First Steps in Formalising Qualitative Systems Dynamics

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Abstract

System dynamics techniques, such as influence diagrams, are used to model poorly understood systems. Qualitative analysis of these models is often extremely vague. More rigorous analysis requires quantitative simulation and can only occur after an additional conversion to a 'stock and flow' model. In this paper we present some augmentations to influence diagrams that allow analysis and simulation to proceed directly from influence diagrams. We describe a new qualitative constraint that is required for this process. In addition, we show how, in some limited situations, reliable and formal qualitative predictions of stability can be drawn from such models using the notion of parametric variation.

Introduction

'System dynamics' is the term given to the study of the dynamic behaviour of a variety of complex systems, generally in the domain of human activity systems such as organisational management (Coyle, 1996; Goodman, 1989). These systems are characterised by a lack of explicit knowledge about the fundamental mechanisms as work in the systems, as well as a lack of quantitative information on how such mechanisms operate.

The main tool of system dynamics is representing the system being studied as an influence graph such as that shown in figure 1. The influence graph indicates the major variables in a system and what influences these have on each other. Traditionally, this model is used solely as a sense-making device, allowing an analyst to organise and communicate his understanding of a complex problem domain. The influence diagram is then manually converted into a 'stock and flow' diagram (figure 2) that shows how the system's components interact. Generally, the stock and flow diagram is more complex than the influence diagram as it includes nodes for each of the model's parameters. The stock and flow diagram is used to develop a set of equations which is used in a numerical simulator to generate the behaviour of the system.

This process suffers from the problem that qualitative reasoning was developed to solve: the system being examined is only known in the most general sense, while the numerical simulation can only accommodate strictly quantitative information. This paper shows a first step in resolving this dichotomy by applying qualitative reasoning techniques to system dynamics.

One existing approach to resolving the qualitative/ quantitative tension has been the development of 'qualitative system dynamics' (Senge, 1990; Senge *et al.*, 1994). The approach here is to compare the original influence graph to a number of 'archetypes'; each archetype exhibits a specific qualitative behaviour. If a given influence diagram resembles that of an archetype, the supposition is that the model will exhibit similar behaviour. However, this process is purely intuitive and has no formal basis, which means that there can be no reliance on the results of qualitative system dynamics.

As a first step towards this goal, we present a method for analysing these models in a more formal way, using some qualitative reasoning techniques. First, we show how some simple augmentations to the influence diagram notation

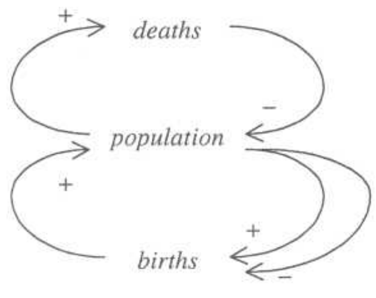


Figure 1: Traditional system dynamics model of controlled population growth.

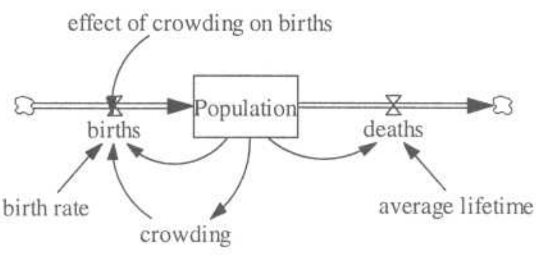


Figure 2: Stock and flow model derived from figure 1.

allow the simple and automated derivation of both quantitative and qualitative state space equations for a model. We also show how feedback caused by parametric variation can be represented in an influence diagram. Finally, we describe how, in some limited circumstances, to evaluate whether this feedback is sufficient to stabilise the model.

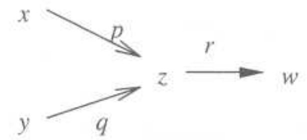
Formalising Influence Diagrams

The traditional approach in system dynamics has been to develop an influence diagram model as a 'sense making' step, where an analyst identifies the major features of the problem situation. This influence diagram is then converted to a 'stock and flow' model, equations are manually developed, and simulation is carried out. Indeed, Wolstenholme (1999) states that "no way has yet been established to directly convert [an influence diagram] representation directly to a simulation model." Figure 1 shows an influence diagram developed for a model of population growth limited by overcrowding and figure 2 shows the corresponding stock and flow model (there is a 'stock' of population and flows caused by births and deaths). The equations used for simulation, derived from the stock and flow model, are shown in figure 3. This process of conversion, from influence diagram to stock and flow model to equations, is performed manually and intuitively. In this section, we show how this process can be simplified and formalised. This is done by augmenting the influence diagram to allow state space equations to be derived automatically from such a model. In this paper, we show how this can be done for arbitrary qualitative systems; we also consider linear quantitative systems. In order to capture the richness inherent in system dynamics influence diagrams, we do not require these models to be parametrically invariant.

The first extension of the influence diagram notation is to differentiate between direct and indirect influences (Forbus, 1984). We do this by following the notation of Rickel & Porter (1994) (figure 4). This allows the identification of state variables in the model, as state variables can only be affected by direct influences, while non-state variables can only be affected by indirect influences. Note that figure 4 shows both the quantitative and qualitative equations that derive from the influence graph fragment shown.

```
carrying capacity = 500
effect of crowding on births lookup([(0,20)-
        (10,0)], (0,20), (3,11), (6,6), (8,2.5),
        (10,0.75))
birth rate = 2
births = Population * birth rate * effect of
        crowding on births lookup
crowding = Population/carrying capacity
deaths = Population / average life
Population = INTEG(births - deaths, population)
```

Figure 3: Equations derived from figure 2



$\dot{w} = r.z$ $\quad$ $\dot{w} = \mathrm{M}(z)$

$z = p.x + q.y$ $\quad$ $z = \mathrm{ADD}(x, y)$

Figure 4: Direct and indirect influences

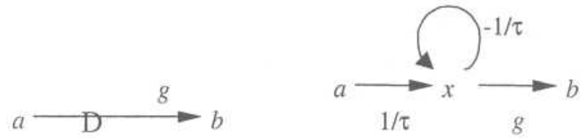
Once this is done, the objective is to eliminate all non-state variables and all indirect influences, combining their effects into direct influences. This simplification is performed directly on the graph, in a manner similar to signal flow diagrams (Wilson & Watkins, 1990; Richards, 1993). However, there are two complications to this approach: some arcs will be annotated with a delay and parametric variation is represented by controlled arcs. Methods for resolving these complications are given below. In addition, the qualitative treatment of controlled arcs requires the development of a new qualitative constraint which is described in the next section. These simplification techniques have been implemented in a program what will take an influence graph and produce the corresponding minimal set of either state space equations or QDEs.

Delay arcs

System dynamics influence diagrams often contain delay arcs (figure 5a), which represent an influence taking effect after some delay. This is used to represent such phenomena as delays caused by the transportation of materials, or organisational inertia in response to changes in management policy. Such delays must be eliminated from the model if linear state space equations are to be derived. This is easily done by realising that delayed effects effectively represent effects via hidden state variables. This equivalence is exploited in a well-known identity (Coyle, 1996) which allows delay arcs to be eliminated from an influence graph (figure 5). It is convenient to eliminate all delay arcs in this way before any other simplifications take place.

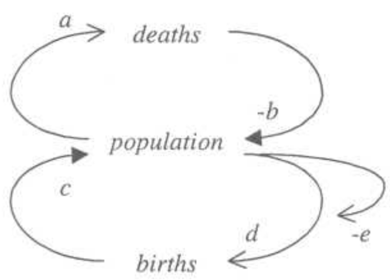
Controlled Arcs

System dynamics influence graphs are traditionally designed to show the 'influences' that can affect a variable. These influences can take many forms which are normally



5a: A delay arc with delay τ $\quad$ 5b: Its undelayed equivalent

Figure 5: Converting delayed arcs



$$\dot{p} = -a.b.p + c(d - e.p)p$$

Figure 6: Population model with a controlled arc

only resolved at the time of the production of the stock and flow model. For instance, in the raw model of controlled population growth (figure 1), two influences affect the number of births. The positive influence represents the notion that a larger population will have a greater number of sexually mature females; as each such female will have a certain probability of reproducing, the size of the population will influence the number of births.

The negative influence on the number of births represents the reduction in fecundity due to population pressures. Figure 1 suggests that this influence affects the number of births in the same manner as the size of the population. However, closer consideration shows that population pressures act to reduce the fecundity of the females. It is this reduced fecundity that causes the lower number of births. This is the relationship shown in the corresponding stock and flow diagram (figure 2). The notion of one influence mediating the effect of another prompts the introduction of controlled arcs as a way of showing this on the influence diagram. The gain of a controlled arc is mediated by the controlling arc or arcs. This extension allows parametric variation to be included in the model. A revised population model, including a controlled arc, is shown in figure 6.

Modified Monotonic Functions

In order properly to discuss controlled arcs and their implications, it is necessary to introduce a new qualitative relationship, the modified monotonic function (M^m). This relationship is a generalisation of the normal monotonic functions described by Kuipers (1986). The modified monotonic function has the following properties:

$$p > 0 \to M^m(x, p) = M^+(x)$$
$$p = 0 \to M^m(x, p) = const\,(y) \text{ for some } y$$
$$p < 0 \to M^m(x, p) = M^-(x)$$

However, the advantage of the M^m constraint is that it allows values of $M^m(x, p)$ to be ordered for different values of p. To allow this, we define the predicates *right* and *left*:

$$\begin{aligned} right\,(M^m, x, p, q) = T \leftrightarrow \\ &\exists x_0 < x\text{: } M^m(x_0, p) = M^m(x_0, q) \\ &\vee p > q \wedge \forall y\text{: } M^m(x_0, p) > M^m(x_0, q) \end{aligned}$$

$$\begin{aligned} left\,(M^m, x, p, q) = T \leftrightarrow \\ &\exists x_0 > x\text{: } M^m(x_0, p) = M^m(x_0, q) \\ &\vee p > q \wedge \forall y\text{: } M^m(x_0, p) < M^m(x_0, q) \end{aligned}$$

(Informally, imagine a *meeting point* (x_0) for which $M^m(x_0, p) = M^m(x_0, q)$. The predicates *left* and *right* indicate whether the current value of x is to the left or right of this point. The second disjunct is to allow the predicate to be used when x_0 does not exist.)

It follows that *right* (M^m, x, p, q) = *right* (M^m, x, q, p) and *left* (M^m, x, p, q) = *left* (M^m, x, q, p)

The use of the functions allows the values of $M^m(x, p)$ to be compared for different values of p:

$$p > q \wedge right\,(M^m, x, p, q) \to M^m(x, p) > M^m(x, q)$$
$$p > q \wedge left\,(M^m, x, p, q) \to M^m(x, p) < M^m(x, q)$$

In addition, the following *controller extension* relations hold:

$$\begin{aligned} p_0 > q_0 \wedge right\,(M^m, x, p_0, q_0) \to \\ &\forall p > p_0 : right\,(M^m, x, p, q_0) \\ &\wedge \forall q < q_0 : right\,(M^m, x, p_0, q) \end{aligned}$$

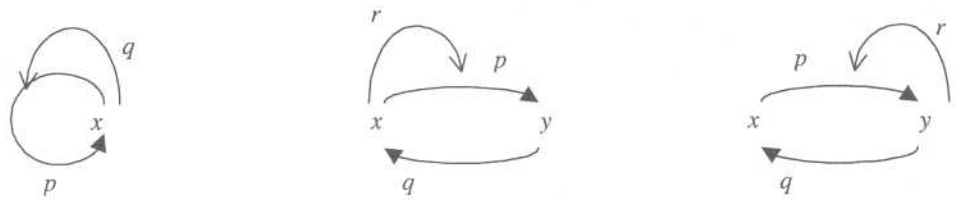
$$\begin{aligned} p_0 > q_0 \wedge left\,(M^m, x, p_0, q_0) \to \\ &\forall p < p_0 : left\,(M^m, x, p, q_0) \\ &\wedge \forall q > q_0 : left\,(M^m, x, p_0, q) \end{aligned}$$

These properties are used to assess how controlled arcs can affect the stability of system dynamics models.

Formal Qualitative Analysis of Influence Diagrams

As described above, there are two major strands of analysis within system dynamics. The quantitative approach can be supplemented with qualitative and semi-quantitative simulation packages such as QSIM and its peers (Kuipers, 1986; Kuipers & Berleant, 1988; Coghill, 1996). However, a more interesting avenue to explore is to use qualitative reasoning techniques to augment the qualitative system dynamics of Senge (1990). This requires formal analysis of the structure of the influence graph to determine, with some degree of rigour, the behaviour that will be expected from the model.

The first and most basic behavioural question to be asked is whether or not the model represents a stable system. System dynamics models are generally autonomous (i.e. without exogenous inputs) and do not necessarily have globally conserved quantities such as energy. Stability is defined as having a bounded behaviour in response to a



7a 7b 7c

Figure 7: Some controlled loops

sufficiently small perturbation[1]. For instance, ignoring the action of the controlling arc, the simple population model of figure 6 is stable around the point $p = 0$ if $a.b > c.d$, but unstable otherwise.

Puccia & Levens (1985) describe how the stability criteria of a model can be determined by applying the Routh-Hurwitz criteria. Each distinct cycle in an (fully simplified) influence graph is identified; its gain is the product of the gains of its constituent arcs. These loop gains can be combined to yield the graph determinant. If various subgraphs are taken, the various graph determinants can be combined to give an evaluation of the overall stability of the model. Such an analysis was performed to yield the stability condition for the population model of figure 6. While this approach is simple and easy to apply, it does have several limitations. The most significant of these is that the Routh-Hurwitz criteria can only be applied to linear or linearised models. The means that a modeller must identify the various equilibrium points and, for each point, linearise the model around that point and assess its stability. The detection of such points is not trivial (Khalil, 1996). Also, the linearisation means that the stability criteria cannot take account of possible changes in the loop gains caused by controlled arcs as the system moves from the equilibrium point.

Stability of Systems with Controlled Loop Gains

Controlling arcs represent feedback mechanisms in the model, and it is well know that feedback of the correct form is capable of ensuring the stability of an otherwise unstable system (D'Azzo & Houpis, 1960). However, the complex systems generally addressed by systems dynamics practitioners do not have clearly defined inputs and outputs and any feedback mechanisms only operate within small regions of the model. These conditions prevent the straightforward application of traditional control theory techniques for determining if feedback stabilises a particular model.

Given that the stability of the uncontrolled model is determined by loop analysis, and the action of controlled arcs is to alter the gains of these loops, a more promising avenue of investigation is to examine the effects if feedback from the point of view of loop gains. Unfortunately, the gain of a controlled arc depends on the value of one or more state variables, and loop analysis is silent on the transient response of these variables (Puccia & Levens, 1985). At present, no general solution to this problem has been identified, but the effect of controlled arcs on the stability of models has been determined when loops of only one or two state variables contain arcs that are controlled by variables within those loops, such as the controlled population model (figure 6). In self-controlled loops of length one or two, the effect of the controlled arc on the loop gain is independent of the direction of movement of the controlling state variables. In such a case, the loop gain will change as the model moves away from its original equilibrium point. By extrapolating this change it is possible to determine if the system will reach another equilibrium point.

Figure 7 shows some self-controlled loops, and figure 8 shows the structure of the qualitative constraints of the controlled and controlling arcs. The gains of the loops shown in figure 7 is $\frac{\dot{y}}{x} \cdot \frac{\dot{x}}{y}$.

Assume some initial values x_0, y_0, z_0. If $z_0 > 0$, then $M^m \in M^+$. If we assume that $x > x_0$, then *right* (M^m, x, z_0, z) holds. If $M_1 \in M^+$, then $M^m(x, z) > M^m(x, z_0)$. Similarly, if $x < x_0$, then *left* (M^m, x, z_0, z) holds; $M_1 \in M^- \rightarrow M^m(x, z) > M^m(x, z_0)$. In both of these cases, the effect of the control arc is to increase the gain of the controlled arc. Similarly, if $M_1 \in M^+$ when *left* (M^m, x, z_0, z) holds, or $M_1 \in M^-$ when *right* (M^m, x, z_0, z) holds, the control arc acts to decrease the gain of the controlled arc. These situations are reversed if $z_0 < 0$. The symmetric nature of the qualitative constraints allow these arguments to be applied to any of the controlled loops shown in figure 7.

In particular, note that the controller extension relations ensure that however the controlling arc starts to act, it will

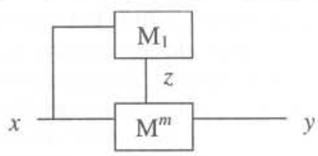


Figure 8: Qualitative constraints on a controlled arc

[1] Generally, the systems being modelled are non-linear and the models are linearised around an equilibrium point. If the initial perturbation is too large, the linearisation approximation will not hold and model will no longer be an accurate representation of the system.

continue to act in the same manner so long as $(x - x_0)$ keeps the same sign. In particular, if $M_1(x)$ reaches zero, the associated loop gain will also become zero.

These results are only applicable to self-controlled loops of length one or two because of the possibility of ambiguity arising from the interaction of several variables on the controlled arc, none of which may be the direct influence on the variable in question. These results show that for self-controlled loops of length one or two, the effect of a controlling arc will increase over time. This allows the effect of the parametric variation to be included in the stability analysis, and therefore show whether the system will ever reach a stable point.

Referring back to the population model (figure 6), recall that the model is stable if the gain of the 'deaths' loop is greater than the gain of the 'births' loop (in the linear case, the model if stable if $a.b > c.d$). If this is not the case, the population will move from its equilibrium value of $p = 0$. As it does so, the controlling arc will act to reduce the gain of the births loop. This action will make the model more stable. It is a reasonable assumption that controlling arc will eventually force the gain of the controlled arc to become zero. The Intermediate Value Theorem then allows us to declare the existence of a value for the population where the gain of the births loop will equal the gain of the deaths loop. At this point, the model will become stable. Note that the controlling arc will not cause the model to stabilise if the population were less than zero: however, populations are constrained to be positive. Quantitative analysis of this model shows that an initially unstable model becomes stable when the population reaches $p^* = \frac{s}{t} - \frac{p.q}{r.t}$. Again, this analysis process has been implemented as a program that will identify if the feedback present in a model is sufficient to ensure the stability of a model.

Further work

The work presented here represents the first steps in the formalisation of qualitative systems dynamics. This work could progress down several avenues. The role of controlled arcs in maintaining stability in more complex situations needs should be addressed; however, this will require an understanding of the transient response of state variables after a perturbation and how these responses interact within and between loops. Ishida (1989) has had some success in this area. More generally, qualitative system dynamics depends on the identification of structural cliches to predict the behaviour exhibited by a model (Senge *et al.*, 1994). The limitations of this approach are obvious and well known (Lane & Smart, 1996) but qualitative reasoning approaches might provide appropriate tools for deriving useful results. For instance, the complexity of identification problem could be reduced through the use of order of magnitude reasoning (Raiman, 1991) to eliminate loops with insignificant gains. The easy identification of such loops is hampered by the action of controlled arcs and the relationship between loop gains and delays.

Conclusions

The major contributions of this work apply to both quantitative and qualitative system dynamics. Firstly, for quantitative system dynamics, we have described some simple augmentations to the influence diagram notation and shown that these augmented diagrams contain all the information needed to produce a set of state equations. This has been demonstrated by deriving such state equations without the need for the intervening stock and flow diagram.

Secondly, we have introduced some rigour into the study qualitative system dynamics. The objective of qualitative system dynamics, to predict the qualitative behaviour of a model from simply inspecting its structure (subject to some assumptions about the magnitude of effects) is an attractive one. However, existing techniques are entirely without rigour; this paper has addressed this issue. We have introduced the concept of controlled arcs to represent feedback mechanisms acting in a model and we have described a new qualitative constraint to represent the behaviour of these arcs. We have shown how the combination of these augmentations can be used to predict qualitatively whether the feedback present in a model is sufficient to ensure the model's stability.

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