Commonsense Inference in Dynamic Spatial Systems
"Phenomenal and Reasoning Requirements"

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Abstract

Spatial changes within an environment are typically a result of interaction— actions and events — occurring within. Reasoning about such changes, when dealt with formally within the context of qualitative spatial calculi and logics of action and change, poses several difficulties along multiple dimensions: (a) phenomenal requirements stemming from the dynamic nature of the spatial system (e.g., appearing and disappearing objects), (b) reasoning requirements (e.g., abductive explanation), (c) domain-independent or epistemological (e.g., persistence, ramification), and (d) aspects concerning the need to satisfy the intrinsic (axiomatic) properties of the spatial calculi (e.g., compositional consistency) being modelled. This paper, encompassing the phenomenal and reasoning aspects in (a) and (b) respectively, presents some instances that demonstrate the role of commonsense reasoning and the non-monotonic inference patterns it necessitates whilst representing and reasoning about dynamic spatial systems in general.

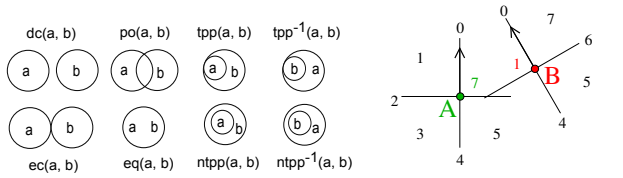
1 Motivation

Dynamic Spatial Systems ($\mathcal{DSS}$) are systems where spatial configurations, denoted by sets of qualitative spatial relations, undergo transformations as a result of actions and events occurring within the environment [Bhatt and Loke, 2008]. The $\mathcal{DSS}$ approach is applicable in a wide-range of application domains as diverse as cognitive robotics, diagrammatic reasoning, architecture design, geographical information systems and even the new generation of ambient intelligence systems involving behaviour or activity monitoring. From the viewpoint of such applications, the basic functionality required from the $\mathcal{DSS}$ approach remains the same, namely, the capability to serve either a predictive (i.e., projection, planning) or an explanatory (e.g., causal explanation) function in the context of high-level qualitative models of space and spatial change. This in turn requires that change in general and spatial change in specific, and its relationship to action, events and other aspects such as causality be taken seriously.

Existing qualitative spatial modelling techniques have primarily remained focused on reasoning with static spatial configurations. In general, research in the qualitative spatial reasoning domain has remained focused on the representational aspects of spatial information conceptualization and the construction of efficient computational apparatus for reasoning over those by the application of constraint-based techniques [Cohn and Renz, 2007, Renz and Nebel, 2007]. For instance, given a qualitative description of a spatial scene, it is possible to check for its consistency along arbitrary spatial domains (e.g., topology, orientation and so forth) in an efficient manner by considering the general properties of a qualitative calculus [Ligozat and Renz, 2004]. However, for applications such as the ones aforementioned, these methods require a realistic interpretation, such as the one provided by the stated $\mathcal{DSS}$ perspective, where sets of spatial relations undergo change as a result of named occurrences in the environment, or broadly, reasoning about space and reasoning about actions and change are consilidate into a 'Reasoning about Space, Actions and Change' (RSAC) paradigm [Bhatt, 2009]. Consequently, the formal embedding of arbitrary spatial calculi – whilst preserving their high-level axiomatic semantics and if necessary, their low-level algebraic properties too – has to be investigated from the viewpoint of formalisms that deal with action and change in general.

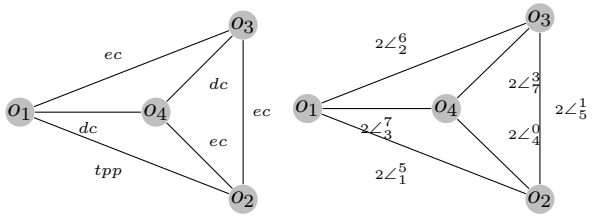
In this paper, we illustrate the utility of commonsense reasoning, and the non-monotonicity it entails, toward realising the suggested embedding of spatial calculi within general formalisms of action and change. Note that the embedding per se is extensive, and not the object of this paper. Rather, we solely focus on *some* commonsense inference patterns that occur whilst achieving the said embedding. These patterns pertain to the following aspects:

- **AI** existential consistency of complete spatial situation descriptions given that fact that the domain of discourse of primitive spatial entities may be incompletely known, i.e., unknown objects may have appeared or known objects may have either temporarily disappeared or may have permanently ceased to exist
- **AII** modelling causal explanation tasks, where given a set of temporally-ordered observations, the objective is to derive an explanation in terms of the (spatial and non-spatial) actions and events that may have caused the observations. Here, modelling causal explanation abductively necesssitates the use of a circumscriptive non-



(a) $\mathcal{RCC}-8$ Primitives (b) The $\mathcal{OPRA}_2$ relation $\vec{A}\ {}_2\angle^1_7\ \vec{B}$

Figure 1: Topological and Orientation Calculi



(a) $\mathcal{RCC}-8$ (b) $\mathcal{OPRA}_2$

Figure 2: Complete N-Clique Descriptions

monotonic approach.

In comparision to the other epistemological and intrinsic spatial calculi related commonsense patterns, which are excluded from this paper, the aspects in (AI–AII) are extrinsic to the process of embedding and concern phenomenal aspects (section 3) of dynamic spatial systems and the computational or reasoning requirements (section 4) expected from an operationalization of the $\mathcal{DSS}$ perspective.

2 Qualitative Spatial Primitives

The objective of this paper is to intuitively present the nature of commonsense reasoning as relevant to aspects (AI–AII; section 1). As such, we do not go into the details of the formal axiomatisation of a theory of change or the details pertaining to the constitution of a qualitative spatial calculus. However, a basic overview of the ontological setup is needed to make the paper self-contained.

The spatial ontology that is required depends on the nature of the spatial calculus that is being modeled. In general, spatial calculi can be classified into two groups: topological and positional calculi. When a topological calculus such as the Region Connection Calculus (RCC) [Randell et al., 1992] is being modeled, the primitive entities are spatially extended and could possibly even be 4D spatio-temporal histories (e.g., in a domain involving the analyses of motion-patterns). Alternately, within a dynamic domain involving translational motion in a plane, a point-based (e.g., Double Cross Calculus [Freksa, 1992], $\mathcal{OPRA}_m$ [Moratz, 2006]) or line-segment based (e.g., Dipole Calculus [Schlieder, 1995, Moratz et al., 2000]) abstraction with orientation calculi suffices. Figure 1(a) is a 2D illustration of relations of the RCC-8 fragment of the region connection calculus. This fragment consists of eight relations: disconnected (dc), externally connected (ec), partial overlap (po), equal (eq), tangential proper-part (tpp) and non-tangential proper-part ($ntpp$), and the inverse of the latter two tpp^{-1} and $ntpp^{-1}$. Similarly, Fig. 1(b) illustrates one primitive relationship for the Oriented Point Relation Algebra (OPRA) [Moratz, 2006], which is a spatial calculus consisting of oriented points (i.e., points with a direction parameter) as primitive entities. The granularity parameter m determines the number of angular sectors, i.e., the number of base relations. Applying a granularity of $m = 2$ results in 4 planar and 4 linear regions (Fig. 1(b)), numbered from 0 to 7, where region 0 coincides with the orientation of the point. The family of $\mathcal{OPRA}_m$ calculi are designed for reasoning about the relative orientation relations between oriented points and are well-suited for dealing with objects that have an intrinsic front or move in a particular direction.

Spatial Situation Descriptions

Spatial situation descriptions consist of a complete n-clique graph for a domain of n objects. Further, there is one such clique for every type of spatial domain (e.g., topology, orientation) that is modelled. Precisely, for a spatial scene description with n domain objects and k spatial calculi being modeled, the scene description involving n objects requires a complete n-clique specification with $[n(n\ -1)/2]$ spatial relationships for each of the respective calculi (Fig. 2). Given such spatial scene descriptions, the following notion of existential consistency is definable:

Definition 2.1 ($\mathcal{E}$-Consistency). *A spatial scene description is $\mathcal{E}$-Consistent, i.e., existentially consistent, if there exists at least one spatial relationship of any spatial domain (i.e., topology, orientation etc) that every existing spatial object participates in with other existing object(s).* □

From the viewpoint of planning and explanation tasks, $\mathcal{E}$-Consistency is necessary and useful toward maintaining the consistency of spatial scene descriptions given the fact that appearance of new objects and disappearance of existing ones may have occurred within the system. In the context of such phenomenal requirements, the significance and use of $\mathcal{E}$-Consistency from Definition 2.1 is further elaborated on in section 3.

3 Phenomenal Commonsense: Appearance and Disappearance of Objects

Appearance of new objects and disappearance of existing ones, either abruptly or explicitly formulated in the domain theory, is characteristic of non-trivial dynamic spatial systems. In robotic applications, it is necessary to introduce new objects into the model, since it is unlikely that a complete description of the robot's environment is either specifiable or even available. Similarly, it is also typical for a mobile robot operating in a dynamic environment, with limited perceptual or sensory capability, to lose track of certain objects because of issues such as noisy sensors or a limited field-of-vision. As an example, consider a '*delivery scenario*' in which a vehicle/robot is assigned the task of delivering '*object(s)*' from one '*way-station*' to another. In the initial situation description, the domain consists of a finite number of '*way-stations*' and deliverable '*objects*' (see Fig.

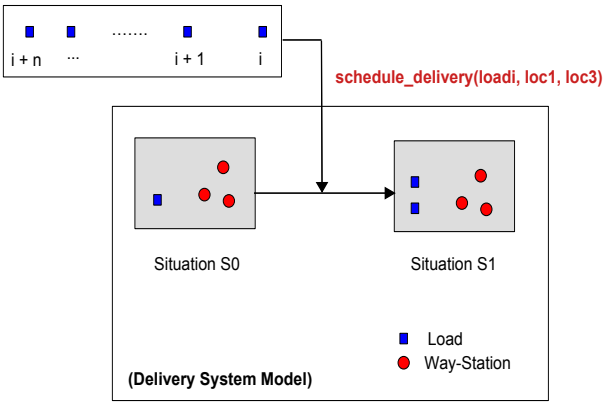


Figure 3: Appearance Events - Delivery Example

3). However, the scheduling of new objects for delivery in future situations will involve introducing new 'objects' into the domain theory. For example, an external event[1] such as '$schedule\_delivery(new\_load,\ loc_1,\ loc_3)$' introduces a new object, namely '*new_load*', into the domain.

Appearance and disappearance events involving the modification of the domain of discourse are not unique to applications in robotics. Even within the projected next-generation of event-based and temporal geographic information systems, appearance and disappearance events are regarded to be an important typological element for the modelling of dynamic geospatial processes [Claramunt and Thériault, 1995, Worboys, 2005]. For instance, Claramunt and Thériault [1995] identify the basic processes used to define a set of low-order spatio-temporal events which, among other things, include appearance and disappearance events as fundamental. Similarly, toward event-based models of dynamic geographic phenomena, Worboys [2005] suggests the use of appearance and disappearance events at least in so far as single object behaviours are concerned. We regard that such phenomena, being intrinsic to a typical dynamic spatial system, merit systematic treatment.

Maintaining and Propagating Existential Facts

The case of disappearance is not problematic, however, for the case of appearance and re-appearance, some questions that need to be addressed include:

- what is the spatial relationship (topological, directional etc) of the newly appearing object with other existing objects?
- given the fact that a newly appearing object is, from a model-theoretic viewpoint, *unknown* in the past, how to make it 'known' and 'not exist' in the past? (this scenario is illustrated model-theoretically in Fig. 5)
- how to make past and present situation descriptions 'compositionally consistent'?[2]

[1]External events are those occurrences that do not have an associated occurrence criteria and may therefore occur abruptly.

[2]Compositional consistency refers to the satisfaction of the global constraints formulated by composition theorems relevant to every spatial calculus that is modelled.

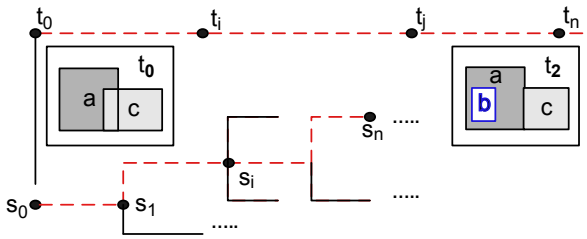


Figure 4: Branching-time Situation History

From a representational viewpoint, introducing new objects in the domain poses a problem since there is no general way to deal with an incompletely known domain of discourse. For instance, let $< s_0,\ s_1,\ s_2, \ldots,\ s_n >$ denote a situation-based linear history or one branch within the branching-tree structure of the overall situation space (see Fig. 4). From a dynamic spatial system perspective, each state corresponding to every situation with this history is primarily a set denoting the spatial configuration of objects in that situation. Further assume that an object 'b', that is unknown or not a part of the dynamic 'spatial configuration set' in the initial situation 's_0', comes into existence (by an appearance event) in a later situation, say 's_2'. At this point, it is necessary to incorporate the non-existence of 'b' in the situations preceding 's_2' by (non-monotonically) propagating its non-existence backwards into the situation-based history. In fact, appearance of previously unknown objects is the only reason '*existential facts*' about objects need to be included as propositional fluents / dynamic properties at a domain-independent level. The case of disappearing objects is trivial and simply involves negating and object's existential status upon the occurrence of disappearance events. Indeed, an object that is known but has disappeared may not participate in spatial relationships with other objects, until such a time when it reappears. The following steps summarise the solution approach for the case where an object's identity is maintained upon reappearance:

S1 firstly, maintain existential facts about objects by way of the propositional fluent $exists(o,\ s_1)$

S2 add special '*appearance*' and '*disappearance*' events that act on the existential fluent through direct effect axioms (i.e., disappearance *causes* an object to *not exist* and so forth)

S3 maintain '*null*' spatial relationships between non-existing objects and all other existing objects. Indeed, this also implies that such null relationships acquire a special status in the situation calculus being modelled. For instance, a calculus such as RCC-8 with eight spatial relationships becomes a calculus with nine primitive relationships.

S4 add a constraint that newly appearing objects must participate in at least one '*non-null*' spatial relationship with an already existing object. The precise relationship is specifiable in domain specific ways.

S5 finally, either apply predicate completion for $exists(\ldots)$ or minimizing it (to close its extensionality) on a situation-by-situation basis. This ensures that newly ap-

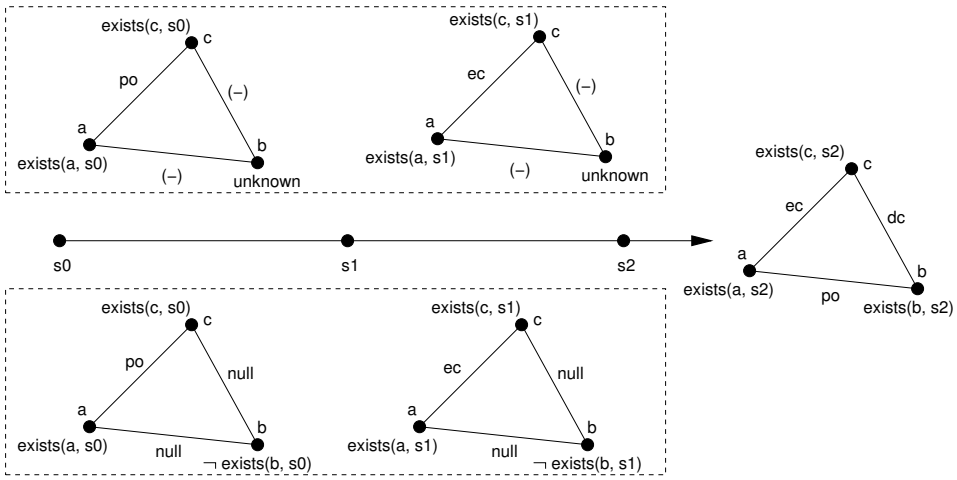


Figure 5: Appearance and Scene Descriptions

pearing objects are by default assumed to 'not exist' in the entire history of the system

Model-Theoretic Implications

In a strictly model-theoretic sense, *appearances* and *disappearances* should respectively correspond to the addition and removal of typed-entities, which in this case are spatial objects, from the underlying domain of discourse.[3] Whereas this is true in the case of the manner in which we model appearances, the same does not hold for disappearances. Strictly speaking, a *disappearance* does not lead to the removal of the entity from the domain in a model-theoretic sense. It simply modifies the object's existential property in a way such that:

- its spatial relationships with all other objects assume the value of $null$. This is easily achievable via the inclusion appropriate effect axioms for the propositional fluent *exists*.
- it cannot participate in subsequent spatial transformations that the system undergoes. This restriction is enforced by compiling the relevant existential preconditions for all relevant/potential occurrences

Note that it is also possible that a previously-disappeared object may re-appear in a later situation. In this case, instead of introducing a new object into the model, this is merely a case of modifying the existential fact about the concerned object. Again, this is achieved via the effect axioms. However, the spatial relationship of the new or re-appearing object with other existing objects cannot be $null$. An alternative approach could have been to not make any assumption with regard to the relationship of the new object with other objects until such information becomes available in a future situation and for as long as such information does not become available, its relationship with other objects will continue to be $null$ because of the default assumption of *inertia*. However, allowing this behaviour leads to '*existential inconsistencies*' where there exist situations in which a object *exists* and does not participate in any (qualitative) spatial relationship with any other object, which is clearly a situation that cannot arise in reality, i.e., the coming into existence of an object has to be based on some real observation (e.g., in robotics applications) or from some othe rsource of data (e.g., GIS dataset).

[3]Whether such a truly general solution is achievable model-theoretically remains doubtful. The approach we suggest is at least applicable in the present context of modelling dynamic spatial systems and modelling the *appearances* and *disappearances* thereof.

4 Explanatory Commonsense: Reasoning Requirement

Explanation tasks constitute a basic reasoning requirement in many application domains. Here, given a set of time-stamped observations or snap-shots (e.g., observation of a mobile-robot or time-stamped GIS data), the objective is to explain which events and/or actions may have caused the resulting state-of-affairs. From a rather general viewpoint, explanatory reasoning encompasses all problems resembling the classic '*stolen-car scenario*'. Explanation, in general, is regarded as a converse operation to temporal projection essentially involving reasoning from effects to causes, i.e., reasoning about the past [Shanahan, 1989]. In the context of the situation calculus formalism [McCarthy, 1977], which is a general formalism for modelling dynamic domains, Shanahan [1993, 1997] proposes a non-monotonic approach that utilises circumscription as a basis of minimization (of effects) and explanation derivation (in terms of potential occurrences). We have specialised this approach toward the formulation of an abductive occurrence-driven causal explanation task, where a set of time-ordered observations (e.g., pertaining to spatial configurations) may be explained in terms of the spatial actions and events that may have caused the observed state-of-affairs.

Let $\mathcal{L}$ denote a first-order many-sorted language with equality and the usual alphabet of logical symbols $\{\neg, \wedge, \vee, \forall, \exists, \supset, \equiv\}$.[4] With $\mathcal{L}$ as a basis, a situation calculus meta-theory Σ_{sit} required from the viewpoint of the causal explanation task in [Bhatt and Loke, 2008] is adopted:

[4]Although the $\mathcal{L}$ requires additional predicates, such details are not relevant here any may be found in [Bhatt and Loke, 2008].

Definition 4.1 (Theory of Space & Change: $\Sigma_{sit} \cup \Sigma_{space}$). *The foundational theory Σ_{sit} of the situation calculus formalism consists of the following set of formulae: the property causation axiom determining the relationship between being 'caused' and being 'true', a generic frame axiom in order to incorporate the assumption of inertia, uniqueness of names axioms for the fluents, occurrences and fluent denotations, and domain closure axioms for propositional and functional fluents. Σ_{space} constitutes a formalisation of the general aspects pertaining to the static and dynamic aspects of spatial calculi. Σ_{space} essentially denotes a general spatial theory that can be re-used in arbitrary dynamic spatial domains.* □

With respect to a basic theory of space and change in Definition 4.1 that accounts for causation, inertia and ramification, and a qualitative spatial theory, we present the general structure of commonsense reasoning involved in abducing an object's appearance for a simple scenario.

Structure of Causal Explanation WRT. $[\Sigma_{sit} \cup \Sigma_{space}]$
We outline the structure of the causal explanation task without going into the details of the underlying/supporting axiomatisation: 'consider again the illustration in Fig. 4 – the situation-based history $< s_0, s_1, \ldots, s_n >$ represents one path, corresponding to a actual time-line $< t_0, t_1, \ldots, t_n >$, within the overall branching-tree structured situational space. Furthermore, assume a simple system consisting of objects 'a', 'b' and 'c' and also that the state of the system is available at time-point t_0 and t_2. Note that the situational-path and the time-line represent an actual as opposed to a hypothetical evolution of the system. From the viewpoint of this discussion, two auxiliary predicates, namely $HoldsAt(\phi, t)$ and $Happens(\theta, t)$, that range over '*time-points*' instead of '*situations*' are needed to accommodate the temporal extensions required to map a path in the situation-space to an actual time-line; complete definitions can be found in Pinto [1994]. Given an initial situation description as in Φ_1 (see (1)), where 'b' is unknown and 'a' and 'c' are partially overlapping, in order to explain an observation sentence such as Φ_2, a formula of the form in Δ needs to be derived'.

$$\left\{\begin{array}{l} \Phi_1 \equiv HoldsAt(\phi_{top}(a, c), po, t_1) \\ \Phi_2 \equiv HoldsAt(\phi_{top}(a, c), ec, t_2) \wedge HoldsAt(exists(b), true, t_2) \\ \qquad \wedge HoldsAt(\phi_{top}(b, a), ntpp, t_2) \\ [\Sigma_{sit} \wedge \Sigma_{space} \wedge \Phi_1 \wedge \Delta] \models \Phi_2, where \\ \Delta \equiv (\exists\, t_i, t_j, t_k).[t_1 \leq t_i < t_2 \wedge Happens(appearance(b), t_i)] \\ \wedge\, [t_i < t_j < t_2 \wedge Happens(tran(b, a, tpp), t_j)] \wedge \\ [t_k < t_2 \wedge Happens(tran(a, c, ec), t_k)] \wedge [t_k \neq t_i \wedge t_k \neq t_j] \end{array}\right\} \quad (1)$$

The derivation of Δ primarily involves non-monotonic reasoning in the form of minimising change and abducing appearance, in addition to making the usual default assumptions about inertia; the details of the derivation may be found in [Bhatt and Loke, 2008].

Domain-Specific Heuristics in Abduction
The non-monotonicity required in modelling explanation tasks is characteristic to modelling explanation problems abductively in general, rather than being peculiar to spatial reasoning tasks. However, one aspect of this non-monotonicity is characteristic to a spatial reasoning task – in deriving minimal models or explanations of observations consisting of changing spatial configurations, it is possible that the derived explanations may be *inadequate*, i.e., may not include domain-specific occurrences that have caused the observed changes. For instance, consider a geographic information system domain / scenario as depicted in Fig. 6. At a domain-independent level (i.e., at the level of a general spatial theory), the scene may be described using topological and qualitative size relationships. Consequently, the only changes that are identifiable at the level of the spatial theory are *shrinkage* and eventual *disappearance* – this is because a domain-independent spatial theory may only include a generic typology (appearance, disappearance, growth, shrinkage, deformation, splitting, merging etc) of spatial change at the most. However, at a domain-specific level, these changes could characterize a specific event (or process) such as, for instance, *deforestation*. The hypotheses or explanations that are generated during a explanation process should necessarily consist of the domain-level occurrences in addition to the underlying (associated) spatial changes (as per the generic typology) that are identifiable. That is to say, that the derived explanations be '*adequate*' and more or less take a form such as: '*Between time-points t_i and t_i, the process of deforestation is abducible as one potential hypothesis*'. To achieve this adequacy, a model-filtration heuristic that disregards those models (i.e., explanations) that do not include any domain-specific (spatial) occurrences (actions or events) leads to explanations that are adequate, if such explanation exists per se – this is because minimal models that only consist of a domain-independent explanation (e.g., in the form of *shrinkage*, *disappearance* and a temporal-order between these two) would be excluded by such a filtration heuristic.

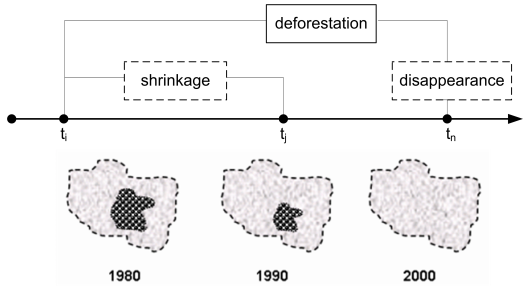


Figure 6: Domain Specific and Independent Abduction

Other potential solution to achieve adequacy is to include high-level or domain-specific predicates that relate the domain-independent occurrences (as per the typology) to arbitrary high-level processes that have a domain-dependent interpretation. Notwithstanding the fact that we regard both potential solutions to the problem of achieving adequacy to be rather rudimentary or ad-hoc solutions, it must be pointed out that the model-filtration approach is more general and does not presuppose any information of the domain-independent typology on the part of a domain modeler.

5 Discussion and Outlook

Qualitative spatial methods have primarily remained focused on reasoning with static spatial configurations. However, for applications such as cognitive robotics, these methods require a more realistic interpretation, where sets of spatial relations undergo change as a result of named occurrences in the environment. Consequently, the formal embedding of arbitrary spatial calculi – whilst preserving their high-level axiomatic semantics and low-level algebraic properties – has to be investigated from the viewpoint of formalisms such as the situation calculus, event calculus and fluent calculus. At a higher level of abstraction, this will result in the (native) incorporation of commonsense notions of space and spatial change within languages such as GOLOG and FLUX for their use in arbitrary robot control domains. In general, the areas of commonsense reasoning, and action and change are mature and established tools, formalisms and languages from therein are general enough to be applied to the case of dynamic spatial systems, where relational spatial models undergo change as a result of interaction in the environment.

The commonsense reasoning patterns pertaining to spatial reasoning illustrated in this paper have been investigated in the context of operationalizing the $\mathcal{DSS}$ perspective within situation calculus [Bhatt and Loke, 2008]. This constitutes one approach to operationalize the reasoning about space, actions and change paradigm [Bhatt, 2009]. Closely related is the work of Davis [2008, 2009] that investigates the use of commonsense reasoning about the physical properties of objects within a first-order logical framework. The key highlight of this work is that it combines commonsense qualitative reasoning about 'continuous time, Euclidean space, commonsense dynamics of solid objects, and semantics of partially specified plans' Davis [2009]. Other formalizations such as within a belief revision framework [Alchourrón et al., 1985], nonmonotonic causal formalizations in the manner of [Giunchiglia et al., 2004] are possible and the subject of ongoing study. Additionally, the suitability of event calculus [Kowalski and Sergot, 1986] and fluent calculus [Thielscher, 1998], vis-à-vis the situation calculus at least for specific reasoning tasks or scenarios is also a topic worth investigating.

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