Qualitative Mechanics: Envisioning the Clock

Paul Nielsen
Qualitative Reasoning Group
Department of Computer Science
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801
U.S.A.
Netmail: nielsen@p.cs.uiuc.edu
Phone: (217) 333-8147

1 Introduction

Qualitative mechanics is the symbolic analysis of the motions and geometric interactions of physical objects. This paper describes a methodology for analysis of rigid body mechanisms, a subset of qualitative mechanics problems. Beginning with drawings of the parts involved we compute a discrete symbolic description showing changes in position and motion of the parts of the mechanism as well as its global behavior.

Understanding the operation of mechanical clocks has been an open problem in qualitative physics for many years. This paper presents the combination of ideas necessary to describe the behavior of many rigid body mechanisms including the mechanical clock. Our theory completes work begun in kinematics by Faltings [2] and combines it with theories of qualitative dynamics, based on work by Forbus [4], to produce a theory that we call qualitative mechanics. Because this theory is grounded on first principles of mechanics it is able to analyze the behavior of common mechanisms as well as new or variant ones.

Throughout this paper we use the mechanical clock to illustrate the stages of analysis. We begin with a discussion of the principles of qualitative physics necessary to understand mechanical devices. Following that we present an algorithm which indicates how the pieces fit together to produce an envisionment. Finally we discuss the envisionment produced by this algorithm and the further research necessary to develop this into a viable tool for assisting engineering analysis.

2 Qualitative Mechanics

2.1 Quantity

In order to reason about quantities in a discrete symbolic way previous researchers in qualitative physics [1] have adopted the method of representing a quantity by its sign. We extend this idea to spatial reasoning by representing vector quantities as ordered lists of their signs in a right handed Cartesian coordinate frame (see [8] for details). For example a motion in the +X +Y direction may be written as **Motion(thing)** $= (+\,+)$. Clockwise rotations are negative and counter-clockwise positive, but we will write these out here to aid understanding.

2.2 Configuration Space

Analysis begins by decomposing the mechanism into "kinematic pairs" of objects.[1] There are entered to the system as diagrams and their relative displacements indicated as well as the restrictions imposed on their motions by lower pairs.

[1] Objects that can interact pairwise.

The type of geometric questions we expect to answer are "How can the objects touch?", "Are there configurations of the objects where they do not touch?", and "What configurations are illegal?" This is done by the program described in [2] that computes configuration space representations of the objects. Essentially, a configuration space is a mapping from each possible motion of each object to a diagram showing whether or not the objects are overlapping. Each axis of the configuration space corresponds to one degree of freedom (allowable motion) of one of the objects. Unconstrained objects will have six degrees of freedom each (three translational motions and three rotational motions), but mechanisms, by definition, are relatively constrained, and typically each object has only one degree of freedom after the lower pair restrictions have been imposed.[2] The boundaries between free and blocked regions in configuration space (CSEG's[3]) represent locations where the objects are in contact.

Figure 3 shows a configuration space representing of the clock escapement of figure 1. Blocked regions are shaded. The horizontal axis represents clockwise rotation of the scape wheel, and the vertical axis represents clockwise rotation of the pallets. Because both dimensions correspond to rotations the configuration space wraps around like a torus. Point **A** in configuration space corresponds to the entry pallet pushed between the teeth of the scape wheel. Point **B** corresponds to the entry pallet touching the top of a tooth on the scape wheel. Point **C** corresponds to the exit pallet pushed between the teeth of the scape wheel. Regions **D** and **E** correspond to configurations where the pallet arbor is inverted from its normal operational position (figure 2).

2.3 Place Vocabularies

The central problem of symbolic spatial reasoning is finding an appropriate representation to describe space as a finite number of regions rather than an infinite number of points. Because the sign information discussed in section 2.1 is not sufficiently rich to capture the geometric information of a diagram (see [5] for more detail) we adopt the terminology of [3] and [2], calling a connected region of space in which all points share relevant common properties a *place* and the set of all places covering the space of interest a *place vocabulary.*

Contact constitutes one relevant property, but contact alone is not enough. When objects are in contact, their places lie along a CSEG. We further distinguish them by the direction of the surface normal of this CSEG. The orientation of the surface restricts the possible motions of the objects and determines the direction of the forces transmitted by contact. Forces change when surface orientations change which consequently changes the mechanism's behavior.

When objects are not in contact we can use information about possible motions to determine the next possible contacts. Free space divisions (FSD's) separate regions that are not in contact according to different next possible contacts. The FSD's are installed along the directions of possible motions or parallel to an axis of the configuration space from the endpoints of the CSEG's. The open regions where the objects are not in contact are referred to as *full faces.* Figure 4 shows the configuration space, for one period of the operating region of the clock escapement, tessilated for a place vocabulary. Solid lines represent CSEG's, and dashed lines represent FSD's. Points **A**, **B**, and **C**, as described in the previous section, illustrate corresponding points between the place vocabulary and the configuration space.

To determine the places of an entire mechanism we could compute the **N** dimensional configuration space of the mechanism (where **N** is the summation of each object times its degrees of freedom). Not only is this an expensive computation, but mechanical engineers do not reason this way. Instead we represent the static state of the mechanism as a place vector consisting of one place from each kinematic pair in the *kinematic chain.*[4] This provides a modular approach to mechanism analysis. When parts are

[2]Lower pair analysis falls outside the scope of our research because there are only six such pairs, and they require a twelve dimensional configuration space for analysis. Recent work by Joskowicz [7] provides interesting suggestions for recognition of lower pairs once this configuration space has been computed, and Gelsey [6] has developed an algorithm for identification of lower pairs from part geometry.

[3]From "constraint segment"

[4]A kinematic chain is composed of kinematic pairs where each object forms part of two pairs. Thus we distinguish objects that may directly or indirectly influence each other's behavior from other objects that just happen to be part of

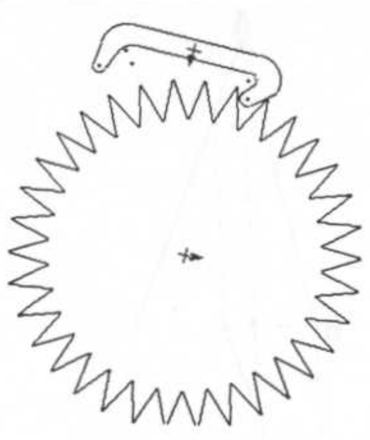

Figure 1: Clock Escapement

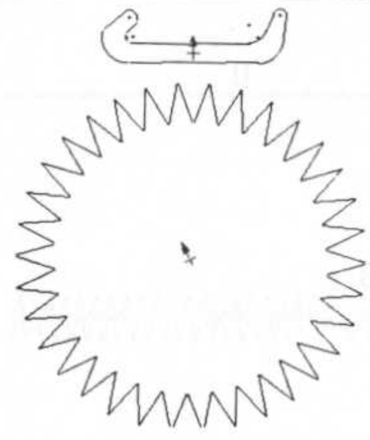

Figure 2: Inverted Clock Escapement

exchanged we need only construct the configuration space of the new parts, not the entire mechanism. It also allows us to create a library of common mechanisms that may be smoothly integrated with new mechanisms.

Movement of the objects in some direction may cause them to transition to the next adjacent place as determined by the metric diagram. For each place and each motion we compute the set of next possible places from the tessilated form of the configuration space. Motions that would enter blocked space[5] are constrained, which means that their next place will be the same as the current place. If the width of the current place is zero in one of the directions of motion, the transition will occur instantaneously. Otherwise the transition will require some interval of time.

Our choice of FSD's corresponding to possible motions of the objects helps minimize the ambiguity of qualitative analysis, but there still may be more than one possible next place, for example, when both objects move. We can eliminate all but a few choices, but without more metric information (rates and distances) there is inherent ambiguity.

2.4 Dynamic Considerations

Our research only considers the causes of forces that arise through contact between rigid bodies, but we may still reason about the effects of forces due to fields (magnetic and gravitational) and non-rigid bodies (springs, motors, and humans) without considering their cause. We accomplish this by allowing an oracle (the user) to associate arbitrary forces with an object (for example a spring always pushes a certain gear clockwise) or a set of places (for example the pendulum in certain positions is pulled counter-clockwise by gravity). Because these considerations fall outside any previous analysis it may be necessary to add further divisions to the configuration space and recompute the place vocabulary to cleanly associate a force with a place.

To transmit forces between two rigid bodies they must be in contact and one object must either have some force acting on it or have motion relative to the other.[6] A force will be transmitted to a second object only if that force or motion is directed into the open half plane centered on the reverse surface normal of the contact. The direction of the resultant force is along the reverse surface normal despite the direction of the original force. (For a formal treatment of this see [8].) At the same time a reaction force is transmitted along the contact surface normal.

the mechanism.

[5] The half plane defined by the reverse surface normal of a CSEG.

[6] A collision.

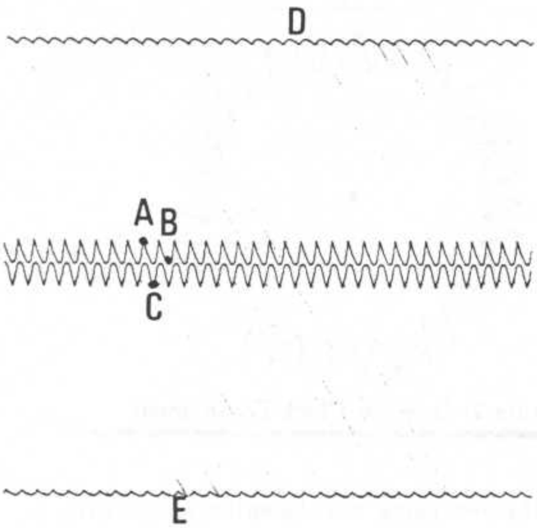

Figure 3: Configuration Space

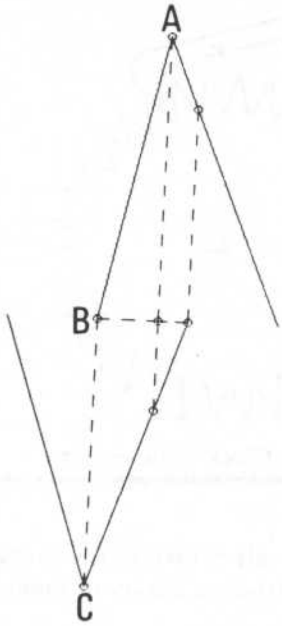

Figure 4: Place Vocabulary

We base our theory of force affecting motion on the Qualitative Process theory [4]. Without knowing the magnitudes of the forces involved we can (with possible ambiguity) determine their affects. If force is qualitatively proportional to acceleration, and acceleration directly influences motion, and force is the only thing affecting either acceleration or motion[7] — we can assert that force directly influences motion. Essentially this means that an object moving in some direction with force applied in the opposite direction may eventually stop moving; an object moving in some direction with force applied in the same direction may continue moving; an object not moving in some direction will instantaneously begin moving in that direction if a force is applied along that direction; and an object with no force applied will not change motion.

2.5 State

Like places in spatial behavior, *states* divide an infinite number of possible behaviors into a finite number of qualitatively distinguished behaviors. A mechanical state consists of two components: a kinematic component (a place or place vector from the place vocabularies) and a dynamic component (the motion of each object).

To develop the full state space of the mechanism we must combine every possible place with every motion of every object which exists in that place. Typically this is an enormous number of states, but [9] shows how this number can be reduced. Table 1 shows a representative state from the qualitative clock. Gear names begin with "G" followed by a number. Place names are composed from the objects constituting that place vocabulary, a "PL" (for place), and a number. Since there is only one degree of freedom there is only one dimension of motion for each object.

Just as state is a combination of dynamic and kinematic information, state transitions are the combination of the changes in the dynamic component and the kinematic component of the current state. Kinematic changes are determined by looking at the resultant place in the place vocabulary for the motion of each object, and dynamic changes are determined by applying consistent forces and

[7] There may be multiple forces, but we assume mass does not change.

Motion(Pallets) = zero	Motion(Scape) = clockwise	Motion(G1) = clockwise
Motion(G2) = c-clockwise	Motion(G3) = c-clockwise	Motion(G4) = clockwise
Motion(G5) = clockwise	Motion(G6) = c-clockwise	Motion(G7) = clockwise
Motion(G8) = clockwise	Motion(G9) = c-clockwise	Motion(G10) = c-clockwise
Motion(G11) = clockwise	Loc(Scape, Pallets) = SP-PL-22	Loc(G1, G2) = G1-2-PL-1
Loc(G3, G4) = G3-4-PL-1	Loc(G5, G6) = G5-6-PL-1	Loc(G6, G7) = G6-7-PL-1
Loc(G8, G9) = G8-9-PL-1	Loc(G10, G11) = G10-11-PL-1	

Table 1: Sample Mechanical State of a Clock

constraints to each motion in the current state as discussed above. When multiple influences are consistent or an influence has ambiguous results there may be several possible next states. Timing information resolves some of this ambiguity since instantaneous changes will occur before those which require some time interval.

3 Algorithm

The following is a high level summary of the algorithm used to construct an envisionment of a mechanism. Beginning with a geometric description of kinematic pairs and a description of the external forces, it produces a discrete symbolic representation of the possible behaviors of a rigid body mechanism in the form of an envisionment.

1. Enter drawings of the kinematic pairs composing the mechanism.
2. For each kinematic pair
    2.1. Compute a configuration space representation
3. For each configuration space
    3.1. Build a place vocabulary
4. For each place in the place vocabulary
    4.1. Determine possible next places under motion
5. For every legal combination of place and motion
    5.1. Assert a mechanical state space of the mechanism
6. Input external forces associated with places or objects in the device
7. For each mechanical state
    7.1. For each force
        7.1.1. Determine forces propagate through connectivity
        7.1.2. Determine the effects on the current motions
    7.2 Determine transitions between the mechanical states

4 Envisionment

The envisionment of a mechanism shows all possible behavioral sequences through changes in position and motion. By computing the next possible transitions from every consistent mechanical state we produce a complete envisionment of the behavior of a mechanical device. Because of space limitations[8] we are unable to include a graph of the clock's envisionment here but will include it as part of the presentation.

[8]The graph consists of 234 vertices.

While this envisionment needs additional information to determine the histories that correspond to acceptable clock behavior, it recognizes that such a behaviors exist and indicates several undesired behaviors, such as the pendulum beating repeatedly without the gear train moving and the pallets both clear of scape wheel allowing the gear train to move freely.

There are also undesirable behaviors which the envisionment recognizes that people might not consider, such as the recoil action of the escapement being powerful enough to drive the clock backwards some number of periods. Additional knowledge, not necessarily metric, can eliminate these behaviors. For example we might include the knowledge that the forces transmitted by contact act slowly enough so that objects will not break contact due to the resultant motions.

Elasticity considerations may eliminate undesired behaviors of the mechanism. The reaction force may slow or stop the speed of a moving object, but not cause it to reverse direction.

5 Summary

This paper represents the culmination of the CLOCK project. We have presented the flow of ideas necessary to produce qualitative, symbolic descriptions of mechanical behavior in the form of an envisionment. While we have used the mechanical clock as an example these ideas seem to be widely applicable to analysis of mechanisms containing pairs that are force coupled or that change the shape of the surfaces that come into contact.

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