# Many-Valued Logic and Qualitative Modelling of Electrical Circuits

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#### Abstract

Several authors have reported previous work on qualitative modelling of electrical systems by assuming steady-state conditions and reducing circuits to networks of resistive elements. This has been very successful in certain applications, particularly for Failure Mode Effects Analysis in automotive electrical systems. In support of this work we have concentrated on simple, minimal models that maintain structural isomorphism with the real system; the motivation being to provide an accessible conceptual structure for ease of use by engineering staff. However, a number of difficulties remain and solutions to these would increase the applicability of the method.

In this paper we introduce further developments that address several of the remaining important issues and provide solutions to some recent obstacles. After describing a series of problems, we draw on the field of many-valued logic for inspiration and investigate some qualitative analogies. The results of a new labelling scheme show the advantages of a 5 valued quantity space for resistance. This is shown to resolve previously ambiguous cases and opens up considerably more modelling possibilities. The interpretation of the labelling scheme can be tailored to the application domain, giving powerful modelling options.

#### Introduction

In a previous conference paper we discussed the relationship between qualitative modelling of electrical circuits and the more conventional, numerical electrical circuit modelling found in science and engineering (Lee 1999b). We highlighted the need for non-numeric, more abstract and less detailed models that can capture the essence of a circuit configuration, and we showed why this is important for the next generation of circuit simulators and engineering tools. We also identified the close similarity of goals and motivation between the ECAD based approach known as "switch-level modelling" and the methods adopted by the qualitative modelling community. In this paper we introduce further developments that address several of the remaining important issues and provide solutions to some recent obstacles. We gain inspiration from ideas in the field of manyvalued logic and demonstrate their value for supporting our qualitative modelling methods.

Our emphasis continues to focus on steady-state analysis and the application domain is automotive electrical systems.

# Current problems with qualitative circuit models

In this paper we build on our work on qualitative circuit modelling for FMEA (Failure Mode Effects Analysis) as applied to automotive electrical systems. This is based on a theory, known as CIRQ, that uses quiescent current models and is described in (Lee 1999a). From this theory (Price et al. 1995) have developed a successful full scale commercial FMEA software tool that outperforms numerical systems and is now in regular use in the automotive industry.

Briefly, CIRQ takes the electrical property of resistance as a first order approximation for any energy absorbing component. A qualitative representation of resistance must map onto the positive reals,  $\mathcal{R}$  and we have used a three-valued finite algebra for resistance,  $[0, +, \infty]$ , corresponding to short circuit, load and open *circuit* respectively. Qualitative voltage and current are modelled similarly although only two values for current are required for FMEA as the question is only whether a component is active or inactive. These qualitative values are a key feature in our approach. They provide primitive but powerful models that abstract the most relevant and significant information about the circuit in an intuitive form; in contrast with the mathematical models and voluminous data produced by numerical simulation.

The design of CIRQ was influenced by the need to address the specific requirements of the FMEA task for electrical systems and this has been satisfied, as discussed in (Lee 1999a). These requirements essentially concern the effects of faults of only two classes: opencircuit faults and short-circuit faults. These can be seen as structural faults and, in most electrical FMEA, there are no requirements for the analysis of faults causing only parameter changes, e.g. increase or decrease in a resistance value. The complexity of the task is such that engineers tend to deal with extreme and worst case scenarios and include lesser cases within this scope. This limitation, of dealing with only structural changes and not component variation, also applies to most other recent qualitative circuit modelling methods (Hotz et al. 1997).

We have also been motivated to adopt simple, minimal models in order to increase accessibility and provide a simple conceptual structure. This seems important for human cognitive processes, such as insight, and helps maintain affinity with engineering concepts. However, when we consider extending the range of our FMEA task, or even other reasoning tasks such as diagnosis, design analysis or verification, then the limitations of our models become evident. The coarseness of the minimal representation suggests further developments or refinements are needed to support other tasks.

For these reasons we have refined our method without losing the features specified by the original motivation. We now list a few problems that frustrate not only CIRQ but also many other similar qualitative resistance modelling systems:

Bridge circuits A bridge is a significant kind of circuit topology that creates ambiguity in any qualitative model. Bridges occur when the ends of a circuit branch can not be resolved into an ordered potential difference and therefore the current magnitude and direction can not be determined. All circuits can be classified into those that can be reduced into a single equivalent value by repeated application of series parallel reduction rules (SP circuits) and those that can not be so reduced. Of the circuits containing bridges, some will be balanced, i.e. current magnitude = zero, and some will be unbalanced, with unknown direction of current flow. Figure 1 shows these classes of circuit.



Figure 1: Circuit taxonomy

Because it is impossible to deduce the state of current flow in bridges, for magnitude or sign, without exact quantitative values of the resistances and voltages involved, this has been a major problem for all qualitative circuit models and no effective treatment of this difficulty has been reported.

#### Diodes and other uni-directional components

Diodes are devices that act as a conductor in one direction but block flow in the other direction. Although diodes are used widely in DC circuits to control the selection of circuit functions and operations, they have not been given much attention by the qualitative modelling community. This may be because directional selectivity is a discontinuity that poses particular problems for models based solely on resistive elements. The CIRQ based FMEA system gets round this problem by performing multiple analyses, one for each possible direction of flow, and then selecting the result that matches the permitted flow direction of the diode. However, better solutions are desirable for more general applications.

- Variable current levels Although a three-valued resistance model can cover a wide range of circuit components (to a first approximation) under steady-state conditions, this only allows a single current level to be represented: either current flows or it does not. There are occasions when an engineer needs to distinguish between different levels of current flow. For example, an electrically active motor might be running free (unloaded), running with normal load, or be in a stalled state. Each of these would give different current levels that can not be modelled in CIRQtype systems. At present, a stalled motor would be modelled as a short-circuit and this is satisfactory for FMEA analysis, but other tasks might require to distinguish between the high load current from stalling and a complete short-circuit (that could have different effects). Similarly, an unloaded motor may take very little current, but still provide an active circuit path. In discussions with engineers we have discovered that three levels of current would provide a sufficient enhancement to cover a large range of applications. This translates into a five-valued quantity space for resistance which seems to offer a good match to engineers' intuitive models, at least as can be deduced from informal descriptions<sup>1</sup>.
- Low current paths Another related problem concerns the distinction between fully active paths and paths with weak current flows. For example, a fault might cause a lamp to be fed from a high resistance source and therefore not actually function. Another case occurs for sneak circuit analysis where live but inactive branches give potential paths for faults in future operation. Previously in CIRQ all paths with any flow are marked as active and the consequences are determined by extra-domain expertise. It would be extremely helpful if any low-level paths could be marked separately. In automotive systems low-level but functionally inactive current flows are quite common.

#### The switch-level modelling approach

As reported previously (Lee 1999b), the QR community has apparently been unaware of the efforts by ECAD

<sup>&</sup>lt;sup>1</sup>We also note that most questionnaire designs, opinion data scores and many forms of psychometric testing use five point scales.

research engineers to build an intermediate grain-size model of electrical circuits. This work was directed at reconciling the detailed numeric data produced by analog simulators with the state information generated by gate-level simulators. Both are properties of electrical circuits but one concerns electrical variables and parameters while the other deals in configurations and states.

A particularly promising approach, called "Switch-Level Models", was based on the framework of manyvalued logic (Hayes 1986). This showed remarkable similarities with the recent resistive qualitative methods as discussed in (Lee 1999b). For example, the voltage quantity space in CIRQ has four values corresponding to the four connectivity possibilities illustrated in figure 2. These are precisely matched in Hayes' representation, with two supply potentials, an intermediate potential, and an open circuit. For current and resistance, however, switch-level models used a finite set of values with a range of intermediate levels. The aim was to support a form of approximate numerical analysis by reducing the grain size as appropriate. Unfortunately this formulation was seriously flawed and gave much worse accuracy than expected because of the error combination effects created when qualitative values are used in operations like subtraction. Similar problems can be experienced with interval arithmetic. These limitations, which killed off this line of modelling, are described in detail in (Cerny et al. 1992).



Figure 2: The four valued voltage space

# Concepts from Many-Valued Logic

Classical logic (first-order predicate calculus) assumes two truth values: true and false. Although the extensive literature on logic has often considered the idea of additional truth values, it was not until 1920 that the first many-valued logic (MVL) was formulated by Lukasiewicz. In an n-valued logic, propositions can be assigned truth values from the range, 0, 1, 2, ..., n-1, where 0 corresponds to false, n-1 corresponds to true and intermediate values give differing degrees of truth. Logical functions can be defined for the operations of and, or, not, xor... etc and the result of a compound assertion can then be computed from the truth values of the components and the truth functions of the connectives. It may be surprising that there are so few reports of MVL systems being used in Artificial Intelligence applications<sup>2</sup>. This is in contrast to other systems of logic that have been explored extensively, e.g. modal logics, nonmonotonic and default logic. For a current view of the field, see the survey by (Hahnle and Escalada-Imaz, 1997) on deduction in MVL.

## Many-valued resistance

Qualitative algebras for systems higher than threevalued are difficult and cumbersome (Struss 1988). Some of the operators are asymmetric and values can have mixed meanings, which makes for awkwardness and therefore less intuitive models. We wish to retain the intuitive character of qualitative circuit models and believe MVL can assist in this respect. However, we must be careful to avoid the multiple current accuracy problems that ruined the switch-level modelling approach. Instead of allowing multiple values for both resistance (impedance) and current (signal level) we have initially investigated a many-valued resistance model.

An obvious start would be to explore the use of MVL operators to implement functions for performing circuit reductions and CIRQ type analysis. However, it is very important to first establish the interpretation of any value system to be used. For example, in Lukasiewicz's 3-valued system, the semantics of the intermediate truth value was taken to mean *indeterminate*, while the 3-valued systems of Kleene and Bochvar interpret this value as *undecided* and *meaningless*, respectively. We must determine the appropriate interpretations for any system we design for circuit modelling.

In the CIRQ formulation the three-valued quantity space,  $[0, +, \infty]$ , corresponds to three topological or connectivity conditions: zero resistance makes nodes electrically identical, positive resistance indicates connectivity and infinite resistance signifies disconnection. We retain this structure but extend the + value to allow a series of different positive values corresponding to different strengths of connectivity. We have experimented with three such values {lo, med, hi} and so the quantity space is:  $[0, lo, med, hi, \infty]$ .

We need to define the interpretation of  $\{lo, med, hi\}$ and specify how they are to be resolved in series and parallel combinations. Previously CIRQ used summation for series reduction, minimum for parallel reduction, and the + case could take an integer value. Resistance label pairs (forward and reverse) were assigned to each node to give the minimum number of

<sup>&</sup>lt;sup>2</sup>Notwithstanding some early attempts, e.g Michalski's use of variable-valued logic for pattern recognition applications.

resistive edges to the positive/negative terminal respectively. We could use this scheme unchanged and simply map the quantity space into a numeric form, i.e.  $[0, lo, med, hi, \infty]$  corresponds to  $[0, 1, 100, 1000, \infty]$ . This would work to some extent but this numeric formulation has some undesirable properties and is cumbersome to read and therefore loses the vividness and directness that we wish to retain. Consequently, we now label nodes with their symbolic resistance values using the operator Max for series reduction and Min for parallel reduction, according to the ordering  $0 < lo < med < hi < \infty$ . In addition, following investigation of various options, we find it useful to record the number of resistance elements in the node labels, as before. Node labels thus have two parts; a qualitative value and a count of elements from a supply terminal.

Note that *med* need not be shown explicitly: any node label consisting of only an integer is assumed *med* by default. For example, parallel reduction of 2lo and 4 will give 2lo but 3hi and 7 will result in 7. Under this scheme a node might be labelled 3hi/2 which indicates it has a path of 3 resistance elements, at least one of which is hi, from the positive terminal and a path of 2 elements, of highest value *med*, from the negative terminal. An illustration of the method is given in figure 3



Figure 3: New labelling scheme

where all edges are of default value med except for the single edge marked hi. The result is a primary path of resistance med and length 5 (= forward + reverse values) and a secondary branch of value hi, of total length 8. The full algorithm is somewhat more complex than can be described here; more detail is available in (Lee 2000).

#### Results

We now present a series of case studies that show how the problems discussed in the earlier section can be solved by the new model.

Diodes Figure 4 shows a circuit containing a diode arrangement that causes difficulties for previous models. In this new solution when a diode is encountered, if it is aligned with the path direction (from positive for forward and conversely for reverse) then its value is taken to be 0, alternatively for the other direction it is assigned the value hi. This produces the node labels seen in figure 4. We have designated hi as reverse diode impedance which is usually very high, consequently any branch containing a hi value has negligible current flow and so the active paths are clearly identified as the upper-left and lower-right diodes and the central load is powered. The meaning of hi is seen as being important in interpreting the results of any particular case. Note that we did not use  $\infty$  for reverse diode resistance because it is useful to distinguish inactivity due to diode action from disconnection.



Figure 4: Diode regulator circuit

- **Bridge circuits** A potential resistive bridge is seen in figure 5. Here all resistors are of value med except for the high impedance indicator and monitor device that are both hi. The results from CIRQ show the main flow path is through the three central resistors with two high resistance branches either side. The relative meaning of hi can be defined by the engineer who will also assess the significance of the lower current levels in the side branches. Without the many-valued distinction the central resistor would form a bridge that could not be resolved.
- Variable current levels Figure 6 shows a motor circuit where the indicator changes from dim to bright according to whether the motor is powered. Because the indicator is linked to the motor, when the main switch is open there is a path for a small current flow through the motor. The previous version of CIRQ would thus record the motor as active, but now with a distinction between main power circuits and signal circuits this can be resolved. The results show the motor to be on a high impedance path and this would be interpreted as being under powered although electrically alive. Note that dead-end branches are now



Figure 5: Bridge circuit

sometimes identified by  $\infty$  labels; unlike the previous scheme.



Figure 6: Motor drive circuit

Low current paths A related example is shown in figure 7 where an electronic monitoring module is connected to a power circuit. It has been assumed that the input to the monitor is of high impedance and this has been represented by an internal resistive network all of value *hi*. Note that the actual function of the monitoring module is not important — it may involve complex electronics or computing functions — all that is needed is an equivalent resistance model that holds for the external view of the circuit during the operation being modelled. The result is that all of the monitoring circuit has different values and does not interfere with the power paths. Notice that previously this would have been another bridge circuit with ambiguous and unsatisfactory results.



Figure 7: Circuit with voltage sensing device

# Discussion

It can be seen that the 5 valued quantity space, with 3 intermediate resistance values, has not used Ohm's law to produce multi-valued current but has instead increased the range of path or circuit-branch properties. These have resolved previously ambiguous cases and open up considerably more modelling possibilities. The interpretations given to the additional resistance values is a key feature of the method and these are determined by the application domain. In our examples we have seen the semantics of hi vary to distinguish: reverse diode leakage current, high impedance circuit sections, and low level signal currents. Others could include the increased impedance of an unloaded (freerunning) motor.

We do not have space to include examples of the use of the value lo but there is a dual relationship between the distinction between lo and med and between medand hi. Paths can be identified, by lo, that carry higher current than "normal" and this offers more benefits for modelling key characteristics of problems. Similar examples can demonstrate cases of lo resolving circuits with overloaded motors, partial shorts, and energy loss in conductors. This latter example is of special interest in automotive applications where all power cables that entail noticeable voltage-drops are to be monitored and distinguished from near ideal conductors such as signal wires.

Orders-of-magnitude relationships (Raiman 1991) have already been shown to be capable of solving some unbalanced bridge circuits (Lee 1999b). We can see that such a relationship is contained within the series/parallel reduction rules because of the nature of the qualitative ordering and the choice of functions. Hence, any number of *med* edges in series will always be considered lower in value than a single hi edge, and similarly with any other pair from the ordering  $0 < lo < med < hi < \infty$ . The Min function has the equivalent effect on parallel circuits. This means the ordering implemented is actually  $0 \ll lo \ll med \ll hi \ll \infty$ .

One remaining problem is the lack of current direction indication. Indeed, the new path labelling method has destroyed the edge current flow direction heuristic that was based on node resistance values. Previously the flow direction between two nodes, u and v, could be estimated by the relative values of

$$F(u) = \frac{f(u)}{f(u) + r(u)}$$

for the two nodes. Although global patterns can not be entirely deduced from local indicators this heuristic has been useful in path following algorithms. Nevertheless, all primary current flow paths between the supply terminals are easily found by following the lowest valued paths and other paths can then be found by elimination. Of course, bridge edges can only be resolved when they are unbalanced and have order of magnitude differences in the relevant branches.

#### Other MVL possibilities

So far we have only explored a few ideas suggested by MVL. We have investigated the modelling benefits of many-valued resistance and (very loosely) used MVL functions to compute series/parallel reductions. We do not require a qualitative version of Ohm's law but note that this can be done either in terms of logic functions or arithmetic operators. In fact, these are closely related mathematically by their use of different t-norms for conjunctors; i.e. to obtain V from I and R we use multiplication for QR and minimum for MVL.

When using MVL there are often several different ways of defining familiar functions, for example, negation has more than one formulation. This opportunity, together with the open interpretation of values, offers various options and benefits for modelling. For example, a three-valued system might correspond to *no-flow*, *lo-flow*, *hi-flow* or it could be interpreted as *no-flow*, *lo-flow*, *unknown/ambiguous*. Thus, we will need to consider both the design of suitable functions and establish their particular semantics in the application domain.

Another role for MVL variables is to represent faults or abnormal situations. This has been done very successfully for digital system design where valid signal values are mixed or interleaved with values indicating error conditions (Hurst 1984). This idea could be used to model given fault classes by separating abnormal cases from other values. It seems this can be achieved more easily in MVL than QR as functions can be specified from their truth tables, rather than by using mathematical operators. Ambiguities, physically impossible cases and other special states can also be incorporated into systems based on MVL functions. Logics that have been used for this include  $K_3$ , the 4-valued system of Belnap (Kaluzhny et al. 1993) and various 5-valued systems (Smith 1981). An interesting possibility is to extend the functionality of our original models to incorporate explicit fault conditions that can then be propagated (as for normal values) using modified algorithms.

Finally, there is a strong motivation to integrate both qualitative and numeric modelling techniques in a combined and coherent modelling environment. It is possible that MVL could offer insight and structure for this integration problem.

# Related work

Many studies in AI have examined electrical circuit analysis but most have been concerned with digital systems or the dynamic aspects of electronics, such as transient behaviour. An early original contribution on linear steady-state systems was the work of (Sussman, 1977). Recent work of relevance has mainly investigated steady-state qualitative resistance models, usually based on a three-valued quantity space. Most such work has dealt with the analysis or effects of structural changes in circuits and we find very little reporting on non-structural or parameter changes.

The work of (Hotz et al. 1997) is particularly relevant and describes the differences between existing (structural) methods, which are based on constraints, connectivity propagation and series-parallel-star tree reduction, and the requirements for analysing non-structural changes. They argue that deviations from a norm are the important factors during diagnosis and develop a method using qualitative deviations. A series of circuit rules are used to propagate current and voltage values and a form of series-parallel-star tree reduction (Mauss and Neumann, 1996) is used to reduce the resistive nets. The main problems with this method are that it relies on topology dependent rules, it can not handle bridges. and it is not general for all circuit topologies. Like (Mauss and Neumann, 1996) this method can carry out some numerical reduction in parallel but it suffers from a loss of user affinity due to the complications involved in processing decomposition trees.

Other work on qualitative deviations has been carried out by (Struss et al., 1996). In the context of diagnosis, these researchers have shown how deviations can capture vital behavioural characteristics that models can employ in diagnosis, analysis and design. These methods are different from ours in that they use connectivity propagation but the important relation to our work is the use of deviations. In our examples we have taken the value med as a default and hi (or, in other cases, lo) has been a variation due to some component effect. If we designate this as a faulty value then hi can represent a deviation. Another example is the identification of cables subject to voltage-drops and distinguishing these from normal current flows. However, not all our examples are of this form; we can designate hi to indicate a different class of circuit, as in the monitoring example. We see our method as a way of classifying branches of a circuit, and the significance of the resulting branch labels will depend upon the application semantics given to the resistance value set.

# Conclusions

We have presented a new version of our successful CIRQ qualitative circuit analysis tool. A number of problems for the old system, with both technical and application urgency, have been solved by the new version. These include diode circuits, variable current levels and some bridge circuits. The changes have retained the vividness of the original model; a feature considered important for acceptance and affinity with practicing engineers.

The new version illustrates how larger quantity spaces for resistance can effectively label circuit nodes with application specific classes for different types of current flow. The modifications rely on an orders-ofmagnitude resistance relation and require a new method of path traversal in the labelling process. The nodes no longer contain values that give minimum distance (in edges) from the terminals. The path traversal method is efficient and general, and does not require star/delta transformation rules.

It is clear that, in general, bridges can not be solved by *any* qualitative method. If the four resistances surrounding a bridge are a, b, c and d, then balance is achieved if:

$$a * d = b * c$$

This requires precise values and the best qualitative methods can do is return the label 'ambiguous'. However, our intrinsic orders-of-magnitude relation does allow us to discern certain cases (not all) that are definitely *unbalanced*. This occurs when

#### Min(a,d) > Min(b,c)

This, at least, is an improvement and it follows that increased numbers of values will provide increasingly finer resolution.

Our method can be viewed as a process that classifies circuit branches into different impedance levels. By not attempting the calculation of any current values we have avoided the combinatorial and accuracy problems that arise from arithmetic on several multi-valued qualitative variables as experienced in the switch-level models of (Cerny et al. 1992). It might seem that we are still some way from ECAD systems that can reconcile numeric models with relevant qualitative circuit information. However, we suggest that, rather than aiming for total integration through convergence of representations, the operation of an analog circuit simulator in tandem with a CIRQ-type model would produce very useful output. Such parallel operation could label each wire, node or component with both actual voltage/current and qualitative labels that signal important application specific conditions. This seems to offer considerable potential for a coherent modelling environment on which to build future engineering reasoning tools.

Many-valued logic has provided inspiration and offers further possibilities for enhancing qualitative modelling work. Future work includes: investigation of a 10valued model, with hierarchical classes, and evaluation of the use of the model in full-scale deviation analysis applications.

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