

Graph Theoretical Analysis of Qualitative Models in Sustainability Science

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Abstract

Solution sets of larger qualitative models tend to explode in the number of possible qualitative states. Moreover, their representation as a tree obscures important properties of the QDE from the modeller. To improve the use of qualitative modelling techniques in sustainability science, a simplified representation, the state transition graph, is introduced. It enables a “gestalt view” of the result and an automated search for general properties and associated structures of the model that are relevant for sustainability issues. Some of these properties are defined and illustrated by an exemplary model on land-use changes in developing countries. Their relationships among each other as well as the connection to management and control is outlined.

Introduction

To an increasing extent qualitative modelling techniques are used in ecology and the emerging domain of sustainability science (Heller & Struss 1996; Struss 1998; Petschel-Held *et al.* 1999; Guerrin & Dumas 2001; Petschel-Held & Lüdeke 2001; Salles & Bredeweg 2001; Eisenack & Kropp 2001; Eisenack, Kropp, & Welsch 2002). The latter field of research aims at understanding the interactions between nature and society to support a global sustainable development (Kates *et al.* 2001). Qualitative modelling addresses at least two typical problems in this context: (i) Knowledge about important interactions is very limited in many cases, and (ii) the broad variety of regional problems is difficult to integrate into typical patterns of global change. Since qualitative differential equations (Kuipers 1994) consider whole classes of models, they provide a formal framework for typifying the variety of mechanisms on the one hand, and to regard knowledge limits on the other.

However, since the number of qualitative solutions tends to increase dramatically when additional variables or ambiguities are introduced, strong efforts were made in the last decade to reduce large solutions, e.g., by the development of chatter box abstraction, global filtering by temporal logic expressions or by the integration of numerical knowledge (Tokuda 1996; Brajnik & Clancy 1996; Clancy 1997; Shults & Kuipers 1997; Berleant & Kuipers 1998; Kay 1998). The paper stands in this line of thought, but complements it with typical questions from sustainability science. These are particularly related to the management

or control of a system. Thus, it is an additional objective of this paper to improve methods for development and assessment of management options by using qualitative models.

One of the limits of the current QSIM implementation is the representation of the solution set as a tree. The shape of the tree does not only depend on the model, but also on the order in which the qualitative states of the model are processed by the algorithm. In particular, this is the case for an envisionment, where the solution is not a tree in the graph theoretic sense. Consequently, important properties of the result of the model can not be seen directly from the simulation output, but have to be discovered by chance or laborious investigation. To improve this situation, Mallory (1996) made an explicit step towards a graph theoretical representation to display projections of the solution set onto user-defined variables of interest. Recently Bernard & Gouze (2002) investigate the phase space of a class of models under uncertainty by means of a transition graph. We go further in this direction by introducing a simplified state transition graph of a QDE to enable a “gestalt view” on model results.

However, efficient computation of qualitative solutions is a prerequisite for a better representation, visualization and interpretation of model results. To tackle models of greater size, we have improved and supplemented an existing C-language implementation of QSIM (Dvorak 1998). Chatter box abstraction is re-implemented and we provide facilities to exclude marginal cases from model results.

Once the state transition graph is computed, it can not only be used for a better visualization, but also well-known concepts and algorithms from graph theory can be applied to detect important properties of the solution, e.g., reachability, connected components or cycles (e.g., Behzad, Chartrand, & Lesniak-Foster 1979). In general, we can test in which regions of the state space the model respects certain specifications, even if the solution graph is too large to be visualized effectively. There is a close relation to model assessment by branching-time temporal logic expressions (Shults & Kuipers 1997), but complementary to this approach we focus on the identification of subgraphs which exhibit certain structural properties.

The paper is organized as follows. In the next section we define the notion of a state transition graph, present how it can be generated from QSIM output, and how it can be further simplified. This is illustrated by results from the new

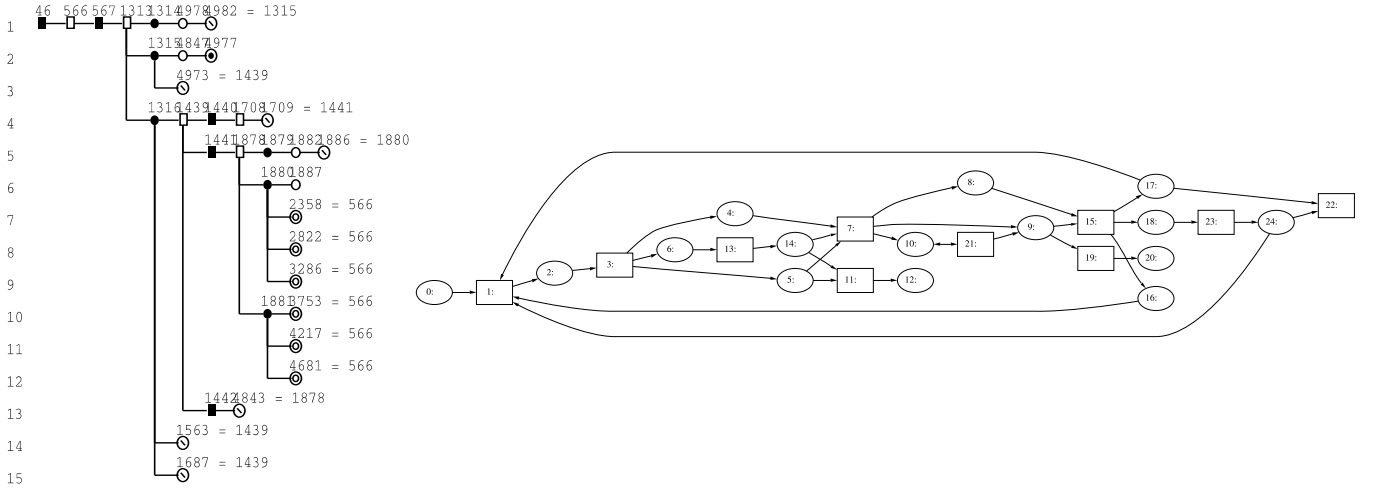


Figure 1: Example for a behavior tree and the resulting behavior graph. Boxes denote time-interval and ellipses time-point states. Generated from a model presented further below in the text.

QSIM implementation. The third section introduces an example from sustainability science, defines important properties of a state transition graph and demonstrates how they can be interpreted and applied. The last section concludes with a discussion of the approach and perspectives for further work using the concepts proposed.

Qualitative Solutions and State Transition Graphs

In this section we introduce the concept of a state transition graph of a QDE, and show how it can be further simplified by eliminating edges which are of little relevance in most applications. Another improvement is the speed-up of the QSIM algorithm, which is achieved by a re-implementation based on the earlier work of Dvorak (1998). Both pave the road for an automated analysis of large qualitative models.

The State Transition Graph of a QDE

Suppose, we have completed an envisionment of a QDE (Kuipers 1994). This gives us a set of qualitative states, each of them having well-defined predecessors and successors. Thus, we already have the structure of a directed graph, where the states are vertices and the non-symmetric successor relation is represented by the edges (see fig.1 for an example). To make a formal definition of this graph, we introduce the notion of a **qualitative state space** by $\Sigma = \Sigma^1 \times \dots \times \Sigma^n$, where $\Sigma^1, \dots, \Sigma^n$ is a system of finite sets, each representing the qualitative magnitude or the qualitative direction of a variable in the QDE. Thus, each possible qualitative state of the model is an element of the qualitative state space and can be regarded as vertex of the graph. Edges are pairs, constituted by states and their successors.

Definition 1 A **behavior graph** of a QDE with qualitative state space Σ is a tuple (Q, T, ι) with vertices $Q \subseteq \Sigma$, edges $T \subseteq Q \times Q$ and a mapping $\iota : Q \rightarrow \{true, false\}$. The following properties have to hold: Each vertex $s \in Q$ is a

qualitative state that occurs in a behavior of the QDE, and $\iota(s) = true$, iff s is a time-interval state. Moreover, for each edge $(s, s') \in T$ the state s' is a qualitative successor of state s in a behavior of the QDE.

The mapping ι is only introduced to discriminate between time-point and time-interval states. It should be remarked that (Q, T) is a bipartite graph, since states of one type are always connected to states of the other type. Qualitative states that have no successor (and therefore never occur in the second component of an edge), are called **final states**. They are equilibria or states where the QDE leaves its region of applicability (e.g., divergent states).

In the next step, we transform the behavior graph to the simpler state transition graph, which has less vertices. This facilitates a better overview over the possible dynamics of the system by omitting information that is usually less important for applications.

Definition 2 The **state transition graph** (V, E) of a behavior graph (Q, T, ι) has vertices V with $\iota^{-1}(true) \subseteq V \subseteq Q$, edges $E \subseteq V \times V$, and is obtained from the behavior graph in the following way:

1. Let $F \subseteq \Sigma$ be the set of final states in (Q, T, ι) . Then $V := \iota^{-1}(true) \cup F$.
2. There is an edge $(s, s') \in E$ iff $s \neq s'$ and there is a vertex $q \in Q$ with $(s, q) \in T$ and $(q, s') \in T$, or if $s' \in F$ and $(s, s') \in T$.

According to this definition the state transition graph is constructed by eliminating all time-point states which have a successor from the behavior graph. Edges are introduced to preserve the reachability of the “surviving” states. Thus, the behavior graph shown in fig.1 is simplified to the graph in fig.3. It should be noted that this simplification can still be regarded as complete when we infer the time point states that are possible between adjacent vertices.

For further use we denote the set of successors of a state $s \in V$ with $succ(s) := \{s' \in V \mid (s, s') \in E\}$. We call

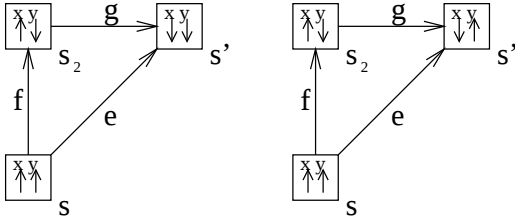


Figure 2: Examples for different types of composed events. Arrows in the boxes symbolize qualitative values of two variables (further explanation in the text).

a finite sequence of states in V , written $s_1 \dots s_m$, a path, if $\forall i = 1, \dots, m-1 : s_{i+1} \in \text{succ}(s_i)$, and no state occurs more than once in the sequence. The only exception from the last restriction is the case, where $s_m = s_1$. Then, the path is called a cycle. The restriction on finite sequences will be sufficient for the concepts introduced below. Once we regard QSIM output as a state transition graph, we can apply concepts and methods from graph theory for further investigations.

Marginal Cases

For some models the number of edges in the state transition graph can be further reduced, since some transitions which occur in many models are of little relevance for application purposes. Moreover, it can be shown that for many systems the set of (quantitative) trajectories featuring some special types of transitions is of measure zero (Bernard & Gouze 2002). On the other hand, the completeness of QSIM guarantees that also the abstractions of such trajectories are computed. By omitting them in the state transition graph we trade off completeness for comprehensibility. The implied loss of information is acceptable, since no features of relevance for the modeler are left out. In this subsection the notion of marginal cases is defined and we present two ways to eliminate them.

Each edge in the state transition graph corresponds to at least one event that shifts the system from one state to the next, i.e., it changes the qualitative values of one or more variables. Some events e in the state transition graph are composed, i.e., there are two other events f and g which change the state in the same way as e when occurring subsequently. In other words, the edge of the composed event e directly connects the predecessor of f with the successor of g . Thus, the event e can be generated by concatenating the events f and g . What happens in the resulting behavior? Two cases for an event f , occurring at time-point t_1 , and g , occurring at time-point t_2 can be distinguished (see fig. 2):

1. In f some variables change their qualitative value, while in g other variables change. In the event e both sets of variables change at the same time-point.
2. In the second event g some variables change back to the qualitative value they had before t_1 , while other variables change to a new value. In the event e only variables that do not change back in g are affected.

The first case is a marginal case, since the time-points t_1 and t_2 converge, but not the second one, since some variables would have to change and change back at the same time. Here, the event e exhibits – compared to f and g – a notable new property, which should not be ignored. Only composed events e of the first type, where events coincide which do not necessarily have to, can be omitted. Usually, they show no special relevance to the modeler, because nothing basically new happens, and they are not likely to be observed in empirical studies.

Definition 3 A *marginal case* in a state transition graph (V, E) is an edge $(s, s') \in E$, for which a path $s_1 \dots s_m$ with $s_1 = s, s_m = s'$ and arbitrary $m \geq 3$ exists, where $Ch(s, s_2), \dots, Ch(s_{m-1}, s')$ are pairwise disjoint. Here $Ch(s_i, s_j) \in \{1, \dots, n\}$ denotes the set of components of the state space Σ which change along the edge (s_i, s_j) .

Note, that it may be useful for applications to set an upper limit for m , because too much information can get lost. However, for long paths the sets of changing components are not very likely to be disjoint. Removing marginal cases results in so-called occurrence debranching, in particular if the events f and g can appear in any temporal order (Tokuda 1996; Clancy 1997).

We have implemented two strategies to avoid marginal cases, one by pre- and one by postprocessing, which can also be combined. The first strategy requires the modeler to define the marginal cases explicitly by introducing so-called correspondence-not constraints into the code of the model, for example

`((cornot x y) (0 0) (1x 1y)),`

where x and y are variables, and 0 , $1x$ and $1y$ are landmarks. Each constraint of this type contains two qualitative variables and one or more pairs of landmarks of their quantity spaces. It acts as a local filter that discards all qualitative states where both variables are at a given pair of landmarks at the same time. This requires to be careful not to ignore important coincidences of events like the transition to equilibrium (e.g., if two *qdirs* are prohibited to become *std* at the same time). However, this strategy can substantially reduce computing costs, since less qualitative states have to be generated during simulation.

The second strategy avoids the problem of the first and is completely automatic, but requires the solution of the QDE to be determined first. Taking this as input, all marginal cases are determined and eliminated from the state transition graph. The algorithm assigns the changing *qmags* and *qdirs*, i.e., $Ch(s, s')$, to every edge (s, s') in the graph. If there is a path where the sets of changing variables are pairwise disjoint, and there is an edge connecting the first and the last vertex of the path, this edge is removed.

Implementation of CQ

Models describing the real world in general become very large. Most recent qualitative models on issues of sustainable development have some 30 variables with up to 50 constraints. Besides problems in representation, visualization and interpretation of model results, which in part have been touched upon in the previous subsection, models of this size

are difficult to be computed within the present implementation of QSIM in LISP. This can be the case even if the tree remains manageable with a rather small number of possible behaviors and states.

These limitations have led to a C-implementation of the basic QSIM-algorithm by Dvorak (1998). We have extended this implementation and included the following features in CQ₂:

1. perform an envisionment,
2. enable some smaller simulation features like no-new-landmarks, unreachable-values and stepwise simulation,
3. a number of constraints missing so far, including S+ and S-, as well as multivariate constraints like (M + . . . -).
4. perform a static chatter-box abstraction following the algorithm of Clancy (1997).

Yet to be implemented are features like model transitions or dynamic chatter abstraction. Due to the very fast computation time, however, the latter can also be performed in a post-processing way.

CQ₂ has been extensively tested on UNIX AIX-4.3 and LINUX 7.3 (SUSE). Within the range of models tested so far it outpaced the LISP implementation by a factor 100, especially for larger models where LISP becomes limited due to memory allocation obstacles.

Application in Sustainability Science

In this section a (simple) exemplary model from sustainability science is sketched and its state transition graph is shown. We use it to illustrate typical questions that can be answered with this representation. We formalize some of these questions to enable an algorithmic investigation of larger models and draw the connection to the management perspective.

Land-Use Changes in Developing Countries

The model studies regional land-use changes due to smallholders agriculture in developing countries (Petschel-Held & Lüdeke 2001). Here the question arises, whether the system develops along the so-called impoverishment-degradation spiral, or recovers from such a situation. The first alternative is characterized by the degradation of the natural basis for production and reproduction and by increasing social disparities, also called the “Sahel Syndrome” (Lüdeke, Moldenhauer, & Petschel-Held 1999).

The outcome depends on how the smallholders achieve their daily income and how this is related to environmental conditions around them. The major functional difference between sources of income relates to their potential impacts on the environment, i.e., whether they have a direct impact or not. Income sources without a direct impact on the environment in particular include off-farm labor, which is considered in the model.

Existential rural poverty drives farmers to overuse their lands, if not enough off-farm labor is available. This leads to soil degradation, which reduces yield and thereby further exacerbates rural poverty. The socio-economic dimension

describes the compulsion of the impoverished rural population to further intensify or expand their agricultural activities, whereas the natural dimension assesses whether such an increase in agricultural activity would damage the natural production basis.

To model these processes we introduce variables for the quality of the resource R , the yield from agricultural activities Y , and the total consumption CT available to smallholders, which is the sum of yield and wage income from off-farm labor. Agricultural activities are employed with the landmark ms (=maximal sustainable agriculture). Below ms the soil regenerates, above it degrades. The socioeconomic dynamics of the model are mainly driven by labor allocation which changes in the direction of the more labor efficient activity. For example, if wages are low and agriculture produces sufficient output, labor is shifted from off-farm labor to agricultural activity. Together with auxiliary variables, the model has 13 quantity spaces and 11 constraints. It is relatively small compared to recent models with several hundreds or even thousands of qualitative states.

The state transition graph of the model is shown in fig.3. It exhibits some features also typical for larger models in the domain:

- A large cycle (here through states #0, #1, #4, #2, #7, #5, #10, #0).
- From any state in the cycle any state in the graph (especially any final state) can be reached.
- There are sets of states, that can not be left once they are entered (here: {#3, #8} and {#6, #11}).
- Some states are branching points in the sense, that they have more than one successor, some of which lead to such a set, and others do not.

The semantics of properties of this kind is important from the perspective of sustainable development, since value judgements can be made about qualitative states. Some final states clearly correspond to situations that are not desirable (e.g., total degradation of the resource and loss of income in #8), henceforth called negative states, while others are more acceptable (e.g., no agricultural yield but higher income and a regenerating resource in #9). Consequently, we want to avoid negative states (or at least predict them as early as possible) and shift the system to positive states. It is a precondition for such an analysis to identify these types of states and their reachability. Since we also want to deal with larger models, there is a need for automatization of this process. To achieve this, some properties of interest are formalized in the next subsection.

Properties of the State Transition Graph

Beside the classification of states as negative or positive, the existence of paths between them is a basic property to elaborate further useful concepts for sustainability science. In this subsection, S, S' are always subsets of V , where (V, E) is a state transition graph. A state s is always an element of S , and s' an element of S' . A qualitative state s' is called **possible** for the state s , if either $s = s'$ or the state transition graph (V, E) contains a path $s_1 \dots s_m$, where $s_1 = s$ and

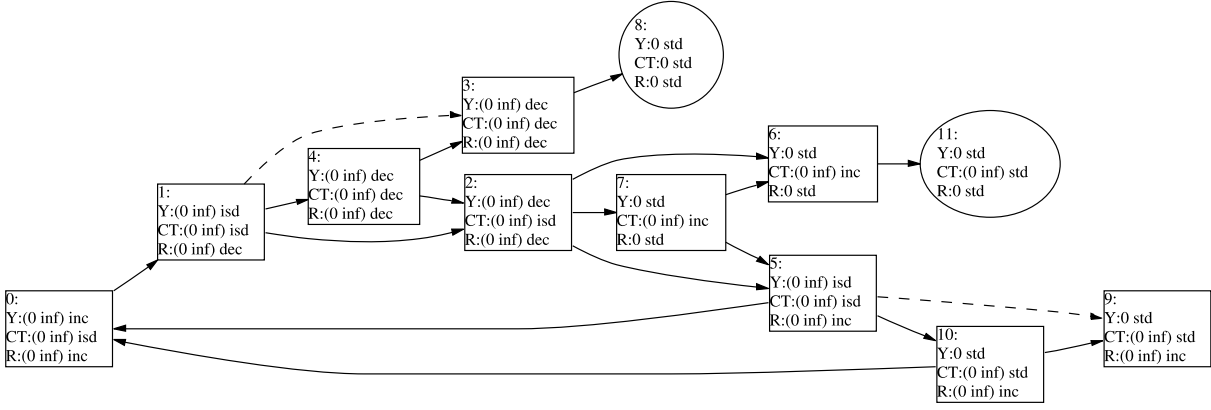


Figure 3: The state transition graph made from the graph in fig. 1, which is the result of the exemplary model on land-use changes. Some of the qualitative variables are depicted and explained in the text. The abbreviation “isd” stands for “increasing, steady or decreasing”, resulting from chatter box abstraction in CQ₂. Dashed arrows are marginal cases.

$s_m = s'$. This simply means, that a system which is in state s possibly arrives at state s' after some time. If s' is positive or negative, respectively, this can be interpreted as a chance or a risk, that s' happens after s has occurred. Here, we see the close relation to modal logic expressions as treated in the model checking algorithm of Shults & Kuipers (1997), since in the definition of possibility we quantify over the set of all behaviors consistent with a qualitative model, and not over the states of a single path. Below, we also specify the notion of necessity in state transition graphs.

In the example, every state is possible for the initial state #0, but there are also the “dead ends” #3, #8, #6, #9 and #11 for which not all states are possible. Thus, it is natural to ask from where a given state is possible, and what is possible for a given set of states.

Definition 4 The *possible set* $ps(S)$ is the set of states that are possible for at least one $s \in S$. A set S is called **weak attractor** of S' if for all $s \in S$ there exists a $s' \in S'$ which is possible for s .

It follows directly, that for $S_1, S_2 \subseteq V$ the equality $ps(S_1 \cup S_2) = ps(S_1) \cup ps(S_2)$ holds, i.e., the union of possible sets is again a possible set. Possible sets and weak attractors can be computed on the basis of efficient algorithms from graph theory to compute the transitive closure (e.g., Goralcikova & Konbek, 1979).

For sustainability issues, we are especially interested in domains where a system possibly sustains for infinite time. We introduce two definitions to capture this property.

Definition 5 A set S is called **viability domain**, if $\forall s \in S : succ(s) \cap S \neq \emptyset \vee succ(s) = \emptyset$. It is called **connected** if for all states $s, s' \in S$ there exists a path $s_1 \dots s_m$ with $s_1 = s, s_m = s'$, and $\forall i = 1, \dots, m : s_i \in S$.

The notion of a viability domain of a state transition graph has a close analogy to the viability domain of a differential inclusion (Aubin 1991). It requires that all states in S which are not final states have successors in S , and therefore the system can be kept inside of S at each position. Connectedness implies, that the system can be shifted from any state in

S to any other state in S without leaving the set. However, both concepts are not equivalent, but it can be shown that all connected sets are viable.

In the example, $\{ \#0, \#1, \#4, \#2, \#7, \#5, \#10, \#9 \}$ is a viability domain. It should be noted, that the definition is neutral with respect to positive or negative states, despite the positive connotation of the term “viable”. In the example, the viability domain contains a state where no agricultural activity appears (#7).

It can be shown that each viability domain contains at least one final state or one cycle. While viability can be checked by testing all states, for a connected set all relevant paths have to be traversed. However, once the transitive closure of the state transition graph is computed, we have a mapping $ps : \mathcal{P}(V) \rightarrow \mathcal{P}(V)$, assigning $ps(S)$ to a set $S \subseteq V$. Then, connectedness can also be characterized by the condition that $\forall s \in S : S \subseteq ps(s)$.

A stronger concept than viability domains are locked sets. While the former correspond to regions in the state space, where the system possibly sustains for infinite time, locked sets can not be left under any circumstances (consistent with the QDE), formally

Definition 6 a set $S \subseteq V$ is called **locked**, if $\forall s \in S : succ(s) \subseteq S$.

In the example (fig. 3), once the system shifts (i) to the set $\{ \#9 \}$, (ii) to the set $\{ \#6, \#11 \}$ or (iii) to the set $\{ \#3, \#8 \}$, this change is irreversible. These sets correspond to three important types of case studies on land-use change: (i) In the end, there is no agricultural yield, while the resource regenerates and wage-oriented labor guarantees a steady income. (ii) There is no agricultural yield, since the resource is totally degraded, but there is still a steady income due to wage-oriented labor. (iii) The resource is overused and there is no income.

It is obvious, that every locked set is viable. For further proofs and the detection of locked sets, the following equivalences are helpful:

Proposition 1 The following statements for a set $S \subseteq V$

are equivalent:

1. S is locked.
2. $\forall s \in S$ and $\forall$ paths $s_1 \dots s_m$ with $s_1 = s$: $s_1, \dots, s_m \in S$.
3. $S = ps(S)$.

It is a corollary from (3), that the union of locked sets is also locked. Moreover, the mapping ps is (with respect to $\subseteq$) a closure operator on the set of the qualitative states, and the locked sets form a closure system. It can be shown from this, that the set of all locked sets of a model has the algebraic structure of a lattice. Thus, locked sets can include other locked sets (as well as, e.g., connected sets or cycles). They demand special attention in sustainability science (Petschel-Held *et al.* 1999).

We are also interested in the “entries” of locked sets. Here it becomes obvious whether the system will be locked or not. Their predecessors can be used as indicators and denote the last chance to prevent or to bring about a locking. Hence, we call a state **branching point**, if it has at least two edges, one of which leading to a locked set and one to a set disjoint from the locked set. The edges leading form a branching point to a locked set can obviously be characterized as irreversibilities. In the example, the states #4, #2, #7 and #10 are branching points. They all belong to the large cycle in the graph. Thus, the system can emerge along this cycle for a long time, but sometimes it may leave this path, irreversibly snapping into one of the locked sets. Since two of these sets are clearly negative, this is more a ride along the edge of an abyss rather than a safe strategy.

Up to now, we defined concepts resting on the existence of paths, i.e., if certain states are possibly reached. Complementary to this is the notion of necessity. As a prerequisite we define a terminating path to be a path $s_1 \dots s_m$, where $s_1 = s_m$ or $succ(s_m) = \emptyset$. It can not be extended to further vertices or forms a cycle. A qualitative state s' is **necessary** for s , if for all terminating paths $s_1 \dots s_m$ with $s = s_1$ there exists an $s_j = s', j \in \{1, \dots, m\}$.

Necessity is used to characterize situations which are unavoidable for all systems that abstract to the QDE and start from a given qualitative state. If a state s' can not be avoided once the system enters s , the latter state can be used as a “predictor”.

Now we can define an important subconcept of weak attractors. A **strong attractor** consists of all states necessarily forcing the system to a given situation. This is formally defined as the set of states S , for which a given $s' \in V$ is necessary. If this is a negative state, we have a significant motivation to change the qualitative structure of the system (cmp. next subsection). The most prominent cases are strong attractors of final states. They are always locked sets (e.g., $\{\#3, \#8\}$ in fig. 1).

The concepts given above can not only be used to describe and find important properties of the underlying system, but also to simplify the graphical representation of larger graphs. Moreover, questions about the management of interacting natural, social and economic systems can be precisely posed and answered with algorithms from graph theory. Possible

consequences for and approaches to management for sustainability are outlined in the next subsection.

The Management Perspective

When the properties of the state transition graph are identified, we want to use this information for the development and assessment of management options under uncertainty. General properties and typical patterns that have to hold even under the weak knowledge specified in the underlying QDE can serve as a guideline for this task. It can be understood as finding means to, e.g., avoid locked sets or strong attractors which contain negative states, or to stay inside a viability domain (Schellnhuber & Kropp 1998). We can think of three possibilities to attain such objectives: (i) Changing the actual state of the system, (ii) changing the likelihood of certain transitions, or (iii) changing the structure of the graph. Under this perspective it has to be clarified how interventions can be analyzed in the framework of (in its roots deterministic) QDEs. The three possibilities already sketched have different pros and cons.

External Interventions: The System is directly shifted to another state by an external intervention, while the mechanism of the QDE is postponed for a while (e.g., by combating impoverishment with short-term financial aid). This can be very expensive and does not change the general patterns of the system which may be inherently problematic. On the other hand, an external intervention can be applied relatively fast and can, e.g., be used to leave a strong attractor.

Micro-Management: To combat the causes of transitions to negative states, the socioeconomic framework of the system may be altered. If this does not change the model qualitatively, i.e., the same state transition graph is produced, we speak of micro-management. Its effect is a change in the “tendency” of the system to shift in one or another direction, for example at branching points. Regrettably, the “tendency” can not be estimated by purely qualitative methods, which have to be complemented by, e.g., semi-quantitative methods or differential inclusions (Berleant & Kuipers 1998; Kay 1998; Aubin 1991). But nevertheless, the qualitative identification of branching points is an efficient guide for investigations in this direction.

Structural Management: If the framework of the system is altered in a more profound way, its qualitative structure changes. In the model some constraints and quantity spaces are replaced or introduced. As a result entries to locked sets may become impossible, new edges may transform strong attractors of negative states to viability domains, etc. This type of management produces fundamental change and can guide to safe, sustainable strategies. Yet, it is the most complex approach to apply, from the scientific as well as the political perspective. The search space for a better framework can become very large, and structural changes are usually difficult to implement in the real world.

Conclusion

In this paper we have introduced the state transition graph of a QDE and simplified it by excluding marginal cases. Motivated by an application from sustainability science, we

defined some important properties state transition graphs may exhibit. Relations between these concepts are shown and their semantics in the domain theory are given. These base on the existence of paths in the graph, which is interpreted as the possibility that a state succeeds another state. Complementary to former approaches using temporal and modal logic, we only regard particular but important structural properties of the graph and do not refer to the qualitative values of states. On the other hand, well-known and efficient algorithms from graph theory can be used to check these properties in large models automatically. In addition, we obtain regions in the qualitative phase space associated with them, which is useful for visualization and further investigation of a model. However, since it is possible that a qualitative model includes solutions that visit all qualitative states in any prescribed order, we have a slight limitation to this approach (Aubin 1996).

Prominent objects in the state transition graph like viability domains, locked sets and branching points, are relevant for the discussion of management measures: We want to find areas in the state space which cannot be left, or where we can keep the system in. Here, we have to take into account that the QDE approach is basically deterministic. Thus, if the model contains no open loop control, only structural change can alter the dynamics of the system. On the other hand, micro-management that does not influence the abstracted form of the model is covered by the approach if there are time intervals between such interventions. Therefore, we can ask how interventions can keep a system in a viability domain, and how the tendency of change can be influenced at branching points.

Investigating tendencies of change could be a fruitful objective for further research in the direction of semi-qualitative simulation or hybrid systems. On the other hand, the purely graph theoretic approach in this paper can be extended by analyzing additional relevant structures and identifying preconditions for their existence and for their entries in the graph. Visualization can be improved by nesting, e.g., locked or connected sets. However, the next steps will be to implement the projection of state transition graphs onto qualitative subspaces of interest, and to examine the graphs of coupled systems, especially how they inherit properties of the graphs of their projections or components.

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