Reasons and Filters for Spurious Behaviors

Peter Struss
SIEMENS Corp.
ZTI INF 22
Otto-Hahn-Ring 6
D-8000 Muenchen 83
West Germany

phone: (89) 636-2414
mail: STRUSS@ZTIVAX.UUCP

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Extended Abstract

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1 Introduction

Current methods in qualitative physics (QP) sometimes predict behaviors of physical systems that do not correspond to any real-valued solution.
The existence of spurious solutions and their origins have been analyzed for the treatment of differential equations [Kuipers 86], [Schmid 88] as well as of simple equations [Struss 87], [Struss 88a]. For the first case, the reason for the prediction of spurious behaviors is the **local** nature of the criteria for determining state changes.
This paper attempts a continuation and a refinement of this analysis and exploits results from the treatment of differential equations by interval mathematics and the qualitative theory of dynamic systems. The problem is: how can we determine the possible continuations of a qualitative behavior taking into account its complete history? Such **global** criteria for the elimination of spurious behaviors are developed for 2nd order differential equations.

2 Outline of the Paper

The following section presents some simple questions for demonstrating limits of the current QP approaches that are mainly used for infering qualitative behaviors. In section 4, their common basis is formally described. Different types of potential problems in behavior generation are identified in section 5. A brief introduction to the analysis of the so-called phase portrait of 2nd order differential equations is given. These techniques are then used in the next section to construct some necessary conditions for filtering out spurious behaviors. The application of these methods is demonstrated by answering some of the questions of section 3.

3 Some Questions

In order to demonstrate typical difficulties of QP, to analyze their reasons, and to present partial solutions, we consider some examples from the problem class "mass on a spring", which is the "Tweety of qualitative physics". Consider a mass on a spring without friction (Fig. 3.1).

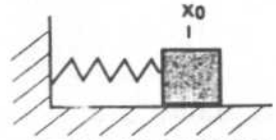


Figure 3.1 Mass on a spring

Question 1: If the mass is moved away from the equilibrium point, $x = 0$, given by the rest length of the spring, to $x = x_0 > 0$, will the mass, after one oscillation, return to x_0, exceed it, or turn back before ?
[Kuipers 86] showed that qualitative simulation in the style of QSIM cannot answer this question based merely on the corresponding differential equation. It derives 3 branches of possible behavior after one oscillation, and worse :

Question 2: What can we tell about the sequence of the maxima during the oscillation ?
Since the above argument for the first oscillation applies to each oscillation, we get further branching in the course of the qualitative simulation (by more than a factor of 3, because new landmarks are introduced). The amplitude is allowed to change arbitrarily over time.
Now, consider the mass on a surface with friction.

Question 3: Which conclusions can be drawn about the oscillating behavior ?
[Schmid 88] demonstrates that systems like ENVISION [de Kleer-Brown 84] do not provide criteria for the number of oscillations performed by the mass. It can be 0 or any natural number. Moreover, there is

Question 4: Can we infer damping to equilibrium ?
No, even a solution with x approaching $-\infty$ cannot be excluded, even if the linearity of the underlying differential equation is exploited by differentiation, and if an infinite number of the respective qualitative equations (called confluences) is added (see [Schmid 88]).
Finally, consider two equal masses at two equal springs, one beside the other, moved away from their common equilibrium position $x = 0$ to the same x_0-position. Imagine they are connected by a thin hair that will tear at a length of $1.5\ x_0$.

Question 5: If we release the masses at the same time, will the hair eventually be torn?
It will be explained in section 5.1 why most QP systems have difficulties in excluding this case.

4 Infering Qualitative Behavior

By (qualitative) behavior we mean the (qualitative) changes in characteristics of a physical system over time. It may imply changes in **structure**, changes in characteristic **parameters**, or global **tendencies and patterns** of such changes, such as oscillation, damping, or approaching equilibrium. Current QP systems are weak in deriving clear statements about such global behavioral characteristics. The general problem under investigation is:

> **Given a description of a system S, and an "admissible" behavior b, determine the "admissible" continuations of b.**

Three subtasks (which are not necessarily separate, subsequent steps) have to be solved:

1. Determine the possible **qualitative states** of a system, i.e. sets of qualitative values that satisfy the **equations** of its description.

2. Determine the possible **state transitions**, i.e. changes from one state to another that are in accordance with the **derivative relations** and **continuity conditions**.

3. Determine the possible **behaviors**, i.e. "correct" sequences of states (or transitions). The existing QP methods offer **no criteria** for checking the global correctness of these sequences, and, hence, they have to assume that each path through the state transition graph is an admissible behavior.

5 Problems in Filtering Behaviors

In this section, we identify different sources of the generation of spurious behaviors. They can essentially be reduced to

- merging temporally distinct behaviors and
- merging different system instances or different behaviors of one system instance.

Note that these sources are active independent of the presence of spurious states or state transitions.

5.1 Problem 1 : The Loss of Temporal Information

A qualitative behavior specifies only a sequence of states. It does not describe how long a system stays in each state. Hence, quite different temporal behaviors of functions may be mapped onto the same qualitative behavior. For the question 5 raised in section 3, this implies that arbitrary combinations of the qualitative values for the location of the two masses are generated, including situations where the masses may collide.

5.2 Making Differential Equations Visible : The Phase Portrait

For the subsequent analysis, we briefly introduce some basic ideas from a mathematical discipline called qualitative theory of dynamic systems (see e.g. [Andronov 66], [Stoker 50]). In this theory, qualitative results about the solution space of differential equations are gained by applying topological methods. This is possible because of a correspondence between sets of differential equations and vector fields.

Consider again the mass on the spring. This system is described by some 2nd order differential equation

(5.1) $d^2x/dt^2 = -M_0^+(x)$

or the equivalent system of first order equations

(5.2) $dx/dt = v$
$dv/dt = -M_0^+(x)$,

where M_0^+ is a monotonic function with $M_0^+(0) = 0$.

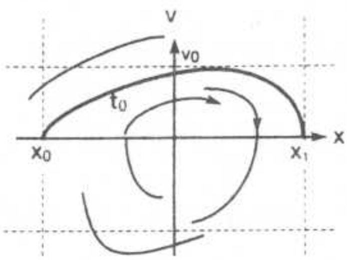
(5.2) defines a vectorfield in the (x,v)-plain by mapping each point (x_0,v_0) of this plain to the vector of the derivatives in this point :

$$(dx/dt|_{(x_0,v_0)}, dv/dt|_{(x_0,v_0)}) = (v_0, -M_0^+(x_0)).$$

Solutions of (5.2) then correspond to those curves ("**trajectories**") in the plain that in each point have a tangent in the direction of the respective vector.

The main charactereristics of the phase portrait of System (5.2) are indicated by Fig. 5.4. It expresses the oscillatory behavior, but does not decide upon the question whether the system really exhibits a cyclic behavior. Starting at an arbitrary point $(x_0, 0)$ on the negative x-axis, the respective trajectory, t_0, first stays in the quadrant $x<0$, $v>0$, then, intersecting the positive v-axis, continues in the quadrant $x>0$, $v>0$ and leaves it by reaching some point $(x_1, 0)$ on the x-axis.

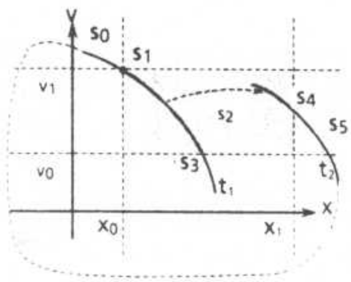
How does this representation relate to the description of the qualitative behaviors derived by QP methods? The quantity spaces for x and v impose a grid on the plain. For each rectangular, its interior, each edge excluding the endpoints, and each corner correspond to qualitative states. Each trajectory defines a behavior in the sense of section 4, namely the sequence of states it crosses. In Fig. 5.4, the behavior corresponding to the arc of the trajectory t_0 is shown.



Sequence of states for t_0

	x	v
s_0	x_0	0
s_1	$(x_0,0)$	$(0,v_0)$
s_2	0	$(0,v_0)$
s_3	$(0,x_1)$	$(0,v_0)$
s_4	$(0,x_1)$	v_0
s_5	$(0,x_1)$	$(0,v_0)$
s_6	x_1	0

Figure 5.4 Definition of a behavior by a trajectory



states

	x	v
s_0	$(0,x_0)$	(v_1,v_2)
s_1	x_0	v_1
s_2	(x_0,x_1)	(v_0,v_1)
s_3	(x_0,x_1)	v_0
s_4	x_1	(v_0,v_1)
s_5	(x_1,x_2)	(v_0,v_1)

Figure 5.5 Merging different behaviors

5.3 Problem 2 : Merging Different Solutions

Consider the pieces of the trajectories, t_1 and t_2, in Fig. 5.5. They introduce the admissible state transitions (s_0, s_1), (s_1, s_2), (s_2, s_3), (s_2, s_4), (s_4, s_5). Having constructed a behavior $b_1 = (s_0, s_1, s_2)$ (which is admissible, since it represents a piece of t_1), we have (at least) two possible continuations for b_1, $b_2 = (s_0, s_1, s_2, s_3)$ and $b_3 = (s_0, s_1, s_2, s_4)$. Neither of them can be ruled out by the step of state transition filtering, although only b_2 corresponds to a trajectory of the **specific system under consideration**, whereas b_3 merges two solutions of the system with different initial conditions. This is the reason why question 1 in section 3 is answered by QP with a branching of the behavior. It was correctly identified in [Kuipers 86] as the merely local nature of the state transition filtering. For question 4, it makes us unable to unambiguously infer a damped behavior for the occurance of friction, unless we supply additional information. Fig. 5.5 indicates that if the damped trajectory t_2 returns to state s_2 as the arc t_1, the existing filters do not forbid a "jump" back to t_2. This establishes a spurious cyclic behavior.

Consider question 3. The system with friction can be described by

$$(5.3)\qquad dx/dt = v$$
$$dv/dt = -x - f * v,$$

where $f * v$ with $0 < f \in \mathbb{R}$ represents (linear) friction.

We understand that, on the current basis, we have no criteria for determining the possible number of oscillations before equilibrium is approached. For the linear system (5.3), however, we know that there are exactly two possibilities : an infinite number of oscillations, if $f < 2$, or merely "half of an oscillation", if $f > 2$

(overdamped case). Note, that these radically different behaviors are distinguished by a **real-valued** threshold of a parameter.

5.4 Problem 3: Merging Distinct Systems

Of course, we can argue that the behavior $b_3 = (s_0, s_1, s_2, s_4)$ may be not admissible for the arbitrary, but fixed system sketched in Fig 5.5. However, the qualitative description covers a whole family of systems (5.3). Could there not be an appropriate choice of a parameter, e.g. in (5.3), such that b_3 is admissible for the corresponding system (and b_2 is not) ? Yes, this might happen, although it is hard to prove it for a specific case. But, subsequent continuations of b_3 then probably require choices between different state transitions that again imply a specific choice for the range of the parameter. Since these choices are only implicit, we have no criteria for determining whether they are consistent (i.e. have a non-empty intersection). Hence, in combining admissible state transitions we are not guaranteed to jump between **different instances of a class of systems and merge their behaviors** thus potentially generating behaviors which are not admissible for any single system.

If we allow the **parameters to vary over time**, i.e. we use inhomogeneous differential equations as our gold standard, then some behaviors may become admissible. The problem is that we currently have no means to express our decision for homogeneous systems and use it as a filter.

5.5 Problem 4: The "Wrapping Effect"

Research in interval mathematics has shown that, in general, using intervals to specify sets of initial conditions for differential equations may lead to strong "rounding errors". The mappings induced by the solutions corresponding to the interval of initial conditions transform this set into sets which can only be "wrapped" into intervals growing larger and larger. In QP, the "wrapping effect" may increase the problems 2 and 3 stated above.

6 Exploiting the Phase Portrait

In this section we construct global criteria for behaviors. We do so mainly by taking advantage of the property that trajectories cannot intersect, because otherwise we would get different solutions for the same initial conditions. The filters apply to 2nd order differential equations. We have to emphasize that this choice has not only been made for the sake of simplicity of the examples. It mainly reflects the fact that only for this case (i.e. vectorfields in the plain), we expect strong results (with the 3rd dimension, chaos starts).

6.1 Avoidance of System Merging

Filters aiming at this goal need to identify behaviors that do not correspond to solutions of the same system. We introduce a symmetric binary relation **exclusive**$\subset B \times B$. exclusive(b_1, b_2) means that there exists no system instance for which both behaviors, b_1 and b_2, are admissible. Here, we try to infer exclusiveness using only general properties of ordinary differential equations. The framework, however, allows us to incorporate further and probably more specific criteria depending on the domain and the case.

We know that different trajectories passing through one point of the plain cannot belong to the same system. The problem is how to recognize the respective behaviors, which run through qualitative states instead of points ?

One case is easily solved, namely if states are involved that correspond to points. For the general case, we analyse whether a behavior b approaches another one, b', "from the left" and leaves it "to the right", we have the relation **crossing**(b,b'). Then we can make use of it by the

Proposition 6.2

Crossing behaviors are exclusive :

$$\text{crossing}(b,b') \Rightarrow \text{exclusive}(b,b').$$

We may detect spurious behaviors with the obvious implication

Proposition 6.3

$$\text{exclusive}(b, b) \Rightarrow \text{spurious}(b).$$

For example, a behavior crossing itself is spurious.

Another filter criterion is given by

Proposition 6.5

If there is no admissible continuation for $b = (\ldots, s_{n-1}, s_n)$, $s_n \neq$ final, then b is spurious: $(\forall b' \quad b \subset b' \Rightarrow \text{spurious}(b')) \quad \Rightarrow \quad \text{spurious}(b)$

Sometimes we can infer the negation of exclusiveness, i.e. for some b, b' there exists a system allowing both b and b' (see section 6.3). We are only allowed to combine behaviors if they belong to the same solution. For $b = (\ldots, s_{-2}, s_{-1}, s_0)$ and $b' = (s_0, s_1, s_2, \ldots)$, we define the **behavior union**

$$b \cup b' = (\ldots, s_{-2}, s_{-1}, s_0, s_1, s_2, \ldots).$$

Proposition 6.6

Let s_0 be a landmark state and $b = (\ldots, s_{-2}, s_{-1}, s_0)$ and $b' = (s_0, s_1, s_2, \ldots)$, then

$$\neg\, \text{exclusive}(b, b') \Rightarrow \neg\, \text{exclusive}(b \cup b', b) \wedge \neg\, \text{exclusive}(b \cup b', b')$$

We now have some criteria for spurious behaviors and for the legal combination or continuation of behaviors. Before demonstrating the use of these criteria, a way to check the crossing relation is provided.

6.2 Detecting Crossing Behaviors

As stated above, we hope to check qualitatively whether a behavior approaches another one from one side and leaves it towards the other side. In between, they may share a sequence of states.

Proposition 6.7

Let $b = (\ldots, s_{-n}, \ldots s_c, \ldots, s_d, \ldots, s_n, \ldots)$ and $b' = (\ldots, s'_{-n}, \ldots s_c, \ldots, s_d, \ldots, s'_n, \ldots)$.

$$\text{convergent-left}(b, b', s_c) \wedge \text{divergent-right}(b, b', s_d) \Rightarrow \text{crossing}(b, b').$$

An algorithm is presented for checking the relation convergent-left denoting that the first behavior joins the second from the left. Since divergent-right can be checked in a similar way, we are then able to detect crossing behaviors.
Although the filter provided by the propositions are far from completely ruling out illegal merging of behaviors, we can demonstrate that some progress is achieved by answering some questions of section 3.

6.3 Symmetry - Infering Cyclic Behavior

Consider again question 1 of section 3. We are now able to infer the cyclic behavior of all solutions to (5.2). The idea is the following: Looking at (5.2), we realize that the application of the transformations $t' = -t$ and $v' = -v$ leads to

$$(6.1) \quad dx/dt' = v'$$
$$dv'/dt' = -M_0^{\cdot}(x) \ ,$$

which is of the same form as (5.2). This means we are able to derive the phase portrait in the half plain $v < 0$ by merely mirroring the $v > 0$ half plain at the x-axis (and reversing the orientation of the trajectories). Hence the trajectory continuing beyond $(x_1, 0)$ is the mirror image of the curve we started with and therefore hitting the x-axis again at $(x_0, 0)$ and establishing a closed curve (Fig. 6.3a). Following this curve with $t \to \infty$ corresponds to oscillation with a constant amplitude.

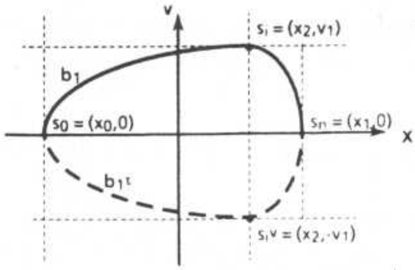


Figure 6.3a Symmetry of behavior

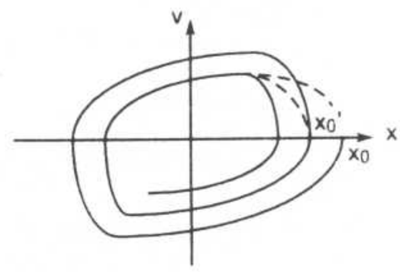


Figure 6.3b Damped oscillation

In the framework introduced in this paper, we can express this method in a general way.

6.4. Infering Steady Damping

Question 2 is concerned with the identification of a global tendency of behavior, namely with the problem of arbitrary changes in the subsequent maxima of the oscillation. This problem is solved for the frictionless case by the result of the previous section. However, it also occurs for the case with friction. Using our filter criteria, we are now able to deduce immediately that if the the oscillation is damped in the first period, it will always be damped: Let $s_0 = (x_0, 0)$ be one maximum, and $s_0' = (x_0', 0)$ with $x_0' < x_0$ the next one (Fig. 6.3b). Since the maximum could only be increased again if the solution crosses itself, Propositions 6.3 and 6.4 detect it to be spurious. The return to s_0' is also ruled out.

7 Summary

Our approach to expressing restrictions imposed by 2nd order homogeneous differential equations is essentially based on the uniqueness of solutions for fixed initial conditions. The criteria can be used to discriminate behaviors that belong to different system instances and to discover spurious behaviors. They enable us to derive cyclic behavior for the frictionless mass-spring system and the principle "once-damped-always-damped" for the case with friction.
Similar methods can be used, for example, to infer damping for the mass with friction [Struss 88b] or properties of systems like the existence of limit cycles etc. (see [Andronov 66], [Struss 75], [Sacks 87]).
The criteria for the detection of spurious behaviors can be implemented as additional filters in systems like QSIM. The hard task is to implement the heuristics for the selection of the appropriate ways to analyze the phase portrait, such as exploiting symmetric properties of the equations.

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