# Homogenising References in Orders of Magnitude Spaces:

# An Application to Credit Risk Prediction

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#### Abstract

This paper presents the concepts underlying the design of an intelligent system based on the absolute orders of magnitude model to define qualitative operators able to deal with a set of variables defined in different reference scales and with different influence degrees. This system is applied to represent and manage some of the factors that take part in the evaluation of credit risk in order to classify firms according to their rating. Most of the variables involved in the rating calculation can be qualitatively described by means of orders of magnitude, but the references that define these orders are different for each variable. We introduce adapted references for qualitative operators (ARQO) and implement a software tool able to perform this classification **Key words**: Qualitative Reasoning, Orders of Magnitude, Qualitative Operators, Credit Risk.

#### 1 Introduction

Calculus with qualitative variables is a common problem in qualitative reasoning. The absolute orders of magnitude model (Piera 1995; Travé-Massuyès 1997) works with a finite set of qualitative labels obtained via a discretisation of the real line. These kinds of structures are useful because the knowledge and relations that human beings

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use to extract conclusions can be modelised by them. Moreover, it is not unusual to have to extract information from variables qualitatively described by means of different discretisations of the real line.

One of the goals of qualitative reasoning is to tackle problems in such a way that the principle of relevance is preserved (Forbus 1988); that is to say, each variable involved in a real problem must be valued with the precision level required. Nevertheless, in orders of magnitude calculus, as in the algebra of signs, it is fairly usual to get ambiguous or indeterminate results. To solve these problems, a method based on the three following strategies is proposed.

First, working with orders of magnitude spaces with variable granularity, in such a manner that all the variables and the result can be described in different qualitative spaces, and so the ambiguity or indetermination of the results is reduced.

Second, unifying the different measurement scales of the order of magnitude spaces where the variables take their values in order to make calculus possible.

Third, building a new reference, where, on the one hand, all the qualitative values of the variables can be seen in different precision levels, and on the other hand the result 2

can be qualitatively described in the most precise way, to get relevant consequences.

The results of this research are applied to predict credit risk of bonds issued by a government or company. This application shows how Qualitative Reasoning techniques, and particularly orders of magnitude calculus, can be useful in the financial domain.

The paper gives a brief introduction to the absolute orders of magnitude model with variable granularity and the qualitative operators defined on this model. Next, the process for homogenising the reference scales of different qualitative variables to be operated is established and adapted references for qualitative operators, ARQO, are defined. In Section 5 this methodology is applied to credit risk evaluation of a firm or an issue of bonds is presented. The paper ends with some conclusions and also with some comments about the implementation of this application and the first results obtained with it.

# 2 The Absolute Orders of Magnitude Models with Granularity n, OM(n)

In this section we give a description of the absolute orders of magnitude model, that is the framework of this paper. The model used is a generalisation of the model introduced in (Travé-Massuyés and Piera 1989). The number of labels we choose for describing our reality depends on the characteristics of each problem (Agell 1998).

The absolute orders of magnitude model with granularity n, OM(n), is built via a real line symmetric partition in 2n+1 classes:

#### Figure 1: The real line discretisation

Each class is named basic description and is represented by a label in the set  $S_1$ .

$$S_1 = \{N_n, N_{n-1}, N_{n-2}, ..., N_2, N_1, 0, P_1, P_2, ..., P_{n-2}, P_{n-1}, P_n\}$$

Finally, the quantity space S is defined by extending  $S_1$ : for all X,  $Y \in S_1$ , with x < y, the label [X,Y] is defined as follows:

$$[X, Y] = \begin{cases} X, & \text{if } Y = 0; \\ Y, & \text{if } X = 0; \\ \text{the smallest interval with respect to} \\ \text{the inclusion containing X and Y.} & \text{if } X \neq 0 \text{ and } Y \neq 0 \end{cases}$$

Moreover, it is possible to define an order relation  $\leq_P$  in S, induced by the inclusion:

Given 
$$X, Y \in S, X \leq_P Y$$
 when  $X \subset Y$ .

The order relation  $\leq_P$  stands for "being more precise than" or "being less general than" and is graphically illustrated in Figure 2.

For all  $X \in S$  the set  $B_x = \{B \in S_1 - \{0\}: B \leq_p X\}$  is named the basis of X, and  $B_x^* = \{B \in S_1: B \leq_p X\}$  the enlarged basis of X. In an OM(n) q-equality is also defined. Given X, Y  $\in$  S they are q-equal, or X $\approx$ Y, if there is a Z  $\in$  S, such that  $Z \leq_P X$  and  $Z \leq_P Y$ . This means they have a common basic element.





It is important to note that the sign algebra is an OM(1) and the orders of magnitude algebra that are defined in (Travé-Massuyés and Piera 1989) are an OM(3).

Finally, in order to define qualitative operators to deal with quantitative and qualitative data, it is necessary to consider the qualitative expression of a set A, denoted by  $[A]_s$  and defined as the smallest element in S with respect to the inclusion that contains A.

# 3 Qualitative Functions and Qualitative Operators

After the order of magnitude spaces have been introduced, functions and operators defined on them are considered.

Let S and S' be spaces with granularity n and m, respectively.

Starting from a real function or a real operator, qualitative functions or operators can be built in the following way:

Given  $f: R \longrightarrow R$ , the qualitative expression [f] of f is defined as the function from S to S' such that, for any  $X_i \in S$ ,

 $[f](X_i) = [f(X_i)]_{S'}$ , where  $f(X_i) = \{f(x) | x \in X_i\}$ .

Thus, the qualitative expression of a real function assigns to each element  $X_i$  in S the qualitative description of the subset containing the images of all the real values in  $X_i$ . Likewise, given a real operator  $\varphi$  defined on R<sup>k</sup> with values in R, i.e. a real function of k real variables, the qualitative expression of  $\varphi$ , is the operator [ $\varphi$ ] defined on S<sup>k</sup> with values in S' such that, for all X<sub>1</sub>, X<sub>2</sub>, ..., X<sub>k</sub>  $\in$  S<sup>k</sup>,

$$[\phi](X_1, X_2, ..., X_k) = [\phi(X_1, X_2, ..., X_k)]_{S'}$$

where  $\phi(X_1, X_2, ..., X_k) = \{ \phi(x_1, x_2, ..., x_k) | x_i \in X_i \forall i \}.$ 

That is to say, the qualitative expression of a real operator with k variables assigns to each k-tuple of elements  $(X_1, X_2, ..., X_k)$  of S<sup>k</sup> the qualitative description of the subset containing the results of the operations of all the real values in  $X_1, X_2, ..., X_k$ .

It is important to note that all the variables are defined in the same orders of magnitude space (same granularity), although each one can have its own discretisation. A method to deal with the cases in which the values of the variables are given in spaces of different granularity is presented in Section 4.

A qualitative function F (a qualitative operator  $\Phi$ ) is consistent with R when there exists a real function f (a real operator  $\varphi$ ) such that F = [f] ( $\Phi$  = [ $\varphi$ ]). In this case, the function f (the operator  $\varphi$ ) is named a real representative of the qualitative function F (qualitative operator  $\Phi$ ).

In order to make the simpler calculations, it is interesting to note that consistent qualitative functions and operators are generable from the basis. That is, the image of any qualitative label is determined from the images of the basic labels in  $S_1$ . Thus, for any F or  $\Phi$  with domain all the set S, and for any X, Y  $\in$  S:

$$F(X) = \left[\bigcup_{B_i \in B_X^*} F(B_i)\right]_S \text{ and}$$
$$\Phi(X_1, \dots, X_k) = \left[\bigcup_{B_i \in B_{X_i}^*} \Phi(B_{i1}, \dots, B_{ik})\right]_S$$

respectively.

# 4 Adapted References for Qualitative Operators (ARQO)

### 4.1 Homogenising Initial References

The problem of having to extract some information from qualitative values represented in heterogeneous references is not unusual, due to the fact that each variable is expressed by its own unit and its own discretisation. In these cases it is necessary to find a common orders of magnitude space where all of the variables can be described. This will allow you to apply to them an adequate operator to them.

If all the variables X1, ..., Xk are described in an orders of magnitude space with the same granularity and given from the same discretisation of the real line, they can be easily operated using qualitative operators consistent with real ones, that is, qualitative operators which are the qualitative expression of a real one (Section 3). The problem arises when the discretisations of the real line that give rise to the orders of magnitude of the variables  $X_1, ..., X_k$  are different.

Let's suppose that each one of these variables is qualitatively described via a different set of labels, which are intervals of the real line, with an odd number of landmarks given by the experts. This allows having a central point  $l_i$  in each set of landmarks, which is necessary to establish a bijection with the set of landmarks of a standard OM(n) space.

In order to be able to apply a qualitative operator consistent with the real line to compose these variables, three steps will first be taken, applied to each one of the discretisations of the variables:

- Step 1 First, a translation  $t_i : R \longrightarrow R$ ,  $t_i(x) = x-l_i$  to transform the central landmark  $l_i$  to 0.
- Step 2 Second, each reference is symmetrised with respect to 0 (which is now the central landmark), by adding the symmetric points with respect to 0 of the landmarks. Thus a symmetric set of landmarks is obtained.
- Step 3 Finally, each one of these sets is extended again, by adding the necessary number of landmarks, to obtain the same cardinal for all of them. These added landmarks are to be chosen by the experts taking into account, for each variable, which would be the most convenient intervals to be subdivided. If the expert has no preference, the added landmarks are chosen at random.

After these three steps, all the values of  $X_1, ..., X_k$  are described in an OM(n) with the same granularity, but the discretisations of the real line that give rise to the orders of magnitude of the variables  $X_1, ..., X_k$  are different.

Let  $\{a_1^i, ..., a_{n-1}^i\}$  be the positive landmarks corresponding to the variable  $X_i$ . A method for homogenising all the values of  $X_1, ..., X_k$  into a new common reference consists of establishing bijections between the real line intervals corresponding to the qualitative labels and the intervals of a new reference. There will a particular bijection for each variable, and all of these bijections will be real representatives of the qualitative function identity in S,  $[f_i] = Id_s$  for all  $i \in \{1, 2, ..., k\}$ .

These functions allow you to preserve the relative position of any real number with respect to the landmarks that define its basic qualitative description; that is to say, if  $r \in [a_{j}, a_{j+1}]$ , the quotient between the distances from r to  $a_{j}$  and to  $a_{j+1}$ , or simple ratio, will be the same in the new reference.

Let's choose  $b_1, ..., b_{n-1} \in \mathbb{R}^+$  as positive landmarks of the unified reference. For each variable  $X_i$ , the bijection  $f_i$  from R to R is determined via a polygonal graphics that connects the coordinate points:

 $(-a_{n-1}^{i}, -b_{n-1}), (-a_{n-2}^{i}, -b_{n-2}), \dots, (-a_{1}^{i}, -b_{1}), (0, 0), (a_{1}^{i}, b_{1}), \dots, (a_{n-2}^{i}, b_{n-2}), (a_{n-1}^{i}, b_{n-1}),$ 

as shown in Figure 3:

3



Figure 3: Graphic representation of fi

After the values of the variables are in the same reference, with landmarks in:

 $B=\{-b_{n-1}, ..., -b_1, 0, b_1, ..., b_{n-1}\},\$ 

qualitative operators consistent with the real line can be applied. These operators can be either internal operators in S, or applications from  $S^k$  to another orders of magnitude space S' with granularity m, OM(m).

Nevertheless, it should be pointed out that some of the elements in B can be *not relevant* for some of the variables. This occurs in the case of a variable  $X_i$  whose reference has required the addition of some landmarks either in Step 2 or in Step 3. These added landmarks are "fictitious", and so their images by the bijection  $f_i$  are *not relevant* for the particular variable  $X_i$ . Thus, the partition of the real line defined by B is a "multi-uses" partition, in which every variable  $X_i$  can be qualitatively described taking into account only its *relevant* landmarks. For example, if  $b_1$  and  $b_3$  are relevant for the variable  $X_j$  but  $b_2$  is not, then the most precise qualitative descriptions for the values of  $X_j$  will be given by:  $P_1$ ,  $[P_2, P_3],...$ , whereas  $P_2$  and  $P_3$  cannot be considered separately for  $X_j$  since they do not come from any initial qualitative description for  $X_i$ .

#### 4.2 The Adapted Final Reference

When applying a qualitative operator  $\Phi = [\phi]$  to a set of qualitative variables defined in an orders of magnitude space built via a partition of the real line B, the proposed *adapted reference* for the qualitative space OM(m) where the operator  $\Phi$  take its values is given by landmarks in the set:

 $\{\varphi(b_{i1}, b_{i2}, ..., b_{ik}), \text{ with } b_{ij} \in B_i\},\$ 

with B<sub>i</sub> is the set of the relevant landmarks for X<sub>j</sub>.

This set of landmarks allows the operator's values to be accurately and qualitatively described, i.e. with the precision level that these values require. Moreover, this choice of landmarks avoids ambiguous or indeterminate results.

This method of homogenising the initial references and adapting the final reference to the qualitative operator to be applied is what we call ARQO (*Adapted References for Qualitative Operators*).

## 5 An application to Credit Risk Prediction

In this section the defined method is used to classify firms according to their credit risk. In this case the orders of magnitude and tendencies of the variables are considered more relevant than their exact numeric values.

## 5.1 Case Study: Rating

The rating is a qualified assessment about the risk of bonds issued by a government or a company. There are specialised rating agencies, the most important of which are Moody's and Standard & Poor's. They classify firms according to their level of risk, using both quantitative and qualitative information to assign ratings to issues. The final rating is the agency's judgement, and reflects the probability of issuer default. Predicting the rating of a firm therefore requires a thorough knowledge of the ratios and values that indicate the firm's situation and, also, a thorough understanding of the relationships between them and the main factors that can alter these values.

The processes employed by these agencies are highly complex. Decision technologies involved are not based on purely numeric models. On the one hand, the information given by the financial data is used, and the different values included in the problem are also influential. On the other hand, they also analyse the industry and the country or countries where the firm operates, they forecast the possibilities of the firm's growth, and its competitive position. Finally, they use an abstract global evaluation based on their own expertise to determine the rating. Moody's ratings are labelled Aaa, Aa, A, Baa, Ba, B, Caa, Ca. From left to right these rankings go from high to low credit quality, i.e., the high to low capacity of the firm to return debt.

The model presented is especially adequate when the goal is to measure the magnitude of a result, based on the qualitative descriptions of the variables that participate. The qualitative descriptions appear when either numerical values are unknown or the experts use only their orders of magnitude.

Qualitative reasoning has been applied recently to business, finance and economics (Alpar and Dilger 1995). But is has to be pointed out that in the 60's some economists (Lancaster 1962) made qualitative modelisations of economic systems and proved that these models (induced by sign algebra) can lead to significant conclusions. In the case of credit risk prediction, there are three main reasons that brought us to a qualitative approach:

- The involved variables have a proper description in qualitative terms. Often the orders of magnitude and tendencies of the variables are more relevant than their exact numeric values.
- The variables involved in credit risk have different relevance or strength in the global calculation. Qualitative operators are able to take into account different degrees of influence.

 The final classification has to be given in a qualitative set of labels.

### 5.2 Prediction Strategy

Let  $V_1,...,V_k$  be the qualitative variables defining the characteristics of a firm, and  $V_1(C),...,V_k(C)$  the qualitative values taken by these variables for a given firm C. Each one of these variables is qualitatively described via a set of labels, which are intervals of the real line, with an odd number of landmarks; provided by the experts.

In order to apply a qualitative operator consistent with R to these variables, the ARQO method is used. Therefore it is first necessary to follow the three steps described in Section 4.1 to each one of the discretisations of the variables:

After these three steps, the values of  $V_1, ..., V_k$  are described in an OM(n) with the same granularity, and can be homogenised into a new common reference with landmarks in a set  $B = \{-b_{n-1}, ..., -b_1, 0, b_1, ..., b_{n-1}\}$ .

Then, in order to obtain a synthesis value that reflects the credit quality of firm C, a qualitative operator  $\Phi$  is applied to the former variables:

 $\Phi: OM(n) \times ... \times OM(n) \rightarrow OM(m)$ 

taking into account their degrees of influence  $\alpha_1,...,\alpha_k$  on the final result. An example of this operator can be the qualitative expression of a weighted sum or a weighted arithmetic mean. If  $\varphi$  is a real operator with qualitative expression  $\Phi$ , the rating of the firm will be obtained as follows:

 $r_{C} = \Phi(V_{1}(C),...,V_{k}(C)) = [\phi(V_{1}(C),...,V_{k}(C))]_{S}.$ 

To define the landmarks of the qualitative space OM(m), where the operator  $\Phi$  takes values, the adapted reference of the real line defined in Section 4.2 is used.

The last step involves taking a new discretisation of the real line in order to express the firm's rating in a qualitative space with eight basic labels, and so emulate Moody's formerly mentioned levels of credit risk. By applying a qualitative function from OM(m) into a space OM(4), which is the qualitative expression of the real identity, the final evaluation of the firm's credit risk is obtained.

The proposed method is currently being introduced Our software tool accepts quantitative and qualitative data defined in an orders of magnitude structure. First the user must specify the set of landmarks for each variable. Because of the possibility of varying the granularity, every data structure in the program is dynamic. The result supplied by our algorithm is twofold; on the one hand, it provides the classification of the firms according to their credit quality, and, on the other hand, it also allows performance simulations to be carried out by modifying the values of the variables.

#### 6 Example

This example shows the classification process in a first approach already implemented. As a first step, the approach will be applied using just five variables, given by accounting ratios, and the strength of their influences on the credit risk. According to the experts' knowledge these ratios are the most relevant in computing credit risk.

The following table shows the ratios and the initial landmarks, which are provided by the experts and will be used to define the orders of magnitude.

Variable	Initial landmarks			
$V_1 = L$	{25, 50, 75}			
$V_2 = ROC$	{I, 2I, 5I}			
$V_3 = IC$	{0, 1, 3,}			
$V_4 = RCF/TD$	{0, 10, 60}			
$V_5 = MV$	{0.5, 1, 1.5}			

Table1: Landmarks of the variables

Each one of the variables has different landmarks. The first variable, the *leverage* of the firm, always takes positive values. It is accepted that a firm's leverage is *very low* when it is under 25, *low* when it is between 25 and 50, *normal* when it is between 50 and 75, and *high* when it is over this value.

The second variable, *return on capital*, is defined as the profit on the capital plus debt, and it is qualitatively measured by comparing it with the interest without risk I of the country in which the firm operates. In the case of a multinational company, the interest paid in the headquarters' country is used. A firm's ROC is *bad* when it is under I, *normal* when it is between I and 2I, *good* between 2I and 5I, and *very good* over 5I.

*Interest coverage* is the quotient between profits and interest. The landmarks 0, 1 and 3 define labels for IC, which are *bad*, *normal*, *good*, and *very good*.

The fourth variable, *cash-flow over debt*, represents nondistributed cash flow over total debt. It is considered *very bad* when it is under 0, *bad* when it is between 0 and 10, *normal* between 10 and 60, and *good* over 60.

Finally, the variable MV represents the firm's *market* value in relative terms, i.e. with respect to the average size of the sector's firms. The landmarks 0.5, 1 and 1.5 define labels for MV, which are *low, normal, high* and *very high*.

Each one of the variables the problem involves has its own description in qualitative terms. Starting from the previously given experts' qualitative descriptions, a symmetric discretisation, and therefore an absolute reference to define an orders of magnitude space, will be considered for each one of the five variables as described in Sections 4.1 and 5.2.

After the three steps, translation, symmetrization, and extension to the same cardinality, the values of the five variables will be able to be expressed in the absolute references in Figure 4, which will represent the ratios qualitative orders of magnitude:



Figure 4: Absolute references of the variables

The homogenisation described in Sections 4.1 and 5.2, allows you to simultaneously handle the qualitative values defined by the above discretisations. The bijections are established in the new absolute reference given in Figure 5.

-2 -1 0 1 2

Figure 5: Common absolute reference

At this point, the five variables are all referenced in an OM(3) with the same landmarks.

The three steps applied to the variables give the successive landmarks shown in Table 2:

Variable	V <sub>1</sub>	V2	V <sub>3</sub>	V <sub>4</sub>	V <sub>5</sub>
Initial	25, 50, 75	I, 2I, 5I	0, 1, 3	0, 10, 60	0.5, 1, 1.5
Step 1	-25, 0, 25	-I, 0, 3I	-1, 0, 2	-10, 0, 50	-0.5, 0, 0.5
Step 2	-25, 0, 25	-31,-1,0,1,31	-2,-1,0,1,2	-50,-10,0,10,50	-0.5, 0, 0.5
Step 3	-25,-10,0,10,25	-31,-1,0,1,31	-2,-1,0,1,2	-50,-10,0,10,50	-0.5,-0.25,0, 0.25,0.5
Homogenised	-2,-1,0,1,2	-2,-1,0,1,2	-2,-1,0,1,2	-2,-1,0,1,2	-2,-1,0,1,2
Not relevant	-1, 1	-2, 1	-2, 1	-2, 1	-1, 1

Table 2: Variation of landmarks

The operator must take into account the influence or strength of each ratio on credit risk. In order to see how these ratios summarise the relevant company information, the following Figure contains a graph that shows the strength of variables considered to define the credit quality.



Figure 6: Influences of the variables.

Using these strengths, a first example would be a simple qualitative operator, which is the qualitative expression of a real weighted sum:

T =[
$$\tau$$
], where:  
 $\tau$  (x<sub>1</sub>, x<sub>2</sub>,..., x<sub>5</sub>)= -2 x<sub>1</sub>+ x<sub>2</sub>+ x<sub>3</sub>+2 x<sub>4</sub>+3 x<sub>5</sub>, with x<sub>i</sub>  $\in \mathbb{R}$ 

Now is the time to compute the landmarks of the adapted reference in order to express the qualitative values of the operator. As described in Sections 4.2 and 5.2, these landmarks are chosen in the set:

{
$$\tau$$
(b<sub>1</sub>, b<sub>2</sub>,..., b<sub>5</sub>)=-2 b<sub>1</sub>+b<sub>2</sub>+b<sub>3</sub>+2 b<sub>4</sub>+3 b<sub>5</sub>  
with b<sub>i</sub>  $\in$  {-2, -1, 0, 1, 2}}.

In this particular example, the exact set of landmarks is:

 $\{-2 b_1 + b_2 + b_3 + 2 b_4 + 3 b_5, \text{ with } b_i \in B_i\},\$ 

where  $B_i$  is the set of *relevant* landmarks for the variable  $V_i$ , i.e, according to table 2:  $B_1 = B_5 = \{-2, 0, 2\}$  and  $B_2 = B_3 = B_4 = \{-1, 0, 2\}$ .

This set of landmarks is the set of integers:

$$L = \{-2 \ b_1 + b_2 + b_3 + 2 \ b_4 + 3 \ b_5, \\ \text{with } b_1, \ b_5 \in \{-2, \ 0, \ 2\}, \ b_2, \ b_3, \ b_4 \in \{-1, \ 0, \ 2\}\}$$

that is:

 $L = \{ z \in \mathbb{Z} / -18 \le z \le 18 \}.$ 

Thus, the set of basic qualitative labels corresponding to the orders of magnitude space S' is:

 $S_1 = \{N_{19}, N_{18}, N_{17}, ..., N_2, N_1, 0, P_1, P_2, ..., P_{17}, P_{18}, P_{19}\}$ 

Finally, choosing in set L three positive and three negative landmarks, gives you a less fine discretisation of the real line, with eight basic qualitative labels {Aaa, Aa,

A, Baa, Ba, B, Caa, Ca}.

#### Figure 7: Rating discretisation

Therefore the final qualitative values for the credit quality will be given in an orders of magnitude space OM(4) with granularity 4.

The software tool has been used for this example. We ran the test on eighty well-known American firms whose Moody's classification is available to the public. The results obtained by the software tool for these firms are shown in Figure 8.



1 4 7 10131619222528313437404346495255586164677073767982

#### Figure 8: Results of tests

The firms are represented on the horizontal axis, their final qualitative label on the vertical one. Numbers from 1 to 8 on the vertical axis represent basic labels from Aaa to Ca. The label of each classified firm is represented with two points: the lighter point is its greatest basic qualitative label, and the darker one is its least basic qualitative label.

For example, firm number 13 has been classified by the software tool as [Aa,Aaa], and firm number 77 has been classified as [Baa,AAa].

Thus, the light polygonal joins all the firms' greatest basic qualitative labels and the darker one joins their least basic qualitative labels.

Figure 9 shows the real Moody's classification of the set of firms considered. Where comparing these two Figures it is quite clear that the ARQO method is effective, even though only five input variables were considered. If the number of variables considered increases, one expects that the classification will be more accurate.



Figure 9: Moody's classification

## Conclusion

This paper presents an on-going work, which provides the concepts and strategies for synthesising qualitative information from variables, each one of which is qualitatively described in a different way (ARQO).

The system is applied in the financial domain to evaluate and simulate credit risk. This approach may also be applicable to problems in other areas where the involved variables are described in terms of orders of magnitude, and the results depend on the given variables and a set of strengths.

The limitations of the method presented cannot be evaluated until the implementation is completed and sufficiently tested.

The proposed method is currently being implemented to be applied to available data referring to the most important American and European firms, whose Moody's rating is known. The work is currently in the initial process of empirical application: the construction of the financial data base for the firms included in the index D.J.500, definition of the involved qualitative variables, as well as validation of the degrees of influence of each variable on the final result.

Some of the on-going tasks consist in:

- 1. Discovering alternative methods for building a homogenised reference.
- Adjusting the weights of the qualitative weighted sum Φ, using an historical set of data. In fact, the weights could be computed initially from simulation from a set of data instead of using the experts' information.
- Determining the most suitable qualitative operator Φ. There exist diverse alternatives to the weighted sum used in the example:
  - a). One of them could be a qualitative weighted mean as defined in (Godo and Torra 1998), that admits operating qualitative values belonging to an ordinal scale, and are based on t-norms and t-co-norms (Schweizer and Sklar 1983).
  - b). Another possibility would be the development of qualitative operators to be directly applied by means of k-input tables.
- 4. It is also intended to compare the obtained results with the results furnished by other classifiers used in artificial intelligence, such as genetic algorithms, neural networks or learning systems, as for example the LAMDA system based on hybrid logic connectives (Aguado 1998, Piera 1987).
- 5. Finally, applying Qualitative Norms (Agell 1998) to compare different orders of magnitude.

Our final words are to note that this work is only the first experiment with a new and simple idea, though one we are convinced is promising: the idea of adapting qualitative references to the operators involved in a real problem (ARQO).

#### References

Agell, N. 1998. Estructures matemàtiques per al model qualitatiu de órdenes de magnitud absoluts. Ph. D. Thesis Universitat Politècnica de Catalunya, Barcelona.

Aguado, J.C. 1998. A mixed qualitative quantitative selflearning classification technique applied to situation assessment in industrial process control. Ph. D. Thesis Universitat Politècnica de Catalunya, Barcelona.

Alpar, P. and Dilger, W. 1995 Market share analysis and prognosis using qualitative reasoning. *Decision Support Systems*, vol. 15, n. 2, 133-146.

Berndsen, R. 1995. Causal ordering in economic models. Decision Support Systems, vol. 15, n. 2, 157-165.

[Dague, 1993] P.Dague. Numeric reasoning with relative orders of magnitude. *Proceedings of AAAI Conference*, Washington, USA, 541-547.

Forbus, K. D. 1988. Commonsense physics. Annals Revue of Computer Science, 197-232.

Godo, L. and Torra, V. 1998. On Qualitative Weighted Means. Actas del VIII Congreso español de tecnologías y lógica fuzzy, Pamplona, 185-192. Goonatilake, S and Treleaven, Ph. 1996 Intelligent Systems for Finance and Business, John Wiley & Sons.

Lancaster, K.1962. The scope of qualitative economics. *Review of Economic Studies*, 29, 99-123.

Maher, J.J. and Sen, T.K. 1997. Predicting Bond Ratings Using Neural Networks: A Comparison with Logistic Regression. *Intelligent Systems in Accounting Finance & Management*, vol 6, 59-72.

Mavrovouniotis, M.L. and Stephanopoulos, G. 1988. Formal Order of Magniture Reasoning. *Process Engineering. Computer Chemical Engineering*, vol. 12, n. 9/10, 867-880.

Piera, N. 1995. *Current Trends in Qualitative Reasoning* and *Applications*, Monografia CIMNE, 33. International Center for Numerical Methods in Engineering, Barcelona. Piera, N. 1987. Connectius de lògiques no estandard com a operadors d'agregació en classificació multivariable i reconeixement de formes. Ph. D. Thesis Universitat Politècnica de Catalunya, Barcelona.

Raiman, O. 1986. Order of Magnitude Reasoning. *Proceedings of AAAI Conference*, Philadelphia, 100-104.

Schweizer, B. and Sklar, A. 1983. Probalistic Metric Spaces, North-Holland, Amsterdam.

[Travé-Massuyès, L. and Piera, N. 1989. The Orders of Magnitude Models as Qualitative Algebras. *Proceedings* 11th IJCAI, Detroit, 1261-1266.

Travé-Massuyès, L. Dague, Ph and Guerrin, F. 1997 Louise Travé-Massuyès, Le Raisonnement Qualitatif pour les Sciences de l'Ingenieur, Ed. Hermès, Paris.