Qualitative Approach for Mobile Robot Path Planning based on Potential Field Methods

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Abstract

The present work deals on the use of qualitative reasoning in order to carry out mobile robot path planning. A qualitative approach, based on potential field like methods has been developed. Qualitative variables, and operators have been established for manipulating, in a qualitative way, the information coming from sensors of mobile autonomous robots. The developed work is focused on a multiple mobile robot environment, and path planning is done in a decentralized way. Each mobile robot will carry out its own path planning without communicating among other robots or elements that are in the environment. The potential fields like schemes used in this paper are obtained from relative positions and velocities.

Mobile robot path planning based on Potential Field Methods

A great number of different techniques has been and are still developed in order to carry out efficient robot path planning. One of the most popular path planning method is based on the Potential Functions utilization where robot is modeled as a moving particle, inside an artificial potential field (U) that reflects free collision space structure into the robot workspace. Oussama Khatib initially developed artificial potential methods in 1980. Such Potential Fields are generated by superposing an attractive potential that attracts the robot to the goal configuration and a repulsive potential, which repulses robot far away from existing obstacles. The negative gradient of the generated global potential field is interpreted as an artificial force acting on the robot and causing variations on its movement. Nevertheless, as a main presented drawback, these methods can result to a trapped robot in local minimums generated by the same potential functions.

There are a large set of studied potential functions that generates artificial potential fields depending only on the distance between robot position and each spatial point belonging to workspace [Khatib 85], [Volpe 87], [Zelinsky 93], [Adams 90], [Warren 90]. Usual potential functions provide artificial potential fields with a symmetric circular or elliptical shape. Nevertheless, other potential functions can be generated using not only a first moment (distance) but also a second moment (velocity), to moving obstacles (including also other robots moving in the same environment). The resulting, artificial potential fields take elongated shapes pointing to the relative moving direction [Fuertes 94], [Martinez 94], [Planas 96], [Planas 00]. Figure 1 shows the obtained geometry of these structures that we had named Dynamic Force Fields (DFFs), where the magnitude at each point can be interpreted as proportional to the probability of collision at that point.

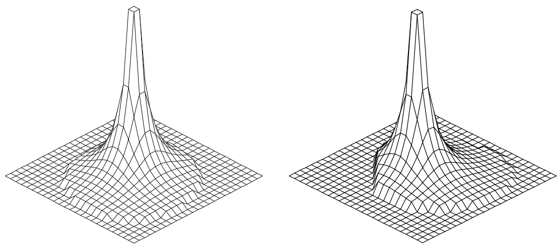


Figure 1. DFF's geometry

Individual DFFs have their maximum value on the position of the detected objects and they decrease with object distance. In order to not associate DFFs in an unnecessary way, they have influence only if the relative distance between the detected object and the mobile robot is below the threshold.

Two more features of our DFFs can be emphasized: they are not static, but they follow the dynamics of the object which they are associated, and they can be applied to static or dynamic objects as we are working with relative distances and velocities.

Each robot associates a different DFF to a given object as there is a different relative position and velocity associated to them. If two or more objects are detected by one mobile robot, a resulting DFF is obtained by addition of individual force field values. So, each mobile robot obtains its own dynamic instantaneous environment model, which we could visualize like a rough landscape. In a multiple mobile robot system we can have very fast dynamics, with strong time requirements. Nevertheless, during robot navigation it could be more important to ensure the safety of the robots than their exact positioning at each time instant. So, Qualitative Reasoning can be a god approach to solve multiple mobile robots path planning, involving faster system operation but at the expense of precision loss.

Qualitative approach to Potential Field Methods: Space Discretisation

To have a qualitative robot workspace representation, and later to apply qualitative path planning, it will be necessary to define qualitative variables, qualitative partition's limits, and qualitative operators associated to defined universe.

Relative distance between robot position and any position in the workspace will be calculated using Euclidean distance, so we always obtain positive values. Based on this relative distance, we construct a qualitative variable Qdist with the associated limits:

$$Qdist = \{[0, l\_zero[, [l\_zero, l\_small[, [l\_small, l\_medium[, [l\_medium, l\_large[\}$$

$$Qdist = \{[0, l\_zero[, [l\_zero, l\_small[, [l\_small, l\_medium[, [l\_medium, l\_large[\}$$

Ordering these intervals on R^+ , the obtained partition and the associated set of labels are shown in Figure 2.

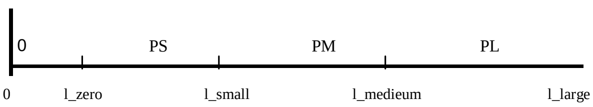


Figure 2. Generated partition and Qdist associated set of labels over R^+

Qdist is a non-equispaced partition, as we seek to emulate the concept of qualitative distance used by humans. The concept of nearby, used by persons, covers a more reduced area than the set of spatial points classified as a far-off distance. In order to define limits of the intervals, we take as a more critical situation those that imply an immediate stop. So, taking in account accelerated movement formulas, using the maximum robot's velocity, and knowing deceleration parameters, we can obtain the necessary time and space to stop the robot, shown in the expressions (1):

$$t = v_{max} / a$$
$$S_{max} = v_{max} * t - ½ * a * t^2 \qquad (1)$$

Spatial zones will be established using the above expressions, but they must fulfill some restrictions:

Qualitative spatial zones should cover the robot size plus the required space to stop it (Smax).

- Qualitative spatial zones related with different labels should be obtained using different deceleration values. As mre far away is the evaluated spatial point, more slowly the robot can brake. So, the far-off spatial zones offer a more wide covered area than the nearby ones.
- Nearest qualitative zone must be calculated using robot maximum deceleration, that can be obtained from datasheets or by means of empirical experimentation.
- If there are different mobile robots, interacting in the same workspace, and with different performances, the more restrictive one will be taken into account for defining space discretisation.
- It will be necessary to obtain an empirical calibration of the calculated qualitative zones, so as to contemplate the inertial robot behavior.

To completely define detected object's position it is necessary a second qualitative variable, since robot workspace is a two dimensional environment. Qdev is defined as a qualitative deviation between detected object position and robot position, and it is defined over real axes, taking positive and negative values. Figure 3 shows Qdev partition and Qdev associated limits and labels:

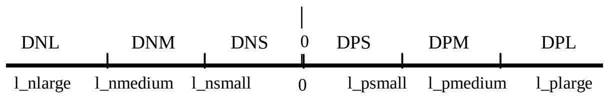


Figure 3. Generated partition and Qdev associated set of labels over R.

Simultaneous representation of Qdist and Qdev gives a discrete representation of the robot workspace. In addition, the reaching area presented by the used sensors will module the obtained workspace representation. Figure 4 displays a qualitative workspace representation with sensors' modulation.

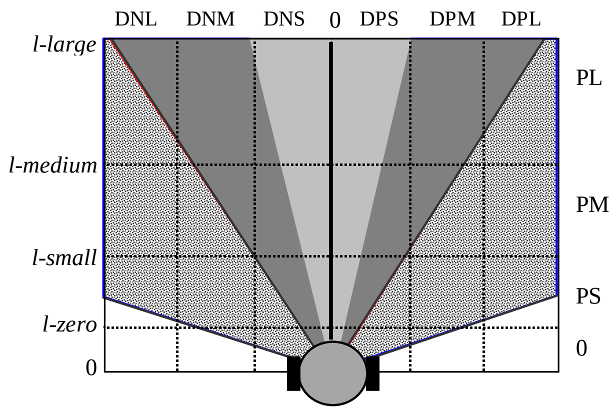


Figure 4. Qualitative workspace representation with different used sensors limitation.

So, robot workspace (W) becomes configured by all pairs (x,y) with:

$$W = \{\forall (x,y) \; / \; x \in Qdist \;\; ; \; y \in Qdev \} \qquad (2)$$

Qualitative Dynamic Force Field (QDFF) generation.

In order to obtain a Qualitative Dynamic Force Field model (QDFF), we propose to collapse the DFF models as a set of elliptical cylinders with different radius and heights. Each one of these elliptical cylinders will represent the repulsive force exercised on all the points over which it projects. Due to its own topology, QDFF projection on the robot workspace (W) becomes a set of concentric ellipses. Figure 5 shows QDFF model and its projection on the workspace.

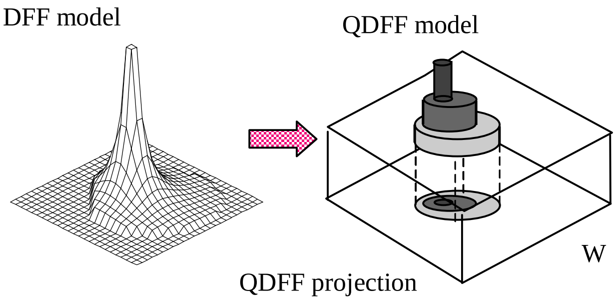


Figure 5. QDFF model and its projection on the workspace.

Each one of the elliptical cylinders or elliptical projected zones will represent a different level of sensed repulsive force. Then, a third variable was defined: Qlevels, which has as many associated labels as repulsive force levels exist in the QDFFs. In a more formal way, QDFF can be described as a set of ellipsis that forms a chain in relation to the inclusion operator. QDFFs will be described as (3)

$$E = \{E_1, E_2, ..., E_k\}; \quad E_k \subset E_{k-1} \subset ... \subset E_1 \tag{3}$$

where: E is the complete QDFF; E_j are the different ellipses (qualitative elliptic zones) and k is the QDFF dimension (number of different levels or ellipses that forms the entire QDFF). It is easy to deduce that k will determine the number of labels associated to the qualitative variable Qlevels.

Previous QDFF definition allows us to create a partition over the real axes R taking in account the inclusion of the ellipses (4). Also, a set of labels is associated to the new qualitative variable (5).

$$Q_e = \{ [0, E_1[, [E_1, E_2[, \cdots, [E_{k-2}, E_{k-1}[, [E_{k-1}, E_k[\} \tag{4}$$

Qlevels ={0, E1, E2 ·· Ek-2, Ek-1, Ek} (5)

Two QDFFs performances must be quoted:

1. They are dynamic models, so they change their representation in time.
2. When the robot detects more than one object, a QDFF is associated to each one of them. Operating with these individual QDFFs, a composed QDFF is obtained. Composed QDFF will be used to realise robot path planning Generated Qualitative Algebra.

Two performances mentioned in the previous paragraph forced us to create a new qualitative algebra that allows to operate with the qualitative variables related with QDFF model. With the purpose to be able to generate the composed QDFF when the robot detects two or more obstacles, a new qualitative operator has been defined: Let it be E and F two QDFFs associated to two detected obstacles, then (•) operator had been defined as (6):

$$S = E \bullet F = \{S_1, S_2, ..., S_k\} \tag{6}$$

Where S represents the composed QDFF. If E and F have the same number of qualitative labels, then composed QDFF has also the same dimension, that is S is represented by the same description universe (7).

$$\dim(S) = \dim(E) = \dim(F) \tag{7}$$

It could be interesting to note that composed QDFFs can present a differentiate performance: they can contain non-connected zones, which are representatives of the same level of repulsive force. Individual QDFFs can't present this topology by its own construction geometry. Let it be S a composed QDFF, and Si the ellipses that forms S, then each new generated ellipse Si can be generated by means of next expression (8):

$$S_j = E_j \cup F_j \cup \varepsilon_j \tag{8}$$

Where ε_j is the polyhedron defined by the convex envelope, as it is shown in expression (9).

$$\varepsilon_j = \left(\bigcup_{i=1}^{k-1} E_k \cap F_i\right) \cup \left(\bigcup_{i=1}^{k-1} E_i \cap F_k\right) \tag{9}$$

(•) operator has some properties that could be interesting to note:

If S_j is non-connected $\Rightarrow$ the convex envelope = 0

If S_j is non-connected $\Rightarrow$ $S_j = E_j \cup F_j$

If S_j is non-connected $\Rightarrow$ S_l ; $l \geq j$ is non-connected

Figure 6 shows composed QDFF generation process from two individual QDFFs. In the composed QDFF we can detect two non-connected zones with same associated label.

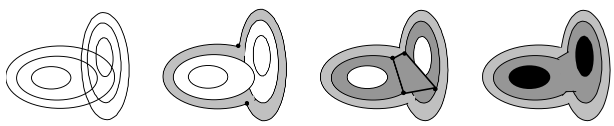


Figure 6. Composed QDFF generation process from two individual QDFFs.

Ellipses Internal or External Points.

From expressions (8) and (9) it can be seen that to generate new ellipses (S_j) that shapes composed QDFF, it is necessary to know which spatial points belong either to individual ellipses (E_j or F_j) or to a generated convex polyhedron (ε_j). In order to classify spatial points as internal to some ellipse or external ones, we propose the following method. Let *R* be a detected obstacle (as a mobile robot) with an associated elliptical QDFF. Let (x_R, y_R) be the estimated position of *R*, and E_j is one of the ellipses that forms the associated QDFF. Then, we can define the descriptive parameters of the ellipse that are shown in the Figure 7.

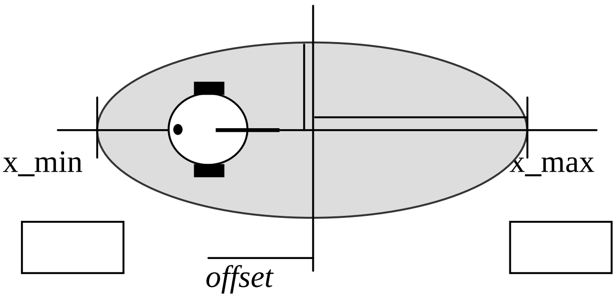


Figure 7. Ellipse descriptive parameters.

As a first deduction we can ensure that any spatial point *(x,y)* referenced to (x_0, y_0) with relative distance *p*, defined as Euclidean distance:

$$p = \sqrt{(x - x_0)^2 + (y - y_0)^2} \quad (10)$$

will be external to the studied ellipse if *((p > x_max) or (p < x_min))*.

Otherwise, no asserts can be done. It is necessary to carry out a more detailed analysis. Figure 8 shows three defined points where it can be noted that if:

$$\begin{aligned} d_{point} &\le d_{ellipse} \;\Rightarrow\; (p,q) \in ellipse \\ d_{point} &> d_{ellipse} \;\Rightarrow\; (p,q) \notin ellipse \end{aligned} \quad (11)$$

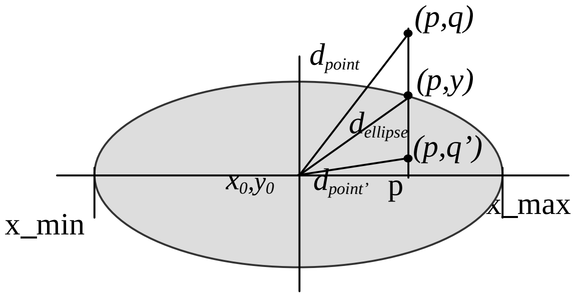


Figure 8. Internal or external point resolution.

Convex Polyhedron's inner or external points.

In order to know if a spatial point belongs to or not belongs to the convex polyhedron generated from the intersection of two ellipses (ej), it will be necessary to be able to construct the mentioned polyhedron. So, it will be indispensable to find the intersection points between the ellipses belonging to two involved individual QDFFs. The problem of finding intersection points between ellipses is a resoluble but complicated question. The problem complexity resolution increases if we want to use a method which takes into account all the possible situations, derived from placing two ellipses in the space. Dave Rusin (rusin@math.niu.edu), presents a method to solve the above-mentioned problem based on the use of quadratic polynomial to describe the ellipses.

Once all the intersection points have been found, we can obtain the convex polyhedron. Nevertheless, the obtained intersection points must be ordered in order to join them in a correct way, otherwise the generated polyhedron can be a non convex polyhedron or a wrong one. In the developed work, we decide to proceed in the following way.

1. Once the intersection points have been obtained, we calculate the center of gravity (x_c, y_c).
2. Taking the center of gravity as the origin of co-ordinates, we will proceed to represent intersection points in polar co-ordinates, and to arrange them according to the angles with respect to X axes.
3. Once the points have been arranged, we will connect each point with their neighbors through a rectilinear segment.
4. In order to be able to classify spatial points as internal or external to the generated polyhedron, it will be necessary to maintain a data structure that stores the information related with the neighboring polyhedron's straight lines.

Figure 9 displays the generated polyhedron after the above mentioned steps.

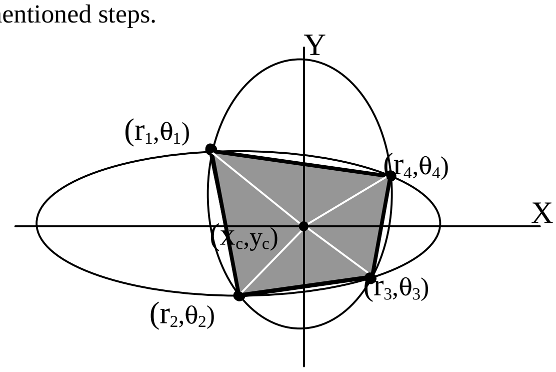


Figure 9. Obtained polyhedron after joining intersection points.

Qualitative Dynamic Force Field (QDFF) evolution.

Working with relative distances and velocities implies that changes either on robot movements or changes on detected obstacles movement, cause variations on the associated QDFF. So, QDFF are variing with time, more over, QDFF can change their position and shape with time.
Let a robot's assigned trajectory be the following sequence of points (12):

$$\left\{ q_{init} = q(t_0), q(t_1), q(t_2), \cdots, q(t_{n-1}), q(t_n) = q_{goal} \right\} \qquad (12)$$

Then, the mobile robot must be positioned at $q(t_i)$ at t_i interval time, with $0 <= i <= n$. QDFFs will follow the trajectory of the objects that they are associated so they are placed on the estimated position of their associated objects. So, we can write:

$$E^i(q(t_i)), E^i(q(t_{i+1})), E^i(q(t_{i+2})), \cdots E^i(q(t_k)) \qquad (13)$$

where $E^i(q(t_i))$ is the QDFF structure assigned to the detected *i*-obstacle at its configuration $q(t_i)$ assigned during trajectory execution. It is important to note that QDFFs appears and disappears dynamically during trajectory execution, They are only generated in order to solve conflictive situations, and then they don't exist during complete trajectory execution. In this way, we can rewrite the expression (3) as:

$$\begin{aligned} E^i(q(t)) &= \left\{ E_1^i(q(t)), E_2^i(q(t)), \cdots, E_k^i(q(t)) \right\} \\ \textit{with} \quad & E_k^i(q(t)) \subset E_{k-1}^i(q(t)) \subset \cdots \subset E_1^i \end{aligned} \qquad (14)$$

Composed QDFFs will become a dynamic and variable structure, as a result to compose dynamic forms. Composed QDFFs will evolve with time, in order to consider the dynamics of the environment. In this way, QDFFs can be used as a mobile robot dynamic path planning tool. Figures 10, 11 and 12 displays in a graphical way, temporal evolution of two individual QDFF as well as the composed QDFF generation process. It can be noticed that as it has been mentioned, composed QDFF modifies its position, its shape and the topology of the zones that form it.

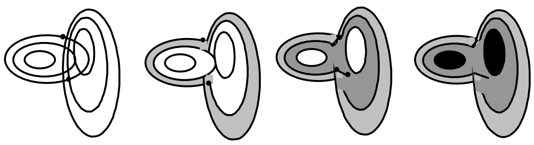


Figure 10. Initial sample time. Composed QDFF corresponding to the initial period of time.

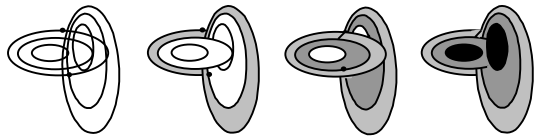


Figure 11. Temporal evolution. Composed QDFF adapting to describe the new situation.

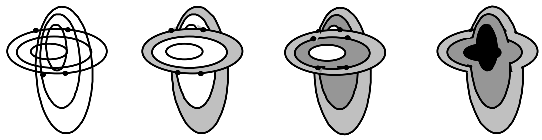


Figure 12. Temporal evolution. Composed QDFF adapting to describe the new situation.

Labeling robot workspace.

In previous sections robot-working space has been discretized in qualitative zones, and DFF has been approached by qualitative methods in order to obtain the QDFFs. Now, it is necessary to carry out a projection of the generated QDFF upon the defined qualitative zones of the workspace with the intention to label them as a safety or risky ones. Later, robot can use this information to realize a safe path planning in a decentralized way.
Each qualitative zone in robot workspace will acquire the risk level inherited to project generated QDFF on it. In a first approach, and in order to simplify the process, each zone will be completely labeled even if QDFF projection covers only a part of the zone. Nevertheless, this approach causes a deformation on QDFF projections, so that results are similar to pixel effects on the original QDFF. Figure 13 shows an original QDFF with three levels of risk, its projection on the qualitative workspace W, and finally the labeling process of the qualitative zones in the workspace W.

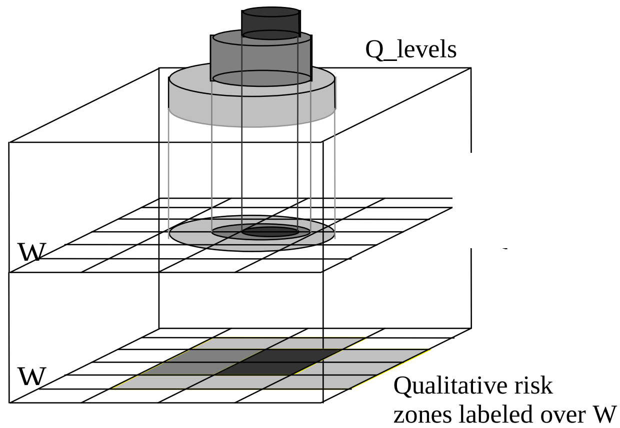


Figure 13 Risk levels of the QDFF, their projection over W and qualitative zones risk labeling process.

Differentiation between projection of the risk levels and labeled risk levels implies a new qualitative variable definition, which we have named Risk. This variable should have, as a universe of description, the set of labels that characterizes the zones of the workspace. In a first problem analysis we can notice that both variables, Risk and Q_levels should be equaled. So, the same labels describe both variables. In this case, it is possible to write (15):

Risk = Q_levels = {0,E1,E2 · ·Ek-2,Ek-1,Ek} (15)

So, a new function of projection of the Risk levels upon qualitative zones from W, can be established.

$$\Lambda_{Risk} : Risk \rightarrow W \qquad (16)$$

Nevertheless, it is possible that two (or more) different projected levels of risk (Q_levels) will cover the same qualitative zone. In this case, and in order to take into account the worst condition, this qualitative zone will be labeled with the higher risk level amongst all of them that projects on it. Previous function (16) will be re-written as,

$$\Lambda_{W(x,y)} = \max \{E_i\} \quad i \in \{m, n\} \qquad (17)$$

where *W(x,y)* is the specific qualitative zone that has its spatial center placed on the co-ordinates (x,y) inside workspace W, and E_i are different levels of risk that projects over *W(x,y)*.
In a second approach we contemplated a different labeling process. So, qualitative zones were labeled taking into account the rate of projection of each QDFF level of risk, which takes place upon them. In this way, it can be established a second function of projection, as it shows the following expression (27)

$$\Gamma_{Risk} : Risk \rightarrow W \qquad (18)$$

In order to carry out the weighing of different QDFF risk levels that project on the same zone, we can proceed as follows.

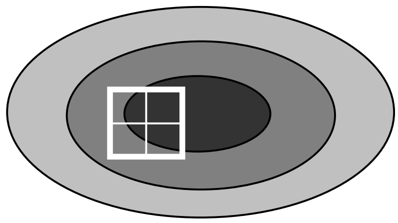
1. It is established a correspondence between labels of Q_levels variable and N^+, so that to each E_i has the associated numeric value *i*.

$$\Psi : risk \rightarrow N^+ \qquad (19)$$

2. Each studied zone *W(x,y)* will be divided in a more fine partition. This fact causes the apparition of *m* sub-zones for each studied zone. Now, the new function of projection will be defined as

$$\Gamma_{W(x,y)} = \max \left\{ \frac{i * z}{m} \right\} \qquad (20)$$

where *i* is the numeric value obtained from Ψ function (19),
z is the number of sub-zones that are projected by the Q-level equivalent to *i* value, and *m* is the number of generated sub-zones due to the new established partition upon *W(x,y)*.
Figures 14.a and 14.b shows in a graphic way, the weighting process described previously. In Figure 14.a and 14.b, it is shown a qualitative zone (white rectangle) and a new partition over it that generates new four sub-zones. Figure shows two possible cases and the obtained labels.

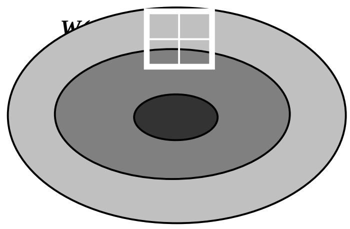


E_i	Ψ
E_1	1
E_2	2
E_3	3

i	*z*	*m*	$\frac{i*z}{m}$
1	0	4	0
2	3	4	1,5
3	4	4	3

$$\Gamma_{W(x,y)} = \max\{0; 1{,}5; 3\} = 3 \;\Rightarrow\; i = 3 \;\Rightarrow\; E_3$$

Figure 14.a. W(x,y) zone will be labeled with a high risk level (E_3).



E_i	Ψ
E_1	1
E_2	2
E_3	3

i	*z*	*p*	$\frac{i*z}{m}$
1	4	4	1
2	2	4	1
3	0	4	0

$$\Gamma_{W(x,y)} = max\{1; 1; 0\} = 1 \;\Rightarrow\; i = 1\, o\, 2 \;\Rightarrow\; E_1 \; o \; E_2$$

Figure 14.b. W(x,y) zone could be labeled indistinctly as a medium (E_2) or low (E_1) risk level.

Applying the described method, Risk and Q-levels variables have the same universe of description. In this way we save a new variable creation.

The obtained results from the Qualitative Dynamic Force Fields (QDFF) formalization and composition allows to extract the robot moving orders, in order to follow a selected global trajectory, conditioned to some cost requirements.

Conclusions

In this paper, a qualitative method for mobile robot path planning has been presented. To obtain the whole description of the qualitative structures, qualitative variables, and their operation in mobile robot workspace have been defined.
In all cases, mobile robot reactive capacity has been pursuit, so methods and procedures with fast computation and easy representation have been hunted. In this way, geometric methods have been more used than analytical ones, although this fact can involve accuracy losses.
The time and computational safety, using developed qualitative structures and path planning, could be more relevant in this qualitative field composed process. The reason of this fact is consequence of using a more reduced number of evaluation points to construct the composed field. The set of points to be evaluated will be dependent of the number of qualitative zones that are being used.
The presented methodology can work as well independently of the number of labels associated to the qualitative variables. Nevertheless as more labels we have, more computation time will be required.

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