nature of the simulation steps: each state of the system is only obtained as a result of the directly preceeding state and cannot be related to the overall development, i.e. the whole sequence of state changes. Although this argument is correct with respect to the given example it fails to identify other, even more fundamental difficulties encountered by the existing approaches. These difficulties even occur when dealing with linear equations rather than general equations or differential equations.

Even when the problems observed in /Kuipers 86/ are unimportant or solved (the appendix shows that, in the given example, this can easily be done using only qualitative arguments), qualitative reasoning of this kind will potentially be unable to rule out impossible states of an analyzed system. Examples drawn from the confluence approach of /deKleer-Brown 84/ as well as from the QSIM algorithm are given in this paper, and it is shown that the errors are inevitable for any qualitative reasoning system which, explicitely or implicitely, is based on a kind of interval arithmetic. Furthermore, it is shown that in this case, the qualitative reasoning models are sensitive to variations in the form of the original mathematical description of the system. Even simple transformations like changing of the order of the terms in an expression may influence the results obtained by the qualitative analysis. One of the main concerns of this paper is to identify the core of these problems: the **"locality problem"**.

In order to avoid misinterpretations, the author wants to emphasize that the very formal, mathematical view is **not** taken because it promises solutions to shortcomings of existing qualitative reasoning methods. It is used to determine precisely which aims cannot be achieved on its basis for principal reasons. Since the uncovered drawbacks turn out to be very severe, this should motivate a search for additional concepts and approaches of a completely different nature.

Overview

The paper starts with a section in which several examples are given using the approaches of causal ordering (/Iwasaki-Simon 86/), ENVISION (/deKleer-Brown 84/), and QSIM (/Kuipers 86/). For the latter, it is shown 1. that it produces wrong solutions, 2. that the algorithm is sensitive to changes of the expressions and 3. that it is even sensitive to a change of the order of terms. The examples are meant to serve as an illustration for the problems we encounter, and they hopefully provide motivation for the following more formal and mathematical approach which is necessary to uncover the ultimate reasons for the failures.

Section 3 provides a formal framework for the description of requirements, algorithms, and criteria for qualitative reasoning methods. The general aim is to analyze the character of the mappings in a diagram which relates quantitative and qualitative descriptions and solutions of a system (Fig. 1)

$$\begin{tikzcd} D_{quant} \arrow[r, "q_d"] \arrow[d, "sol_{quant}"'] & D_{qual} \arrow[d, "sol_{qual}"] \\ P(\mathbb{R}^n) \arrow[r, shift left, "q'"] & P(Q^n) \arrow[l, shift left, "p'"] \end{tikzcd}$$

Figure 1

This can serve as a basis for asking the important and frequently discussed questions "Does the qualitative method M_1 miss real solutions?" and "Can every qualitative solution be realized?" more precisely (but, emphasized again, remaining in the world of formal models and ignoring the physical situation, i.e. reality).

The following problem is practically important: Let

$$T = \{ t \mid t : D_{quant} \rightarrow D_{quant} \}$$

be some set of transformations on quantitative descriptions (e.g. adding two equations or linearizing a differential equation etc.) and

$$\sim_T \subset D_{quant} \times D_{quant}$$

a binary relation of quantitative descriptions defined by

$$d_1 \sim_T d_2 \;:\Leftrightarrow\; \exists\, t_1, t_2, \ldots, t_n \in T \quad d_1 = t_1 \circ t_2 \circ \ldots \circ t_n(d_2)$$

(i.e. one description can be derived from the other one by applying a number of transformations). We are interested in answering the question: What happens to the qualitative solution when we transform a quantitative description d_{quant} by applying some $t_o \in T$?:

(3.15) $$sol_{qual}(q_d(d_{quant})) \quad ?\rightarrow \quad sol_{qual}(q_d(t_0(d_{quant})))$$

The most interesting cases are, of course, sets of transformations which do not change the solutions for the descriptions (like multiplying both sides of an equation with a non-zero constant) or change them in a controlled way (parameter transformations). Choosing T appropriately (namely T should be a group with respect to composition) we can make $\sim_T$ an equivalence relation and investigate if and how far $sol_{qual} \circ q_d$ respects this relation and what we can tell about

$$sol_{qual}(q_d(d^{\sim}_{quant}))$$

the set of qualitative solutions for all the members of some equivalence class $d^{\sim}_{quant} \in D_{quant} / \sim_T$.

In this paper, the investigation is mainly focussed on D_{quant} being the space of (even a special kind of) equations.

A first approach based on a kind of interval arithmetic is introduced in section 4. It suffers from the fact that some qualitative values do not have invert elements and that qualitative solutions are dependent on the formulation of the single equations. A modification of the concept of a solution (section 5) leads to a kind of interval arithmetic whose solutions are more robust w.r.t. certain transformations applied to the underlying equations. It is shown that the confluences used in ENVISION are an instance of this approach and that the results still may change when other simple transformations are applied to the underlying set of equations.

In section 6, a further modification of the simple interval arithmetic is presented in order to achieve the separation of the real number line by a finite number of landmark values including values different from zero. This approach corresponds to the QSIM algorithm and allows to provide deeper explanations for its drawbacks as a result of the loss of associativity of this kind of interval addition.

In both cases, the origin of the problem lies in restrictions imposed by the locality of the representational entities: equations and constraints.

The conclusions discussed in section 7 attempt to uncover this locality problem and suggest to emphasized the importance of physical knowledge (including knowledge about the component structure of a physical system) as opposed to the attempt of merely analyzing mathematical descriptions.

Three qualitatively different appearances of the general locality problem are identified and discussed:

1. The locality of equations and constraints, leading to the detection of wrong states and state transitions.
2. The locality of state transitions, causing problems in determining possible sequences of state transitions (i.e. a global behavior).
3. The locality of a description in a family of descriptions (as it is treated in bifurcation theory and catastrophy theory),