RCC11: A Finer Topological Representation for the Alignment of Regions in Sketch Maps

Sahib Jan, Angela Schwering, Carl Schultz and Malumbo Chipofya
Institute for Geoinformatics, University of Muenster, Germany
sahib.jan|schwering|schultzc|mchipofya|@uni-muenster.de

Abstract

During the last two decades, dozens of qualitative representations have been proposed. These representations are motivated by a wide variety of applications of spatial data processing such as Geographical Information System (GIS), robotic navigation, and high level vision. For topological reasoning, the *Region Connected Calculus* (RCC) is perhaps the best-known formalism. The two algebras, RCC5 and RCC8 distinguish five and eight different topological relations. The different levels of granularity provide flexibility in the selection of representations suitable for different applications. In this paper, we propose RCC11, finer versions of the RCC in which topological relations are refined using the geometric point-set approach. If one region is tangential proper part of another or if two regions externally connect then their topological relation is further distinguished by the dimension of the intersection of their boundaries: line or point contact. This is an important spatial distinction for users of GIS to query and retrieve information from databases. The qualitative qualifier is used to compute the RCC11 topological relation between closed regions. The composition table for base relations is computed using the declarative spatial reasoning system CLP(QS). The proposed representation is evaluated within the application of sketch map alignment: We compute regions (city-blocks) in sketch and geo-referenced maps. First, RCC11 relations between city-blocks are extracted in the form of qualitative constraint networks. Afterwards, the evaluation of RCC11 is done by matching the qualitative constraint networks from sketch and geo-referenced maps.

Keywords: qualitative representations, sketch maps, qualitative alignments, geo-referenced maps, region connection calculus

Introduction

The main goal of qualitative representation and reasoning is to represent our everyday commonsense knowledge about the physical world. It provides mechanisms which characterize essential properties of elementary objects (e.g. *points*, *lines*, and *regions*) and spatial configurations between them (e.g. *adjacent*, *on the left of*, and *included in*).

In the area of Qualitative Spatial Reasoning (QSR), dozens of spatial representations have been proposed focusing on different aspects of the space such as representations for topology (Randell, Cui, and Cohn 1992; Cohn et al. 1997), directions (Frank 1996; Renz and Mitra 2004), relative position of points (Moratz, Dylla, and Frommberger 2005; Renz and Mitra 2004; Moratz, Renz, and Wolter 2000) and others, each of which introduces a finite number of basic spatial relations. The qualitative representations and methods for reasoning are motivated by a wide variety of application areas such as Geographical Information System (GIS), robotic navigation, engineering design, and commonsense reasoning about the physical world.

In the context of GIS, the spatial relations between the geographic objects play a central role in query specification, query processing and for retrieval tasks. For modelling topological relations in GIS and spatial databases the three models: the 9-intersection (9IM) (Egenhofer, Franzosa, and Ranzosas 1991), RCC-family (Randell, Cui, and Cohn 1992; Cohn 1997), and Calculus Based Method (CMB) (Clementini, Felice, and Oosterom 1993) play an important role both in terms of theoretical developments and practical applications. In many geo-spatial applications (Egenhofer 1996; Forbus et al. 2011; Volker and Michael 1997; Nedas and Egenhofer 2008), these representations are used to make qualitative distinctions and process spatial information on a qualitative level.

In our previous work (Schwering et al. 2014; Chipofya, Wang, and Schwering 2011), we propose a theoretical framework to use freehand sketches as a visual interaction mechanism to process spatial information qualitatively. In freehand sketches, street segments are linear features. They are connected to other street segments at junctions. Landmarks represent all other geographical objects such as water bodies, landmarks, and parks. City-blocks are the smallest regions. They are delimited by a lineal representation of connected street segments. As typically only qualitative relations between these spatial objects are persevered in sketch maps, processing spatial information on a qualitative level has been suggested (Egenhofer 1996; Chipofya, Wang, and Schwering 2011). In (Jan et al. 2014a; Jan et al. 2014b; Jan et al. 2014c; Schwering et al. 2014), we proposed a set of plausible qualitative representations to formalize the spatial aspects preserved in sketch maps.

This study extends our previous work on qualitative representations for the alignment of spatial objects. In this paper, we propose a finer representation, called RCC11[1], to formalize topological relations between city-blocks. The RCC11 distinguishes 11 topological relations between closed regions in the plane $\mathbb{R}^2$ based on the dimension of the intersection of boundaries which we distinguish into **l**ine or **p**oint **c**ontact. The RCC8 captures a very general notion of connectivity, which may be useful for various purposes. However, the representation appears too weak to formalize the important topological distinctions of connectivity such as regions being connected by lines or points. Using the proposed representation, we capture the important topological distinctions that are preserved in sketch maps such as city-blocks being externally connected by street segments (i.e. line segment contact) or being connected diagonally at junctions (i.e. point contact), which are important distinctions for qualitative alignment. The composition table specifies the relations obtained by composing the 11 base relations. It provides the basis for composition-based reasoning, extensively used in relation algebraic qualitative spatial reasoning.

The proposed representation is evaluated by aligning city-blocks from sketch maps with the corresponding city-blocks in geo-referenced maps. The qualitative matching of spatial objects requires structures called Qualitative Constraint Networks (QCNs), which represent pairs of spatial objects and relations among objects for a particular aspect of space. We use the qualitative qualifier (Jan and Chipofya 2011) to compute QCNs using RCC11 from the geometric representations of both sketch and geo-referenced maps. Afterwards, the QCNs are used as inputs in qualitative matching algorithms (Wallgrün, Wolter, and Richter 2010; Chipofya, Schwering, and Binor 2013). Both matching approaches use constraint-based techniques for the matching of spatial objects, where the composition table of the representation helps to prune the search space and perform the consistency check during constraint–based reasoning. The overall evaluation shows that the finer topological relations using RCC11 are suitable for the alignment of spatial objects in both sketch and geo-referenced maps.

The remainder of this paper is structured as follows: in the following section, we introduce the background and related work on sketch maps representation and RCC theory. In section 2, we describe the RCC11 relations, and give an illustration of topological relations between closed regions. In section 3, we discuss the evaluation of the proposed representation against the RCC8 representation through qualitative matching. Section 4 concludes the paper with an outlook on future work.

Background and Related Work

Sketch Maps Representation

People capture the spatial configurations of a scene in a hand-drawn map, called a sketch map. As this information is based on observations rather than measurements, it is distorted, schematized, incomplete and generalized (Tversky 1992). People draw only a few significant objects such as landmarks and the street network composed of street segments, junctions, and city-blocks.

During the last two decades, several approaches attempt to process spatial information from sketch maps at a qualitative level. Egenhofer (Egenhofer 1996) proposed "*Spatial-Query-by-Sketch*", an approach to query spatial databases using a sketch-based interface. The approach uses topological relations (9-intersection model) and cardinal relations to formalize the sketched scene. Volker et al. (Volker and Michael 1997) propose the *visual query system—VISCO*. It integrates geometric and topological querying with deductive spatial reasoning. Forbus et al. (Forbus et al. 2011) develop a *sketch understanding system—nuSketch*. It uses both qualitative topological reasoning and quantitative information to construct spatial configurations between depicted objects. Nedas et al. (Nedas and Egenhofer 2008) propose a similarity measure to compare two spatial scenes by identifying similarities between (i) objects in scenes, (ii) relations among objects, and (iii) the ratio of matched objects.

In empirical studies (Wang, Muelligann, and Schwering 2011; Schwering et al. 2014; Wang, Mülligann, and Schwering 2010), we determined a set of qualitative relations between depicted objects that are not affected by schematization and distortions, and are usually represented correctly in sketch maps. The topological relation between city-blocks is one of these qualitative relations that is of

[1] The topological relations presented in this paper are different from the 11 relations introduced in (Düntsch 1999).

particular interest for this paper; the complete list of qualitative relations can be found in (Schwering et al. 2014). In sketch maps, city-blocks have significant topological distinctions such as disconnect, externally connected by line (sharing common street segments), and externally connected by point (sharing common junctions). However, city-blocks never overlap each other as they are delimited by connected street segments.

People do not always sketch complete city-blocks, in particular at the edge of the sketch medium. We define city-blocks as areas either bounded by the street segments, or bounded by street segments and the sketch-boundary (Jan et al. 2014a). As shown in the Figure 1b, all incomplete street segments are extended towards the sketch-boundary. We extracted all depicted spatial objects from sketch maps using the segmentation procedures proposed in (Broelemann 2011; Broelemann, Jiang, and Schwering 2011).

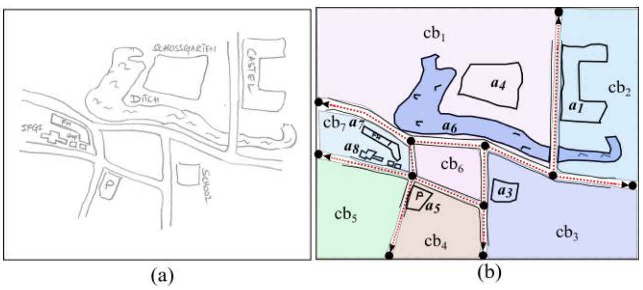

Figure 1: (a) Sketch map, (b) the depicted street segments, landmarks (a_1, a_2,... a_8), and city-blocks (cb_1, cb_2, cb_3,... cb_7) bounded by the street segments and medium-boundary in sketch map.

In order to formalize the topology of city-blocks, we (Jan et al. 2014a) analyze and evaluate different representations of RCC (Randell, Cui, and Cohn 1992; Cohn 1997) and string-based topological relations (Li and Liu 2010). As a preliminary result, we find string-based topological relations as a reliable model to capture the topological relations between depicted city-blocks. The representation distinguishes the external connectivity of city-blocks by street segments and junctions, which is an important distinction for qualitative alignment. Using RCC8 (Randell, Cui, and Cohn 1992; Cohn 1997), we lose the distinction as the single relation "**EC**" of the RCC8 represents both scenarios.

However, string-based topological relations (Li and Liu 2010) are restricted to convex regions and each topological scenario can be represented using multiple sets of circular strings. The qualitative alignment of spatial objects involves spatial reasoning. Therefore, we need a set of tractable relations which are mutually exclusive. In freehand sketches, city-blocks are mixed closed regions (concave and convex). Therefore, we need a representation which supports both concave and convex regions.

An Overview of RCC Theory

The formal definition of geometric objects and relations are based on the point-set approach, where features are sets and points are elements of the sets (Clementini, Felice, and Oosterom 1993). In this way, all the geometric features are closed sets where each feature contains all its accumulation points. The spatial features used in GIS are simple points, lines, and regions in the plane $\mathbb{R}^2$.

The focus of qualitative representation and reasoning on these features started with the evaluation and implementation of formal axioms for space and time (Clarke 1981) and expressed in the many sorted logic LLAMA (Cohn 1987). Based on "*calculus of individuals based on connection*" (Clarke 1981), the RCC theory is developed throughout a series of research work (Randell, Cui, and Cohn 1992; Cohn 1995; Cohn 1997; Cohn et al. 1997; Bennett, Isli, and Cohn 1998). The most distinctive feature between Clarke´s theory and RCC theory is that RCC theory considers extended regions rather than points as fundamental.

The basic part of the RCC theory assume a primitive dyadic relation called $\mathcal{C}(x,y)$, which represents the connectivity of two regions x and y and the relation $\mathcal{C}(x,y)$ is reflexive and symmetric. Using the $\mathcal{C}(x,y)$, further dyadic relations are defined such as: DC (disconnect), P (part of), PP (proper part), PO (partially overlap), EC (externally connected), TPP (tangential proper part), NTPP (non-tangential proper part), EQ (equal), O (overlay), DR (discrete) and the converse relations of the P, PP, TPP, and NTPP.

A region is a set A in the plane $\mathbb{R}^2$ if it is non-empty and regular closed, i.e. $A = \overline{A^\circ} \neq \emptyset$, where x° and $\overline{x}$ represent the interior and respectively, the closure of a set x. If we have two regions A and B, the RCC8 base relations will be as follows:

- $(\mathrm{A},\mathrm{B}) \in \mathbf{DC}$ if $\mathrm{A} \cap \mathrm{B} = \emptyset$
- $(\mathrm{A},\mathrm{B}) \in \mathbf{EC}$ if $\mathrm{A}^\circ \cap \mathrm{B}^\circ = \emptyset$ but $\mathrm{A} \cap \mathrm{B} \neq \emptyset$
- $(\mathrm{A},\mathrm{B}) \in \mathbf{PO}$ if $\mathrm{A}^\circ \cap \mathrm{B}^\circ \neq \emptyset$ and $\mathrm{A} \nsubseteq \mathrm{B}$ and $\mathrm{B} \nsubseteq \mathrm{A}$
- $(\mathrm{A},\mathrm{B}) \in \mathbf{TPP}$ if $\mathrm{A} \subseteq \mathrm{B}$ but $\mathrm{A} \nsubseteq \mathrm{B}^\circ$
- $(\mathrm{A},\mathrm{B}) \in \mathbf{NTPP}$ if $\mathrm{A} \subseteq \mathrm{B}^\circ$
- $(\mathrm{A},\mathrm{B}) \in \mathbf{EQ}$ if $\mathrm{A} = \mathrm{B}$

The above basic relations together with the converses of **TPP** and **NTPP** are jointly exhaustive and pairwise disjoint (JEPD).

Other Topological Representations

In (Egenhofer and Herring 1991; Egenhofer, Franzosa, and Ranzosas 1991), Egenhofer et al. propose a formal approach to define the binary topological relations between objects based on point-set theory. A drawback of the approach is that it distinguishes only between empty and non-empty intersections of boundaries and the interior of the geometries and the method also results in too many different topological relations for end users.

Clementini et al. (Clementini, Felice, and Oosterom 1993) propose an approach called *Calculus Based Method* (CBM) for representing a small set of topological relations based on the dimension of the intersecting geometries. The resulting relations are grouped together into a few general topological relations such as *touch*, *in*, *cross*, *overlap*, and *disjoint*.

RCC11 Base Relations

In the context of Geographic Information Systems (GISs), the spatial relationships existing between geographic objects play a central role both at the spatial queries definition and processes. The two algebras (RCC5 and RCC8) of RCC make simple and general classifications, however, these classifications may not be precise enough when fine grained information is required, for example, when information about the dimensionality of intersections between regions is required.

In this paper, we propose RCC11, a new representation combining the ideas of finer topological relations in (Clementini, Felice, and Oosterom 1993), the concept of strong connection and congruence in (Borgo, Guarino, and Masolo 1996), and string-based topological relations between convex regions (Li and Liu 2010). For each pair of regions (A, B) in the plane, RCC8 classifies topological relations based on empty and non-empty intersections without distinguishing certain cases of intersections of the boundaries of two regions leading to refinements of the RCC8 relations **EC**, **TPP**, and **TPPi**. In our approach, we take into account the dimension of intersections (Clementini, Felice, and Oosterom 1993) and line-point contact instead of only considering empty or non-empty intersections. In 2D-dimensional space, the intersection set (I) can be either empty ($\emptyset$), point (0D), line (1D), or region (2D). The dimension of the intersection cannot be higher than the lowest dimension of the two intersecting operands: $dim\ (\partial A) = 1$ and $dim\ (A°) = 2$. For the topological relations between two closed region A and B (with their interiors°, and boundaries ∂), we distinguish the following intersections:

- $\mathrm{I} = (\partial\mathrm{A}, \partial\mathrm{B}) : \emptyset,\ 0\mathrm{D}, 1\mathrm{D}$ (3 cases)
- $\mathrm{I} = (\partial\mathrm{A}, \mathrm{B}°) : \emptyset,\ 0\mathrm{D}, 1\mathrm{D}$ (3 cases)
- $\mathrm{I} = (\mathrm{A}°, \partial\mathrm{B}) : \emptyset,\ 0\mathrm{D}, 1\mathrm{D}$ (3 cases)
- $\mathrm{I} = (\mathrm{A}°, \mathrm{B}°) :\ \emptyset,\ 2\mathrm{D}$ (2 cases)

Using the dimension of the intersections between regions, we have 11 possible topological relations. The "*p*" and "*l*" in relations represent the dimension (*dim*) of intersection by point or line, respectively.

- $(\mathrm{A}, \mathrm{B}) \in \mathbf{DC}$ if $\partial\mathrm{A} \cap \partial\mathrm{B} = \emptyset$
- $(\mathrm{A}, \mathrm{B}) \in \mathbf{EC_p}$ if $\mathrm{A}° \cap \mathrm{B}° = \emptyset$ but $dim\ (\partial\mathrm{A} \cap \partial\mathrm{B}) = 0$
- $(\mathrm{A}, \mathrm{B}) \in \mathbf{EC_l}$ if $\mathrm{A}° \cap \mathrm{B}° = \emptyset$ but $dim\ (\partial\mathrm{A} \cap \partial\mathrm{B}) = 1$
- $(\mathrm{A}, \mathrm{B}) \in \mathbf{PO}$ if $dim(\mathrm{A}° \cap \mathrm{B}°) = 2 \wedge \mathrm{dim}\ (\partial\mathrm{A} \cap \partial\mathrm{B}) = 0 \wedge dim(\mathrm{A}° \cap \partial\mathrm{B}) = 1 \wedge dim(\partial\mathrm{A} \cap \mathrm{B}°) = 1$
- $(\mathrm{A}, \mathrm{B}) \in \mathbf{TPP_p}$ if $dim\ (\mathrm{A}° \cap \mathrm{B}°) = 2 \wedge dim\ (\partial\mathrm{A} \cap \partial\mathrm{B}) = 0 \wedge dim(\mathrm{A}° \cap \partial\mathrm{B}) = \emptyset \wedge dim(\partial\mathrm{A} \cap \mathrm{B}°) = 1$
- $(\mathrm{A}, \mathrm{B}) \in \mathbf{TPP_l}$ if $dim(A° \cap B°) = 2 \wedge dim\ (\partial\mathrm{A} \cap \partial\mathrm{B}) = 1 \wedge dim(\mathrm{A}° \cap \partial\mathrm{B}) = \emptyset \wedge dim(\partial\mathrm{A} \cap \mathrm{B}°) = 1$
- $(\mathrm{A}, \mathrm{B}) \in \mathbf{NTPP}$ if $dim(A° \cap B°) = 2 \wedge dim(\partial\mathrm{A} \cap \partial\mathrm{B}) = \emptyset \wedge dim(A° \cap \partial B) = \emptyset \wedge dim(\partial A \cap B°) = 1$
- $(\mathrm{A}, \mathrm{B}) \in \mathbf{EQ}$ if $dim(A° = B°) = 2 \wedge \mathrm{dim}(\partial A = \partial B) = 1$

The above basic relations together with the converses of $\mathbf{TPP_p}$, $\mathbf{TPP_l}$, and **NTPP** are the set of finer topological relations. Using the above definitions, the RCC11 has the following mutually exclusive relations (see Figure 2):

Name of the RCC11 relations	Symbol
Disconnect	**DC**
Equal	**EQ**
Partially Overlay	**PO**
Externally Connected by point	$\mathbf{EC_p}$
Externally Connected by line	$\mathbf{EC_l}$
Tangential Proper Part by point	$\mathbf{TPP_p}$
Tangential Proper Part by line	$\mathbf{TPP_l}$
Non-Tangential Proper Part	**NTPP**
Tangential Proper Part by point inverse	$\mathbf{TPP_pi}$
Tangential Proper Part by line inverse	$\mathbf{TPP_li}$
Non-Tangential Proper Part inverse	**NTPPi**

Table 1: The 11 topological relations in RCC11.

Composition Table

Originating from Allen´s interval Algebra (IA), composition-based reasoning (Guesgen 1989) has been widely acknowledged as the most popular reasoning technique in the area of QSR. Given a fixed vocabulary of relations, the composition table enables us to infer implicit qualitative knowledge. A difficult problem in QSR is the computation of a composition table, the verification that all the closure properties are met, and the determination of the computational complexity.

Deriving the composition table manually is a challenging and error-prone process, particularly if the representation contains many base relations. Thus, in order to automatically generate the composition table, we use the declarative spatial reasoning system CLP(QS) (Schultz and Mehul 2014; Schultz and Mehul 2012; Bhatt, Lee, and Schultz 2011) refer to *Appendix A* for further details. The composed relations in the composition table (see *Appendix B*) are refined relations $\mathcal{R}(x, y)$ that hold between any two variables x and y occurring in the network via the following operation.

$$\mathcal{R}_{(x,y)} \leftarrow \mathcal{R}_{(x,y)} \cap (\mathcal{R}_{(x,z)} \circ \mathcal{R}_{(z,y)})$$

The composition table enables topological reasoning at the conceptual level, rather than having to calculate all relations from the geometrical representation of the spatial objects in the $\mathbb{R}^2$. We use the composition table as a computational model to assess the consistency of the topological relations between city-blocks during the alignment.

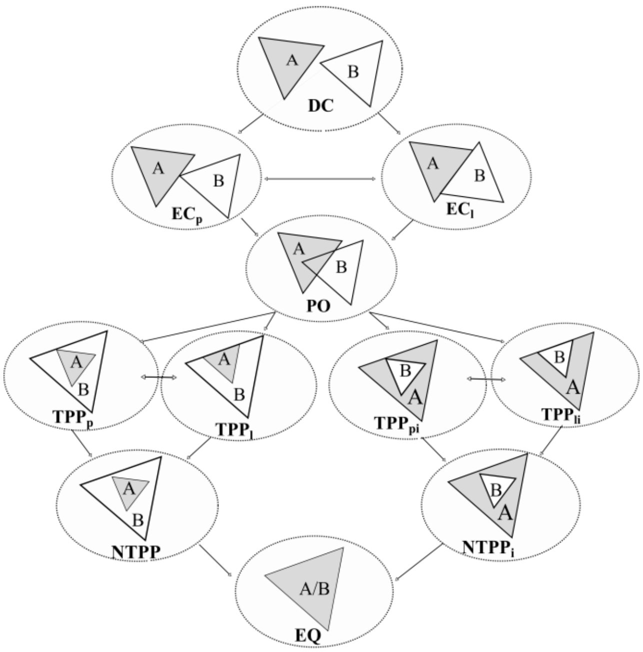

Figure 2: Illustration of the 11 possible topological relations between region A and B.

Implementation of the Qualifier

In the area of qualitative spatial reasoning (QSR), the spatial relations and certain operations on them constitute a qualitative representation (calculus). Using the operations of the representation, one can infer new knowledge (Renz and Nebel 2007).

In order to compute the qualitative relations in the form of QCNs, we implement a qualifier (Java-based plugin) for the OpenJUMP GIS software (Jan and Chipofya 2011). Applying the definitions of the RCC11 relations, the qualifier computes the finer topological relation between polygons from sketch and geo-referenced maps. As shown in the Figure 3a, we have city-blocks surrounded by the street-segments and medium-boundary. The qualifier computes RCC11 relations between city-blocks from sketch map in the form of QCN (see Figure 3b). The refined topological relations such as "$\mathbf{EC_p}$" and "$\mathbf{EC_l}$" preserve the important distinctions of connectivity, i.e. city-blocks being externally connected by street segments from being externally connected by junctions. The computed QCNs from maps are used as inputs for the alignment of city-blocks.

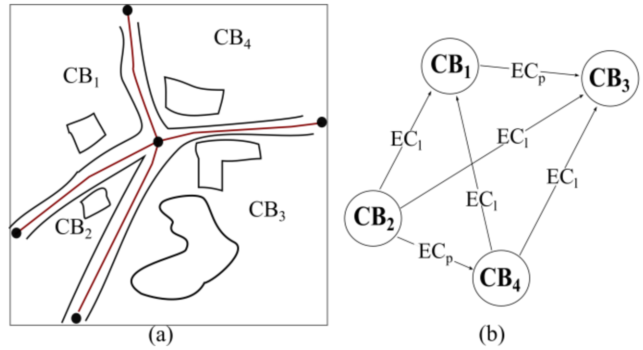

Figure 3: (a) City-blocks in sketch map, (b) the possible topological relations between city-blocks using the RCC11.

Evaluation through Matching

Qualitative matching of spatial scenes is a task that involves finding correspondences between an input map and a target map. The qualitative matching of spatial objects requires QCNs representing pairs of objects and relations among objects for a particular aspect of the space. Matching qualitative spatial scene descriptions can therefore be considered as the task of matching QCNs.

The evaluation of RCC11 is done by testing the accuracy of qualitatively matched city-blocks from sketch maps with corresponding city-blocks in geo-referenced maps (generated from OpenStreet Map[2]). We compare the RCC11 with the RCC8 using two different matching algorithms: the *Interpretation Tree* (Wallgrün et al. 2010) and *TABU Search metaheuristic* (Chipofya, Schwering, and Binor 2013). The QCNs are computed from geometric representations of sketch and geo-referenced maps. Afterwards, the QCNs are used as inputs for the matching algorithms.

For this evaluation, we considered freehand sketch maps (20 in total) from two different locations of area about 1.04km^2 and 2.10 km^2 in Münster, Germany. When people draw sketch maps they often abstract away unnecessary detail and aggregate spatial objects. Oftentimes, more than 60% of all street segments are aggregated. During the evaluation, we handle aggregation of street segments manually. After drawing the sketch maps, the participants were asked to indicate the corresponding street segment for every sketched street segment in geo-referenced maps and use this as ground truth for our evaluation. We also got information on how streets were aggregated and the connectivity of aggregated street segments forms the aggregated city-blocks in sketch and geo-referenced maps.

The compiled results show that RCC11 gives higher accuracy of matches in comparison to the RCC8 for both qualitative matching approaches (see Table 2). The

[2] http://www.openstreetmap.org

evaluation shows that RCC11 is an effective representation for the alignment of extended objects. The refined topological relations preserve important topological distinctions. While using the RCC8, we lose these distinctions, which affect the overall qualitative matching of city-blocks.

Locations	Interpretation Tree Search		TABU Search Metaheuristic	
	RCC8	RCC11	RCC8	RCC11
	% Corr. Matches	% Corr. Matches	% Corr. Matches	% Corr. Matches
Location-I	32,20	41,23	61,55	87,66
Location-II	28,00	35,52	68,39	82,98
Average	30,10	**38,38**	64,97	**85,32**

Table 2: Accuracy of qualitative matching of city-blocks using the RCC8 and RCC11 relations.

Conclusions

In this paper, we introduced a finer version of the RCC family, RCC11. Based on 11 relations, RCC11 is an extension of RCC8. It models topological relations based on the line-point contacts between boundaries and interiors of the closed regions in the space. We use this representation to formalize the topological relations between city-blocks in sketch maps. The representation captures important topological distinctions that users preserve in sketch maps, which is not possible with RCC8 or any other family members of RCC. The evaluation of the proposed representation is done by testing the accuracy of qualitative matches; the results show the importance of the qualitative distinctions that RCC11 preserves.

We compute a composition table for the base relations on which the usual Qualitative Spatial Reasoning mechanism relies. The composition table has been automatically generated using the declarative spatial reasoning system CLP(QS) based on a tubular spatial domain model for weak composition of RCC11 relations. We evaluate the representation by aligning spatial objects from sketch and geo-referenced maps. Future work will also explore other application areas of RCC11 and its importance in other domains.

Acknowledgements

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Appendix A: Tubular Model for Deriving the RCC11 Composition Table

In order to derive the RCC11 composition table automatically we use the declarative spatial reasoning system CLP(QS) (Schultz and Mehul 2014; Schultz and Mehul 2012; Bhatt, Lee, and Schultz 2011). CLP(QS) provides sound and complete spatial reasoning, which is necessary for deriving the composition table. The system is based on analytic geometric reasoning: spatial relations are encoded as systems of real polynomial constraints; determining spatial consistency is then equivalent to determining satisfiability of this polynomial constraint system. Given polynomial constraints over real variables X, the constraints are satisfiable if there exists some real value for each variable in X such that all the polynomial constraints are simultaneously satisfied. Within CLP(QS), polynomial solving is achieved using a range of dedicated solvers such Sat Modulo Theories (SMT), real quantifier elimination, and CLP(R) (Schultz and Mehul 2014).[3]

One approach for generating the RCC11 composition table is to define polynomial constraints that encode spatial relations between polygons. However, sound and complete reasoning about polygons is a difficult task. Thus, we propose a more abstract spatial domain as a model for RCC11 that is tractable to reason about: a restricted tubular 3D model based on topological relations between circles. This tractable spatial domain enables us to determine what relations can and cannot hold between triples of objects in more complex and expressive domains, such as polygons, that are relevant to our sketch map application area.

The spatial domain of circles is a model for the weak composition of RCC8 relations (Düntsch 1999) (that is, strong composition of the so-called *closed circle algebra* corresponds exactly to weak composition of RCC8; it is also referred to as a *representation* of RCC8). The polynomial encodings for RCC8 topological relations between circles are given below. A circle c_i is defined as a centre point $p_i = (x_i\ ,\ y_i)$ and a radius r_i. The predicate $\Delta\ (c_i,\ c_j) \equiv_{\text{def}} (x_i - x_j)^2 + (y_i - y_j)^2$ is the squared distance between the centres of c_i, c_j.

When deriving entries in the composition table, we treat each triple of relations $R1$, $R2$, and $R3$ as a theorem: there exists regions a, b, and c such that $R1_{ab} \wedge R2_{bc} \wedge R3_{ac}$. If the theorem is true, then $R3$ is placed in the cell corresponding to row $R1$ and column $R2$ in the composition table; if the theorem is false, $R3$ is omitted from that table cell.

$$
\begin{array}{ll}
DC_{ab} & \equiv_{\text{def}} \Delta(c_a,c_b) > (r_a + r_b)^2 \\
EC_{ab} & \equiv_{\text{def}} \Delta(c_a,c_b) = (r_a + r_b)^2 \\
PO_{ab} & \equiv_{\text{def}} (r_a - r_b)^2 < \Delta(c_a,c_b) < (r_a + r_b)^2 \\
TPP_{ab} & \equiv_{\text{def}} \Delta(c_a,c_b) = (r_a - r_b)^2 \wedge (r_a < r_b) \\
NTPP_{ab} & \equiv_{\text{def}} \Delta(c_a,c_b) < (r_a - r_b)^2 \wedge (r_a < r_b) \\
TPPi_{ab} & \equiv_{\text{def}} TPP_{ba} \\
NTPPi_{ab} & \equiv_{\text{def}} NTPP_{ba} \\
EQ_{ab} & \equiv_{\text{def}} x_a = x_b \wedge y_a = y_b \wedge r_a = r_b
\end{array}
$$

[3] http://www.spatial-reasoning.com/

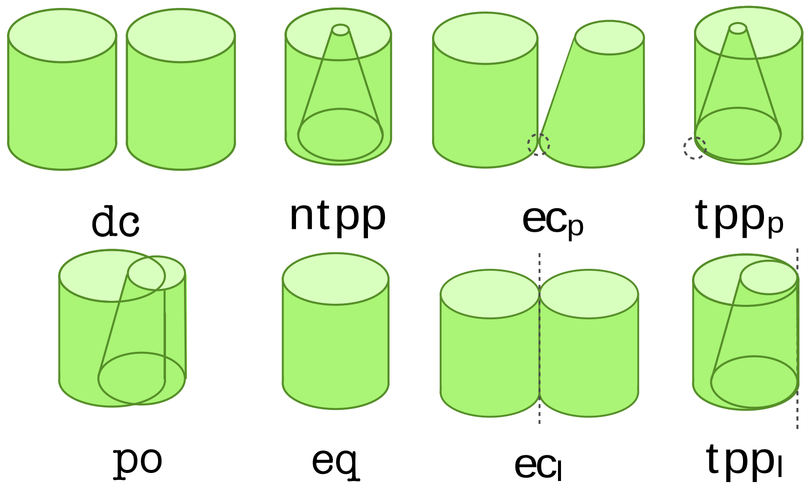

Figure 4: Tubular model for RCC11 weak composition.

For example, consider proving the theorem: there exists circles a, b, and c such that $DC_{ab} \wedge TPPi_{bc} \wedge EC_{ac}$. This corresponds to the constraint system: there exists reals x_a, y_a, r_a, x_b, y_b, r_b, x_c, y_c, r_c such that:

$$(x_a - x_b)^2 + (y_a - y_b)^2 > (r_a + r_b)^2 \qquad (DC_{ab})$$
$$(x_c - x_b)^2 + (y_c - y_b)^2 = (r_c - r_b)^2 \qquad (TPPi_{bc})$$
$$r_c < r_b$$
$$(x_a - x_c)^2 + (y_a - y_c)^2 = (r_a + r_c)^2 \qquad (EC_{ac})$$

CLP(QS) determines that this system of constraints is unsatisfiable, and therefore the spatial relations are inconsistent. Thus, *EC* must *not* appear in the RCC8 composition table cell for row *DC* and column *TPPi*.

As circles are a representation for RCC8, they can be used to determine the ways that three objects can have contact without specifying the *nature* of that contact. To distinguish between *line* and *point* contact we build on the circular model by proposing our 3D tubular model for RCC11 weak composition. Let tube t_{ab} be a pair of circles a, b such that b is part of a (i.e. P_{ba}):

$$P_{ba} \quad \equiv_{\text{def}} \Delta(c_a,c_b) \leq (r_a - r_b)^2 \wedge (r_b \leq r_a)$$

Topological relations are defined by comparing the base circles and top circles between tubes as illustrated in Figure 4. The interpretation is that the tubes are hollow, they do not have base nor lid covers, they are sitting on the same plane, they all have the same height, and all tops lie in a plane parallel to the base plane. The tubes can be negatively tapered, i.e. they taper inwards, and the axis does not need to be at right angles to the plane (formally, they are axis-aligned open circular cylinders that can inwardly taper to become cones). The contact between the surfaces of tubes distinguishes between line and point contact; the topological relations between the bases distinguishes the standard RCC8 contact and containment relations.

The topological relations between tubes t_{ab}, $t'_{a'b'}$ are as follows:

$$DC_{t\,t'} \equiv_{\text{def}} DC_{a\,a'}$$
$$ECp_{\,t\,t'} \equiv_{\text{def}} EC_{a\,a'} \wedge DC_{b\,b'}$$
$$ECl_{\,t\,t'} \equiv_{\text{def}} EC_{a\,a'} \wedge EC_{b\,b'}$$
$$PO_{t\,t'} \equiv_{\text{def}} PO_{a\,a'}$$
$$TPPp_{\,t\,t'} \equiv_{\text{def}} TPP_{a\,a'} \wedge NTPP_{b\,b'}$$
$$TPPl_{\,t\,t'} \equiv_{\text{def}} TPP_{a\,a'} \wedge TPP_{b\,b'}$$
$$NTPP_{t\,t'} \equiv_{\text{def}} NTPP_{a\,a'} \wedge NTPP_{b\,b'}$$
$$TPPip_{\,t\,t'} \equiv_{\text{def}} TPPp_{\,t'\,t}$$
$$TPPil_{t\,t'} \equiv_{\text{def}} TPPl_{\,t'\,t}$$
$$NTPPi_{t\,t'} \equiv_{\text{def}} NTPP_{t'\,t}$$
$$EQ_{t\,t'} \equiv_{\text{def}} EQ_{a\,a'} \wedge EQ_{b\,b'}$$

It is straightforward to show that these topological relations between tubes satisfy our RCC11 base relation definitions (in particular, with respect to the dimension of contact). CLP(QS) is used to prove (or disprove) all $11^3 = 1331$ composition table theorems.

Appendix B: The RCC11 Composition Table

A (R1) B B (R2) C	DC	ECp	ECl	EQ	PO	TPPp	TPPl	TPPpi	TPPli	NTPP	NTPPi
DC	.-?-.	DC, ECp, ECl PO, TPPp, TPPl, NTPP	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	DC	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	DC	DC	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	DC
ECp	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	DC, ECp, ECl, EQ, PO, TPPp, TPPl, TPPpi, TPPli	DC, ECp, ECl, PO, TPPp, TPPl	ECp	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	ECp, ECl, PO, TPPp, TPPl, NTPP	ECp, ECl, PO, TPPp, TPPl, NTPP	DC, ECp	DC, ECp	PO, TPPp, TPPl, NTPP	DC
ECl	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	DC, ECp, ECl, PO, TPPpi, TPPli	DC, ECp, ECl, EQ, PO, TPPl, TPPli	ECl	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	PO, TPPp, TPPl, NTPP	Ecl, PO, TPPp, TPPl, NTPP	DC, ECp	DC, ECp, ECl	PO, TPPp, TPPl, NTPP	DC
EQ	DC	ECp	ECl	EQ	PO	TPPp	TPPl	TPPpi	TPPli	NTPP	NTPPi
PO	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	PO	.-?-.	PO, TPPp, TPPl, NTPP	PO, TPPp, TPPl, NTPP	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	PO, TPPp, TPPl, NTPP	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi
TPPp	DC	DC, ECp	DC, ECp	TPPp	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	TPPp, NTPP	TPPp, NTPP	DC, ECp, ECl, EQ, PO, TPPp, TPPl, TPPpi, TPPli	DC, ECp, ECl, PO, TPPp, TPPl	NTPP	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi
TPPl	DC	DC, ECp	DC, ECp, ECl	TPPl	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	TPPp, NTPP	TPPp, TPPl, NTPP	DC, ECp, ECl, PO, TPPpi, TPPli	DC, ECp, ECl, EQ, PO, TPPl, TPPli	NTPP	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi
TPPpi	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	ECp, ECl, PO TPPpi TPPli NTPPi	PO, TPPpi, TPPli, NTPPi	TPPpi	PO, TPPpi, TPPli, NTPPi	EQ, PO, TPPp, TPPl, TPPpi, TPPli	PO, TPPpi, TPPli	TPPpi, NTPPi	TPPpi, NTPPi	PO, TPPp, TPPl, NTPP	NTPPi
TPPli	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	ECp, ECl, PO, TPPpi, TPPli, NTPPi	ECl, PO, TPPpi, TPPli, NTPPi	TPPli	PO, TPPpi, TPPli, NTPPi	PO, TPPp, TPPl	EQ, PO, TPPl, TPPli	TPPpi, NTPPi	TPPli, TPPli, NTPPi	PO, TPPp, TPPl, NTPP	NTPPi
NTPP	DC	DC	DC	NTPP	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	NTPP	NTPP	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	DC, ECp, ECl, PO, TPPp, TPPl, NTPP	NTPP	.-?-.
NTPPi	DC, ECp, ECl, PO, TPPpi, TPPli, NTPPi	PO, TPPpi, TPPli, NTPPi	PO, TPPpi, TPPli, NTPPi	NTPPi	PO, TPPpi, TPPli, NTPPi	PO, TPPpi, TPPli, NTPPi	PO, TPPpi, TPPli, NTPPi	NTPPi	NTPPi	EQ, PO, TPPp, TPPl, TPPpi, TPPli, NTPP, NTPPi	NTPPi

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