A QUALITATIVE APPROACH TO STRUCTURAL MECHANICS

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ABSTRACT

In the field of Civil Engineering, Knowledge Engineering techniques have been applied to safety evaluation, damage assessment and design of structures, but applications are usually limited to specific cases. In this paper, a general approach to structural mechanic problems, based on Qualitative Physics, is discussed, providing the way to model and reason about a wide class of structures as physical devices. Some aspects of the theory proposed by De Kleer and Brown, recognized to be particularly suitable for structural mechanic problems, have been partially reformulated, taking into account the vectorial nature of the problem. The results obtained by the analysis of some plane frame cases provide suggestions to improve the qualitative model of the structures and the process of finding their behavioural description.

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INTRODUCTION

Knowledge Engineering (KE) is that subfield of Artificial Intelligence concerned with the acquisition, representation and manipulation of human knowledge in symbolic form. Feigenbaum defines the activity of knowledge engineer as [9] :

"...practices the art of bringing the principles and tools of AI research to bear on difficult applications problems requiring experts' knowledge for their solution".

The theoretical and technical issue of acquiring, representing, and using this knowledge appropriately to construct and explain lines of reasoning, have entered various fields. In the field of civil engineering, KE techniques have been applied to safety evaluation, damage assessment, and design of structures, but applications are usually limited to specific cases (for example, bridges or tall buildings [2,11,17,20]). In this paper a more general approach to Structural Mechanics is presented, based on Qualitative Physics (QP). The models obtained by means of QP theories are suitable for a wide class of physical systems and, in the field of Structural Mechanics, they allow to analyse structures in general and to reason over them.

Generally, Structural Mechanics concerns with deformable, continuous or discrete systems, which can be analyzed under dynamic or static loads. This work deals whit discrete, statically determined or indetermined systems, which are examined in static conditions. More specifically, the analysis of plane frames in the elastic range is the scope of this application as the most common case in civil engineering.

As a result of a comparative analysis among different theories of QP [3,10,15], the basic concepts proposed by De Kleer and Brown in [3,7] are recognized to be particularly suitable because of the characteristics of the problem at hand.

Modeling, based on components and their interconnections (topology) [3], follows very closely the construction of the classical physical models used for this kind of structures and, more generally, for discretized continuous systems (finite element method [22]).

Device−centered ontology [3] is very suitable for structural problems, in which the agents of change are the components of the device, identifiable with the elements of the structure (e.g. beams). The change consists in the deformation of the elements over time, caused by load applied to the previous equilibrium configuration. Deriving this change from structure (e.g. components−beams and interconnections−nodes) is coherent with the common point of view in structural engineering.

Causal accounts of the behaviour of the analysed system is a useful result of QP applied to Structural Mechanic domain. More particularly the kind of causality proposed in [3] reflects certainly one important reasoning technique performed by a civil engineer during the analysis of a device (e.g. the qualitative sketch of the bending moment diagram for the elements of a frame) : causality in structural systems intuitively depends on the interactions among physical constituents, rather than on the mathematical structure of the model [7,13,14].

STRUCTURAL MECHANICS AND QUALITATIVE PHYSICS

Due to the intrinsic vectorial nature of the behavioural parameters (forces and displacements), numerical modeling of a linearly elastic plane frame can be obtained through matrix formulations [18] (displacement or force methods) allowing a complete description of the structural scheme be given only on the basis of components (beams) and their interconnections (nodes). Three kinds of information are usually processed in conventional solution procedures (Fig. 1a) :

- Mechanical : the matrix relationships between generalized force and displacement vectors for any element (beam) within the structure and the distribution of the applied load vectors.
- Topological : the interconnections among elements at the nodes.
- Geometrical : the orientation of the elements and the spatial position of the nodes.

The basic choice in building the qualitative model has been to derive the "confluences" [3] from a quantitative formulation of the above type, and to improve the model, if necessary, adding *commonsense knowledge* about the domain. In particular, the formulation of matrix methods for structural analysis presented by Spillers [21] has been taken as reference, because it emphasizes the

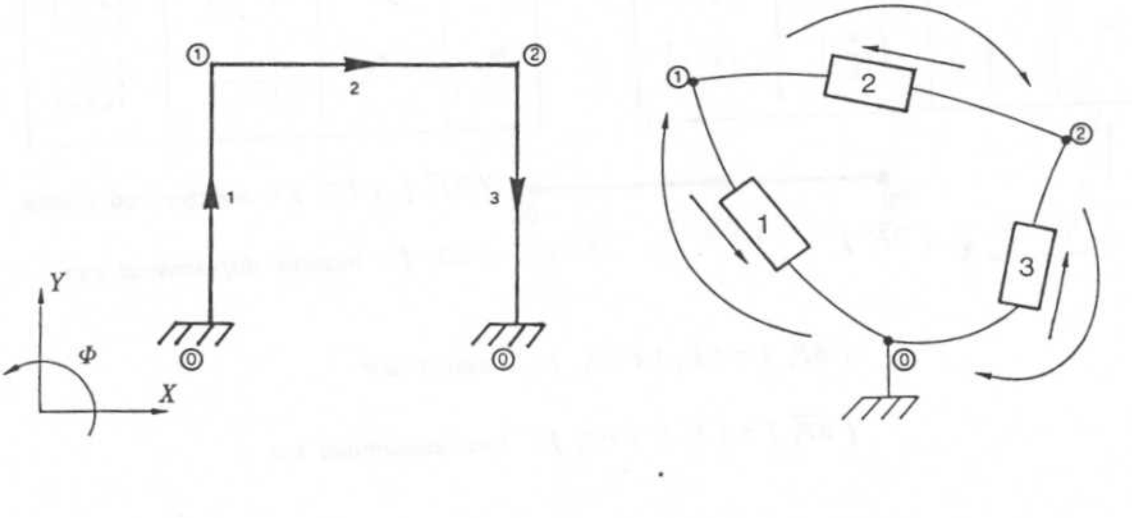


Fig. 1

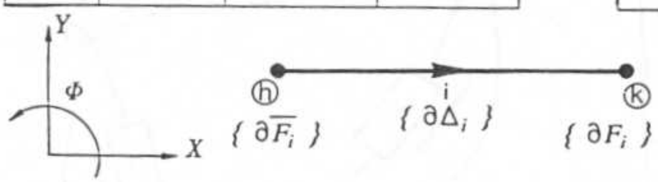
topological aspect of the problem. More precisely, the construction of the force and of the displacement methods is derived in this approach from an equivalent graph−theoretic representation (Fig. 1b) of the structure, respectively following the mesh or the node solution method for a generalized network problem. The topology is represented by an *incidence matrix* which is explicitly used in assembling the equations of the problem. The other kinds of information are represented by the *stiffness matrix* (mechanical information) and by the *rotation matrix* (geometrical information). However, in order to obtain a suitable representation in terms of component and connection laws, the component model should contain not only mechanical information about beams, but geometrical information as well. The original displacement formulation has been therefore modified to insert geometrical information in the behavioural model of beams; to achieve this goal all the matrices and vectors have been referred to a unique global coordinate system. In this way the qualitative translation of the stiffness matrix of the beams, expressing member end forces in terms of the member end displacements, allows to get a component library, in which every beam is characterized by both mechanical and geometrical properties (see Fig.2). Different orientations of the same beam (from

COMPONENT LIBRARY
Fixed end beams

Case	Scheme	$[\Gamma_i]$	$[\Lambda_i]$
N1		$\begin{bmatrix} + & 0 & 0 \\ 0 & + & - \\ 0 & - & + \end{bmatrix}$	$\begin{bmatrix} - & 0 & 0 \\ 0 & - & - \\ 0 & 0 & - \end{bmatrix}$
N2		$\begin{bmatrix} + & 0 & + \\ 0 & + & 0 \\ + & 0 & + \end{bmatrix}$	$\begin{bmatrix} - & 0 & + \\ 0 & - & 0 \\ 0 & 0 & - \end{bmatrix}$
N3		$\begin{bmatrix} + & 0 & 0 \\ 0 & + & + \\ 0 & + & + \end{bmatrix}$	$\begin{bmatrix} - & 0 & 0 \\ 0 & - & + \\ 0 & 0 & - \end{bmatrix}$
N4		$\begin{bmatrix} + & 0 & - \\ 0 & + & 0 \\ - & 0 & + \end{bmatrix}$	$\begin{bmatrix} - & 0 & - \\ 0 & - & 0 \\ 0 & 0 & - \end{bmatrix}$

COMPONENT LIBRARY
Fixed end beams

Case	Scheme	$[\Gamma_i]$	$[\Lambda_i]$
N5		$\begin{bmatrix} + & + & + \\ + & + & - \\ + & - & + \end{bmatrix}$	$\begin{bmatrix} - & 0 & + \\ 0 & - & - \\ 0 & 0 & - \end{bmatrix}$
N6		$\begin{bmatrix} + & - & + \\ - & + & + \\ + & + & + \end{bmatrix}$	$\begin{bmatrix} - & 0 & + \\ 0 & - & + \\ 0 & 0 & - \end{bmatrix}$
N7		$\begin{bmatrix} + & + & - \\ + & + & + \\ - & + & + \end{bmatrix}$	$\begin{bmatrix} - & 0 & - \\ 0 & - & + \\ 0 & 0 & - \end{bmatrix}$
N8		$\begin{bmatrix} + & - & - \\ - & + & - \\ - & - & + \end{bmatrix}$	$\begin{bmatrix} - & 0 & - \\ 0 & - & - \\ 0 & 0 & - \end{bmatrix}$



$\{ \partial \overline{F_i} \}, \{ \partial F_i \}$ = *member end forces*

$\{ \partial F_i \}$ $\{ \partial \Delta_i \}$ = *member displacement gap*

$\{ \partial F_i \} = [\Gamma_i] \{ \partial \Delta_i \}$ *elasticity law*

$\{ \partial \overline{F_i} \} = [\Lambda_i]^t \{ \partial F_i \}$ *local equilibrium law*

Fig.2

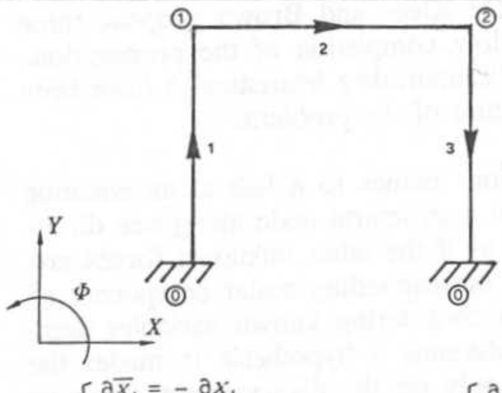
the mechanical point of view) correspond to different components. The number of different orientations, infinite in the quantitative field, can be reduced to eight, assuming the quantity space $\{+,0,-\}$ for qualitative variables.

The set of confluences of the qualitative model is represented by:

• Component Model Confluences (CMC): the qualitative translation of the local equilibrium law (CMC1) and of the elasticity law (CMC2) for the beams.

• Network laws: the qualitative translation of the Equilibrium of Forces Law (EFL) and of the Compatibility of Displacements Law (CDL).

Because of the nature of the problem, qualitative states for component−beams are not defined; the behaviour of an elastic beam is completely described (from the static point of view) by a set of "pure" confluences, without introducing qualitative states. The qualitative model, i.e. the confluences, for a plane frame is shown in Fig.3.



$$\{\partial F_i\} = \begin{Bmatrix} \partial X_i \\ \partial Y_i \\ \partial M_i \end{Bmatrix} \quad \{\partial \Delta_i\} = \begin{Bmatrix} \partial \Delta x_i \\ \partial \Delta y_i \\ \partial \Delta \phi_i \end{Bmatrix}$$

$$\{\partial \overline{F_i}\} = \begin{Bmatrix} \partial \overline{X_i} \\ \partial \overline{Y_i} \\ \partial \overline{M_i} \end{Bmatrix} \quad \{\partial \delta_j\} = \begin{Bmatrix} \partial \delta x_j \\ \partial \delta y_j \\ \partial \delta \phi_j \end{Bmatrix} = \textit{node displacement}$$

$$CMC1 \begin{cases} \partial \overline{X}_1 = -\partial X_1 \\ \partial \overline{Y}_1 = -\partial Y_1 \\ \partial \overline{M}_1 = \partial X_1 - \partial M_1 \end{cases} \quad CMC2 \begin{cases} \partial X_1 = \partial \Delta x_1 + \partial \Delta \phi_1 \\ \partial Y_1 = \partial \Delta y_1 \\ \partial M_1 = \partial \Delta x_1 + \partial \Delta \phi_1 \end{cases} \quad CDL \begin{cases} \partial \Delta x_1 = \partial \delta x_1 \\ \partial \Delta y_1 = \partial \delta y_1 \\ \partial \Delta \phi_1 = \partial \delta \phi_1 \end{cases}$$

$$CMC1 \begin{cases} \partial \overline{X}_2 = -\partial X_2 \\ \partial \overline{Y}_2 = -\partial Y_2 \\ \partial \overline{M}_2 = \partial Y_2 - \partial M_2 \end{cases} \quad CMC2 \begin{cases} \partial X_2 = \partial \Delta x_2 \\ \partial Y_2 = \partial \Delta y_2 + \partial \Delta \phi_2 \\ \partial M_2 = \partial \Delta y_2 + \partial \Delta \phi_2 \end{cases} \quad CDL \begin{cases} \partial \Delta x_2 = \partial \delta x_1 - \partial \delta x_2 \\ \partial \Delta y_2 = \partial \delta y_1 - \partial \delta y_2 + \partial \delta \phi_2 \\ \partial \Delta \phi_2 = \partial \delta \phi_1 - \partial \delta \phi_2 \end{cases}$$

$$CMC1 \begin{cases} \partial \overline{X}_3 = -\partial X_3 \\ \partial \overline{Y}_3 = -\partial Y_3 \\ \partial \overline{M}_3 = \partial X_3 - \partial M_3 \end{cases} \quad CMC2 \begin{cases} \partial X_3 = \partial \Delta x_3 + \partial \Delta \phi_3 \\ \partial Y_3 = \partial \Delta y_3 \\ \partial M_3 = \partial \Delta x_3 + \partial \Delta \phi_3 \end{cases} \quad CDL \begin{cases} \partial \Delta x_3 = \partial \delta x_2 \\ \partial \Delta y_3 = \partial \delta y_2 \\ \partial \Delta \phi_3 = \partial \delta \phi_2 \end{cases}$$

$$EFL \begin{cases} \partial X_1 + \partial X_2 = \partial PX_1 \\ \partial Y_1 + \partial Y_2 = \partial PY_1 \\ \partial M_1 + \partial M_2 = \partial PM_1 \end{cases} \quad EFL \begin{cases} \partial \overline{X}_2 + \partial X_3 = \partial PX_2 \\ \partial \overline{Y}_2 + \partial Y_3 = \partial PY_2 \\ \partial \overline{M}_2 + \partial M_3 = \partial PM_2 \end{cases}$$

Fig.3

DETECTION OF THE BEHAVIOUR

In order to obtain a causal description of its behaviour, a structure must be loaded: if the system remains at equilibrium indefinitely, no causal action will take place and it will be impossible to discover the dependency relations among variables. Initially the device is assumed to be at equilibrium; this equilibrium is disturbed by a change in the value of one scalar component of a load vector, which acts on a node of the structure. The choice of this procedure has been made because of simplicity of the analysis and because it allows to consider separately the effects of each scalar input. The behaviour of a structure loaded by a full vector input force can be achieved by merging the effects of each component force.[1] The effect of the initial disturbance is then propagated through the constraint network (scalar confluences) until all the variables are assigned new values and equilibrium is restored. This propagation is done considering the inferences among the scalar components of the vector variables (scalar propagation).

When sufficient local information is not available to propagate a disturbance at any point, the propagation process comes to a halt. Then new premises need to be introduced in order to continue propagation. De Kleer and Brown propose three heuristic rules to introduce new premises to allow completion of the propagation. In the structural domain these heuristic rules ("canonicality heuristics") have been reformulated taking into account the vectorial nature of the problem:

Conduit heuristic. It applies when the propagation comes to a halt at an equation that states the equilibrium of the forces acting on a structural node along one direction (x, y, ϕ). A known force acts on a node as if the other unknown forces are negligibles and causes an according value of the corresponding scalar component of the displacement variable for the node. Besides considering known variables dominant with respect to unknown ones, in this inference a hypothesis is made: the scalar force acting on a node has direct effect only on the displacement along its same direction. This corresponds to consider a preferred direction of propagation of the material (forces) through the conduits (structural nodes).
For example, consider Fig.4. Let $\partial\, P\, M_1 = +$ the input disturbance. The propagation stops immediately at the equilibrium condition of the moments for node 1. Applying the conduit heuristic rule it is possible to get the value of the displacement of the node: $\partial \delta\ \phi_1 = +$ and continue the propagation process.

Component heuristic. It applies to the equations that state the compatibility of the displacements for a beam. A known displacement causes an according displacement gap across the beam for the same scalar component of the known displacement, as in the previous heuristic.
Consider, for example, the frame whose mathematical model is shown in Fig.3. In the CDL confluence for beam 2, if the unique known variable is the displacement $\partial \delta\ \phi_1$ of node 1, then the heuristic rule can be applied to provide the value $\partial \Delta\ \phi_2$, under the assumption that the known displacement variable is the cause of the difference of displacements between the structural nodes (displacement gap) at

[1]The possibility of multiple inputs in different nodes of the structure involves reasoning about orders of magnitude [8,19] and this fact introduces side effects on the causality (interactions among different causal paths starting from different input nodes).

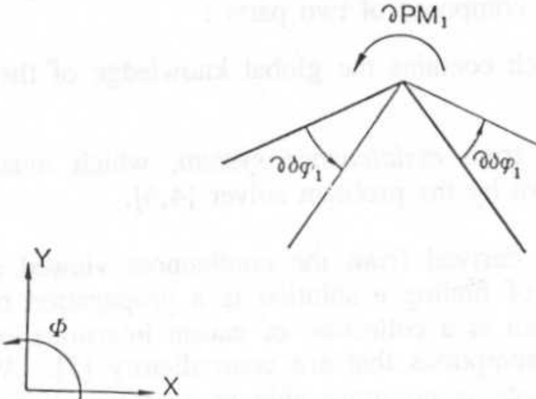


Fig.4

which the beam is linked.

Confluence heuristic. It applies to the equations that represent the model of a beam. This kind of heuristic rule simply considers negligible an unknown variable within one scalar confluence, regardless to its physical meaning. For example, in the first CMC2 confluence for beam 1, if the known variable is the displacement gap $\partial\Delta\, x_1$, it is possible to infer the value either of the force transmitted by node 1 to the beam or of the other displacement gap in the confluence: $\partial\Delta\phi_1$.
In this heuristic inference there is no preferred direction depending on the known scalar component or on the kind of variable involved (force, displacement, displacement gap): it is impossible to choose the variable to be implied without introducing domain dependent considerations. Moreover it should be noted that the inferences which can be drawn about beam models heavily depend on all its confluences.[2] Reasoning about one confluence at a time and considering negligible in turn all the variables without any physical consideration, could affect realizability of solutions. Reasoning about only one confluence could affect realizability of solutions because of a component being actually represented by three confluences. To avoid these problems it is possible to modify the introduction of such an heuristic rule to obtain useful and correct inferences: for example, attempting to propagate the known variables not only in one scalar confluence, but in all the confluences describing the vectorial relation of the component. In other words, a variable is to be considered "known" only when all its scalar components are known, and the implied variable must be considered with all its components as well. Of course such a formulation of the confluence heuristic limits its use.

The system previously described is composed by two modules; a module which produces the qualitative model (a set of confluences) starting from a

[2]Every component model is represented by three confluences in the present work, dealing with plane frames only.

topological description of a plane frame (which could be easily extended to deal with other kinds of structures), and a second module which takes the output of the previous one and performs the qualitative reasoning about the provided model. This latter module is logically composed of two parts :

- a *problem solver*, which contains the global knowledge of the problem and the inference procedures;

- an *assumption−based truth maintenance system*, which must ensure consistency of the inferences drawn by the problem solver [4,6].

The inference procedures are derived from the confluences viewed as constraints for the device. The process of finding a solution is a propagation of constraints. The output of the entire system is a collection of causal interpretations which are discerned by means of the assumptions that are contradictory [5]. When the process of satisfying the constraints is no more able to continue, it is necessary to introduce heuristic inferences.

The architecture of the "reasoner" is depicted in Fig.5. A frame representation has been adopted to codify knowledge about structures. A confluence is represented by inference procedures ("consumers" [6]), codified in terms of rules and attached as attributes to the variables participating in it. At every step an inference procedure is scheduled and triggered as possible (SCHEDULER), or an

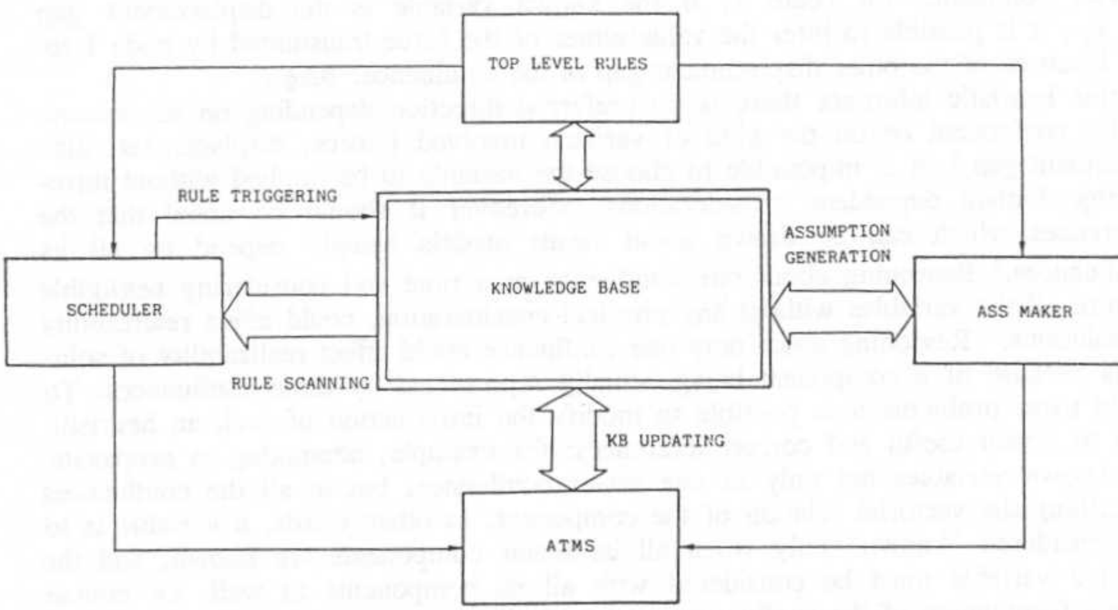


Fig.5

assumption is formulated and a heuristic inference is introduced (ASS−MAKER). Both actions produce new information that cannot be directly stored in the Knowledge Base, but they must be processed in advance by the ATMS module in order to maintain the knowledge base updated and consistent.

The system has been implemented on an EXPLORER lisp machine using the KEE tool [12].

BASIC RESULTS AND CONCLUDING REMARKS

Two kinds of results can be achieved by means of qualitative analysis of structures: a qualitative behavioural description, i.e. the possible values of all the variables of the system, and a causal explanation for it, i.e. the causal relations among variables established during the propagation process.

The behavioural descriptions obtained from the analysis of the frame of Fig.3 are shown in Fig.6. There are three interpretations of the behaviour of the device (only two are shown in the figure for clarity: the third one being simply the bound case between them), loaded by a positive moment in node 1. These three interpretations correspond to three values for the moment transmitted from the fixed joint to beam 1 { +,0,− }, which depend on the relative stiffnesses of the beams. This

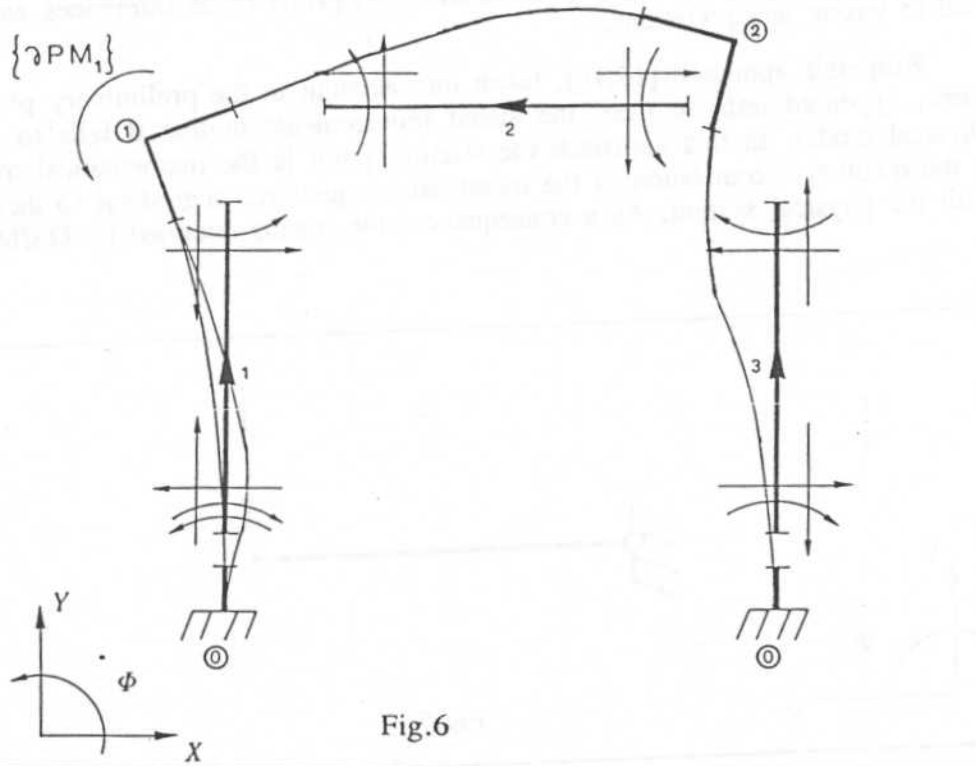


Fig.6

result, produced by the reasoner, means to get "realizability" and "completeness" for the provided model, that is it includes all the possible behaviours that the structure can manifest.

Another important result in the structural domain, is the detection of particular physical situations, such as the statical unadmissibility of the modeled system. This means that the analysed structural arrangement cannot sustain the applied load (see Fig.7) and correspond to the detection of contradictions involving data supposed to be true. The validity of the solution highly depends on the model provided, that can be improved by the addition of conditions, which are redundant in the quantitative domain, but not in the qualitative one. This is especially true when the structure becomes complex. In the domain of structures, redundant confluences can be built from the "mesh law" [21], that states the continuity of node displacements along a structural loop. To build this kind of conditions algebraic manipulation of qualitative matrices is necessary. Because of the ambiguity which can arise from qualitative matrix products such conditions can be often unusable: a choice could be the formulation of only those relations which can be drawn without ambiguity. Considering this difficulty, other ways to add confluences to the model, deriving from commonsense knowledge, are possible. For example, imposing the global equilibrium of the device, or imposing qualitative relationships between the unique input variable and the force variables for *every* beam connected to the loaded node.

Moreover, the inferences deriving from the application of the confluence heuristic rule appear not always reliable. In particular, it seems necessary, in order to ensure realizability of the results, to modify the formulation of this heuristics introducing domain dependent considerations to prune those inferences which could lead to wrong interpretations.

Kuipers's approach [15,16], taken into account in the preliminary phase of the work, appeared unfit to meet the stated requirements in that it fails to provide a physical model. In this approach the starting point is the mathematical model, that is the qualitative translation of the quantitative equations, regardless to their relation with the physical system. As a consequence, the results obtained by QSIM [16] are

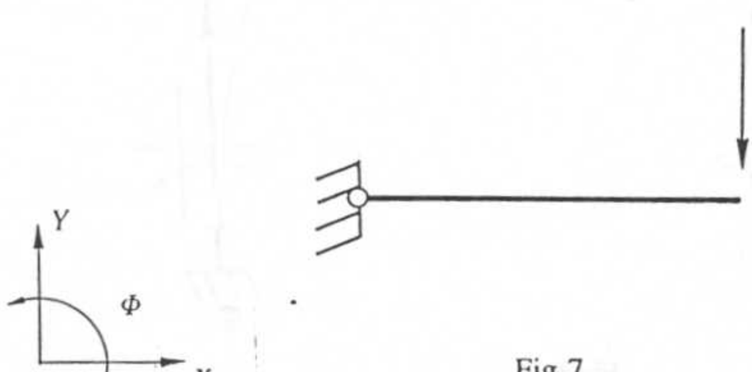
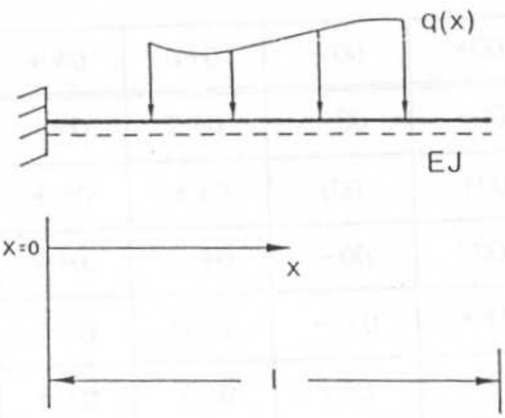


Fig.7



$$EJ \frac{d^4}{dx^4} v(x) = q(x)$$

Fig.8

the time history of each variable, instead of the explanation of the interactions among physical constituents, as in De Kleer and Brown's theory. Anyway, interesting developments can arise from the integration of the two approaches of QP. It is possible, indeed, to exploit the ability of solving differential qualitative equations of Kuipers's QSIM, applying it to the differential equation of a beam, which is the component of a frame. This equation is a fourth order differential equation with respect to the independent variable x (see Fig.8). The necessary boundary conditions can be supplied by an analysis with De Kleer and Brown's approach performed over the whole frame. In this way global analysis of the structure is performed by the module here discussed, and local analysis of the behaviour of the components can be performed by means of QSIM, in order to obtain a more detailed behavioural description, including strains and internal forces along beam axis.

Considering the difficulties previously described arising from the introduction of the confluence heuristics (CMC), depending on the vectorial nature of the system, and aiming to get a far more simpler causal explanation, the formalization of a Vectorial Qualitative Physics is presently under way [1]. In such a new formalization the components act on vector variables; consequently a variable can assume 27 qualitative values corresponding to the possible combinations of the values of each component of a vector (+,0,−). To deal with vectors, a qualitative vectorial algebra is required (see Fig.9). This algebra takes into account in a different manner ambiguity: while an addition between two scalar variables is ambiguous or not, the same operation, with vector operands, can produce an ambiguous, not ambiguous or partially ambiguous result. In the third case the inference drawn still supplies information, because it limits the number of values assumed by the sum. For example, adding $\{\partial X\} = [+, 0, -]^t$ to $\{\partial Y\} = [+, +, +]^t$ the

∂Y \ ∂X	000	00+	00–	0+0	0++	0+–	...
000	000	00+	00–	0+0	0++	0+–	...
00+	00+	00+	00?	0++	0++	0+?	...
00–	00–	00?	00–	0+–	0+?	0+–	...
0+0	0+0	0++	0+–	0+0	0++	0+–	...
0++	0++	0++	0+?	0++	0++	0+?	...
0+–	0+–	0+?	0+–	0+–	0+?	0+–	...
0–0	0–0	0–+	0––	0?0	0?+	0?+	...
..	...	...	...	...	...	...	...

Fig.9

qualitative sum is $\{ \partial Z \} = \{ \partial X \} + \{ \partial Y \} = [+, +, ?]'$. In this case ambiguity affects only the third scalar component of the sum, so that the provided information allows to reduce the possible values to three ($[+, +, 0]'$, $[+, +, -]'$, $[+, +, +]'$), instead of 27 as in the case of full ambiguity. As in the scalar approach , in order to discover causal relations among variables, a disturbance must be applied. Such a disturbance is a vector, whose effects are propagated along topological paths of the structure. A beam , at this level of analysis, has not internal topological paths and when the front of the causal wave reaches a component terminal, its model is used to infer the whole vector to which the known variable is linked[3]. Such a vector could not have a unique value because of ambiguity, in this case all possible vector values are propagated on[4]. It should be noted that in structural problems where components are beams joined to other beams at the two extreme ends, the vectorial confluences which represent the behaviour of the beams, cannot have more than two variables. As a consequence, it is not necessary to introduce heuristics to propagate through halted component confluences (i.e. CMC heuristics).

[3]These inferences are drawn using the results of the scalar causal analysis of the behaviour of the beams (pushing down the analysis inside it).

[4]In order to reduce the number of values to propagate in case of ambiguity, it is possible to make use of specifical knowledge about the domain of application.

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