On Qualitative Probabilities for Legal Reasoning about Evidence

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Abstract

A crucial aspect of evidential reasoning in crime investigation involves comparing the support that evidence provides for alternative hypotheses. Recent work in forensic statistics has shown how Bayesian Networks (BNs) can be employed for this purpose. However, the specification of BNs requires conditional probability tables describing the uncertain processes under evaluation. When these processes are poorly understood, it is necessary to rely on subjective probabilities provided by experts, which are difficult to describe in a manner that is both accurate and precise. Recent work in qualitative reasoning has developed methods to perform this type of reasoning using coarser representations. However, the latter types of approaches are too imprecise to compare the likelihood of alternative hypotheses. This paper examines the shortcomings of the qualitative approaches when applied to the aforementioned problem, and identifies and integrates techniques to refine them.

Introduction

While legal reasoning about evidence is mostly similar to abductive diagnostic problem solving (Keppens & Schafer 2006), there are some crucial differences. The most important of these is that there is no need for decision making systems on legal evidential reasoning problems. Decisions of this nature must be made by humans, be they judges or jury panels. As such, decision support support systems for legal evidential reasoning must be primarily concerned with explaining the extent to which pieces of evidence support alternative plausible scenarios (Keppens, Shen, & Lee 2005).

So-called Bayesian techniques for testing hypotheses have proven to be a particularly influential approach to this problem. On the one hand, and unlike conventional symbolic inference mechanisms, the Bayesian approach to evidence evaluation enables the use of the well-understood classical probabilities in order to obtain a precise assessment of the relative strength of support of a piece of evidence for certain hypotheses. On the other hand, and unlike conventional techniques for statistical hypothesis testing, the Bayesian approach supports causal reasoning on how a piece of evidence can materialise as a consequence of a hypothetical scenario. This makes it particularly suitable for modelling situations that occur relatively infrequent and are difficult to synthesise. Moreover, it provides a foundation from which explanations for the results of a probabilistic analysis can be derived.

However, the Bayesian approach is not without its drawbacks. In legal evidential reasoning, it normally necessitates the use of subjective probabilities (as does any probabilistic reasoning approach). These are numbers that express a person's believe in the proposition of interest. Such probabilities are more prone to inaccuracy than ones that express the proportions of outcomes of a frequently repeated experiment in which the proposition of interest is true. Moreover, the acquisition of a set of precise expert beliefs in propositions that is consistent with the axioms of classical probability theory is a substantial problem in its own right. A rigourous evaluation of the impact of these potential inaccuracies and inconsistencies, by such techniques as sensitivity analysis, may help overcome the problem associated with these drawbacks. But this may make the technique inaccessible for many people responsible for evidence evaluation, such as crime investigators, juries and judges.

A substantial part of the difficulty of applying the Bayesian approach is due to the amount of knowledge required to acquire precise and accurate numerical probabilities. But in this domain, precise numerical probabilities are not required. Indeed, in the evaluation of forensic evidence, the objective is normally merely to produce a justifiable indication of the difference in magnitude of support for one hypothesis over another, given the available evidence. Therefore, various approaches for qualitative Bayesian inference have been developed, such as qualitative probabilistic networks (QPNs) (Wellman 1990), qualitative certainty networks (QCNs) (Parsons & Mamdani 1993) and linguistic Bayesian networks (Halliwell & Shen 2002).

Generally speaking, abductive diagnostic systems require an approach to compose complete models from partial ones. While earlier work has shown that models in the form of numerical Bayesian networks can be composed from descriptions of influences between variables, strong and somewhat unrealistic independence assumptions have to be made in order to enable the composition of influences (Keppens, Shen, & Lee 2005). However, the qualitative approaches to Bayesian inference, such as QPNs and QCNs, provide a means to reason explicitly about independent influences and, with certain extensions such as those developed by Renooij et. al. (Renooij, van der Gaag, & Parsons 2002), influences upon influences. As such, these approaches be easily adapted to perform abductive reasoning.

The main limitation of the qualitative approaches is that they are too vague to provide any useful information regarding the relative support of evidence for hypothetical scenarios, in all but the most obvious cases (Biedermann & Taroni 2006). This is due to qualitative overabstraction. Parsons (Parsons 1995) has suggested the incorporation of order of magnitude reasoning (Raiman 1991) in QPNs and QCNs, which refines the precision of these qualitative probabilistic reasoning approaches while maintaining their composability.

This paper aims to identify how qualitative probabilistic reasoning techniques can be employed to perform legal reasoning about evidence. It will examine the nature of typical relationships between variables in this domain and the limitations of the basic QPN/QCN methods in representing them. Various extensions that may address some of these issues, and which have been developed independently in the literature, are identified. By means of simple but realistic examples, the paper shall demonstrate how these individual extensions can be integrated with one another in order to produce a rich qualitative approach to Bayesian inference, which is sufficiently precise to help human decision makers assess the relative support of evidence for alternative hypotheses while retaining composability.

Bayesian Evidential Reasoning

Underlying Bayesian methods for evidence evaluation lies the notion that statistical hypothesis testing constitutes a suitable paradigm for this purpose. Cook et. al. propose that a piece of evidence e be evaluated by determining its likelihood under alternative hypotheses h_1 and h_2 (Cook *et al.* 1998). If the probability of e's occurrence assuming hypothesis h_1, denoted $Pr(e|h_1)$, is substantially higher than the probability of e's occurrence assuming hypothesis h_2, denoted $Pr(e|h_2)$, then the investigator can conclude that e provides stronger support for h_1 than for h_2.

Bayesian Networks

A Bayesian network (BN) is a representation that facilitates the computation of joint probability distributions over a set of variables $\mathbf{X} = \{X_1, \ldots, X_n\}$. Reasoning with joint probability distributions over a large set of variables $\{X_1, \ldots, X_n\}$ is problematic because the number of variable assignment combinations that need to be considered increases exponentially with n. A BN simplifies these calculations by considering the independencies between variables.

A BN consists of a directed acyclic graph (DAG) that describes the independencies between variables, and a set of probability distribution tables that quantify the relations between variables. Figure 1 is an example of such a BN, which is partially based on work by Aitken et. al. (Aitken, Taroni, & Garbolino 2003). The DAG contains a node for each variable. And intuitively, each arc in the DAG from a variable A to a variable B represents the notion that A influences B.

Independencies are defined formally in a BN by means of the concept of *d-separation*. More specifically, a chain

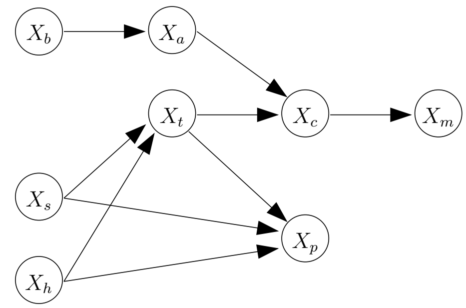


Figure 1: Sample Bayesian network

Table 1: The variables of the sample BN

Symbol	Meaning
X_b	background of suspect involves handling blood
X_a	suspect may have background blood splatters
X_s	the suspect stabbed the victim
X_h	the suspect examined the victim's body
X_t	blood was transferred from victim to suspect
X_c	investigator chooses blood splatter from victim on suspect for investigation
X_m	a blood splatter matching the victim's dna was found on the suspect
X_p	the blood splatter is a projected stain

of variables from A to B, formed by following arcs in the DAG in either direction, is said to be *blocked* by a set **C** of observed variables, if it contains

- a variable $D \notin$ **C** with two incoming arcs in the chain (e.g. given no observations the chain $X_t \rightarrow X_c \leftarrow X_a$ is said to be blocked), or
- a variable D that is either observed (i.e. $D \in$ **C**) or that has an observed descendant (e.g. if X_p is observed, the chain $X_s \rightarrow X_t \rightarrow X_c$ is blocked).

Two variables A and B are said to be d-separated by a set **C** of observed variables if all chains between A and B are blocked by **C**. Variables that are d-separated in a BN by an observation, are deemed conditionally independent given that observation.

The latter feature of BN models helps simplify the calculations of joint probability distributions. This is perhaps best explained by means of the example in Figure 1. Let each variable X_i in this BN have a boolean domain $\{x_i, \overline{x_i}\}$, where x_i denotes that X_i =true and $\overline{x_i}$ that X_i =false. Then, the joint probability that all variables are true can be computed by:

$$
\begin{aligned}
P(x_b, &x_a, x_s, x_h, x_t, x_c, x_m, x_p) \\
&= P(x_m|x_c) \times P(x_p|x_t x_s x_h) \times P(x_c|x_t x_a) \times \\
&\quad P(x_a|x_b) \times P(x_b) \times P(x_t|x_s x_h) \times P(x_s) \times P(x_h)
\end{aligned}
$$

Generally speaking, $2^8 - 1$ or 255 distinct probabilities are required to specify the joint probability distribution of the 8 variables in the BN of figure 1. With the BN, that number

Table 2: Conclusions drawn from likehood ratios by the Forensic Science Service

Likelihood Ratio	Support of evidence to prosecution claim over defense claim
1 to 10	limited
10 to 100	moderate
100 to 1,000	moderately strong
1,000 to 10,000	strong
over 10,000	very strong

can be reduced to $2^1 + 2^3 + 2^2 + 2^1 + 1 + 2^2 + 1 + 1$ or 23 distinct probabilities. These probabilities are specified in so-called conditional probability tables and prior probability distributions.

For each non-root node X in its DAG, a BN contains a conditional probability table that, in turn, contains a conditional probability distribution for each set of assignments of parent nodes of X. For each root node in its DAG, a BN contains a prior probability distribution. For example, the nodes X_t and X_c could have the following probability distributions:

$P(x_t|x_s x_h) = 0.99$ $\quad P(x_c|x_t x_a) = 0.3$
$P(x_t|x_s \overline{x_h}) = 0.99$ $\quad P(x_c|x_t \overline{x_a}) = 1$
$P(x_t|\overline{x_s} x_h) = 0.7$ $\quad P(x_c|\overline{x_t} x_a) = 0$
$P(x_t|\overline{x_s x_h}) = 0.01$ $\quad P(x_c|\overline{x_t x_a}) = 0$

The Likelihood Ratio Approach to Evidence Evaluation

The likelihood ratio approach evaluates a piece of evidence e with respect to two or more hypotheses. Let h_p be the hypothesis corresponding to the claim made by the prosecution and h_d be the one made by the defence in a trial. Then, the likelihood ratio (LR):

$$LR = \frac{Pr(e|h_p)}{Pr(e|h_d)}$$

expresses the degree to which the evidence is more plausible under the prosecution's claim than under the defence's claim. For instance, the Forensic Science Service, a major forensic laboratory in England and Wales, employs the likelihood ratio to report their findings in court. Table 2 summarises the way they report their conclusions.

A BN is a natural representation to help compute the conditional probabilities in the numerator and the denominator of a LR. BNs are particularly suitable to represent uncertain causal relations between relevant variables in a domain. In the context of evidence evaluation in crime investigation, they are used to describe how hypothetical situations and events may lead to observable evidence. In such a model, the hypotheses of interest typically correspond to one or more root nodes and a piece of evidence to a leaf node.

Consider, for example, an investigation of suspicious death where the victim died from a stab wound. The crime investigators have arrested a suspect, whom they believe has stabbed the victim to death. And, an examination of the suspect's clothes revealed blood splatter matching the victim's dna. The suspect's defence attorney claims that the suspect did not stab the victim, but instead discovered the victim's body and tried to revive him. It is assumed that the suspect does not come into contact with blood on a regular basis, under either hypothesis. In this case, the prosecution hypothesis is specified by $\{x_s, \overline{x_h}, \overline{x_b}\}$, the defense hypothesis is $\{\overline{x_s}, x_h, \overline{x_b}\}$, and the evidence can be represented as x_m. The likelihood ratio $\frac{P(x_m|x_s\overline{x_h}\overline{x_b})}{P(x_m|\overline{x_s}x_h\overline{x_b})}$ can be calculated easily by means of BN software, such as Hugin.

Discussion

While there is no universally accepted approach to evidence evaluation within the forensic science and crime investigation community, there are some important benefits to Bayesian evidence evaluation, which stem from combining the advantages of statistical and logic-based approaches.

As a statistical method, a BN can be employed to compare the relative support for alternative hypotheses by given pieces of evidence. This differentiates the Bayesian approach to evidence evaluation from logic-based ones as the latter are typically restricted to roles such as abductive reasoning about plausible hypotheses, reasoning about the implication and validity of arguments and explanation generation. And while the latter roles are important, their remains a crucial need for decision support in the area of comparing the plausibility of hypotheses under consideration (Cook *et al.* 1998).

Similar to logic-based approaches and contrary to most conventional statistical inference methods, a BN can model causal explanations for available evidence (Lacave & Díez 2002; Pearl 1988). Such causal structures are important because they enable the forensic expert to justify the results of evidence evaluation in court and identify how it relates to the plausible crime scenarios under consideration. Eventually, these structures can be transformed into arguments that constitute a basis for legal proceedings in criminal cases (Bex *et al.* 2003; Schum 1994).

There are some important objections to the Bayesian approach, however. Although, as explained in Section , BNs reduce the requirement for probabilistic knowledge, collecting sufficient and suitable conditional probability distributions remains an important stumbling block in many practical applications of BNs (Druzdzel & van der Gaag 2000). There are processes that produce certain types of evidence, which are well understood: e.g. the effect of mixtures of DNA material on the corresponding profile (Mortera, Dawid, & Lauritzen 2003). However, for many types of hypothesis and evidence, it is difficult to identify the conditional probability distributions from the underlying physical processes. For example, it is very difficult to categorise and relate types of contact between two people and the amount of trace material that is transferred between those people as a consequence (Aitken, Taroni, & Garbolino 2003).

In the latter case, experts may be able to provide estimates of the conditional probabilities based on their knowledge and experience. Such probabilities are called *subjective probabilities* because they reflect the personal opinion of one expert. The difficulty of obtaining point estimates of probabilities from experts has been widely reported (Kahneman,

Table 3: Sign operations

$\otimes$	+	0	-	?	$\oplus$	+	0	-	?
+	+	0	-	?	+	+	+	?	?
0	0	0	0	0	0	+	0	-	?
-	-	0	+	?	-	?	-	-	?
?	?	0	?	?	?	?	?	?	?

Slovic, & Tversky 1985; Zimmer 1983). Moreover, it has been reported that verbal expressions of probabilistic uncertainty were more accurate than numerical values in estimating the frequency of multiple attributes (Druzdzel & Henrion 1993; Zimmer 1986). This has led to the development of a range of qualitative approaches to perform Bayesian inference.

Qualitative Bayesian Inference

Qualitative Certainty Networks (QCNs) (Parsons & Mamdani 1993) constitute qualitative abstractions of various probabilistic and possibilistic reasoning techniques. Similar to a BN, a QCN consists of a DAG that describes the independencies between variables. But instead of a conditional probability table, each arc $A \rightarrow C$ between two variables A and C in the DAG is associated with a set of so-called qualitative derivatives, one for each pair of values a_i and c_j from the domains of A and C.

Definition 1 *Given a QCN containing two variables A and C connected by an arc $A \rightarrow C$, a value a_i of A and a value c_j of C, the qualitative derivative $[\frac{dP(c_j)}{dP(a_i)}]$ has the value $+$ iff for all values $a_k \neq a_i$ of A and all assignments $\mathbf{x}$ of the parent variables of C in the DAG other A.:*

$$P(c_j|a_i, \mathbf{x}) \geq P(c_j|a_k, \mathbf{x}) \quad (1)$$

Informally, the qualitative derivative $[\frac{dP(c_j)}{dP(a_i)}]$ is said to be positive iff C is more likely to take c_j when A is more likely to take a_i. The definitions for $[\frac{dP(c_j)}{dP(a_i)}] = 0$ and $[\frac{dP(c_j)}{dP(a_i)}] = -$ can be specified in the same way as Definition 1 by replacing $\geq$ with $=$ and $\leq$ respectively. If $[\frac{dP(c_j)}{dP(a_i)}]$ does not equal $+$, 0 or $-$, it is said to be ambiguous and takes value $?$.

Let $[\Delta P(a_i)]$ denote a change in the sign of the probability of variable A taking value a_i. Then, such a change can be propagated along an arc by:

$$[\Delta P(c_j)] = [\frac{dP(c_j)}{dP(a_i)}] \otimes [\Delta P(a_i)]$$

where $\otimes$ denotes sign multiplication. The effect of multiple sign changes are combined with sign addition $\oplus$. Both operations are defined in Table 3.

In the remainder of this paper, all variables are assumed to have boolean domains. In this case, the domain of a variable Y can be denoted as $\{y, \overline{y}\}$. This allows the notation to be simplified substantially because a single qualitative derivative implies all the others in these circumstances. That is, when C takes values c and $\overline{c}$ and A takes values a and $\overline{a}$,

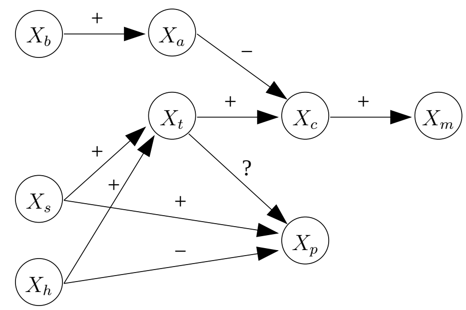


Figure 2: Sample QPN/QCN

then $[\frac{dP(c)}{dP(a)}] = +$ implies that $[\frac{dP(c)}{dP(\overline{a})}] = -$, $[\frac{dP(\overline{c})}{dP(a)}] = -$ and $[\frac{dP(\overline{c})}{dP(\overline{a})}] = +$ because $P(c|a, \mathbf{x}) \geq P(c|\overline{a}, \mathbf{x})$ implies that $P(c|\overline{a}, \mathbf{x}) \leq P(c|a, \mathbf{x})$, $P(\overline{c}|a, \mathbf{x}) \leq P(\overline{c}|a, \mathbf{x})$ and $P(\overline{c}|\overline{a}, \mathbf{x}) \geq P(\overline{c}|a, \mathbf{x})$. In what follows, the sign of qualitative derivative $[\frac{dP(c)}{dP(a)}]$ will be denoted by $[S(A, C)]$. Note that, as shown by Parsons (Parsons 1995), the qualitative derivative $[\frac{dP(c)}{dP(a)}]$ of variables A and C with a boolean domains $\{a, \overline{a}\}$ and $\{c, \overline{c}\}$ equals the sign of the qualitative influences in Qualitative Probabilistic Networks (Wellman 1990).

It follows from the above that $[S(A, C)] = [S(C, A)]$. This property has enabled Druzdzel and Henrion (Druzdzel & Henrion 1993) to devise an algorithm to propagate a change in sign of any assignment h in these QPN/QCNs. In a nutshell, this algorithm first initialises the change in likelihood for every assignment to 0. Then, the algorithm propagates the sign change of h to every other node in the QPN/QCN, via every path, from H to other nodes, that is not blocked. The sign change in a node A that is directly connected to a node C via an arc $A \rightarrow C$ or $A \leftarrow C$ on a path that is not blocked is propagated by:

$$[\Delta P(c)] = [S(A, C)] \otimes [\Delta P(a)]$$

Given that the sign change has been provisionally set to $[\Delta P(c)]_{\text{current}}$ by the algorithm and that a newly considered sign change equals $[\Delta P(c)]_{\text{influence}}$, then the combined sign change equals:

$$[\Delta P(c)] = [\Delta P(c)]_{\text{current}} \oplus [\Delta P(c)]_{\text{influence}}$$

Figure 2 shows a QPN/QCN describing a qualitative version of the BN of Figure 1. This model indicates that both the hypothesis that the suspect stabbed the victim (x_s) and the hypothesis that the suspect tried to determine whether he could help the victim by examining the body (x_h) justify the observation of a blood splatter matching the victim's dna on the suspect (x_m). It also suggests that the blood splatter is more likely to be a projected bloodstain (x_p) if the suspect stabbed the victim and less likely to be a project bloodstain ($\neg x_p$), i.e. a contact stain, if the suspect examined the victim's body.

Consider, for instance, that blood splatter on the suspect matching the victim's dna is observed. This corresponds to $[\Delta P(x_m)] = +$. Druzdzel and Henrion's algorithm propagates this change as follows: $[\Delta P(x_c)] = +$, $[\Delta P(x_a)] =$

$-$, $[\Delta P(x_b)] = -$, $[\Delta P(x_p)] =?$, $[\Delta P(x_s)] = +$ and $[\Delta P(x_h)] = +$. Note that the change $[\Delta P(x_p)] =?$ is not propagated to X_s and X_h because the paths $X_t \rightarrow X_p \leftarrow X_s$ and $X_t \rightarrow X_p \leftarrow X_h$ are blocked.

Evidentiary reasoning with a QCN

The likelihood ratio approach is not directly applicable to QCNs as they do not provide any numerical information with which to calculate the fraction. However, the spirit of the approach can be applied by comparing the effect of different sets of hypotheses on a given piece of evidence. Let $\Delta_d P(e)$ denote the change of obtaining a piece of evidence e if a given defence scenario is true and let $\Delta_p P(e)$ denote the change of obtaining a piece of evidence e if a given prosecution scenario is true. The relative effect of the defence scenario on $P(e)$ compared to that of the prosecution scenario indicates how much more or less the evidence supports the defence scenario compared to the prosecution one.

Because QCNs can only provide information on the signs of $\Delta_d P(e)$ and $\Delta_p P(e)$, which indicates a negative, zero, positive or ambiguous change in $P(e)$, their comparisons rarely yield useful information in practice. Effective defendants and prosecutors tend to hypothesise scenarios that provide seemingly reasonable explanations for the available evidence. In such situations, the hypotheses being compared both render the available evidence more likely: i.e. $[\Delta_d P(e)] = [\Delta_p P(e)] = +$.

One approach to address this issue is the use of so-called product synergy in QPNs. Let A, B and C be variables connected by arcs $A \rightarrow C$ and $B \rightarrow C$ in a QPN. Then, there is said to be negative product synergy between A and B for a value c of C iff:

$$P(c|ab\mathbf{x})P(c|\bar{a}\bar{b}\mathbf{x}) - P(c|a\bar{b}\mathbf{x})P(c|\bar{a}b\mathbf{x}) \leq 0 \qquad (2)$$

It has been shown that negative product synergy enables hypotheses to be explained away (Wellman & Henrion 1993). Clearly, given (2), the observation of c implies that an increase in the likelihood of a makes b less likely and vice versa. Thus, if there is negative product synergy between A and B for a value c of C, the observation of c entails that $[S(A, B)] = -$, thereby enabling evidence that confirms a to be used to undermine b. But while negative product synergies provide the mechanism to infer counter-arguments within the framework of a QPN, it leaves much room for ambiguity. Indeed, the negative qualitative derivative implied by a negative product synergy works both ways and evidence that confirms *either* hypothesis undermines the other.

In the QPN/QCN of Figure 2, the prosecution hypothesis corresponds to $\{[\Delta P(x_s)] = +, [\Delta P(x_b)] = -\}$ and the defense hypothesis to $\{[\Delta P(x_h)] = +, [\Delta P(x_b)] = -\}$. Both hypotheses yield $[\Delta P(x_m)] = +$. As such, a QPN/QCN is not able to differentiate between both hypotheses.

QCN with orders of magnitude

Another approach to refine the reasoning that can be accomplished with a QCN involves the use qualitative or semi-quantitative representations of the magnitudes of changes in probabilities and qualitative derivatives. In such an approach, each direction of change of a probability $[\Delta_i P(a)]$ and each qualitative derivative $[S(A, C)]$ is also associated with a magnitude of change. These are denoted as $|\Delta_i P(a)|$ and $|S(A, C)|$ respectively. Note that while $[\frac{dP(c)}{dP(a)}] = [\frac{dP(a)}{dP(c)}]$ in a QCP with an arc $A \rightarrow C$, it is not necessarily the case that $|\frac{dP(c)}{dP(a)}|$ equals $|\frac{dP(a)}{dP(c)}|$. Therefore, the propagation mechanisms discussed in the remainder of the paper only apply in the direction of the arcs.

Table 4: Multiplication of relative orders of magnitude

rel_1 \ rel_2: $\otimes$	$\approx$	$\sim$	$\simeq$	$\ll$	$\gg$
$\approx$	$\approx$	$\sim$	$\simeq$	$\ll$	$\gg$
$\sim$	$\sim$	$\sim$	U	$\ll$	$\gg$
$\simeq$	$\simeq$	U	U	U	U
$\ll$	$\ll$	$\ll$	U	$\ll$	U
$\gg$	$\gg$	$\gg$	U	U	$\gg$

A range of order of magnitude reasoning (OMR) techniques has been devised to express magnitudes in a qualitative manner (Raiman 1991). There are two types of OMR: relative OMR and absolute OMR. *Relative OMR* defines orders of magnitudes of variables by relating them to one another. For example, Dague's Relative OMR, named ROM, relates variables to one another using four types of ordering relations between pairs of quantities: x is close to y (denoted $x \approx y$), x is comparable to y ($x \sim y$), x is distant from y ($x \simeq y$) and x is negligible compared to y ($x \ll y$) (Dague 1993). Parsons (Parsons 2003) has devised a method that can be employed to propagate such order of magnitude information in a QCN. Let A, B, C and D be variables in a QCN in which A and B are connected by an arc $A \rightarrow B$ and C and D by an arc $C \rightarrow D$. Then, it can be shown that if $|S(A, B)| rel_1 |S(C, D)|$ and $|\Delta_i P(A)| rel_2 |\Delta_j P(C)|$, then rel_3 in $|S(A, B)|.|\Delta_i P(A)| rel_3\ |S(C, D)|.|\Delta_j P(C)|$ is given by Table 4.

The sample QCN of Figure 2, for instance, may include the ordering relation $|S(X_s, X_t)| \gg |S(X_h, X_t)|$. When comparing the effects of the prosecution and defence hypotheses, it can be assumed that the strength of the hypotheses are of a similar order of magnitude: i.e. $\Delta_p P(x_s) \approx \Delta_d P(x_h)$ and $\Delta_p P(x_b) \approx \Delta_d P(x_b)$. Based on these inputs, ROM based QCN infers that

$$\begin{aligned} &|\Delta_p P(x_s)| \otimes |S(X_s, X_t)| = \\ &\qquad |\Delta_p P(x_t)| \gg |\Delta_d P(x_t)| \\ &\qquad\qquad = |\Delta_d P(x_h)| \otimes |S(X_h, X_t)| \end{aligned}$$

because according to Table 4, $\approx \otimes \gg$ yields $\gg$. Similarly, ROM based QCN infers $|\Delta_p P(x_c)| \gg |\Delta_d P(x_c)|$ and $|\Delta_p P(x_m)| \gg |\Delta_d P(x_m)|$. Thus, QCN extended with ROM computes that the discovery of transfer evidence supports the prosecution hypothesis more strongly than the defence hypothesis.

A difficulty arises, however, when the model were to be extended with an additional node, say, X_o, which describes

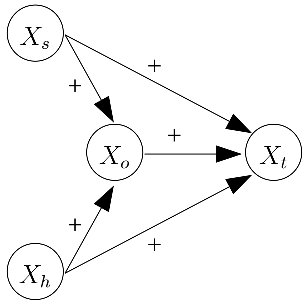


Figure 3: A more difficult application of ROM

Table 5: Intervals for qualitative derivatives

Symbol	Name	Definition
SP	Strong positive	$]1, \alpha]$
WP	Weak positive	$[\alpha, 0[$
Z	Zero	0
WN	Weak negative	$]0, -\alpha]$
SN	Strong negative	$[-\alpha, -1[$

whether or not the suspect touched the victim. The affected part of the QPN/QCN is shown in Figure 3. This change complicates the application of ROM considerably. This does not only introduce a requirement of additional pairwise ordering information regarding qualitative derivatives affecting the same node, but also between qualitative derivatives affecting different nodes in order to determine whether there are causal chains between the hypothesis nodes and X_t that have a dominant effect. In this way, the need for ordering information can grow exponentially with the network, thereby making this method impractical.

Absolute OMR defines magnitudes as numerical intervals rather than by means of ordering relations between individual magnitudes. Parsons (Parsons 1995) has briefly examined the use of absolute OMR for integration in QCNs and suggested the interval distribution shown in Table 5 for each qualitative derivative magnitude $|S(A, C)|$ and the interval distribution shown in Table 6 for each variable change $\Delta_i P(a)$. He has also shown, using conventional interval calculus, that if $\alpha = 0.5$ and $\beta = \frac{1}{3}$, the results for multiplication and addition of intervals are given in Table 7 and Table 8 respectively. Note that in these tables, $[I_1, I_2]$ refers to the combined interval containing both I_1 and I_2, and that empty cells refer to impossible combinations of values.

Figure 4 shows a version of the QPN/QCN of Figure 1 with the aforementioned absolute order of magnitude scale. In this approach, the prosecution scenario corresponds to $|\Delta_p P(x_s)| = CP$ and $|\Delta_p P(x_b)| = CN$ and the defence scenario to $|\Delta_d P(x_h)| = CP$ and $|\Delta_d P(x_b)| = CN$. These values can be propagated as follows:

Table 6: Intervals for variable changes

Symbol	Name	Definition
CP	Complete positive	1
BP	Big positive	$]1, 1-\beta]$
MP	Medium positive	$[1-\beta, \beta]$
LP	Little positive	$[\beta, 0[$
Z	Zero	0
LN	Little negative	$]0, -\beta]$
MN	Medium negative	$[-\beta, \beta-1]$
BN	Big negative	$[\beta-1, -1[$
CN	Complete negative	-1

Table 7: Interval multiplication

$\otimes$	SP	WP	Z	WN	SN
CP	$[BP, MP]$	$[MP, LP]$	Z	$[MN, LN]$	$[BN, MN]$
BP	$[BP, MP]$	$[MP, LP]$	Z	$[MN, LN]$	$[BN, MN]$
MP	$[MP, LP]$	$[MP, LP]$	Z	$[MN, LN]$	$[MN, LN]$
LP	LP	LP	Z	LN	LN
Z	Z	Z	Z	Z	Z
LN	LN	LN	Z	LP	LP
MN	$[MN, LN]$	$[MN, LN]$	Z	$[MP, LP]$	$[MP, LP]$
BN	$[BN, MN]$	$[MN, LN]$	Z	$[MP, LP]$	$[BP, MP]$
CN	$[BN, MN]$	$[MN, LN]$	Z	$[MP, LP]$	$[BP, MP]$

Table 8: Interval addition

$\oplus$	CP	BP	MP	LP	Z
CP					CP
BP				$[CP, BP]$	BP
MP			$[CP, MP]$	$[CP, MP]$	MP
LP		$[CP, BP]$	$[CP, MP]$	$[BP, LP]$	LP
Z	CP	BP	MP	LP	Z
LN		$[BP, MP]$	$[MP, Z]$	$[LP, Z]$	
MN		$[BP, Z]$	$[MP, Z]$		
BN		$[LP, Z]$			
CN					

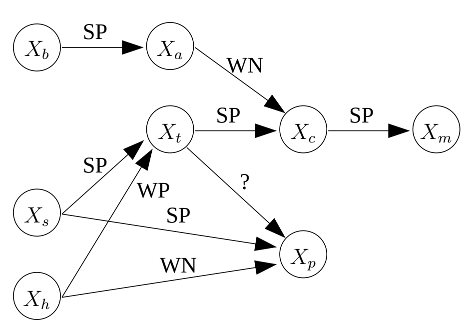


Figure 4: Sample QPN/QCN with absolute orders of magnitude

$$
\begin{aligned}
|\Delta_p P(x_t)| =& SP \otimes CP = [BP, MP]\\
|\Delta_p P(x_a)| =& SP \otimes CN = [BN, MN]\\
|\Delta_p P(x_c)| =& (WN \otimes [BN, MN]) \oplus (SP \otimes [BP, MP])\\
=& [MP, LP] \oplus [BP, LP] = [CP, LP]\\
|\Delta_p P(x_m)| =& SP \otimes [CP, LP] = [BP, LP]
\end{aligned}
$$

The outcome of this analysis (i.e. $|\Delta_p P(x_m)| = [BP, LP]$) is that there is a small to substantial increase in likelihood to obtain a blood splatter from the suspect matching the victim's dna under the prosecution scenario. The hypotheses of the defence scenario can be propagated in the same way, resulting in $|\Delta_d P(x_m)| = [BP, LP]$. As such, this approach is not able to differentiate between both scenarios in this case.

Note that a basic QPN/QCN as defined in Section is a special case of a QPN/QCN with orders of magnitude. It employs the following intervals for both changes in likelihood of variables and qualitative derivatives:

$$0 = [0] \qquad + = [0, 1] \qquad - = [-1, 0] \qquad ? = [-1, 1] \tag{3}$$

Reducing over-abstraction

The survey of qualitative Bayesian inference methods has shown how these techniques can be applied to evidence evaluation in crime investigation. An important limitation of the approaches discussed in the survey is that, even with the introduction of order of magnitude calculi, they tend to produce very abstract results. Even in simple examples, such as the one used throughout this paper, the propagation of likelihood changes quickly yields intervals that are too wide for comparison. This problem of over-abstraction is inherent to all types of interval calculi, including those involving orders of magnitude and fuzzy sets. However, the nature of influences between variables in models for evidential reasoning in crime investigation exhibits certain features that enable the knowledge engineer to reduce the effects of qualitative abstraction.

Cause v context

The order of magnitude approaches discussed herein presume that each arc $A \rightarrow C$ implies that the likelihood of c is either proportionate or inversely proportionate to the likelihood of a. As such, these approaches are particularly well suited to model processes whereby A adds to or removes from the likelihood of$C$ (or vice versa). These are situations in which A is a direct *cause* of C (or vice versa). However, in evidential reasoning, this is not always the case. Certain variables, which will be called *context* variables, affect the process rather than the consequence. Figure 5 illustrates this distinction.

The crucial difference between a causal and a context variable of an influence is that the causal variable always affects the consequent, whereas the effect of the context on the consequent determines the magnitude with which the process takes place. Four types of context variables can be

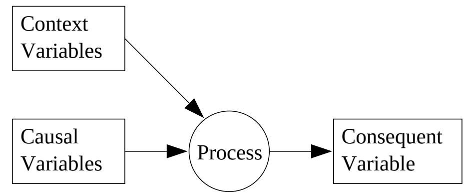


Figure 5: Cause v context

identified: *enablers*, which are conditions for the process to take place; *disablers*, which prevent the process from taking place; *amplifiers*, which increase the effect of the process; and *inhibitors*, which decrease the effect of the process.

The example used throughout this paper contains two context variables. Firstly, X_a is an inhibitor to $X_t \rightarrow X_c$. Here, the transfer of blood from the the victim to the suspect (X_t) generates blood traces that are relevant to the crime and may be retrieved by the investigators (X_c). Blood splatter on the suspect from a source unrelated to the crime (X_a) makes it less likely that the investigators will retrieve blood splatter related to the crime. Thus, the likelihood of x_a has a negative effect on the likelihood of x_c, but only if x_t is true to begin with. Secondly, X_t is an enabler to $X_s \rightarrow X_p$ and to $X_h \rightarrow X_p$. Both hypotheses, i.e. the suspect stabbed the victim (x_s) and the suspect examined the body of the victim (x_h), affect the pattern of the blood splatter that is transferred X_p. However, blood must be transferred from suspect to victim for there to be a pattern to examine.

Context variables can be identified in a conventional numerical BN as follows. Let A be a cause of C and B be a context variable, such that

$$P(c|ab) = \alpha \quad P(c|a\overline{b}) = \beta \quad P(c|\overline{a}b) = \epsilon_1 \quad P(c|\overline{a}\overline{b}) = \epsilon_2.$$

where $\epsilon_1 \approx 0$ and $\epsilon_2 \approx 0$. Then, B is an enabler if $\alpha > \beta = 0$, a disabler if $\beta > \alpha = 0$, an amplifier if $\alpha > \beta$ and an inhibitor if $\alpha < \beta$. As such, the likelihood of context variables assignments is not proportional to the likelihood of consequent variable assignments, which, in turn, makes the order of magnitude approaches unsuitable.

Renooij et. al. (Renooij, van der Gaag, & Parsons 2002) have extended the basic (i.e. sign-only) QPN approach with so-called *non-monotonic* influences. The sign of a non-monotonic influence changes with the assignment of another variable. For example, in the QPN of Figure 2, the signs of the qualitative derivatives $S(X_s, X_p)$ and $S(X_h, X_p)$ could be specified as follows:

$$[S(X_s, X_p)] = \begin{cases} + & \text{if } x_t \\ 0 & \text{if } \overline{x_t} \end{cases} \quad \text{and} \quad [S(X_h, X_p)] = \begin{cases} - & \text{if } x_t \\ 0 & \text{if } \overline{x_t} \end{cases}$$

Let $S_{\mathbf{x}}(A, C)$ denote the qualitative derivative in effect given the set of assignments $\mathbf{x}$ of the context variables of $A \rightarrow C$. Then, the above derivative signs can be denoted as: $[S_{x_t}(X_s, X_p)] = +$, $[S_{\overline{x_t}}(X_s, X_p)] = 0$, $[S_{x_t}(X_h, X_p)] = -$ and $[S_{\overline{x_t}}(X_h, X_p)] = 0$.

When the assignments of the context variables are not all know, then the smallest possible range of effects that includes all plausible contexts must be assumed (using the interval definitions of (3)). Thus, given an influence $A \rightarrow C$

and two sets of assignments $\mathbf{x}$ and $\mathbf{y}$ of the context variables of $A \rightarrow C$, then:

$$[S_{\mathbf{x}\cap\mathbf{y}}(A,C)] \subseteq [S_{\mathbf{x}}(A,C)] \cup [S_{\mathbf{y}}(A,C)]$$

It follows that if $\mathbf{x}'$ denotes a partial specification of the context variables of $A \rightarrow C$, then

$$[S_{\mathbf{x}'}(A,C)] = \bigcup_{\mathbf{x}'\subseteq\mathbf{x}} [S_{\mathbf{x}}(A,C)]$$

where the $\mathbf{x}$ are all assignments of the context variables of $A \rightarrow C$ such that $\mathbf{x}' \subseteq \mathbf{x}$. Note that it follows from (3) that:

$$0 \subset + \qquad 0 \subset - \qquad + \subset ? \qquad - \subset ?$$

Therefore, if in the example, the assignment of the context variable X_t of $X_s \rightarrow X_p$ and $X_h \rightarrow X_p$ is unknown, then:

$$[S(X_s,X_p)] = [S_{x_t}(X_s,X_p)] \cup [S_{\overline{x_t}}(X_s,X_p)] = + \cup 0 = +$$
$$[S(X_h,X_p)] = [S_{x_t}(X_h,X_p)] \cup [S_{\overline{x_t}}(X_h,X_p)] = - \cup 0 = -$$

which is identical to the original specification of the network in Figure 2.

Clearly, this idea can be generalised to absolute orders of magnitudes by using more precise interval definitions than those of (3). Generally speaking, given an influence $A \rightarrow C$ and two sets of assignments $\mathbf{x}$ and $\mathbf{y}$ of the context variables of $A \rightarrow C$, the context specific magnitudes of qualitative derivatives must adhere to:

$$|S_{\mathbf{x}\cap\mathbf{y}}(A,C)| \subseteq |S_{\mathbf{x}}(A,C)| \cup |S_{\mathbf{y}}(A,C)|$$

Categorical influences

Because Bayesian inference models in general, and qualitative abstractions of such models in particular, have an explanatory role in addition to a computational one, it is important that the structure of the network matches the way the human user would organise his/her arguments. For example, in practice, the arcs in BNs often describe causal relations between variables, even though that is not necessary. However, causal relations are often the most natural way of justifying analyses.

To improve the explanatory benefits of a QPN/QCN, additional variables that do not introduce any source of uncertainty may need to be introduced. In the original version of the BN shown in Figure 1, X_a is such a variable (Aitken, Taroni, & Garbolino 2003). Its probability distribution is defined as $P(x_a|x_b) = 1$ and $P(x_a|\overline{x_b}) = 0$. The variable describes that a certain background of a suspect (x_b) may constitute an alternative source of blood splatter on the suspect (x_a), which in turn inhibits the discovery of blood splatter matching the victim's dna on the suspect (in the hypothetical case that blood has been transferred from victim to suspect).

Categorical influences represent precisely this type of information in the restricted setting of a sign calculus (Parsons 1995; 2004). Table 9 displays sign multiplication in a setting where a qualitative derivative $[S(A,C)]$ can take values $++$ and $--$, indicating a categorical influence. Variable change signs $[\Delta P(a)]$ can take values $++$ and $--$, which describe that the variable increases to 1 or decreases to 0 respectively.

Table 9: Sign multiplication with categorical influences

⊗	++	+	0	-	--	?
++	++	+	0	-	--	?
+	+	+	0	-	-	?
0	0	0	0	0	0	0
-	-	-	0	+	+	?
--	--	-	0	+	++	?
?	?	?	0	?	?	?

The approach can be generalised and integrated into the context of the absolute order of magnitude reasoning. In essence, a categorical influence $A \rightarrow C$ propagates any changes in $P(a)$ directly to $P(c)$. Formally, given a magnitude change $\|DeltaP(a)|$ and a categorical influence $S(A,C) = ++$, the magnitude change $|\Delta P(c)| = |\Delta P(a)|$. Similarly, given a magnitude change $|\Delta P(a)|$ and a categorical influence $S(A,C) = --$, $|\Delta P(c)| = -|\Delta P(a)|$.

Magnitudes

As illustrated by the example, the absolute order of magnitude scale discussed in Section is too abstract to derive a firm conclusion. The main problem with the use of absolute orders of magnitude is that every propagation of probability changes along an arc on a path from a hypothesis node to an evidence node involves an interval multiplication. And, every interval multiplication produces a result that tends to be wider than the constituent factors. Context variables and categorical influences can, to some extent, alleviate these issues. However, in more complex models, a more refined order of magnitude scale has to be employed.

One approach of defining absolute order of magnitude scales, which limits the amount of interval size expansion and facilitates flexible definition of absolute order of magnitude scales, is NAPIER (Nayak 1992). In NAPIER, the order of magnitude of a quantity is defined as the nearest lowest integer of the logarithm of that quantity. That is:

$$om(p) = \lfloor \log_b |p| \rfloor \qquad (4)$$

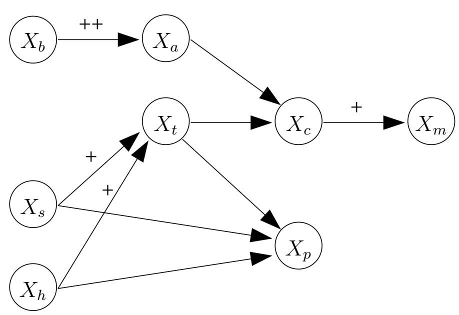
where b is the base of the logarithm. Thus, in this approach, magnitudes are defined by integers, where low integers indicate values closer to 1. In addition to the integer id of the order of magnitude, each qualitative derivative and magnitude change remains associated with a sign indicating the direction of change. An example of an order of magnitude scale and corresponding verbal qualifications of the corresponding values is shown in Table 10. Note that this table also includes the additional magnitudes $++$ and $--$ to denote categorical changes and influences discussed in Section
.

Using this approach, orders of magnitude for multiplication of two quantities p_1 and p_2 is given by (see (Nayak 1992)):

$$om(p_1 \times p_2) = [om(p_1) + om(p_2), om(p_1) + om(p_2) + 1]$$

Table 10: A sample absolute order of magnitude scale

Id	Description	Range
++/--	complete positive/negative	1 or -1
0	very strong positive/negative	$[0.8, 1[$
1	strong-very strong positive/negative	$[0.64, 0.8[$
2	strong positive/negative	$[0.51, 0.64[$
3	moderate-strong postive/negative	$[0.41, 0.51[$
4	moderate positive/negative	$[0.33, 0.41[$
5	weakly moderate positive/negative	$[0.26, 0.33[$
6	weak-weakly moderate postive/negative	$[0.21, 0.26[$
7	weak positive/negative	$[0.17, 0.21[$
8	very weak postive/negative	$[0.13, 0.17[$



Magnitudes and non−monotic influence signs:

$|S(X_s, X_t)| = 0$
$|S(X_h, X_t)| = 2$

$|S_{x_a}(X_t, X_c)| = 5$
$|S_{\overline{x_a}}(X_t, X_c)| = ++$

$|S(X_c, X_m)| = 0$

$[S_{x_t}(X_s, X_p)] = +$ $\quad |S_{x_t}(X_s, X_p)| = 0$
$[S_{\overline{x_t}}(X_s, X_p)] = 0$
$[S_{x_t}(X_h, X_p)] = -$ $\quad |S_{x_t}(X_h, X_p)| = 5$
$[S_{\overline{x_t}}(X_h, X_p)] = 0$

Figure 6: QPN/QCN with logarithmic order of magnitude scale, categorical influences and non-monotonic influences

Integrating the refinements

The refinements discussed above can now be combined in the ongoing example. Figure 6 shows the resulting QPN/QCN with integrated non-monotonic influences, categorical influences and the logarithmic order of magnitude scale of Table 10. With this approach, the prosecution scenario corresponds to $|\Delta_p P(x_s)| = (++)$ and $|\Delta_p P(x_b)| = (--)$ and the defence scenario to $|\Delta_d P(x_h)| = (++)$ and $|\Delta_d P(x_b)| = (--)$. These values can be propagated as follows:

$$|\Delta_p P(x_t)| = (++) \otimes (0) = (0)$$
$$|\Delta_p P(x_a)| = (++) \otimes (++) = (++)$$

The latter result implies that the magnitude $|S_{x_a}(X_t, X_c)| = ++$ is in effect. Therefore,

$$|\Delta_p P(x_c)| = (++) \otimes (0) = (0)$$
$$|\Delta_p P(x_m)| = (0) \otimes (0) = (0, 1)$$

Similarly,

$$|\Delta_d P(x_t)| = (++) \otimes (2) = (2)$$
$$|\Delta_d P(x_a)| = (++) \otimes (++) = (++)$$
$$|\Delta_d P(x_c)| = (++) \otimes (2) = (2)$$
$$|\Delta_d P(x_m)| = (2) \otimes (0) = (2, 3)$$

This result indicates that obtaining blood matching the victim's dna from the suspect's clothes is somewhat more likely under the prosecution scenario than under the defence scenario, which is consistent with our intuition. Note that this outcome does not entail a claim regarding the extent to which the case of the prosecution is shown, as that is ultimately to be decided in court. However, in combination with the supporting network providing causal explanations, it captures all the information that is relevant regarding this piece of evidence and its support of the alternative hypotheses. As such, qualitative representations of this type constitute a suitable basis upon which decision support systems for legal evidential reasoning may be built.

Conclusions and Future Work

This paper has discussed qualitative approaches to probabilistic reasoning. It has examined the need for qualitative probabilistic reasoning and shown how qualitative probabilistic inference methods can be employed to perform analyses similar to that of Bayesian evidence evaluation. However, it has been clarified that while the level of abstraction employed in the qualitative probabilistic reasoning enables the generation of intuitive explanations to justify decisions, it also prevents Bayesian-like evidence evaluation. A critical survey has presented a range of extensions designed to improve the precision of qualitative probabilistic reasoning techniques while maintaining their explanation generation ability. This has identified number of features that Bayesian models designed for evidentiary reasoning exhibit. By means of small examples, it has been shown that the each of the extensions can effectively describe some of these features. Last but not least, the paper has shown how these extensions can be integrated with one another.

The ideas discussed herein can form the foundation for future applications that model the lines of inquiries of crime investigators, both for educational and case management software. As such, the development and analysis of such software constitutes an interesting area of future research. Apart from this, some important theoretical concerns remain. One of these relates to the potential availability of knowledge of varying degrees of precision about the probability distributions involved in the models. Especially at the early stages of an investigation, the investigators can employ a broad range of evidence types, including some that can benefit the investigation but may not be admissible in court. These include: hearsay, witness testimony, records and recordings, psychological profiles and, of course, the entire spectrum of physical evidence. The lack of knowledge regarding reliability and accuracy varies considerably between these different types of evidence. Thus, future works should examine if and how these can be represented and integrated in a single model.

Acknowledgments

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References

Aitken, C.; Taroni, F.; and Garbolino, P. 2003. A graphical model for the evaluation of cross-transfer evidence in dna profiles. *Theoretical Population Biology* 63:179–190.

Bex, F.; Prakken, H.; Reed, C.; and Walton, D. 2003. Towards a formal account of reasoning about evidence, argument schemes and generalizations. *Artificial Intelligence and Law* 11:125–165.

Biedermann, A., and Taroni, F. 2006. Bayesian networks and probabilistic reasoning about scientific evidence when there is a lack of data. *Forensic Science International* 157(2–3):163–167.

Cook, R.; Evett, I.; Jackson, G.; Jones, P.; and Lambert, J. 1998. A model for case assessment and interpretation. *Science and Justice* 38(3):151–156.

Dague, P. 1993. Symbolic reasoning with relative orders of magnitude. In *Proceedings of the 13th International Joint Conference on Artificial Intelligence*, 1509–1514.

Druzdzel, M., and Henrion, M. 1993. Efficient reasoning in qualitative probabilistic networks. In *Proceedings of the 11th Conference on Artificial Intelligence*, 548–553.

Druzdzel, M., and van der Gaag, L. 2000. Building probabilistic networks: where do the numbers come from? *IEEE Transactions on Knowledge and Data Engineering* 12(4):481–486.

Halliwell, J., and Shen, Q. 2002. Towards a linguistic probability theory. In *Proceedings of the 11th International Conference on Fuzzy Sets and Systems*, 596–601.

Kahneman, D.; Slovic, P.; and Tversky, A. 1985. *Judgement Under Uncertainty*. Cambridge University Press.

Keppens, J., and Schafer, B. 2006. Knowledge based crime scenario modelling. *Expert Systems with Applications* 30(2):203–222.

Keppens, J.; Shen, Q.; and Lee, M. 2005. Compositional bayesian modelling and its application to decision support in crime investigation. In *Proceedings of the 19th International Workshop on Qualitative Reasoning*, 138–148.

Lacave, C., and Díez, F. 2002. A review of explanation methods for bayesian networks. *The Knowledge Engineering Review* 17:107–127.

Mortera, J.; Dawid, A.; and Lauritzen, S. 2003. Probabilistic expert systems for dna mixture profiling. *Theoretical Population Biology* 63:191–205.

Nayak, P. 1992. Order of magnitude reasoning using logarithms. In *Proceedings of the International Conference on Principles of Knowledge Representation and Reasoning*, 201–210.

Parsons, S., and Mamdani, E. 1993. On reasoning in networks with qualitative uncertainty. In *Proceedings of the 9th Conference on Uncertainty in Artificial Intelligence*.

Parsons, S. 1995. Refining reasoning in qualitative probabilistic networks. In *Proceedings of the 11th Conference on Uncertainty in Artificial Intelligence*.

Parsons, S. 2003. Qualitative probability and order of magnitude reasoning. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* 11(3):373–390.

Parsons, S. 2004. On precise and correct qualitative probabilistic inference. *International Journal of Approximate Reasoning* 35(2):111–135.

Pearl, J. 1988. *Probabilistic reasoning in intelligent systems: networks of plausible inference*. Morgan-Kaufmann.

Raiman, O. 1991. Order of magnitude reasoning. *Artificial Intelligence* 51:11–38.

Renooij, S.; van der Gaag, L.; and Parsons, S. 2002. Context-specific sign-propagation in qualitative probabilistic networks. *Artificial Intelligence* 140:207–230.

Schum, D. 1994. *Evidential foundations of probabilistic reasoning*. John Wiley and Sons.

Wellman, M., and Henrion, M. 1993. Explaining explaining away. *IEEE Transactions on Pattern Analysis and Machine Intelligence* 15(3):287–292.

Wellman, M. 1990. Fundamental concepts of qualitative probabilistic networks. *Artificial Intelligence* 44:257–303.

Zimmer, A. 1983. Verbal vs. numerical processing of subjective probabilities. In Scholz, R., ed., *Decision making under uncertainty*. North-Holland, Amsterdam.

Zimmer, A. 1986. What uncertainty judgements can tell about the underlying subjective probabilities. In Kanal, L., and Lemmer, J., eds., *Uncertainty in Artificial Intelligence*. North-Holland, Amsterdam.