Qualitative Representation of Kinematic Robots

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Abstract

This paper proposes a qualitative representation for kinematic robots. First, qualitative geometric primitives are introduced by combining a qualitative orientation component and qualitative translation component using normalisation. A position in Cartesian space can be mathematically described by the scalable primitives. Secondly, the qualitative positions of the components of a robot are derived in terms of qualitative geometry primitives. Thirdly, the representation shows how to connect both quantitative and qualitative representation of the robot. On the one hand, the integration of normalisation and domain theory generates normalised labels to introduce the cognitive parameters into the proposed representation. On the other hand, the normalized labels of this representation can be converted to a quantitative description using a proposed generator, whose numeric outputs provide a connection to numeric techniques (e.g. interpolation).

1 Introduction

There exists an interesting gap between traditional robotics and cognitive robotics, or robot motion and human perception. The reason is two-fold. On the one hand, research in robotics has traditionally emphasized low-level sensing and control tasks including sensor processing, path planning and robot design and control; on the other hand, research in cognitive robotics is concerned with endowing robots and software agents with higher level cognitive functions that enable them to reason, act and perceive in changing, incompletely known, and unpredictable environments. This gap is one of crucial issues for interdisiplinary research in the engineering community, robotics community and AI community. It emphasises the goal of robotics research that "robotics is the intelligent connection of perception to action" [Brady, 1985].

Research on qualitative reasoning & model-based technology can be found in [Weld and de Kleer, 1990; Williams and de Kleer, 1991; Faltings and Struss, 1992; Bredeweg and Struss, 2003]. Generally speaking, there are two approaches to qualitative spatial representations [Forbus, 1996; Blackwell, 1988]. One is to explore what aspects do lend themselves to qualitative representation, the other is to use a quantitative representation as a starting point and compute problem-specific qualitative representations to reason with. Cohn and Hazarika [Cohn and Hazarika, 2001] gave sufficient overview of qualitative spatial representation and reasoning techniques by investigating the main aspects of the representation of qualitative knowledge including ontological aspects, distance, orientation and shape, and qualitative spatial reasoning including reasoning about spatial change. The representation of qualitative kinematics is the best developed field in qualitative spatial representation. Its history can be covered by the following research work. Firstly, The possible motions of objects are represented by qualitative regions in configuration space representing the legitimate positions of parts of mechanisms [Faltings, 1992]. Faltings built upon Nielsen and Forbus' earlier work on qualitative kinematics [Nielsen, 1988], and developed a first principles algorithm for analyzing planar mechanisms. However, this work suffered from the limitation that certain problems could not be solved without including quantitative information. Secondly, Olivier *et al* proposed a qualitative kinematic reasoning method based upon the use of occupancy arrays [Olivier *et al.*, 1995]. This approach works simply on the constraint that no two objects occupy the same occupancy array position and can be extended to include semi-quantitative information. Thirdly, Kramer [Kramer, 1992] proposed 'The Linkage Assistant' kinematic simulator which demonstrated that mechanism kinematic analysis did not solely have to rely on exact geometric mechanism information. Fourthly, Liu [Liu, 1998] presented a qualitative representation and reasoning approach based upon the formalism of qualitative trigonometry, qualitative arithmetic, and qualitative spatial inference. However, developing a general approach to the representation of qualitative kinematics is still an open problem. This study aims to develop a general qualitative representation for kinematic robots, the approach also can be extended to general mechanisms.

The rest of this paper is organized as follows: Section 2 presents qualitative geometric primitives in Cartesian space. Section 3 derives a qualitative representation for qualitative robot kinematics. Section 4 addresses how the representation connect both qualitative states and robot motion. Section 5 concludes this paper.

2 Qualitative geometric primitives

The degrees of freedom of a robotic system can be simply viewed as the number of coordinates that it takes to uniquely specify the position of the system. Consider a rigid block $\mathcal{A}$ that is free to move in a two dimensional plane. Its motion along its two degrees of translation and around its one degree of orientation can be described in terms of its degrees of freedom, whatever coordinates are used to describe its position. The position representation of a system consists of two components: translation and orientation components. The position of the block $\mathcal{A}$ is denoted by $\mathcal{A}_p(C_t,\ C_o)$ in general coordinates, where C_t stands for translation component, C_o for orientation component. The formula $\mathcal{A}_p(C_t,\ C_o)$ can be used to describe the position in quantitative or qualitative terms. Its quantitative representation is $\mathcal{A}_p(p_l,\ p_\theta)$, while its qualitative representation is given by $\mathcal{A}_p(qp_l,\ qp_\theta)$. In order to connect robot motion and perception, we have to define the mathematical description of $\mathcal{A}_p(C_t,\ C_o)$ for qualitative analysis.

Further, let us consider the facts. First, qualitative knowledge is relative knowledge where the reference entity is a single value rather than a whole set of categories. Secondly, qualitative knowledge is obtained by comparing features within the object domain. Hence, the qualitative position description is mapped into a unit circle using normalisation technique, the position on its radius is given by a scalable rectangle, the scalable description is shown in Figures 1 and 2. Hence the general representation of the block $\mathcal{A}$ can be

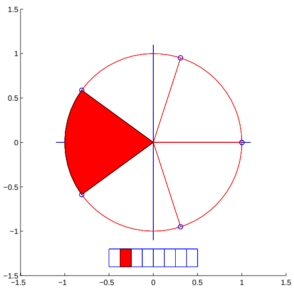

Figure 1: The separate qualitative description of $QS(2,\ 3)$

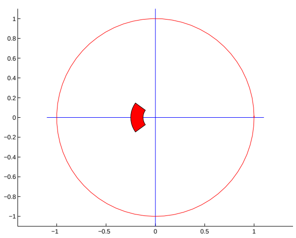

Figure 2: The integrate qualitative description of $QS(2,\ 3)$

given by $\mathcal{A}_p(C_t(s),\ C_o(r))$, where the $s,\ r$ are defined as the mapping parameters over quantization. As $s \to \infty$ and $r \to \infty$, the limits of $C_t(s)$ and $C_o(r)$ in Equation 1 are approaching the set of real numbers $\mathcal{R}$, that is, quantitative description $\mathcal{A}_p(p_l, p_\theta)$.

$$\begin{aligned} \lim_{s\to\infty} C_t\,(s) &= QS(p_l) \\ \lim_{r\to\infty} C_o\,(r) &= QS(p_\theta) \end{aligned} \qquad (1)$$

On the other hand, as $s \to s_0$ and $r \to r_0$, $QS(qp_l)$ and $QS(qp_\theta)$ in Equation 2 are the quantity spaces of translation and orientation components in qualitative terms, respectively. It should be noted that the mapping parameters $s,\ r$ are independent, though the items of quantity spaces are application-dependent. For example, s_0 is set as 8 and r_0 is set as 5 in Figure 1, its qualitative position $QS(2,3)$ is shown in Figure 2.

$$\begin{aligned} \lim_{s\to s_0} C_t\,(s) &= QS(qp_l) \\ \lim_{r\to r_0} C_o\,(r) &= QS(qp_\theta) \end{aligned} \qquad (2)$$

3 Qualitative Robot Kinematics

3.1 Qualitative robotic primitives

Roughly speaking, there are two types of robotic primitives: revolute and prismatic joints, see Figures 3 and 4. They construct a variety of robotic systems. For the former, l_0 denotes the length of its rigid link, θ denotes its orientation variable; for the latter, l denotes its translational variable, θ_0 denotes its starting anglar state. Note that the symbols $[qualitative\ states]$ are described by quantity spaces here. The qualitative representation of the end effector of a revo-

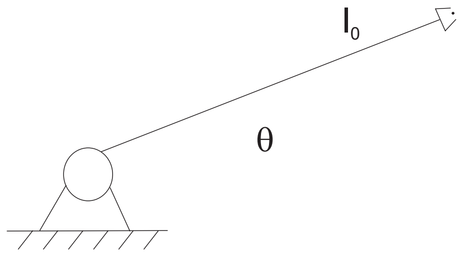

Figure 3: A revolute robotic primitive

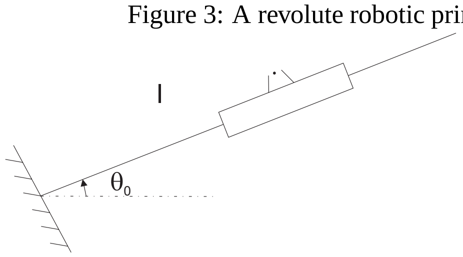

Figure 4: A prismatic robotic primtive

lute primitive in Figure 3 is given in Equation 3. The translation component is a one-item quantity space $[l_0]$, the quantity space of the orientation component $QS(qp_\theta)$ is on the closed range $[0\ 2\pi]$.

$$\left\{ \begin{array}{c} qp_l|qp_l \in [l_0] \\ qp_\theta|qp_\theta \in [0, 2\pi] \end{array} \right. \qquad (3)$$

The qualitative representation of a prismatic primitive shown in Figure 4 is given in Equation 4. The quantity space of its translation component $QS(qp_l)$ belongs to the closed range $[0\ l]$, its $QS(qp_\theta)$ is a one-item quantity space, $[\theta_0]$.

$$\left\{ \begin{array}{c} qp_l|qp_l \in [0, l] \\ qp_\theta|qp_\theta \in [\theta_0] \end{array} \right. \qquad (4)$$

The general qualitative representation of hybrid joint robots shown in Figures 5, 6 is given in Equation 5. The difference from those single primitives in Figures 3 and 4 is that a constraint function C_{dof} is introduced to confine the order of degrees of freedom of a robot from its base. The robotic structures in Figures 5 and 6 are distinguished by value assignment of their C_{dof}, whose entries, qp_θ, qp_l in Fig 5 are assigned to 1 and 2, those in Figure 6 are assigned the other way around.

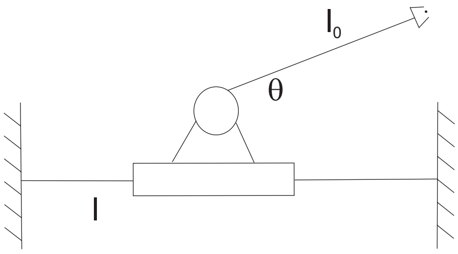

Figure 5: A hybrid joint robot (1)

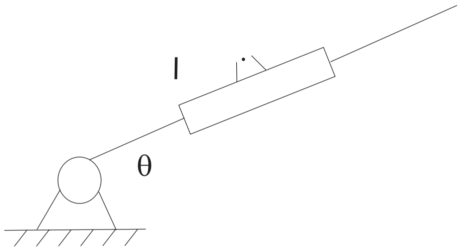

Figure 6: A hybrid joint robot (2)

$$\begin{cases} qp_l | qp_l \in [0, l_0] \\ qp_\theta | qp_\theta \in [0, 2\pi] \\ C_{dof} = \{qp_l, qp_\theta\} \end{cases} \tag{5}$$

Furthermore, the quantity spaces, $QS(qp_l)$ and $QS(qp_\theta)$, can be defined in many terms such as ordinal, interval, ratio scale. Equation 2 can be rewritten in terms of the intervals of the length and orientation angle of a link segment. In this paper the quantity spaces are given by average division by mapping parameters in the following,

$$\begin{cases} QS(qp_l) = \left[\frac{l_1}{l}, \frac{l_2}{l}, , \cdots, \frac{l_{s-1}}{l}, 1\right] \\ QS(qp_\theta) = \left[\frac{q\theta_1}{2\pi}, \frac{q\theta_2}{2\pi}, \cdots, \frac{q\theta_{r-1}}{2\pi}, 1\right] \end{cases} \tag{6}$$

where

$$0 \leq \frac{l_1}{l} \leq \frac{l_2}{l} \leq \cdots \leq \frac{l_{s-1}}{l} \leq 1$$
$$0 \leq \frac{q\theta_1}{2\pi} \leq \frac{q\theta_2}{2\pi} \leq \cdots \leq \frac{q\theta_{r-1}}{2\pi} \leq 1$$

For example, a three-link planar robot, its qualitative representation can be found in Figure 7 given s & r as 20 and 19, where the ellipse is the trajectory of the end-effector, the inside continuous black areas are the qualitative description.

$$\begin{cases} qp_l | qp_l \in \left[\frac{l_1}{l}, \frac{l_2}{l}, , \cdots, \frac{l_{s-1}}{l}, 1\right] \\ qp_\theta | qp_\theta \in \left[\frac{q\theta_1}{2\pi}, \frac{q\theta_2}{2\pi}, \cdots, \frac{q\theta_{r-1}}{2\pi}, 1\right] \\ C_{dof} = \{qp_\theta = 1, \; qp_l = 2\} \end{cases} . \tag{7}$$

3.2 Qualitative representation of kinematic robots

The components of a robot are described at a crude but important level by just two attributes: their position and their

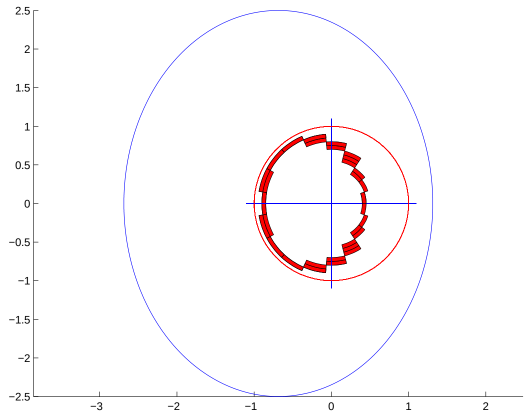

Figure 7: The qualitative representation of the end-effector of a three-link planar robot ($s = 20, r = 19$)

orientation. The aim of robot qualitative representation is the manner in which we qualitatively represent these quantities and manipulate them mathematically. Consider the quantitative description of a n-link serial revolute robot combined by links and joints shown in equation 8 in Cartesian coordinates. Let the length and absolute joint angle of the ith link component be l_i and θ_i, respectively.

$$P(\Theta) = \begin{bmatrix} \sum\limits_{i=1}^{n} p_x^i(\theta) \\ \sum\limits_{i=1}^{n} p_y^i(\theta) \end{bmatrix} = \begin{bmatrix} \sum\limits_{i=1}^{n} l_i \cos\theta_i \\ \sum\limits_{i=1}^{n} l_i \sin\theta_i \end{bmatrix} \tag{8}$$

Where $p_x^i(\theta)$, $p_y^i(\theta)$ are quantitative description of each component in its local coordinates, $P(\Theta)$ is that of the end effector in global coordinates. Compared with quantitative representation, the qualitative description of a robot needs both two types of coordinates to describe its local and global behaviours. The robot is decomposed into n link-based segments, each of which can be described in its local coordinate system by robot primitives, the position and orientation of its end effector can be qualitatively described in global reference coordinates. The local representation of the ith link can be described by $\mathcal{A}_p^i(C_t(s_i)), \; C_o(r_i))$ as follows,

$$\begin{cases} qp_l^i | qp_l^i \in \left[qp_1^i, qp_2^i, \cdots, qp_{s_i-1}^i, l_i\right] \\ qp_\theta^i | qp_\theta^i \in \left[q\theta_1^i, q\theta_2^i, \cdots, q\theta_{r_i-1}^i, 2\pi\right] \end{cases} \tag{9}$$

where

$$qp_j^i = \frac{l_i j}{s_i}, \quad q\theta_j^i = \frac{2\pi j}{r_i}$$
$$0 \leq qp_1^i \leq qp_2^i \leq \cdots \leq qp_{(s_i-1)}^i \leq l_i$$
$$0 \leq q\theta_1^i \leq q\theta_2^i \leq \cdots \leq q\theta_{(r_i-1)}^i \leq 2\pi$$

The qualitative position of the free end of the ith component can be described by a pair of qualitative position and qualitative orientation. Mapping the representation of the ith segment into a unit circle using normalisation in its local coordinates, equation 9 can be rewritten as,

$$\left\{ \begin{array}{c} qp_l^i \left| qp_l^i \in \left[\frac{qp_1^i}{\sum\limits_{i=1}^{n} l_i}, \frac{qp_2^i}{\sum\limits_{i=1}^{n} l_i}, \cdots, \frac{qp_{(s_i-1)}^i}{\sum\limits_{i=1}^{n} l_i}, \frac{l_i}{\sum\limits_{i=1}^{n} l_i} \right] \right. \\ qp_\theta^i \left| qp_\theta^i \in \left[\frac{q\theta_1^i}{2\pi}, \frac{q\theta_2^i}{2\pi}, \cdots, \frac{q\theta_{(r_i-1)}^i}{2\pi}, 1 \right] \right. \end{array} \right. \tag{10}$$

The representation of the end effector in global coordinates is key part in robotics. The qualitative representation of the end effector can be derived in the following,

$$\left\{ \begin{array}{c} qp_l = \overset{n}{\underset{i=1}{\oplus}} qp_l^i \left| qp_l^i \in UC_{ql} \right. \\ qp_\theta = \overset{n}{\underset{i=1}{\oplus}} qp_\theta^i \left| qp_\theta^i \in UC_{q\theta_i} \right. \\ C_{dof} = UC_{dof} \end{array} \right. \tag{11}$$

where

$$\begin{array}{c} UC_{qp_l} = \left[\frac{qp_1^i}{\sum\limits_{i=1}^{n} l_i}, \frac{qp_2^i}{\sum\limits_{i=1}^{n} l_i}, \cdots, \frac{qp_{(s_i-1)}^i}{\sum\limits_{i=1}^{n} l_i}, \frac{l_i}{\sum\limits_{i=1}^{n} l_i} \right] \\ UC_{qp_\theta^i} = \left[\frac{q\theta_1^i}{2\pi}, \frac{q\theta_2^i}{2\pi}, \cdots, \frac{q\theta_{(r_i-1)}^i}{2\pi}, 1 \right] \\ UC_{dof} = [qp_\theta^i = i], \ i \in (0, \ 1, \cdots, \ n) \end{array}.$$

Here $UC_{qp_l^i}$ stands for the translation component of the ith link segment in a unit circle, $UC_{qp_\theta^i}$ for the orientation component. The constraint function C_{dof} is employed to define the degrees of freedom constraints between components. $qp_\theta^0 = 0$ stands for the base of the robot when i is equal to zero. The qualitative representation of the end effector is a qualitative addition of translation and orientation components of each link segment based on its constraints on degrees of freedom. The role of qualitative addition can be played by a variety of qualitative techniques. For example, fuzzy arithmetic can be employed given the components are described by fuzzy numbers.

3.3 Description of the change of qualitative states

In terms of the representation of robot qualitative position, $\Delta\mathcal{A}(\Delta C_t(s), \Delta C_o(r))$ is used to denote the change of qualitative states, which consists of two components for the change of the translation and orientation. The state change, $\Delta C_t(s_i), \Delta C_o(r_i)$ of the ith link segment from time instant t to t' are given as follows,

$$\Delta C_t(s_i) = sign\left(qp_l^i(t') - qp_l^i(t)\right) = \left\{ \begin{array}{ll} + & \Delta qp_l^i > 0 \\ 0 & \Delta qp_l^i = 0 \\ - & \Delta qp_l^i < 0 \end{array} \right.$$

$$\Delta C_o(r_i) = sign\left(qp_\theta^i(t') - qp_\theta^i(t)\right) = \left\{ \begin{array}{ll} + & \Delta qp_\theta^i > 0 \\ 0 & \Delta qp_\theta^i = 0 \\ - & \Delta qp_\theta^i < 0 \end{array} \right. \tag{12}$$

The state change of an end effector, $\Delta\mathcal{A}_p(\Delta C_t(s), \Delta C_o(r))$, can be derived based on equation 11,

$$\begin{array}{c} \Delta C_t(s) = sign\left(\overset{n}{\underset{i=1}{\oplus}} \Delta qp_l^i(t') - \overset{n}{\underset{i=1}{\oplus}} \Delta qp_l^i(t) \right) \\ \Delta C_o(r) = sign\left(\overset{n}{\underset{i=1}{\oplus}} \Delta qp_\theta^i(t') - \overset{n}{\underset{i=1}{\oplus}} \Delta qp_\theta^i(t) \right) \end{array} \tag{13}$$

Generally speaking, it is impossible to compare two symbolic labels in different representation scales. For instance, if there are two quantity spaces $[lowest, lower, medium, faster, fastest]$ and $[lower, medium, fast]$ for qualitative descriptions of their state change, no one can tell the label $fastest$ from the former quantity space whether or not change quicker than the label $fast$ in the latter. The reason is that there is no reference standards for the labels that are used to reflect the perception, without which there is no way to carry out the qualitative arithmetic. That is a crucial problems in AI research, sharing labels across subsystems. The power of the proposed approach is the introduction of normalised labels as the reference for relationship construction of robotic link segments. On the one hand, the items of $UC_{qp_l^i}$ and $UC_{qp_\theta^i}$ in equation 11 correspond to the normalised symbols of the ith link segment. On the other hand, they also have relative quantitative description of knowledge features with a unit circle.

4 Qualitative states and robot motion

This section shows how the proposed approach connects to qualitative states and their motion.

4.1 Qualitative states generation

Almost all reasoning systems are based on symbols, so it is crucial to generate scalable symbols, which can properly reflect their cognitive meaning. The adjustable primitive components which robot cognition will operate on are normalised labels. The qualitative workspace of a link segment in equation 10 can be mathematically described in a matrix $\mathcal{W}_q^i$,

$$\mathcal{W}_q^i = \left[\begin{array}{ccc} \left(\frac{qp_1^i}{\sum\limits_{i=1}^{n} l_i}, \frac{q\theta_1^i}{2\pi} \right) & \cdots & \left(\frac{qp_{r_i}^i}{\sum\limits_{i=1}^{n} l_i}, \frac{q\theta_1^i}{2\pi} \right) \\ \vdots & \vdots & \vdots \\ \left(\frac{qp_1^i}{\sum\limits_{i=1}^{n} l_i}, \frac{q\theta_{s_i}^i}{2\pi} \right) & \cdots & \left(\frac{qp_{r_i}^i}{\sum\limits_{i=1}^{n} l_i}, \frac{q\theta_{s_i}^i}{2\pi} \right) \end{array} \right]$$

And the qualitative workspace of its end-effector can be roughly derived by the union of qualitative workspaces of link segments, the union operation can be taken by a variety of reasoning techniques, (e.g., interval computation).

$$\mathcal{W}_q = \bigcup_{i=1}^{n} \mathcal{W}_q^i \tag{14}$$

Each entry of the $\mathcal{W}_q^i$ is comprised of qualitative states of the orientation and translation of the ith link segment, which actually normalised symbols. The dimension of the envisionment of the end-effector of a n-link robot, $\mathcal{E}^d$, is given by the following,

$$\mathcal{E}^d = \prod_{i=1}^{n} (r_i \times s_i). \tag{15}$$

An example of a 2-link planar robot shown in Figure 8 is presented to calculate its end-effector qualitative position using a toolbox *Xtrig*. The *XTrig* is 4-tuple fuzzy number based implementation of fuzzy qualitative trigonometry (FQT) [Liu and Coghill, 2004]. The fuzzy qualitative trigonometry is the trigonometry of fuzzy qualitative version . Let the compo-

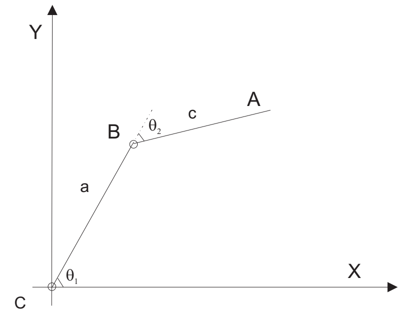

Figure 8: A 2-link robot

nents of links 1 and 2 have the same mapping indexes, (i.e., $r_1 = r_2 = 16$ & $s_1 = s_2 = 21$), we have,

$$UC_{ql_i} = \left[\tfrac{1}{21}, \tfrac{2}{21}, \cdots, \tfrac{20}{21}, 1\right]^T$$
$$UC_{q\theta_i} = \left[\tfrac{1}{16}, \tfrac{2}{16}, \cdots, \tfrac{15}{16}, 1\right]$$

and their qualitative workspace $\mathcal{W}_q^i$, where $i = 1, 2$,

$$\mathcal{W}_q^i = \begin{bmatrix} \left(\frac{1}{21}, \frac{1}{16}\right) & \left(\frac{1}{21}, \frac{2}{16}\right) & \cdots & \left(\frac{1}{21}, \frac{15}{16}\right) & \left(\frac{1}{21}, 1\right) \\ \left(\frac{2}{21}, \frac{1}{16}\right) & \left(\frac{2}{21}, \frac{2}{16}\right) & \cdots & \left(\frac{2}{21}, \frac{15}{16}\right) & \left(\frac{2}{21}, 1\right) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \left(\frac{20}{21}, \frac{1}{16}\right) & \left(\frac{20}{21}, \frac{2}{16}\right) & \cdots & \left(\frac{20}{21}, \frac{15}{16}\right) & \left(\frac{20}{21}, 1\right) \\ \left(1, \frac{1}{16}\right) & \left(1, \frac{2}{16}\right) & \cdots & \left(1, \frac{15}{16}\right) & (1, 1) \end{bmatrix}$$

Given the scenario of position side a, side c and angle $\angle ABC$ (b is the distance between A and C), we have their fuzzy descriptions generated by the toolbox *XTrig*,

$$P_a\left(QS_a\left(\tfrac{3}{16}\right)\right) = \begin{bmatrix} 0.1263 & 0.1789 & 0.0105 & 0.0105 \end{bmatrix}$$
$$P_{\theta_1}\left(QS_d\left(\tfrac{5}{21}\right)\right) = \begin{bmatrix} 0.4068 & 0.4915 & 0.0169 & 0.0169 \end{bmatrix}$$
$$P_c\left(QS_a\left(\tfrac{4}{16}\right)\right) = \begin{bmatrix} 0.5085 & 0.5932 & 0.0169 & 0.0169 \end{bmatrix}$$
$$P_{\angle ABC}\left(QS_a\left(\tfrac{6}{16}\right)\right) = \begin{bmatrix} 0.3158 & 0.3684 & 0.0105 & 0.0105 \end{bmatrix}$$

Where $QS_d()$ and $QS_a()$ are qualitative length and angle.

Applying FQT SAS and AAA theorems and arcsin function [Liu and Coghill, 2004], the position of the end-effector, $P\left(QS_a\left(\angle ACX\right), QS_d\left(\bar{AC}\right)\right)$, are given in the following and Table1,

$$\begin{bmatrix} QS_d(\frac{20}{21}) \\ QS_d(1) \end{bmatrix} = \begin{bmatrix} 0.8136 & 0.8983 & 0.0169 & 0.0169 \\ 0.9153 & 1.000 & 0.0169 & 0 \end{bmatrix}$$

$$\begin{bmatrix} QS_a(\frac{1}{16}) \\ QS_a(\frac{2}{16}) \end{bmatrix} = \begin{bmatrix} 0 & 0.0526 & 0 & 0.0105 \\ 0.0632 & 0.1158 & 0.0105 & 0.0105 \end{bmatrix}$$

4.2 Connection to robot motion

Section 4.1 presents the generation of normalised qualitative states of the proposed robotic representation. A robotic behaviour is a sequence of qualitative states occurring over a particular span of time, e.g., a behaviour $QS(1) \rightarrow QS(2) \rightarrow QS(3) \rightarrow QS(4)$, see Figure 9. In order to connect to robot motion, qualitative states have to be described in quantitative terms. For this purpose some methods have been developed to extract qualitative state from a set of quantitative data, and vice versa, such as particle filter [Verma *et al.*, 2003], quantised modelling [Schroder, 2003]. It should be noted that nearly all motion planning algorithms suffer intractable computational complexity when they convert the positions of robots and obstacles to its configuration space [Latombe, 1991]. And we propose a normalised symbol-based algorithm, which not only connects qualitative states to robot motion, but also overcomes the problem of computational complexity. This method only converts mapping indexes of normalised symbols of a robot into a configuration space rather than positions in Cartesian space. That is, a configuration is expressed as a vector of qualitative position/orientation parameters. Then it generates its behaviour plan and converts it back to un-normalised parameters.

Table 1: Qualitative position description of the end-effector

	QS_1	QS_2	QS_3	QS_4
$QS_a\,(\angle ACX)$	$QS_a(\frac{1}{16})$	$QS_a(\frac{1}{16})$	$QS_a(\frac{2}{16})$	$QS_a(\frac{2}{16})$
$QS_d\,(b)$	$QS_d(\frac{20}{21})$	$QS_d(1)$	$QS_d(\frac{20}{21})$	$QS_d(1)$

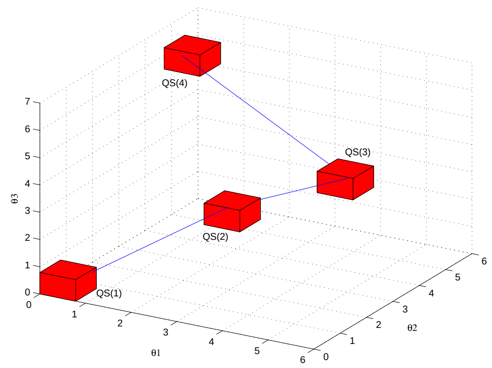

Figure 9: A behaviour of the end-effector of a three-link robot

For instance, consider a three-link robot with a behaviour shown in Figure 9. And a qualitative state $QS(i)$ in its local joint coordinates $O_i(J_{q1}^i, J_{q2}^i, J_{q3}^i)$. The desired joint position J_{dj}^i of the qualitative state is described as the addition of local origin J_{qj}^i and randomly generated positions within its mapping indexes on both orientation and translation components. Its mathematical description is given in equation 16.

$$J_{dj}^i = J_{qj}^i + D \cdot \left(\frac{2\pi}{r_j} + \frac{l_j}{s_j}\right) \tag{16}$$

Where D is a function used to generate random numbers whose elements are uniformly distributed in the interval (0,1), l_j, r_j and s_j are translation limit, qualitative mapping indexes of jth component, respectively. For the behaviour in Figure 9, the method generates a set of quantitative trajectories in Figures 10 and 11. There are two trajectories of the end-effector in Figure 10 for the behaviour. It clearly shows that a behaviour could have a completely different motion for a specific robot. The positions labelled $\diamondsuit$ are produced using equation 16. The joint trajectories are given in Figure 11. The dashed-line joint trajectories correspond to the dashed Cartesian trajectory in Figure 10. Trajectories can be refined more precisely by either symbolic systems or adding more qualitative states. The precision of motion description is determined by normalised symbols. The bigger the mapping indexes of Cartesian components, the higher the precision.

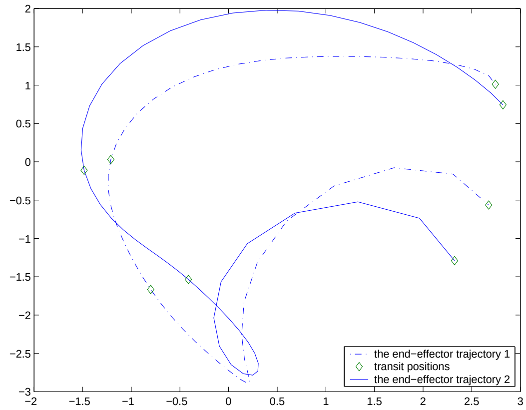

Figure 10: Trajectories of the end effector

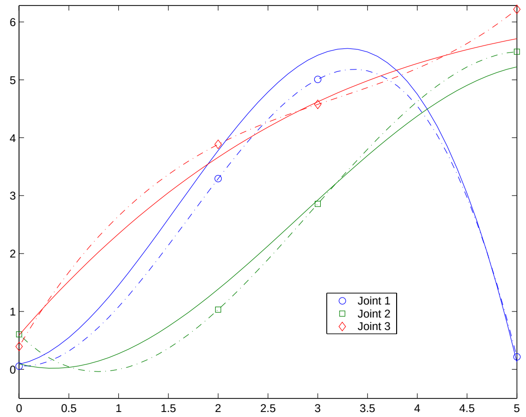

Figure 11: Joint trajectories of those in Figure 10

5 Conclusion

This research has proposed a novel qualitative representation for a robot. This method has first presented qualitative primitives for robotic components, then gradually constructs qualitative representation for a complex robot. This representation works as a converter between low-level control & sensing (i.e., robotic control modules) and high-level cognitive functions (i.e., symbolic systems). The two advantages of the proposed method should be noted. One is the proposed normalised technique, it not only allows sharing normalised symbols across multiple robots or subsystems, but also overcomes the problem of computational complexity that most planning systems suffer. The other is scalable mapping indexes for Cartesian motion components. It naturally provides a facility for the negotiation of connection between the qualitative and quantitative descriptions.

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