Qualitative Reasoning at Multiple Resolutions

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Abstract

In this paper we describe an approach to unify the various quantity spaces that have been proposed in qualitative reasoning with numbers. We work in the domain of physical devices, such as electrical circuits using lumped parameter models. We show how changing the quantity space can be achieved in the course of analysis and how this is similar to dynamically changing the resolution in analysis. We demonstrate the utility of this approach with two examples in the domain of circuit analysis.

1. Introduction

One of the chief aims of Qualitative Reasoning is to provide a broad picture of the functioning of the world by taking a step back from the details. In this paper we show that in reasoning with numbers the aim is to break the real number line into broad, qualitatively distinct classes and describe the working of a device in terms of these classes.

[Joha85a] defines the qualitative values a variable can have $A_0 \ldots A_z$ as representing disjoint abutting intervals that cover the entire number line. I define the set of values $\{A_0 \ldots A_z\}$ as the Q-space.[1]

The aim of Qualitative Reasoning is to reduce the cardinality of the Q-space while still retaining the information available from doing the analysis using quantitative values. This has two benefits.

- Complete quantitative information is not always available about the variables being analyzed. For example in design, one may not know the exact values of all parameters in the design. Yet one has to make decisions using this partial information. In this case the partial information can be used by representing the variables in a qualitative form. By using the smallest possible Q-space in which to perform the analysis we are able to deal better with incomplete information.
- By using a qualitative description of the variables we can form a description of the working of a device that has a smaller number of states. Thus one can get a better understanding of the workings of the device, at the desired level of detail. In essence using a small Q-space gives a broader picture of the workings of a device.

It is therefore intuitively clear that the best approach is to use the smallest Q-space possible that will describe the working of the device. Unfortunately however, the expressive power of a Q-space depends on the number of elements it contains. This paper describes a scheme to carry out analysis in the smallest possible Q-space.

We show that depending on the problem at hand it is advantageous to perform the analysis in different Q-spaces. We propose a set of 4 Q-spaces which represent different resolutions on the number line. We show that with this judicious choice of Q-spaces we can switch dynamically between Q-spaces while performing the analysis. In the process we perform each operation in the analysis at the smallest resolution. We show how to switch to a Q-space with a higher resolution when the results of an operation are ambiguous. Different parts of the analysis can be carried out at different resolutions and the final result is a description of the device that is close to optimal. This is illustrated with the help of two examples in linear circuit design.

2. Q-spaces

The following set of Q-spaces are proposed:

1. (±) (0, non-zero)

   This Q-space is identical to the one described in [Joha85a] The following relationships between variables can be expressed in this Q-space.

   $a > b$ if $[a - b] = +$

   $a = b$ if $a - b$ is 0

   The converse of these relationships can also be expressed.

   In addition we can express relationships between quantities based on the relations =, > and <

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1 I would have liked to use the term Quantity Space but that has a different meaning[Forb85]. I am willing to accept suggestions for better names.

a is increasing if

$a(t2) > a(t1)$ and $t2 > t1$

The rules for arithmetic are described in [Joha85a] It is to be noted that if [a] ≠ [b] then [a + b] is indeterminate. Magnitude information is also absent. This ambiguity can be resolved by moving to the next Q-space.

2. (±) (0, infinitesimal, large)

   This Q-space is identical to the one described in [Raim86] All relations that can be expressed in Q-space 1 can be expressed in this Q-space. In addition, the following relationships can be expressed[Raim86].

   $a \gg b$ if a is large and b is infinitesimal.

   $a \cong b$ if $a = b(1 + \varepsilon)$.

   $a \sim b$ if a and b are both infinitesimal or large.

   [Mavr87] shows how to tie this Q-space to the real number line. This is done by choosing a value e that is the minimum ratio between a large and a small number.

   Q-space 2 splits the positive half of the real number line into two halves that are separated by a threshold. The threshold is different for different types of variables e.g. impedance and frequency. Even for the same type of variable the threshold depends on the particular comparison being made. For example when we say two places are far apart it depends on whether the journey is being made by car or on foot.

   In the following a_t and b_t are the thresholds for a and b. The rules for addition are[2] described in [Raim86]. It is to be noted that these rules holds only if $a_t = b_t$.

   Multiplication in this Q-space retains the sign information.

   a × b is large if a is large and b is large
   a × b is small is a is small and b is small
   Here the threshold is $a_t \times b_t$
   The product is ambiguous is all other cases.

   It is significant that the threshold changes during multiplication. We show in the examples how this can result in ambiguity. These ambiguities can resolved by using Q-space 3.

3. (±)(0, y^z) where y is the base, (e.g. 2 or 10), and z is an integer. Here 1 is y^0.

   If $|a| = y^z$, then $\log(a) = z$.

   In this Q-space it is possible to express all the relations that can be expressed in Q-spaces, 1 and 2. In addition it is possible to describe the logarithmic distance , LD, between two numbers

   LD(a,b) = log(a) - log(b)

   For multiplication.

   $[a][b] = [ab]$

   $\log(a*b) = \log(a) + \log(b)$

   For addition the rules are.

   If log(a) > log(b) or ([a] = [b] and log(a) = log(b)) then[2]

   $\log(a + b) = \log(a)$

   and

   $[a + b] = [a]$

   If log(a) = log(b) and [a] ≠ [b] then

   $\log(a + b) \leq \log(a)$

   and $[a + b]$ is indeterminate. To resolve the ambiguity we need to go to a finer level of resolution, i.e. the next Q-space.

4. (±)(x * y^z), y and z as before and x is a number with n significant digits. As n increases the veracity of the description increases till at n=infinity this Q-space approaches the real number line. The rules for addition and subtraction are similar to that in machine arithmetic with fixed precision.

3. Relationship to previous work.

In this section we illustrate the use of the 4 Q-spaces, with two examples from the domain of circuit analysis.

[2] We run into Zeno's paradox here. This can be resolved by going to the next finer resolution if necessary.

The (±) (0, non-zero) Q-space [Joha85a, Forb85] has the lowest resolution. It is excellent for describing the working of the circuit in Figure 1 if we merely wish to discover whether the current I flowing in the circuit increases with V.

$I = V_o/R_L$

$I1 = V1/R$

$I2 = V2/R$

$I2 - I1 = (V2 - V1)/R$

If $V2 > V1$ then $[V2 - V1] = +$

Therefore $[I2 - I1] = +$ and I increases with V.

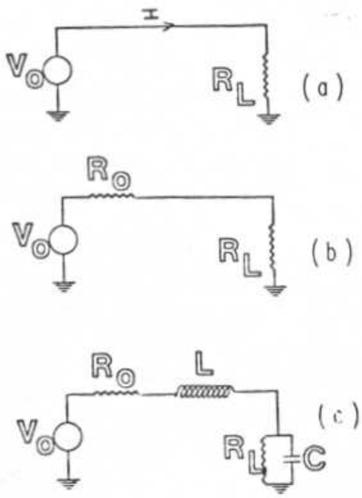

Figure 1. Figure a is a simplified model of a voltage source in series with a load resistance R_L. In figure b the voltage source is represented as an ideal voltage source in series with an output resistance R_o. In figure c the load is represented by a resistor R_L in parallel with a capacitance C. The whole unit is in series with an inductance L.

Other examples of reasoning in the (±)(0, non-zero) space can be found in [Joha85b, Will85] [3] The main problem with reasoning in this space is that addition of two numbers of different signs results in ambiguity. Also it is not possible to neglect small influences w.r.t. big ones. This is a very important part of Qualitative Reasoning in humans. To achieve this capability we need to move to Q-space 2.

[3] Using the signs of partials as the elements of an implicit Q-space is a common technique in economics.

If we started out with a more complicated model of a voltage source that includes an output resistance R_o as in Figure 1b we can use the (±)(0, infinitesimal, large) Q-space [Raim86] to reason about the quantities. To determine the current flowing in the circuit we use Ohm's law to find

$I = V/(R_o + R_L)$

If $R_o \ll R_L$ then R_o can be neglected w.r.t R_L. i.e.

$R_o + R_L = R_L$

Therefore

$I = V/R_L$

Reasoning in the (±)(0, infinitesimal, large) Q-space can bring about ambiguity if two quantities are multiplied. Consider the example of Figure 1c. Here we represent the load by a capacitor C in parallel with the load resistance R_L. The combination is in series with an inductance L.

$\text{Admittance}(R_L \parallel C) = \omega C + 1/R_L$[Purc65]

Each type of variable in this equation has its own threshold. That is because different types of variables have different units. For example, it does not make sense to compare frequency and resistance. If we know that frequency has a threshold ω_t, resistance has a threshold R_t, and capacitance has a threshold C_t it is not necessary that

$C_t\omega_t = 1/R_t$

even though they have the same units. It is therefore not possible to compare ω C and 1/R in this Q-space. It is also not possible to compare R and ωL, the impedance of the inductance L. Hence it is not possible to know if any of the quantities in the admittance can be neglected.

A threshold must be chosen for each comparison that is made. In order to do this we need to move to Q-space 3.

If we know that $\omega \sim 10^5$, and $C \sim 10^{-12}$, then $\omega C \sim 10^{-7}$. Similarly if

$R_L \sim 10^3$, then $1/R_L \sim 10^{-3}$

. If we set the threshold at 10^{-4}, we find that

$1/R_L \gg \omega C$

$1/R_L + \omega C = 1/R_{L}$

The impedance of $R_L \parallel C$ is R_L, and the capacitance C can be deleted from the model. Hence the current I flowing through the circuit is

$V/(R_o + \omega L + R_L)$

Here again it is not possible to compare R_L and R_o. If we move back to Q-space 3 we find that $L \sim 10^{-10}$ and its impedance $\omega L \sim 10^{-5}$. If $R_o \sim 10^{-3}$ then we can set the threshold at 10^4.

$R_L \gg \omega L$

and

$R_L \gg R_o$

Hence these two quantities can be neglected w.r.t. R and

$I = V/R_L$

Let us now consider an example that has more components. Figure 2 shows the circuit for a positive voltage follower.

The model for the operational amplifier has the following parameters:

Bias current	$I_b \sim 10^{-10}$ A
Input resistance	$R_i \sim 10^{12}\,\Omega$
Input capacitance	$C_i \sim 10^{-12}$ F
Cutoff frequency	$\omega_c \sim 10^7$Hertz
Output voltage	$V_o \sim 10^1$ Volts
Gain	$K \sim 10^2$
Output resistance	$R_o \sim 10^{-2}\,\Omega$
Biasing resistors	R_1 and $R_2 \sim 10^5\,\Omega$
Load resistor	$R_L \sim 10^3\,\Omega$.
The voltage source has a	
Voltage	$V_i \sim 10^0$ Volts,
Output resistance	$R_{i1} \sim 10^5\,\Omega$
Frequency	$\omega \sim 10^4$ Hertz

On analyzing this circuit we find that Q-space 2 is not sufficient to remove ambiguities. We need to go to Q-space 3 like in the previous example. We find that

$1/R_i \ll \omega C_i$

Therefore R_i can be dropped from the model.

$V_i \omega C_i \gg I_b$

therefore I_b can be dropped from the model.

$R_{i1} \ll 1/\omega C_i$

With these simplifications to the model, the voltage at the input to the operational amplifier is the same as V_i Similar reasoning reduces the circuit to the one shown in Figure 3

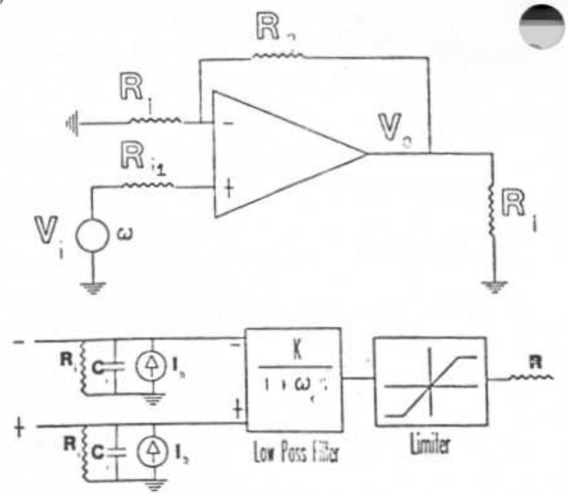

Figure 2. A positive voltage follower. The top figure shows the circuit using an operational amplifier and the bottom is the model of the operational amplifier.

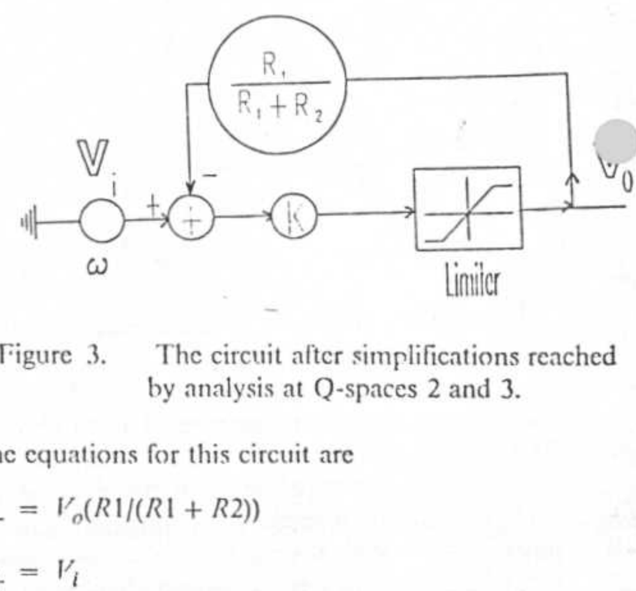

Figure 3. The circuit after simplifications reached by analysis at Q-spaces 2 and 3.

The equations for this circuit are

$V_- = V_o(R1/(R1 + R2))$

$V_+ = V_i$

$V_o = k(V_+ - V_-)$

Hence

$V_+ - V_- \sim (V_o/k) \sim 10^{-1}$

and

$V_+ \cong V_-$

If V_+ and V_- are represented in Q-space 3, or lower, the difference is indeterminate.

Therefore to simulate the circuit we need to represent all the variables in Figure 3 in Q-space 4 with at least 3 significant digits.

4. Discussion.

The qualitative values a variable can have $A_0 \ldots A_z$ as representing disjoint abutting intervals that cover the entire number line[Joha85a]. I define the set of values $\{A_0 \ldots A_z\}$ as the Q-space. In this paper we have proposed a set of 4 Q-spaces that are useful in engineering problem solving. They allow us to represent the sort of relations that are useful in making engineering approximations.

The Q-spaces that we describe are chosen because relationships that hold between quantities in Q-space with lower resolution hold in a Q-space with a higher resolution. If the results are indeterminate going to a Q-space with a higher resolution may resolve the conflict. Thus $>$, $<$ and equal can be represented in all 4 spaces. $\gg$, $\sim$ and $\cong$ can be expressed in Q-spaces 2, 3 and 4. In Q-space 3 and 4 the logarithmic distance between two number can be expressed. In Q space 4 with n significant digits we can express the difference of two numbers q_1 and q_2 where $q_1 - q_2 \sim 10^{-n}$

Q-space 4 has the advantage that it is similar to the way numbers are represented on machines. There is a calculus for obtaining error bounds with such arithmetic. As the number of significant digits increases this Q-space approximates the real line.

It is possible to have a different break up of the number line. For example the temperature, We also advocate choosing the threshold in Q-space 2 dynamically. Each comparison involves different quantities and by moving from Q-space 3 to 2 we are able to set our threshold dynamically.

There is a many-one mapping from Q-space 4 to 3. One just ignores the significant digits. To go from Q-space 3 to 2 one needs to compare the variable to the appropriate threshold If

$\log(q) > \log(\mathit{threshold})$ implies q is large.

$\log(q) < \log(\mathit{threshold})$ implies q is infinitesimal.

Moving from Q-space 2 to 1 is trivial. Only the sign is retained.

A device is analyzed at the lowest possible resolution. If ambiguities result, we move to a higher resolution Q-space till the ambiguity is resolved. Using this technique we get as general a description of the device as possible.

5. Conclusions

We describe a scheme to analyze devices at multiple levels of resolution. We propose that 4 Q-spaces be used in qualitative analysis. These smoothly span the range form $(\pm)$ (0, non-zero) to the real-number line. Analysis is performed at the lowest possible resolution until ambiguities occur. To resolve ambiguities in a Q-space with a lower resolution, we move to a Q-space with a higher resolution. This paradigm allows us to obtain the most general description of the working of a device.

6. Acknowledgements

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