PDL for Qualitative Reasoning about moving objects. A first step

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Abstract

We propose an initial approach to the development of a general logic framework for a qualitative description of the movement of objects. To this end, we represent the movement of an object with respect to another with different qualitative labels such as velocity, orientation, relative movement, allowed directions, relative longitude and relative latitude. The use of PDL allows us to construct new relations between moving objects and the use of a language very close to programming languages. Some of the advantages of our approach are explained on the basis of examples.

1 Introduction

Qualitative reasoning, QR, has many advantages for raising or answering questions about moving objects. Human beings do not need quantitative information for moving themselves, even for moving other objects such a cars, though this information could be helpful. This seems to imply that the combination of quantitative and qualitative information could be the key to deal with moving objects. Sometimes the use of many quantitative information may cause information overload, that is, more information has to be handled than can be processed [Delafontaine *et al.*, 2011].

Several papers have been published about moving objects from a qualitative point of view (see, for example [Cohn and Renz, 2007; Liu *et al.*, 2009; Liu *et al.*, 2008; Zimmermann and Freksa, 1996]) which try to make progress in the development of qualitative kinematics models, as studied in [Forbus *et al.*, 1987; Nielsen, 1988; Faltings, 1992]. The problem of the relative movement of one physical object with respect to another can been treated by the Region Connection Calculus [Randell *et al.*, 1992] and the Qualitative Trajectory Calculus [Van de Weghe *et al.*, 2005; Delafontaine *et al.*, 2011]. However, to the best of our knowledge, the only paper with introduces a logic framework to manage qualitative velocity is [Burrieza *et al.*, 2011].

The use of logic in QR, as in other areas of AI, improves the capability of formal representation of problems and provides insights into their most suitable solving methods. As examples of logics for order of magnitude reasoning in the sense of [Raiman, 1991; Travé-Massuyès *et al.*, 2005], see [Burrieza *et al.*, 2007; Burrieza *et al.*, 2010]; a theorem prover for one of these logics can be seen in [Golińska-Pilarek and Muñoz-Velasco, 2009], which has been implemented in [Golińska-Pilarek *et al.*, 2008].

In this paper, we continue the line of [Burrieza *et al.*, 2011] in order to construct a logic framework for reasoning with qualitative movement. Propositional Dynamic Logic, PDL, provides the possibility of constructing complex relations from simpler ones and the use of a language very close to programming languages. Some applications of PDL in AI can be seen in [van Benthem *et al.*, 2006; Bugaychenko and Soloviev, 2007; Bollig *et al.*, 2007]. We propose the representation of the movement of an object with respect to another with different labels such as velocity, orientation, relative movement, possible directions and relative position. The values of these labels are given by different qualitative values, and the granularity can be changed depending on the problem in question. As in [Burrieza *et al.*, 2011], we consider labels for the (module of the) velocity and its orientation but we enrich this information by adding other labels. Thus, we use the ideas presented in [Delafontaine *et al.*, 2011] for representing the relative movement of an object with respect to another with pairs of type $(+.-)$, meaning, in this case, that the first object is moving *away* from the second one, and the second object is moving *towards* the first one. We include also a temporal component to inform about what is happening in the present and what *could/should* happen in the future, and some labels to represent the allowed movements, the relative longitude and the relative latitude. The use of all these labels together is original in this paper, and provides very important information, as we will see below. As stated above, PDL is useful for constructing complex relations from simpler ones, for example, by using the composition operation, if we have information about object A_i moving with respect to A_j, and A_j with respect to A_k, we can *infer* information about the movement of A_i with respect to A_k. Another advantage of PDL, the use of a language very close to programming, can be exploited for dealing with automatized movements, such as robots or for navigation systems, such as GPS.

The paper is organized as follows. In Section 2, we introduce informally the definitions needed to construct our logic. In Section 3, the formal presentation of the proposed logic is given. Finally, some conclusions and future works are discussed in Section 4.

2 Preliminary definitions

We represent the movement of an object A_i with respect to A_j by a tuple $(x_1; \ldots; x_8) \in L$, being $L = L_1 \times \ldots \times L_8$ defined as follows. As some of the sets L_i are defined also by a cartesian product, for an easy reading we will eliminate some parenthesis by using ";" to indicate the eight components of our label, while we will use "," for the components of each L_i. The set $L_1 = \{\mathsf{P}, \mathsf{F}\}$ represents a temporal component, where P represents the situation at the *present*, and F the situation at the (closest) *future*. A set $L_2 = \mathcal{N} \times \mathcal{N}$, being $\mathcal{N} = \{A_1, \ldots, A_k\}$, with $k \in \mathbb{N}$, where the pair (A_i, A_j) represents the movement of object A_i with respect to object A_j. Let us consider also the set of qualitative velocities $L_3 = 2^{\{\mathsf{z},\mathsf{v}_1,\mathsf{v}_2,\mathsf{v}_3\}}$ [1], where $\mathsf{z}, \mathsf{v}_1, \mathsf{v}_2, \mathsf{v}_3$ represent *zero*, *slow*, *normal* and *quick*, respectively; and the set of qualitative orientations is $L_4 = 2^{\{\mathsf{n},\mathsf{o}_1,\mathsf{o}_2,\mathsf{o}_3,\mathsf{o}_4\}}$, where $\mathsf{n}, \mathsf{o}_1, \mathsf{o}_2, \mathsf{o}_3, \mathsf{o}_4$ represent, respectively, *none*, *North*, *South*, *East* and *West* orientations. For example, $(\mathsf{v}_1\mathsf{v}_2, \mathsf{o}_4)$ means a slow or normal velocity towards the West. Following the ideas from [Delafontaine *et al.*, 2011], we consider the set $M = \{\varnothing, 0, -, +\}$ and $L_5 = M \times M$ in order to represent the relative movement, where $\varnothing, 0, -, +$ means *lack of information*, *stable*, *moving towards* and *moving away*. For example, $(0, +)$ means A_i is stable with respect to A_j, while A_j is moving away from A_i [2]. We also consider the set L_6 to represent the allowed movements, and the sets L_7 and L_8 for representing the qualitative latitude and longitude of A_i with respect to A_j. The set L_6 coincides with L_4, but it has now a different interpretation: it represents the possible directions that object A_i can follow, this is suited for movements in a network, as presented in [Delafontaine *et al.*, 2011]. For example, label $\mathsf{z}\mathsf{o}_1\mathsf{o}_2\mathsf{o}_3$ means that A_i can only move either towards to the North, or the South, or the East, because the West direction is not allowed. Set $L_7 = 2^{\{\mathsf{n},\mathsf{o}_1,\mathsf{o}_2\}} \times 2^{\{\mathsf{z},\mathsf{d}_1,\mathsf{d}_2,\mathsf{d}_3\}}$ means the *North-South* position and the distance, where $\mathsf{z}, \mathsf{d}_1, \mathsf{d}_2, \mathsf{d}_3$ mean *zero, close, normal*, and *distant*. For example, $(\mathsf{o}_1, \mathsf{d}_1)$ means that A_i is close and to the North from A_j, and $(\mathsf{o}_1\mathsf{o}_2, \mathsf{d}_2)$ that A_i can be either close and to the North or close and to the South from A_j. Similarly, $L_8 = 2^{\{\mathsf{n},\mathsf{o}_3,\mathsf{o}_4\}} \times 2^{\{\mathsf{z},\mathsf{d}_1,\mathsf{d}_2,\mathsf{d}_3\}}$ means the *East-West* position, for example, $(\mathsf{n}, \mathsf{d}_2)$ means that A_i is at a normal distance but neither to the East nor to the West from B.

The *composition* of a movement of A_i with respect to A_j and a movement of A_j with respect to A_k provides information about the movement of A_i with respect to A_k, and will be defined below.

In the following examples, we present informally some of the advantages of our approach. The situation of some policemen chasing a gangster inspired in the example presented in [Delafontaine *et al.*, 2011].

Example 1 *Consider the situation of Figure 1, where two policemen A_1 and A_2 are chasing a gangster A_3. Suppose that A_1 and A_2 know their relative position with respect to each other, whereas only A_2 has information about the movement of A_3. Label $(\mathsf{P}; \mathsf{A}_3, \mathsf{A}_2; \mathsf{v}_3; \mathsf{o}_3; +, -; \mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_1, \mathsf{d}_1; \mathsf{n}, \mathsf{z})$ represents that, at the present, A_3 is moving with respect to A_2 with a quick velocity (v_3) towards the East (o_3), being A_3 moving away A_2 ($+$), while A_2 is moving towards A_3 ($-$). Moreover, A_3 can move only towards South, East and West ($\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4$), because the North street is closed off. A_3 is close and to the North from A_2 ($\mathsf{o}_1, \mathsf{d}_1$), and it is neither to the East nor to the West from A_2 (n, z). Analogously, label $(\mathsf{P}; A_2, A_1; \mathsf{v}_3; \mathsf{o}_1; -, 0; \mathsf{o}_1\mathsf{o}_4; \mathsf{o}_2, \mathsf{d}_2; \mathsf{o}_4, \mathsf{d}_2)$ represents that, at the present, A_2 is moving with respect to A_1 with a quick velocity towards the North, being A_2 moving towards A_1, and A_1 is stable with respect to A_2. Moreover, A_2 can move only towards North, East and West. A_2 is at a normal distance and to the South, and at a normal distance to the West with respect to A_1. In this case, the composition of both movements should be $(\mathsf{P}; A_3, A_1; \mathsf{v}_3; \mathsf{o}_3; -, 0; \mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_2, \mathsf{d}_1\mathsf{d}_2; \mathsf{o}_4, \mathsf{d}_2)$. As A_1 is chasing A_3, and the street to the South of A_1 is closed off, the future movement of A_1 with respect to A_3 has to be $(\mathsf{F}; A_1, A_3; \mathsf{v}_3; \mathsf{o}_4; -, -; \mathsf{o}_4; \mathsf{o}_1, \mathsf{d}_1\mathsf{d}_2; \mathsf{o}_3, \mathsf{d}_2)$, that is, a quick velocity towards the West, A_1 and A_3 moving towards each other, A_1 can move only towards the West, A_1 is at the close or at a normal distance and to the North, and at a normal distance to the East with respect to A_3.*

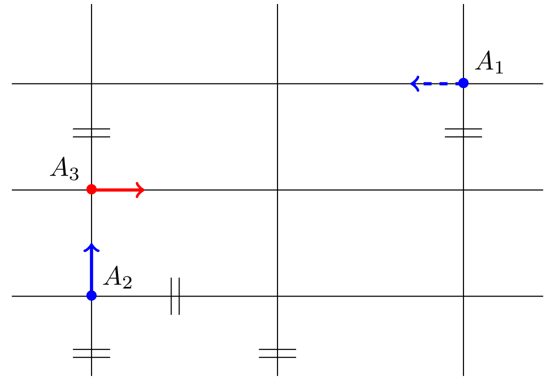

Figure 1: A_1 and A_2 chasing A_3

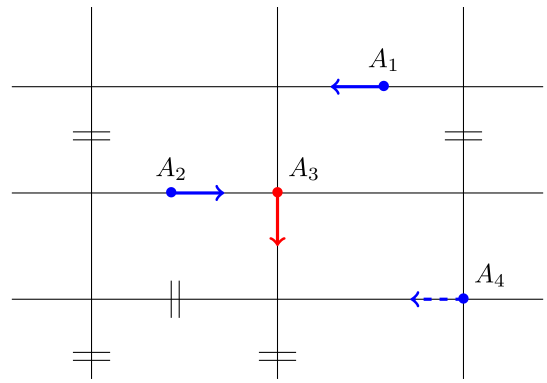

Figure 2: A_1, A_2 and A_4 chasing A_3

Example 2 *Suppose now that we have the situation of Figure 2, where A_3 is in a crossroad and decides to go to*

[1] For any set X, we denote by 2^X the set of subsets of X.

[2] This consideration suggests an underlying external reference system.

the South; moreover, there is a new policeman A_4 awaiting for orders. The movement of A_3 with respect to A_2, is given by $(\mathsf{P}; A_3, A_2; \mathsf{v}_3; \mathsf{o}_2; +, -; \mathsf{o}_1\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{n}, \mathsf{z}; \mathsf{o}_3, \mathsf{d}_1)$. *The movement of A_2 with respect to A_1 is given by* $(\mathsf{P}; A_2, A_1; \mathsf{v}_3; \mathsf{o}_3; -, -; \mathsf{o}_3; \mathsf{o}_2, \mathsf{d}_1\mathsf{d}_2; \mathsf{o}_4, \mathsf{d}_1\mathsf{d}_2)$; *and the movement of A_4 with respect to A_2 is given by* $(\mathsf{P}; A_4, A_2; \mathsf{z}; \mathsf{n}; 0, -; \mathsf{o}_1\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_2, \mathsf{d}_1; \mathsf{o}_3, \mathsf{d}_2)$. *We can compose these movements to obtain information about the movement of A_3 with respect to A_1, being* $(\mathsf{P}; A_3, A_1; \mathsf{v}_3; \mathsf{o}_2; +, -; \mathsf{o}_1\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_2, \mathsf{d}_2; \mathsf{o}_4, \mathsf{d}_1\mathsf{d}_2)$, *and the movement of A_4 with respect to A_1:* $(\mathsf{P}; A_4, A_1; \mathsf{z}; \mathsf{n}; 0, +; \mathsf{o}_1\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_2, \mathsf{d}_2; \mathsf{o}_3, \mathsf{zd}_1)$. *Then, as a consequence of the previous assumptions, in the next crossroad (future), the movement of A_1 with respect to A_3 has to be* $(\mathsf{F}; A_1, A_3; \mathsf{v}_3; \mathsf{o}_2; +, -; \mathsf{o}_1\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_1, \mathsf{d}_1\mathsf{d}_2; \mathsf{n}, \mathsf{z})$, *the future movement of A_4 with respect to A_3 has to be* $(\mathsf{F}; A_4, A_3; \mathsf{v}_3; \mathsf{o}_4; -, -; \mathsf{o}_1\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_2, \mathsf{d}_1; \mathsf{o}_3, \mathsf{d}_1)$, *and the movement of A_2 with respect to A_3 in the following crossroad has to be* $(\mathsf{F}; A_2, A_3; \mathsf{v}_3; \mathsf{o}_2; -, +; \mathsf{o}_1\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_1, \mathsf{d}_1; \mathsf{n}, \mathsf{z})$ *and the chase is over, because the distance between A_2 and A_3 will be either zero or small and, as A_4 is moving to the West, there are no possibilities to run away for A_3.*

3 The logic

In this section, we introduce formally our logic. As it is an extension of PDL, some of the information presented is general for this type of logic [Harel *et al.*, 2000]. However, we will try to focus our attention in the specific components of our approach.

3.1 Syntax and Semantics

In order to introduce the language of our logic, we consider a set of formulas Φ and a set of programs Π, which are defined recursively on disjoint sets Φ_0 and Π_0, respectively. Φ_0 is called the set of *atomic formulas* which can be thought of as abstractions of properties of states. Similarly, Π_0 is called the set of *atomic programs* which are intended to represent basic instructions.

Formulas:

- $\Phi_0 = \mathbb{V} \cup \mathbb{C}$, where $\mathbb{V}$ is a denumerable set consisting of propositional variables and $\mathbb{C} = L_1 \times \ldots \times L_k$, where $L_1, \ldots, L_k$ are intended to represent finite sets of labels.
- If φ and ψ are formulas and a is a program, then $\varphi \to \psi$ (propositional implication), $\perp$ (propositional falsity) and $[a]\varphi$ (program necessity) are also formulas. As usual, $\vee$ and $\wedge$ represent logical disjunction and conjunction, respectively; whereas $\langle a \rangle$ represents program possibility.

Notice that, as the elements of $\mathbb{C}$ have k components, we could consider different spatial components, such as position, distance, cardinal directions, etc. In our approach, we consider $k = 8$, but this situation could be modified depending on the problem in question.

Programs:

- $\Pi_0 = \{\otimes_\star \mid \star \in \mathbb{C}\}$, a set of specific programs.
- If a and b are programs and φ is a formula, then $(a; b)$ ("do a followed by b"), $a \cup b$ ("do either a or b, nondeterministically"), a^* ("repeat a a nondeterministically chosen finite number of times") and $\varphi?$ ("proceed if φ is true, else fail") are also programs.

Example 3 *In the situation of Figure 1, if we denote* $\mathsf{C}^{\mathsf{P}}_{3,2} = (\mathsf{P}; \mathsf{A}_3, \mathsf{A}_2; \mathsf{v}_3; \mathsf{o}_3; +, -; \mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_1, \mathsf{d}_1; \mathsf{n}, \mathsf{z})$, $\mathsf{C}^{\mathsf{P}}_{2,1} = (\mathsf{P}; A_2, A_1; \mathsf{v}_3; \mathsf{o}_1; -, 0; \mathsf{o}_1\mathsf{o}_4; \mathsf{o}_2, \mathsf{d}_2; \mathsf{o}_4, \mathsf{d}_2)$, *and* $\mathsf{C}^{\mathsf{P}}_{3,1} = (\mathsf{P}; A_3, A_1; \mathsf{v}_3; \mathsf{o}_3; -, 0; \mathsf{o}_2\mathsf{o}_3\mathsf{o}_4; \mathsf{o}_2, \mathsf{d}_1\mathsf{d}_2; \mathsf{o}_4, \mathsf{d}_2)$, *we can use our language to express the composition of the movements of A_3 with respect to A_2 and A_2 with respect to A_1 by the formula* $\mathsf{C}^{\mathsf{P}}_{3,2} \to [\otimes_{\mathsf{C}^{\mathsf{P}}_{2,1}}]\mathsf{C}^{\mathsf{P}}_{3,1}$. *This formula means that if the movement of A_3 with respect to A_2 is represented by $\mathsf{C}^{\mathsf{P}}_{3,2}$, for every composition with a movement of A_2 with respect to A_1 represented by $\mathsf{C}^{\mathsf{P}}_{2,1}$, we obtain a movement of A_3 with respect to A_1 represented by $\mathsf{C}^{\mathsf{P}}_{3,1}$. The conclusion for the future movement of A_1 with respect to A_3 can be expressed by the formula* $\mathsf{C}^{\mathsf{P}}_{3,1} \to \mathsf{C}^{\mathsf{F}}_{1,3}$, *being* $\mathsf{C}^{\mathsf{F}}_{1,3} = (\mathsf{F}; A_1, A_3; \mathsf{v}_3; \mathsf{o}_4; -, -; \mathsf{o}_4; \mathsf{o}_1, \mathsf{d}_1\mathsf{d}_2; \mathsf{o}_3, \mathsf{d}_2)$.

We now define the *semantics* of our logic. A *model* $\mathcal{M}$ is a tuple $(W_1 \times \ldots \times W_k, m)$ where each $W_i \subseteq L_i$, being L_i as defined above. In order to simplify the notation, from now on, we will use W instead of $W_1 \times \ldots \times W_k$. The states $u = (u_1, \ldots, u_8) \in W$ referred to, are to be understood as states of objects moving with respect to other objects. For example, the state $u = (\mathsf{P}, A_4, A_1, \mathsf{z}, \mathsf{n}, 0+, \mathsf{o}_1\mathsf{o}_2\mathsf{o}_3\mathsf{o}_4, \mathsf{o}_2, \mathsf{d}_2, \mathsf{o}_3, \mathsf{zd}_1)$ represents the movement of A_4 with respect to A_1 at the present. By abuse of notation, we will use the same symbols to represent the qualitative classes and its corresponding formulas. On the other hand, m is a meaning function such that $m(p) \subseteq W$, for every propositional variable, $m(\star) = \star$, for every $\star \in \mathbb{C}$ and $m(a) \subseteq W \times W$, for all atomic program a. The semantics of the specific programs in Π_0 depends on the required properties and will be explained later. Moreover, if φ and ψ are formulas and a, b are programs, then we have the following:

- $m(\varphi \to \psi) = (W \smallsetminus m(\varphi)) \cup m(\psi)$
- $m(\perp) = \varnothing$
- $m([a]\varphi) = \{w \in W : \text{for all } v \in W, \text{ if } (w, v) \in m(a) \text{ then } v \in m(\varphi)\}$
- $m(a \cup b) = m(a) \cup m(b)$
- $m(a; b) = m(a); m(b)$
- $m(a^*) = m(a)^*$ (reflexive and transitive closure of relation $m(a)$).
- $m(\varphi?) = \{(w, w) : w \in m(\varphi)\}$

Given a model $\mathcal{M} = (W, m)$, a formula φ is *true* in $u \in W$ whenever we have that $u \in m(\varphi)$. We say that φ is *satisfiable* if there exists $u \in W$ such as φ is true in u. Moreover, φ is *valid in a model* $\mathcal{M} = (W, m)$ if φ is true in all $u \in W$, that is, if $m(\varphi) = W$. Finally, φ is *valid* if φ is valid in all models.

The informal meaning of some formulas is given below. Let φ be any propositional formula, then:

- $\langle \mathsf{C}^{\mathsf{P}}_{3,2}? \rangle \varphi$ is true in u iff u is at the present a movement of A_3 with respect to A_2 labeled by $\mathsf{C}^{\mathsf{P}}_{3,2}$, and φ is true in u.
- $[\otimes_{\mathsf{C}^{\mathsf{P}}_{3,2}}; \otimes_{\mathsf{C}^{\mathsf{P}}_{2,1}}]\varphi$ is true in u iff for every movement u' obtained by composing u with a movement labeled by $\mathsf{C}^{\mathsf{P}}_{3,2}$, followed by a movement labeled by $\mathsf{C}^{\mathsf{P}}_{2,1}$, φ is true in u'.
- $[(\mathsf{C}^{\mathsf{P}}_{3,1}?; \mathsf{C}^{\mathsf{P}}_{1,3})^*; \neg\mathsf{C}^{\mathsf{P}}_{3,1}?]\neg\mathsf{C}^{\mathsf{P}}_{3,1}$ says that *while* the movement of A_3 with respect to A_1 is labeled by $\mathsf{C}^{\mathsf{P}}_{3,1}$, the movement of A_1 with respect to A_3 has to be $\mathsf{C}^{\mathsf{P}}_{1,3}$, because A_1 is chasing A_3.

Observe that, in the last two formulas, we use the advantages of PDL for expressing the programming command *while . . . do*.

We can construct the desired logic depending on the granularity and the specific properties required. Thus, we can express the properties for the composition as follows. Suppose that the movement of A_i with respect to A_j is represented by the label $(x_1, \dots, x_8) \in \mathbb{C}$, being $x_1 = y_1$ and $x_2 = A_i, A_j$. The movement of A_j with respect to A_k is represented by $(y_1, \dots, y_8)$, being $x_1 = y_1$ and $y_2 = A_j, A_k$. Then, the composition is the movement of A_i with respect to A_k represented by a label $(z_1, \dots, z_8)$, such that $z_1 = x_1$, $z_2 = A_i, A_k$, $z_3 = x_3$, $z_4 = x_4$, and $z_6 = x_6$. If $x_7 = (\mathsf{o_l}, \mathsf{d_r})$ and $y_7 = (\mathsf{o_m}, \mathsf{d_s})$ then z_7 can be obtained as follows:

1. If $l \neq m$, then
   (a) If $r < s$, $(\mathsf{o_m}, \mathsf{d_r d_s})$
   (b) If $r = s$, $(\mathsf{no_l o_m}, \mathsf{d_r})$
   (c) If $r > s$, $(\mathsf{o_l}, \mathsf{d_r d_s})$
2. If $l = m$, then
   (a) If $r < s$, $(\mathsf{o_l}, \mathsf{d_s d_{s+1}})$ [3]
   (b) If $r = s$, $(\mathsf{o_l}, \mathsf{d_r d_{r+1}})$
   (c) If $r > s$, $(\mathsf{o_l}, \mathsf{d_r d_{r+1}})$

For example, if $l = r = 1$ and $m = s = 2$, A_i is close to the North from A_j, and A_j is at a normal distance to the South from A_k. Hence, A_i is close or at a normal distance to the South from A_k. On the other hand, if $l = m = 2$ and $r = s = 3$, it means that if A_i is distant to the South from A_j, and A_j is at a distant to the South from A_k, then A_i is distant to the South from A_k. The same reasoning can be applied for obtaining z_8 from x_8 and y_8.

Finally, let us denote $x_5 = (x_5^1, x_5^2)$, and similarly for the rest of components. In order to obtain z_5 we reason as follows:

(1) If $x_5^1 = 0$, then $z_5^1 = 0$. Similarly, if $y_5^2 = 0$, then $z_5^2 = 0$.

(2) If $x_4 = \mathsf{o_r}$ with $\mathsf{r} \in 1, 2$ and $x_7^1 = \mathsf{o_s}$ then:
   (a) if $\mathsf{r} \neq \mathsf{s}$, then $z_5^1 = -$
   (b) if $\mathsf{r} = \mathsf{s}$, then $z_5^1 = +$

(3) If $x_4 = \mathsf{o_r}$ with $\mathsf{r} \in 3, 4$ and $x_8^1 = \mathsf{o_s}$ then:
   (a) if $\mathsf{r} \neq \mathsf{s}$, then $z_5^1 = -$
   (b) if $\mathsf{r} = \mathsf{s}$, then $z_5^1 = +$

[3] where $\mathsf{d_s d_{s+1}} = \mathsf{d_s}$ in the case $s = 3$, similarly for r.

The explanation of (1) is clear: for example, $x_5^1 = 0$ means that A_i is stable, so $z_5^1 = 0$. For (2) and (3), consider as an example $x_4 = \mathsf{o_1}$ and $x_7^1 = \mathsf{o_2}$, this means that A_i is moving to the North and it is to the South from A_k, this implies that A_i is moving towards A_j, as a consequence $z_5^1 = -$.

For simplicity in our approach, we consider that in the rest of cases $z_5 = (\varnothing, \varnothing)$, that is we do not have enough information. However, more information could be obtained, for example if we have information about the movement of A_k with respect to either A_i or A_j.

As stated in the introduction, our approach is flexible with respect to the different levels of granularity. That is, if we consider other qualitative classes to represent velocity and orientation, the same intuitive properties above may hold, because they are very general. As a consequence, the system will be easily extensible. If the system requires more properties, these have to be reflected in the syntax and semantics as well. The previous properties of the specific programs can be expressed semantically in the following general form. Given a model (W, m), for every $\mathsf{C_{i,j}} = (x_1, \dots, x_8)$, $\mathsf{C_{j,k}} = (y_1, \dots, y_8)$ and $\mathsf{C_{j,k}} = (z_1, \dots, z_8)$ defined as above we have:

$$m(\otimes_{\mathsf{C_{j,k}}})(\mathsf{C_{i,j}}) \subseteq m(\mathsf{C_{i,k}})$$

From a syntactical point of view, the conditions reflecting the required properties have to be included as axioms of our system. This situation is considered in the following section.

3.2 Axiom system

We introduce an axiom system in order to deal with the required properties presented in the previous section. Let us consider the following specific axioms.

Specific axiom schemata:

For every $\mathsf{C_{i,j}} = (x_1, \dots, x_8)$, $\mathsf{C_{j,k}} = (y_1, \dots, y_8)$ and $\mathsf{C_{j,k}} = (z_1, \dots, z_8)$ defined as above we have, and $\mathcal{F}$ a finite set of indexes determined by specific properties considered above:

S$_{\mathcal{F}}$ $\mathsf{C_{i,j}} \to [\otimes_{\mathsf{C_{j,k}}}]\mathsf{C_{i,k}}$

QE $\bigvee_{\star \in L} \star$ for every $\star \in L = L_1 \times \ldots \times L_8$

QU $\star \to \neg\#$ for every $\star \in L$ and $\# \in L \setminus \{\star\}$

The previous axioms have the following intuitive meaning:

- Family of axioms **S**$_{\mathcal{F}}$ reflect the specific properties assumed above.
- **QE** and **QU** mean the existence and uniqueness of the qualitative classes, respectively.

The rest of axioms are those specific to PDL.

Axiom schemata for PDL:

A1 All instances of tautologies of the propositional calculus.

A2 $[a](\varphi \to \psi) \to ([a]\varphi \to [a]\psi)$

A3 $[a](\varphi \land \psi) \to ([a]\varphi \land [a]\psi)$

A4 $[a \cup b]\varphi \to ([a]\varphi \lor [b]\varphi)$

A5 $[a; b]\varphi \to [a][b]\varphi$

A6 $[\varphi?]\psi \to (\varphi \to \psi)$

A7 $(\varphi \wedge [a][a^*]\varphi) \to [a^*]\varphi$

A8 $(\varphi \wedge [a^*](\varphi \to [a]\varphi)) \to [a^*]\varphi$ (induction axiom)

Inference Rules:
(MP) $\varphi, \varphi \to \psi \vdash \psi$ (*Modus Ponens*) **(G)** $\varphi \vdash [a]\varphi$ (generalization)

4 Conclusions and Future Work

We presented a first step in the development of a general logic framework for reasoning with qualitative movement. The movement of an object with respect to another has been represented with different qualitative labels such as velocity, orientation, relative movement, allowed directions relative longitude and relative latitude. Some of the advantages of PDL have been exploited, such as the use of composition for obtaining information about the relative movement of A_i with respect to A_k, from the information about the movements of A_i with respect to A_j and A_j with respect to A_k and the use of a language very close to programming.

As a future work, we are studying Soundness and Completeness of our system in the line of [Burrieza *et al.*, 2011]. We also consider the extension of our system by considering for example, temporal axioms which allow us to reason about the future from the information in the present. Last, but not least, we consider the construction of a theorem prover for our logic, as in [Golińska-Pilarek *et al.*, 2008].

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