# Limitations of the Confluence Model for Circuit Analysis

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## Abstract

Confluences are used in QR as a mean to deal with physical systems where we want to know how the system reacts in the presence of changes. In this paper we derive a confluence-based qualitative model for the star-mesh transformation; this can be done by taking the quantitative model to its correspondent qualitative model using sign algebra and its axioms. Once the qualitative model is derived, we proceed to show that constraint propagation using the confluence-based model is complete in the sense that it produces useful results for reducible Series-Parallel (SP) circuits. We show that propagation becomes incomplete for non-reducible SP circuits yielding ambiguous results.

## 1 Introduction

In electrical circuits, questions like *What would happen with current in element x if element y is allowed to vary?* have always been of interest. These questions have been answered for SP reducible circuits [Flores97], not being able to deal with the more general case. The main characteristic of SP circuits is that its reduction graph is a binary tree. An example of this kind of circuits is shown in figure 1, whose reduction graph is shown next to it; the reduction graph illustrates one way to cluster this circuit.

On the other hand, the clustering graph obtained from the reduction of a non SP circuit is not a tree anymore. Figure 2 shows an example of a non SP reducible circuit and its reduction graph.

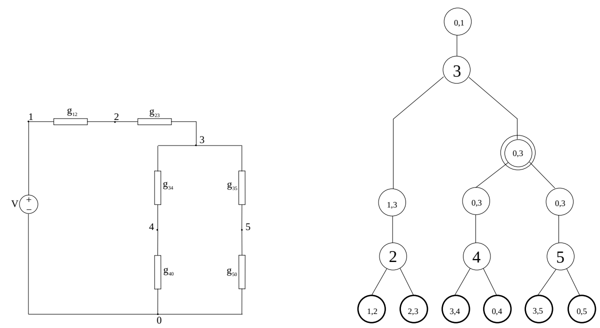

Figure 1: An SP reducible circuit and its correspondent clustering graph.

As a consequence of this, the answer to the question in the first paragraph of this section is not clear; in fact, if we try to answer it without knowing the quantitative values of the circuits elements, we may derive some contradictions. An example of this kind of circuit is the bridge circuit which has been qualified by other authors as a creator of ambiguity in any qualitative model [Lee00]. In this paper we prove that for non SP reducible circuits, constraint propagation using a confluence-based model always yields ambiguous results.

In section 2, we derive a Confluence-based Qualitative Model. Section 3 shows the limits of the confluence model in qualitative circuit analysis. In section 4 we propose some possible ways to perform qualitative analysis of non SP reducible circuits. Finally, in section 5 we conclude this work.

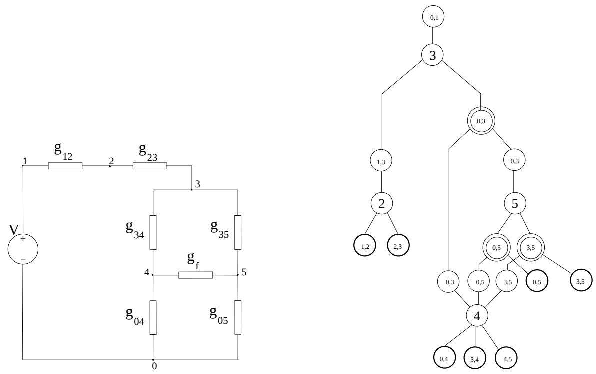


Figure 2: A non SP reducible circuit and its correspondent clustering graph.

## 2 Qualitative Circuit Model

The qualitative model derived in this section is based in the confluence model developed by Johan deKleer [deKleer, deKleer84a]. Qualitative variables can take on values from a small set, as opposed to quantitative variables which can take on values from an infinite set. This set of values is determined by the variable's quantity space. $[.]_Q$ is used to express the qualitative value of the expression inside the parenthesis with respect to the quantity space Q. Each qualitative value corresponds to some interval in the line of the real numbers. These regions are typically disjoint. The quantity's most important property is its qualitative change rate. A sign quantity space, has only three values: $-, 0, +$. In general, the quantity space of $x < a, x = a$, and $x > a$ is denoted as $[.]_a$. So, $[x]_0 = +$ iff $x > 0, [x]_0 = 0$ iff $x = 0$ and $[x]_0 = +$ iff $x < 0$. Through this paper, $[.]_0$ will be written as $[.]$.

The addition and multiplication in the sign algebra, are defined by table 1, where the symbol ? is used to denote an ambiguous qualitative value (not determined).

In this formalism "$x$ is increasing" is denoted by $\left[\frac{dx}{dt}\right] = +$. This notation, tends to get very tedious, so $\partial x$ is used as an abbreviation for $\left[\frac{dx}{dt}\right]$.

Although confluences can be derived from the common sense knowledge of the modeling domain, most of them are an adaptation of the conventional physical models.

| $[X]$ | $[Y]$ | $[X]+[Y]$ | $[X]*[Y]$ |
|---|---|---|---|
| $+$ | $+$ | $+$ | $+$ |
| $+$ | $0$ | $+$ | $0$ |
| $+$ | $-$ | ? | $-$ |
| $0$ | $+$ | $+$ | $0$ |
| $0$ | $0$ | $0$ | $0$ |
| $0$ | $-$ | $-$ | $0$ |
| $-$ | $+$ | ? | $-$ |
| $-$ | $0$ | $-$ | $0$ |
| $-$ | $-$ | $-$ | $+$ |

Table 1: Addition and multiplication operations in the sign algebra.

Generally, a quantitative equation can be transformed into its qualitative version by using the sign algebra axioms, listed in table 2. (Axiom I assumes all quantities are positive, which in this domain represent admittances).

| | | | |
|---|---|---|---|
| **A-I** | $[e_1 + e_2]$ | $=$ | $[e_1] + [e_2]$ |
| **A-II** | $[e_1 e_2]$ | $=$ | $[e_1]\,[e_2]$ |
| **A-III** | $[0] + [e]$ | $=$ | $[e]$ |
| **A-IV** | $[0]\,[e]$ | $=$ | $[0]$ |
| **A-V** | $[+]\,[e]$ | $=$ | $[e]$ |
| **A-VI** | $[-]\,[e]$ | $=$ | $-[e]$ |
| **A-VII** | If $e$ is a constant, or always preserves the same sign, this one substitutes its value (i.e. $[g] = +$) | | |

Table 2: Sign algebra axioms.

Now we will derive a qualitative version of the star-mesh transformation. We define $\Gamma_n$ as the set of nodes connected to node $n$. Now, let $i, j \in \Gamma_n$. Equations 1 and 2 are the expression for the qualitative change rate for the star-mesh transformation. See [Flores00] for details on the qualitative models for the star-mesh transformation.

$$\partial g_{ij} = \left[\frac{dg_{ij}}{dt}\right] = \left[\frac{d}{dt}\left(\frac{g_i g_j}{\sum\limits_{r \in \Gamma_n} g_r}\right)\right] \qquad (1)$$

$$\partial g_{ij} = \left[ \frac{\sum_{r\in\Gamma_n} g_r \left( g_i \frac{dg_j}{dt} + g_j \frac{dg_i}{dt} \right) - g_i g_j \left( \sum_{r\in\Gamma_n} \frac{dg_r}{dt} \right)}{\left( \sum_{r\in\Gamma_n} g_r \right)^2} \right] \quad (2)$$

Regrouping the summation members from 2 and applying **A-I**:

$$\partial g_{ij} = \frac{1}{\left( \sum_{r\in\Gamma_n} g_r \right)^2} \Bigg[ \left( \sum_{\substack{r\in\Gamma_n \\ r\neq j}} g_r g_i \frac{dg_j}{dt} + g_i g_j \frac{dg_j}{dt} \right) + \left( \sum_{\substack{r\in\Gamma_n \\ r\neq i}} g_r g_j \frac{dg_i}{dt} + g_i g_j \frac{dg_i}{dt} \right) - \left( g_i g_j \sum_{\substack{r\in\Gamma_n \\ r\neq i \\ r\neq j}} \frac{dg_r}{dt} + g_i g_j \frac{dg_i}{dt} + g_i g_j \frac{dg_j}{dt} \right) \Bigg] \quad (3)$$

Reducing terms in 3 leads to:

$$\partial g_{ij} = \frac{\left( \sum_{\substack{r\in\Gamma_n \\ r\neq j}} g_r g_i \right) \frac{dg_j}{dt} + \left( \sum_{\substack{r\in\Gamma_n \\ r\neq i}} g_r g_j \right) \frac{dg_i}{dt}}{\left( \sum_{r\in\Gamma_n} g_r \right)^2} - \frac{g_i g_j \sum_{\substack{r\in\Gamma_n \\ r\neq i \\ r\neq j}} \frac{dg_r}{dt}}{\left( \sum_{r\in\Gamma_n} g_r \right)^2} \quad (4)$$

Applying **A-I y A-II** to 4, we get:

$$\partial g_{ij} = \frac{\left[ \sum_{\substack{r\in\Gamma_n \\ r\neq j}} g_r g_i \right] \partial g_j + \left[ \sum_{\substack{r\in\Gamma_n \\ r\neq i}} g_r g_j \right] \partial g_i}{\left[ \left( \sum_{r\in\Gamma_n} g_r \right)^2 \right]} - \frac{[g_i g_j] \left[ \sum_{\substack{r\in\Gamma_n \\ r\neq i \\ r\neq j}} \frac{dg_r}{dt} \right]}{\left[ \left( \sum_{r\in\Gamma_n} g_r \right)^2 \right]} \quad (5)$$

Now, from 5 and the sign algebra axioms, we can deduce

$$\begin{aligned}
[g_i] &= +, && \mathbf{A-VII} \\
[g_j] &= +, && \mathbf{A-VII} \\
[g_i g_j] &= +, && \mathbf{A-II} \\
\left[ \sum_{\substack{r\in\Gamma_n \\ r\neq i}} g_r g_i \right] &= +, && \mathbf{A-I}, \mathbf{A-II} \\
\left[ \sum_{\substack{r\in\Gamma_n \\ r\neq j}} g_r g_j \right] &= +, && \mathbf{A-I}, \mathbf{A-II} \\
\left[ \left( \sum_{r\in\Gamma_n} g_r \right)^2 \right] &= +, && \mathbf{A-I}, \mathbf{A-II}
\end{aligned}$$

so, equation 5 can be written as

$$\partial g_{ij} = \partial g_j + \partial g_i - \sum_{\substack{r\in\Gamma_n \\ r\neq i, r\neq j}} \partial g_r \quad (6)$$

Now, let us assume that element $k$ of the star changes. If $k = i$ then

$$\partial g_j = 0$$

$$\sum_{\substack{r \in \Gamma_n \\ r \neq i, r \neq j}} \partial g_r = 0$$

so equation 6 becomes

$$\partial g_{ij} = \partial g_k$$

If $k = j$ then:

$$\partial g_i = 0$$

$$\sum_{\substack{r \in \Gamma_n \\ r \neq i, r \neq j}} \partial g_r = 0$$

so equation 6 becomes

$$\partial g_{ij} = \partial g_k$$

On the other hand, if $k \neq i, k \neq j$, then

$$\partial g_i = 0$$
$$\partial g_j = 0$$
$$\sum_{\substack{r \in \Gamma_n \\ r \neq i, r \neq j}} \partial g_r = \partial g_k$$

so equation 6 becomes

$$\partial g_{ij} = -\partial g_k$$

Given that, equation 6 can be expressed as:

$$\partial g_{ij} = \begin{cases} \partial g_k & \text{if } k = i \vee k = j \\ \text{-}\partial g_k & \text{otherwise} \end{cases} \tag{7}$$

To illustrate this, let us take figure 3 as reference

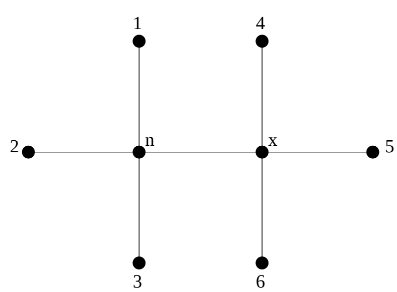


Figure 3: Original Circuit

from this figure we define

$\Gamma_n = \{1, 2, 3, x\}$
$\Gamma_x = \{4, 5, 6, n\}$
$\Psi = \Gamma_x \setminus \{n\} = \{4, 5, 6\}$

We will assume that node $n$ is eliminated before node $x$. And node $x$ is eliminated before nodes $1, 2, 3$.

Let us assume that element $g_{nx}$ varies. Applying equation 6 (eliminating node $n$), figure 3 becomes the circuit shown in figure 4

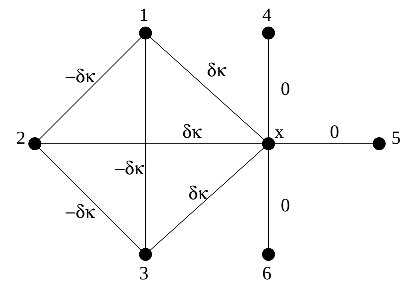


Figure 4: Variation analysis when element $g_{nx}$ varies.

Now, let us assume that element $g_{nk}, k \neq x$ varies. For instance, if $x = 2$, applying equation 6, figure 3 becomes the circuit shown in figure 5.

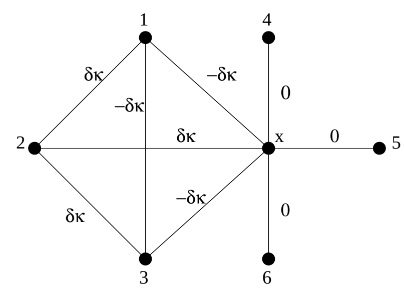


Figure 5: Confluences derivation when element $g_{nk} k \neq x$ variate.

## 3 The Limitations of Confluences

As we can deduce from equation 7, there will be cases where we will not be able to analyze a circuit from the qualitative point of view and this will depend greatly on the topology of the clustering graph.

As a particular case, we have the class of SP reducible circuits, where $k = i$ or $k = j$, so just the first and second cases apply, meaning that the qualitative constraint propagation is complete for this kind of circuits. On the other hand, this equation yields this model to be incomplete for stars composed of three or more elements. This is because it generates multiple changes, with different signs in the same element. Moreover, equation 6 involves all of the star's elements connected to node $n$; so it suffices that one of the elements be ambiguous for the entire expression to become ambiguous. We can characterize an important property of these clustering graphs, which gives us certainty about limits of the confluence-based qualitative reasoning, when applied to non reducible SP circuits.

We will show that in the presence of a variation of one of the star elements associated to node $n$ where $\Gamma_n > 2$, when we apply the star-mesh transformation at node $n$ and, following we apply this transformation to a node $x \in \Gamma_n$, yields nodes $(\Gamma_x \cup \Gamma_n) \setminus \{x, n\}$, to an ambiguous configuration, because they have at least one ambiguous element.

**Proposition 1** *If in the process of qualitative constraint propagation we find a star-mesh transformation associated to the node $n$ elimination, where $|\Gamma_n| > 2$, the qualitative constraint propagation yields ambiguous results for nodes $x \in \Gamma_n$.*

***Proof.***

Without loose of generality, we will use the example in figure 3 to develop the proof. Let us assume that we have eliminated node $n$, and the next node to be eliminated is $x \in \Gamma_n$. The star at node $x$ is formed by the nodes $\Gamma_x = \Gamma_n \setminus \{x\} \cup \Psi$, where $\Psi$ is the set of elements which are not connected to nodes $\Gamma_n$. From a topological point of view, figure 3 becomes a complete graph with nodes $\Phi = (\Gamma_n \cup \Gamma_x) \setminus \{x, n\}$ As shown in figure 6.

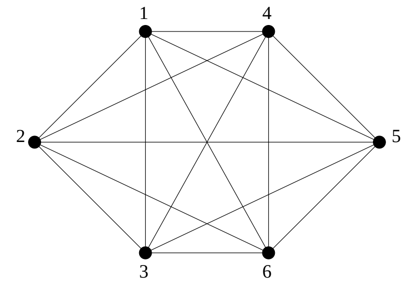


Figure 6: Complete graph after eliminating nodes $n$ and $x$

From equation 6, we note that it is enough to have an element with an ambiguous value (?), for the entire expression to become ambiguous. So all we need to show is that at least one of the elements of each node in figure 6 has an ambiguous value. Formally: $\forall i \in \Phi, \exists j \in \Phi \setminus \{i\} : \partial g_{ij} = ?$

We will consider two cases, the first one where $x = k$, the node where the changing element was connected and the other one where $x \neq k$. 1. In this case, we have $x = k$, and we proceed to analyze elements $g_{ij}, i \in \Psi, j \in \Gamma_n \setminus \{k\}$, as shown in figure 7. The confluence expression is:

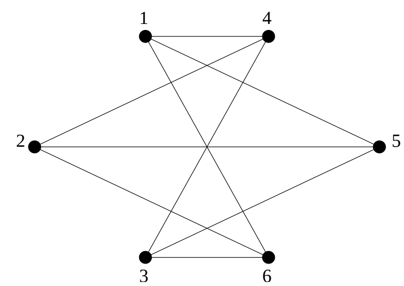


Figure 7: Elements analyzed in case 1

$$\begin{aligned} \partial g_{ij} &= 0 + \partial g_k \\ &\quad - [(\partial g_k + \partial g_k + \ldots) - (0 + 0 + \ldots)] \\ \partial g_{ij} &= \partial g_k - \partial g_k \\ \partial g_{ij} &= ? \end{aligned}$$

2. In this case, $x \neq k$, here we will analyze two cases

2.a. Here, we have $g_{ij}, i \in \Psi, j \in \Gamma_n \setminus \{k\}$, as shown in figure 8. The confluences expression will be

$$\begin{aligned} \partial g_{ij} &= 0 - \partial g_k - \\ &\quad [(\partial g_k - \partial g_k - \ldots) - (0 + 0 + \ldots)] \\ \partial g_{ij} &= -\partial g_k - [(\partial g_k - \partial g_k)] \\ \partial g_{ij} &= -\partial g_k - ? \\ \partial g_{ij} &= ? \end{aligned}$$

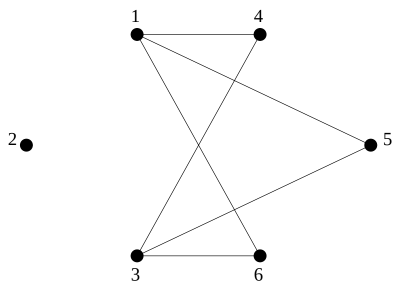


Figure 8: Analized elements in case 2.a

2.b. I this case, we have $g_{ij}, i = k, j \in \Gamma_n \setminus \{k\}$ as shown in figure 9. The confluence expression will be

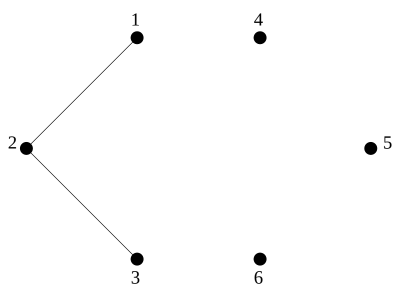


Figure 9: Analyzed elements in case 2.b

$$\partial g_{ij} = \partial g_k - \partial g_k$$
$$-[(-\partial g_k - \partial g_k - \ldots) - (0 + 0 + \ldots)]$$
$$\partial g_{ij} = \partial g_k - \partial g_k - [-\partial g_k]$$
$$\partial g_{ij} = \partial g_k - \partial g_k + \partial g_k$$
$$\partial g_{ij} = \partial g_k - \partial g_k$$
$$\partial g_{ij} = ?$$

Both cases 1 and 2 involve nodes $i \in \Phi$, so we guarantee that at least one of the elements connected to each one of them will be ambiguous, this in turn will make the qualitative constraint propagation become ambiguous. □

# 4 Discussion

In the preceding sections we have shown that the confluence model is not useful in qualitative analysis of electrical circuits that are not series-parallel reducible. Now the question is what remains to be done. We are proposing two approaches to overcome this problem: the first one is to produce a quantitative model of the change rates, and perform propagation, using those constraints; the second idea is to use Order of Magnitude Reasoning to disambiguate the propagation of qualitative values. The first idea will be presented here, the second one is left as future work, or as an exercise to the adventurous reader.

## 4.1 Quantitative Models

In the first idea, we can use equation 2 for each star-mesh transformation. In performing constraint propagation, we can produce results as accurate as the available information allows. Using exact real numbers, of course there will be no ambiguity, and all results will also be real. As an example, we will use a simple star-mesh conversion where a node of degree three is eliminated (see Fig. 10).

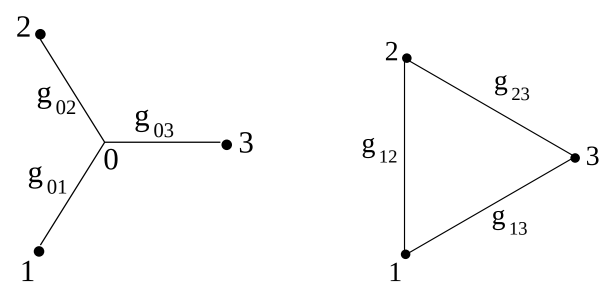


Figure 10: Quantitative example. Application of equation 2.

In that example we can assume the following values for admittances and changes in them: $g_1 = 1, g_2 = 2, g_3 = 3, \frac{dg_1}{dt} = 0.5, \frac{dg_2}{dt} = 0, \frac{dg_3}{dt} = 0$. Using those values, we can compute the corresponding delta or mesh elements $g_{12} = 1/3, g_{13} = 1/2$, and $g_{23} = 1$, and the following change rates:

$$\partial g_{12} = \frac{\sum_{r\in\Gamma_n} g_r \left( g_1 \frac{dg_2}{dt} + g_2 \frac{dg_1}{dt} \right) - g_1 g_2 \left( \sum_{r\in\Gamma_n} \frac{dg_r}{dt} \right)}{\left( \sum_{r\in\Gamma_n} g_r \right)^2}$$

$$\partial g_{12} = \frac{5}{36}$$

Similarly,

$$\partial g_{13} = \frac{7.5}{36}$$
$$\partial g_{23} = -\frac{3}{36}$$

Using intervals to represent incomplete or uncertain information, we can see the rate changes at different granularities, ranging from reals to qualitative values. For instance, for the example of Figure 10 (same as in the previous paragraph), we could use the following values for admittances and their derivatives: $g_1 = [0.5, 3]$, $g_2 = [1, 2]$, $g_3 = [2.5, 6]$, $\frac{dg_1}{dt}$=[0.1, 0.3], $\frac{dg_2}{dt}$=0, $\frac{dg_3}{dt}$=0. Applying the same constraints, and using interval algebra, constraint propagation yields the following results: $\frac{dg_{12}}{dt} = [-0.0875, 0.4093]$, $\frac{dg_{13}}{dt} = [-0.2750, 1.2296]$, $\frac{dg_{23}}{dt} = [-0.2250, -0.0020]$.

The good idea about intervals is that the results can be refined as new, more precise, information arrives. In one extreme, we have real numbers, expressed as point intervals, where ambiguity is never present. On the other extreme, when all intervals are [-INFINITY, 0], 0, or [0, INFINITY], equivalently to -, 0, and +, respectively, propagation yields the same ambiguous results as with the qualitative model.

# 5 Conclusions

In this paper we developed a qualitative model based on confluences, by means of which one would expect to be able to draw inferences about the circuit behavior in the presence of changes in some of its elements.

Experience has shown that the result of the propagation process became ambiguous in many cases. That fact lead us to prove that in fact it will always become ambiguous for star-mesh reductions involving nodes of cardinality greater than two. That proof reveals the limits of the confluence model as applied to circuit analysis.

To solve this problem, we have used a quantitative model, which used along with an interval representation for numbers, allows us to still get useful information from the circuit analysis. Besides, using intervals allows us to perform circuit analysis at different levels of granularity.

More work needs to be done to find other solutions; for instance, reasoning about orders of magnitude of elements and changes, or perhaps with richer sets in the quantity spaces for the change rates.

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