CHAOS, QUALITATIVE REASONING, AND THE PREDICTABILITY PROBLEM

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Abstract

Qualitative Physics attempts to predict the behavior of physical systems based on qualitative information. No one would consider using these methods to predict the dynamics of a tennis ball on the frictionless edge of a knife. The reason: a strong instability caused by the edge which makes two arbitrarily small different initial conditions have very different outcomes.

And yet, research on chaos suggests that sharp edges are not so unique; most nonlinear systems can be shown to be as unstable as the ball on the cutting edge. As a result, prediction is often impossible regardless of how simple the deterministic system is. We analyze the predictability problem and its origins in some detail, show how it is generic, and discuss the impact of these results on commonsense reasoning and Qualitative Physics. This done by presenting an example from the world of billiards and attempting to uncover the underlying assumptions that may lead physics, commonsense reasoning, and Qualitative Physics to generate different results.

1 Introduction

Qualitative Physics (QP) aims at describing, explaining, and predicting the behavior of physical systems based only on qualitative information ([Bobrow 84], [Hobbs-Moore 85]). Predicting the future development of some part of the world given knowledge of its current situation is also a central problem when attempting to formalize the commonsense by some sort of non-monotonic logic (eg. [Shoham 88]). Obviously, envisioning the possible behavior of a system (a natural phenomenon or an artifact) is reasonable and feasible only if it exhibits stable behavior, i.e. the system under consideration is predictable as opposed to chaotic. This appears to be a tautological and useless consideration, for we often seem to successfully predict the development of our environment. As a result, setting aside the weather or the economy, the world seems to be mainly predictable. Recently, in the physical sciences, many systems which were taken to be regular in their dynamics were discovered to be chaotic and unpredictable in spite of their deterministic nature. These systems are not just exotic or artificial situations but quite common, such as the motion of the pendulum, circuits like the phase-locked-loop, and devices based on the Josephson junction[D'Humieres 82, Huberman 83].

Is this a shocking observation in the context of QP? In a way, commonsense knowledge also allows for unpredictable mechanisms and consciously relies on them when deciding to use a flipping coin to generate random decisions. People tend to avoid predicting such events, and so do the various attempts to formalize commonsense or causal reasoning. They do not deal with a ball on a knife's edge, but rather with balls rolling or jumping on a smooth surface ([de Kleer 77], [Forbus 80], [Kuipers 86], [Shoham 88])

And yet, we will show that even the smooth, restricted world of billiards, used for a discussion of the prediction problem by Shoham, provides a scenario in which prediction is impossible and chaotic dynamics is the norm. Given these startling discoveries, one may ask whether and in which respect they are relevant to QR. One reason for their consideration is that research on chaos has uncovered that most nonlinear systems can be chaotic, and that regular, integrable motion might actually be the exception rather than the rule.

In this paper we analyze the implication of chaotic behavior for qualitative physics. We do so by introducing the notion of sensitivity to initial conditions through a simple example. We next contrast this problem with notion of predictability as it enters QP, and discuss the issue of approximating reality. We conclude by showing the range of regimes for which QP can effectively predict phenomena by incorporating some of the principles that are captured by commonsense reasoning about the physical world.

2. Chaos

2.1 The initial condition syndrome.

A hallmark of chaotic behavior for non-linear dynamical systems is their sensitive dependence on initial conditions. Stated briefly, this means that if one were to observe the evolution of a dynamical system when started from two slightly different initial conditions, chaotic dynamics would imply that the outcomes are vastly different from each other, whereas regular behavior implies that they are not. In order to make this statement clear, let us take the example of the game of billiards. Imagine starting a game with all the balls at rest and hitting the black one towards any of the rest. After some time, the initial collision between the black ball and one of the others sets in motion a complicated pattern of movement which can be analyzed in terms of the positions and velocities of all the balls. If after the initial impact one allowed the balls to move and collide with each other for a certain amount of time and then wrote down their positions and velocities, that listing would be very different from the one generated by the same initial disposition of balls on the table but with the black ball having started with a slightly different velocity.

Notice that since any dynamical experiment samples many different initial conditions over its course, a time record of its development would look erratic in spite of the deterministic nature of all the forces at play. The interesting fact from our point of view is that systems with very few degrees of freedom can also display this behavior. Thus even though one has a deterministic set of equations for a few interacting particles, sensitivity to initial conditions implies the appearance of behavior which cannot be distinguished from random outcomes.

2.2 An example

In order to make these ideas more concrete we analyze a example that both fits into the favorite domain of rolling billiard balls and has been analyzed in a mathematical way [Bleher88]: a billiard with two pockets (which we'll call exits), one ball, and a cylindrical obstacle in the middle (Fig. 1). Given an initial position and velocity for the ball, one can follow its perfectly deterministic trajectory and ascertain which of the exits it went though. The problem however, lies in the outcome of a same experiment differing by an arbitrarily small amount from the previous initial condition. Short of flipping a coin, it is impossible to predict which pocket will the ball exit through. This is seen in Figures 2a and 2b, where we depict the set of initial conditions that map into either one of the exits.

Figure 2a shows the computation performed by Bleher et al. for a set of 512 by 512 initial conditions on the dashed line (displayed in Fig. 1) parametrized by the x coordinate and the angle of initial velocity. The black lines correspond to points exiting through exit A and the white regions those points exiting through B. Even more interesting, if one were to magnify a piece of this-region by a factor of 105, one obtains Figure 2b, which shows the same characteristics as Figure 2a. What these results show is that the boundaries separating initial conditions mapping into either exit contain both fractal regions and smooth, non fractal ones, and that these two types of boundary behavior are intertwined on arbitrary fine scales.

This example illustrates the fact that even systems with very simple dynamics may exhibit unpredictable behavior. There are, to be sure, initial conditions that lead to an exit immediately or after one or two collisions. These regions can be easily identified and their respective behaviors are predictable in both senses:

- we can easily determine their exit in advance, and

- the result is reproducable in experiments.

However, for most of the initial conditions it is impossible for a human observer to foresee the eventual exit. They are likely to lie in a region that excludes predictability. Since such regions are very "dense", and we do not know their location (neither qualitatively nor quantitatively), the whole system becomes unpredictable except for the obvious regions of immediate or early exit. Althought no one would try to predict the unstable behavior of a ball on a sharp edge, this example demonstrates that it is not always obvious whether or not we are in a situation with similar instability. In this case, reliable predictions are also impossible.

3 Qualitative Physics

3.1 Different "Gold-Standards" for QP

The field of Qualitative Physics (QP) emerged out of two major roots: one being the attempt to analyze and formalize commonsense reasoning ("naive physics") applied by humans in there everyday lifes' environment ([Hayes 78], [Forbus 84]). The other important motivation is modeling more or less advanced techniques used in technical or scientific domains. (e.g. [Brown 76], [deKleer 79]). The latter has led to systems that try to adopt elaborate mathematical techniques, such as higher order derivatives ([de Kleer-Bobrow 84], [Kuipers-Chiu 87]), algebra ([Aubin 88], [Struss 88a], [Williams 88]), or phase portrait analysis of ordinary differential equations ([Lee-Kuipers 88], [Sacks 88], [Struss 88b, c], [Yip 88]).

Almost the whole spectrum between the modeling of a child's knowledge about liquids in [Hayes 85] and the phase space analysis for area-preserving mappings [Yip 88] is covered by existing theories or systems. But we hardly find any approach that systematically relates formal mathematical methods and commonsense reasoning. [Struss 88a] contains only first steps of an analysis as to whether the properties of the mathematical techniques underlying the majority of existing QP systems match our intuitions (e.g. a non-associative addition). One of the goals of this paper is to raise and discuss problems in the relation of mathematical models and QP models of the physical world from the following perspectiv:

1. So far, this discussion was mainly performed by matching results of QP with classical methods of mathematics and physics, as suggested by the framework of Fig. 3a, which was first used in [Kuipers 86], adopted by [de Kleer 87], and formalized by [Struss 88a, b]. In this paper, we change the perspective by reintroducing two "gold-standards" which are both important and admittedly vague to the same extent: a) the real physical world, and b) human commonsense reasoning about this reality (Fig. 3b). If we find that results of mathematical physics and QP do not coincide as desired or expected, this does not say too much about their appropriateness. After all, a certain paradigm may correctly formalize commonsense reasoning and also contradict evidence of the physical world, thus revealing deficiencies in the commonsense-based treatment of the world. Alternatively, the mathematical model may contain idealizations that prevent it from capturing essential characteristics of the modelled system. The aim should be to discover implicit presumptions underlying the various ways of modeling the physical world and, thus, identify potential restrictions of their applicability.

2. Rather than analyze properties of single solutions of either modeling (such as spuriousness, branching, etc.) we will discuss intrinsic global characteristics of the respective models focusing on such aspects as continuity or linearity.

3.2 QP Assumes Predictability

In most QP approaches, systems are characterized in terms of parameters that take on "qualitative values". Such qualitative values may be obtained from the respective quantitative (i.e. real-valued) description by considering significant thresholds which discriminate qualitatively between different states or modes of behavior (e.g. the freezing point and the boiling point of a fluid object, or the maximal opening of a valve). Even if such landmarks are not precisely known, the assumption underlying these approaches is that there exists a finite set of intervals that can serve as a sufficient characterization of the various qualitatively different states of a system or of its components. Sufficient for the purposes of QP means in particular that this set of intervals captures the distinctions necessary for making correct predictions about the possible behavior(s) of a system. This assumption, in turn, is grounded on the presupposition that the overall result or behavior is continuously dependent on the initial conditions, or, stated differently, under perturbations small enough the system will not change its behavior qualitatively. This property of "structural stability" is essential for a system to be observable and predictable. We stress the fact that "predictable" is not meant in a theoretical sense ("Knowing the initial conditions exactly, one is able to compute the further development"), but in a pragmatic sense: "If we repeat an experiment carefully enough several times, it will always yield the same result".

Obviously, chaotic dynamics not only contradicts simple technical details of some QP systems, but some of their fundamental assumptions. In the billiards example, assuming that the initial conditions may be characterized by some qualitative values (=intervals) which lead to the same qualitative behavior (i.e. same exit) contradicts the analysis of section 2.

Note that the problem raised by these considerations is even more fundamental than the frame problem. The latter states that our predictions about the future may become invalid if something that has not been considered or expected influences the subject of our predictions. For instance, a rolling billiard ball is prevented from following the expected straight line if a sudden strong wind enters the billiard room, or if another ball moves in the way and causes a collision. But in our example, a prediction about the exit of the ball is not disturbed by some external cause or unanticipated event. Even if we consider the scenario to be a closed world and its description to be complete, the exit is unpredictable.

We may rephrase the problem and relate it to the extended prediction problem studied in [Shoham 88]. Shoham uses the concept of a "potential history" which "represents an arbitrary course of events which is 'the way things would happen naturally' " (Do not get confused by the fact that "history" may refer to a development in the future). Potential histories manifest themselves, "become real" as long as they do not interact with other histories.

In our example, although everything but the ball is static, "the rules that govern the world" generously supply the ball with an infinite set of potential histories, and our problem is the predictability problem, i.e. selecting one of this set, rather than the extended prediction problem i.e. determining how long a potential history is allowed to manifest itself until it is affected by some other history.

4 Discussion

4.1 Approximations to Reality

In order to discuss the issues raised in the previous sections and their impact on QP in particular, we will first describe the problem and our point of view more carefully.

Obviously, there seems to exist a contradiction between some fundamental assumptions of most QP approaches and the results presented in section 2. Can one conclude that QP contradicts the real billiard ball system? Not at all. First of all, the results about the exit as a function of the initial conditions were not obtained by experimenting with a real instance of the scenario. Those results are properties of a specific model of this scenario, a model using mathematical concepts, such as points, circles, angles, equations etc. in order to approximate things that would occur if one really carries out such an experiment. Hence, all we can tell at this point is that there is a contradiction between this model and a QP approach. So far, one cannot make a statement about how QP relates to the real world unless one finds or assumes that mathematics and physics match exactly physical reality. We emphasize this point because the relationship of QP and mathematics and physics has been subject to many claims and analyses, and is without any doubt an interesting question.

At times however, there appears the misconception that qualitative descriptions are a weaker substitute for the world when lacking "exact information". Descriptions of physical systems, e.g. in terms of equations or differential equations, are exact in that they contain real-valued parameters or specific functions, but those equations do not describe the real situation "exactly". Real physical systems are permanently influenced and changed by an arbitrarily large number of other phenomena. If they, despite all the perturbations acting on them, remain in some identifiable state for a period long enough to observe them, we may be able to find a mathematical description that agrees with the observed quality of the behavior. In this case, we may use this quantitative description as a good approximation to the "qualitative reality", to put it in a provocative way.

From this point of view, we can treat mathematics/physics, commonsense reasoning, and QP as three different ways of approximating processes and events occurring in the real world. Since at least some QP approaches aim at modeling commonsense reasoning, whereas traditional physics do not, we have again motivated the analytical framework in Fig. 3b.

These three ways to model the physical world claim and have different scopes of applicability, but there is some overlap among them. If their results are different, it makes sense to analyze which (perhaps implicit) assumptions and idealizations cause this discrepancy, and, if possible, to determine which are more appropriate for certain types of physical systems. Based on the example, we will try to take a step towards this goal.

4.2 Idealizations and Assumptions

4.2.1 Geometry

The first idealization that probably comes to our minds concerns the description of the geometrical shapes of the objects in the mathematical model. The walls and the surface are treated as planes, the obstacle is a "perfect" cylinder, and (probably the most unreasonable assumption from the commonsense viewpoint) the billiard ball is shrunk to a point. Mathematicians would give up, if the walls were washboards, the ball an ıpple, and the obstacle a cactus. However, so would commonsense, and it may eventually turn out that humans, in order to model the example, make almost the same geometrical abstractions as the mathematical model, except, perhaps, for modeling the ball as a sphere rather than a point. Any QP system adressing a problem of this kind would use similar idealizations potentially in a "compiled" form, e.g. hidden in the laws of reflection.

4.2.2 Continuity

If while playing minigolf one narrowly misses the hole in a given trial, one slightly changes the direction of the hit at the next trial. Commonsense tells us that the result of such an action will only change by a small amount if the action itself is carefully modified. The outcome does not "jump". We exploit the same fact when choosing an angle between the first and the second trial if in the second case the ball bounced back slightly left of the hole. This leads commonsense to predict that, for repeated experiments in our billiard example, a ball will always exit through the same hole. Ordinarily, the same continuity principle is used in QP systems. For, instance, a variable, "temperature", is not allowed to skip the value "freezing point" when progressing from "below freezing" to "above freezing". As already stated, this continuity principle is the ultimate grounding method for modeling qualitative values by intervals. However, rather than being explicitly represented as a property of parameters, it is hardwired and implicit in the algorithms of most QP systems.

In mathematics, this property is captured by the notion of continuous functions. One way to define a real-valued function, $f(x)$, is to state that given a quantity $\delta > 0$, there exists an $\varepsilon > 0$, so that the amount $(x_0 - x) < \varepsilon$ implies $(f(x_0) - f(x)) < \delta$, a fact which is close to our intuition.

Consider the type of functions involved in the computational treatment of the billiard example. They can be mainly reduced to the computation of the location and incidence angle of one collision from the previous one. In the example situation shown in Fig. 4, the functions

$$\theta_{i+1} = f_\theta (x_i, \theta_i)$$

$$x_{i+1} = f_x (x_i, \theta_i)$$

are continous (in the ranges considered). However, the analysis reported in section 2 states that the resulting function which maps the initial conditions to the exit exhibits discontinuities.

The concept of discontinuity is also present in commonsense reasoning. If a ball is rolling towards the (concave) edge of an obstacle, a small deviation causes a significantly different result (Fig. 5). However, billiards seem smooth and continuous. What is causing the discontinuity?

4.2.3 Infinity

Commonsense may construct the following argument by repeatedly exploiting the continuous dependency of a collision on the previous one: Consider an orbit, O_1, that after several collisions with the obstacle and the walls exits through hole A. By continuity, other orbits with initial point and angle of the last collision sufficiently close to those of O_1 will stay close to O_1 and, hence, also exit through hole A. This condition can be guaranteed if the collisions preceding the last one are sufficiently close. Finally, we can find a neighborhood of the initial conditions of O_1 such that all other orbits starting in this environment also exit through hole A. This argument can also be turned into a rigorous mathematical proof, and the restriction for the initial conditions could even be computed exactly. So, what remains of the unpredictability problem?

One point is that the neighborhood of the initial conditions gets very small if the number of collisions increases, and it will shrink to a point if this number gets arbitrarily large. At this point, we find one of the most important distinctions in the basic assumptions. What does "arbitrarily large" mean? Commonsense tells us that, in each experiment, the ball will stop after a certain number of collisions having lost its energy due to friction and elastic collisions. In our physical experience, the observation time is limited, and so is the period of activity for many processes or systems. On the contrary, the mathematical model implies an unbounded progression of time unless it is modified appropriately. Chaos is defined by divergent behavior as time goes to infinity.

If we add a restriction to the mathematical treatment of the billiard example (either by limiting the time of motion through friction, the number of collisions, or the distance), then in addition to the previously discussed scenarios the ball can now stop somewhere on the table. The corresponding mathematical model would show nice isolated regions for initial conditions leading to either hole and the space between them corresponding to the stopping ball. There is no chaos in this system, it is predictable, at least in the mathematical sense. There is a certain limited precision required to guarantee that the experiment can be repeated with the same outcome. The question remains, however, whether this precision is really achievable in reality. The boundaries for the initial conditions depend on the strength of the imposed temporal restriction. They may become too narrow and infeasible if the period is too long, for example. Additionally, the experiment might become predictable if made up of wooden parts but unpredictable if constructed of glass.

Clearly, the mere existence of a temporal limit as is implied by commonsense reasoning (and physical reality), is not sufficient to exclude chaotic behavior. The question is whether the "natural" time of activity or observation is so short that the observed system does not exhibit its intrinsic chaos.

4.2.4 Linearity

In our billiards example we do not find an edge like the one shown in Fig. 5 that would make a discontinuous behavior plausible for commonsense reasoning. We find instead something like a "very smooth edge": the surface of the cylinder. A ball hitting the cylinder orthogonally will return on the same path, whereas any small displacement of the starting point to one side or the other will cause the ball to escape in the reflected direction (Fig. 6). This is basically the same behavior as for the edge case, only in milder form. However, the whole surface of the cylinder is a continuum of such "mild edges", and such that as the ball hits this surface several times, each collision amplifies the divergence of the orbits. Fig. 7 illustrates this phenomenon (b) and contrasts it with the case where the cylindrical surface is replaced by a plane surface (a).

In other words, only a small number of collisions with the cylinder suffices to add up (or rather, multiply) to almost the effect of a knife's edge. We are forced to use the precision of the real numbers if we aim at reliable predictions. (This replicates the results of the analysis in [Struss 88a] which state that a space of "qualitative values" closed w.r.t. multiplication and division can only be represented by the real numbers.)

The non-linearity of the obstacle's shape is correctly captured by the mathematical model. On the other side, there is strong evidence, supported by the example, that commonsense is very bad at estimating non-linear effects. We hypothesize that commonsense reasoning is mainly based on an implicit linearity assumption. Thus, the domain of its applicability is limited to systems which can be sufficiently approximated by linear descriptions, or to ranges of parameter values where this is the case. For instance, we have a good estimate of the distance we need to come to a stop when walking, running, or driving a car slowly. Exceeding this speed by an order of magnitude, e.g. when driving on a freeway, makes our commonsense fail predicting this distance. With a constant negative accelaration applied by the brakes, this distance grows quadratically, whereas our commonsense tends to prefer a linear extrapolation.

Having hypothesized the linearity assumption as one of the major distinctions between commonsense reasoning and the mathematical physics modeling of the world, we now ask how QP methods reflect this problem. The answer seems to be that this is done a) only implicitly and b) not in a coherent way. On the one hand, as we already pointed out, predictability and the absence of knife's edges, i.e. discontinuities, are deeply involved in the major approaches, e.g. hidden in the fundamental hypothesis that qualitative values with a finite granularity or even of a finite number suffice. Our example shows that knife's edges may be hidden in the description and become effective only in the course of time.

On the other hand, QP uses rather weak expressions for the functional dependencies. For instance, "qualitative proportionality" which is used in different incarnations (e.g. in Envision [de Kleer 84], QPT [Forbus 84], and QSIM [Kuipers 86]) subsumes linear as well as non-linear dependencies. Since qualitative envisionments are complete, they accordingly include the whole spectrum of awkward phenomena potentially exhibited by non-linear systems. We might therefore get not only an explanation for the occurance of intractable branching of qualitative behaviors, but also the idea that the effect of additional "immanent" filters can only be very limited. In implicitly assuming linearity, QP methods seem to approximate commonsense reasoning, including its defects. Where the weak descriptions cover also non-linear systems, qualitative simulation may mirror their possible chaotic behavior in a mess of envisionments.

If the world is really massively non-linear, how can commonsense succeed and survive at all? We have already provided two answers to this question:

- there are ranges of parameters where linearity is a sufficient approximation

- the period of activity or observation can be too short for the chaotic behavior to develop or to become obvious.

There is another aspect, that of granularity, which we now consider.

4.2.5 Granularity

An existing chaotic behavior may be hidden below a certain threshold imposed by our interest or by perceptional capabilities. Drinking a beer, we are not bothered by the chaotic behavior of the molecules in the liquid. If we shake our piggy bank, we do not have to predict or to know the movements and collisions of coins inside in detail in order to be able to exploit them and achieve the desired and likely effect of some coins falling out. Or, modifying the billiard example, the specific exit may become totally irrelevant, depending on our goal. For instance, if we sieve pebbles, we can still reliably predict the overall result, namely that pebbles smaller than the holes of the sieve will fall through. For this, it does not matter through which hole an individual pebble exits. In such cases, the chaotic structure vanishes behind a certain granularity or level of observation and reasoning.

5 Conclusions

Throughout this paper, we tried to reveal some of the idealizations and assumptions underlying different attempts to model the physical world and to make predictions about it: commonsense reasoning, physics, and Qualitative Physics. This was done by confronting them with something both common and "extreme": chaotic behavior.

We noticed that commonsense reasoning about the physical world can successfully guide our behavior in a certain domain. It is not a new issue that this domain is limited. Indeed, commonsense leads to the realizaton that certain events cannot be predicted, like the case of a ball on a knife. We hypothesize a limited capability of commonsense in dealing with non-linear phenomena, the reason being that non-linear systems bear the potential for chaotic behavior whereas commonsense (almost by definition) can only develop with regard to stable or repetitive process. Thus, it is confined to systems that

- may (at least in a certain range) be sufficiently approximated by linear descriptions, or

- are strongly damped, bounded in time and, thus prevented from unfolding their intrinsic chaotic dynamics, or

- are observed only for periods too small to reveal their chaotic behavior (for us, the chaotic cosmos appears to be a most reliable system).

Assumptions about such boundaries for the development of chaos seem to be built into commonsense reasoning, but it is hard for it to determine when they are exceeded.

The mathematical methods of physics provide means for analyzing non-linear systems beyond their "almost linear regions" and beyond time threshold. Mathematical models (or, rather, those who apply them) tend to assume infinite domains and are not always able to determine their own boundaries of applicability. They are suitable for revealing types behaviors for certain abstractions of model systems. Determining whether a real physical instance of an abstraction will exhibit the predicted behavior requires experiments, experience, or commonsense guidance.

For QP, this means that it should not be reduced to form a "coarse version" of the traditional mathematical methods (replacing real numbers by intervals and particular functions by huge classes of functional dependencies). Such an approach can only mirror the unbounded space of non-linear behaviors, including chaos. Since mathematical methods do not contain a gold-standard by themselves, they are not very appropriate to serve as a gold-standard for QP. The best result QP could achieve on this basis, is finding all and only possible behaviors including those of very odd chaotic non-linear systems. This would not tell much about physically manifested behaviors. QP has to incorporate at least some of the principles that are captured by commonsense reasoning about the physical world as, for instance, boundaries of domains and, in particular, of time. Some approaches do so only implicitly, somewhere in their algorithms, if at all, rather than being able to represent these assumptions and reason about them. Still, in succeeding to do so, QP systems may correctly model commonsense reasoning together with its capabilities and its faults.

In the attempt to deal with systems beyond the domain of commonsense, e.g. with complex artifacts, they have to be able to reason about the appropriateness of specific models and of commonsense assumptions. In particular, there appears to be a need for some sort of meta-level reasoning about global features of systems, such as linearity and its implications. At least part of the required skills should be present in the theories, or perhaps in the practice, of scientific and engineering domains. Though being partially set in contrast to commonsense reasoning, they ought to be grounded on our basic concepts of the physical world. Analyzing the relation between commonsense reasoning and the grounding of advanced scientific and engineering methods, is a challenging task which is crucial for progress in QP. We intend this paper to be a step in this direction.

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Figure Captions

Fig. 1: Billiard example: hard disk in box with two holes (after Bleher et al.)

Fig. 2: a)Plot of 512 by 512 uniformly distributed initial conditions (x0, q0). b) Enlargement by a facgtor of 105 of a subregion of Fig. 2a).

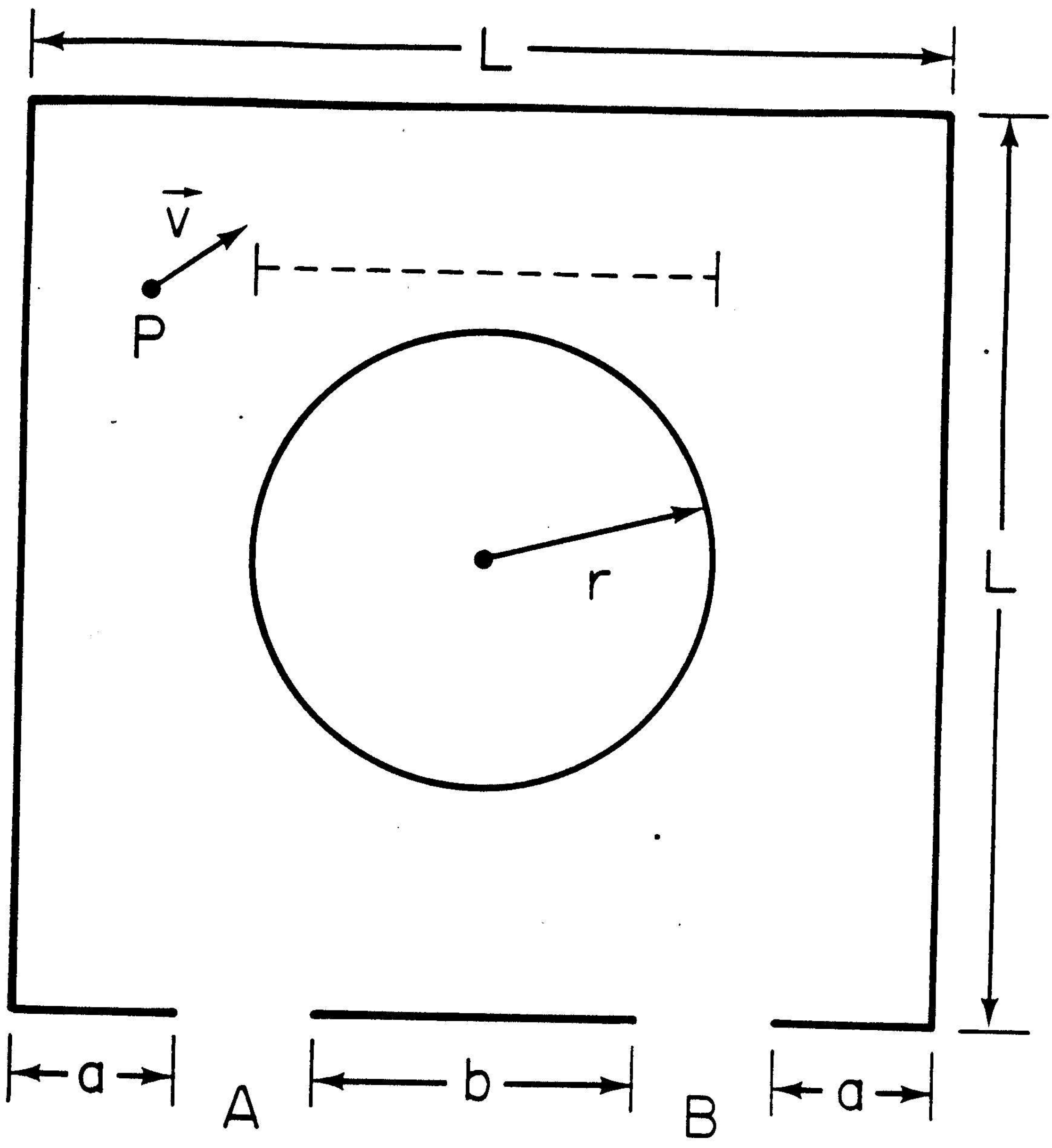
Fig. 3: a) The matching of results from QP with those of mathematics, and physics. b) A change of perspective brought about by introducing two "gold-standards"; 1) the real physical world, and 2) commonsense reasoning about this reality.

Fig. 4: Sequence of collisions between a ball and two obstacles.

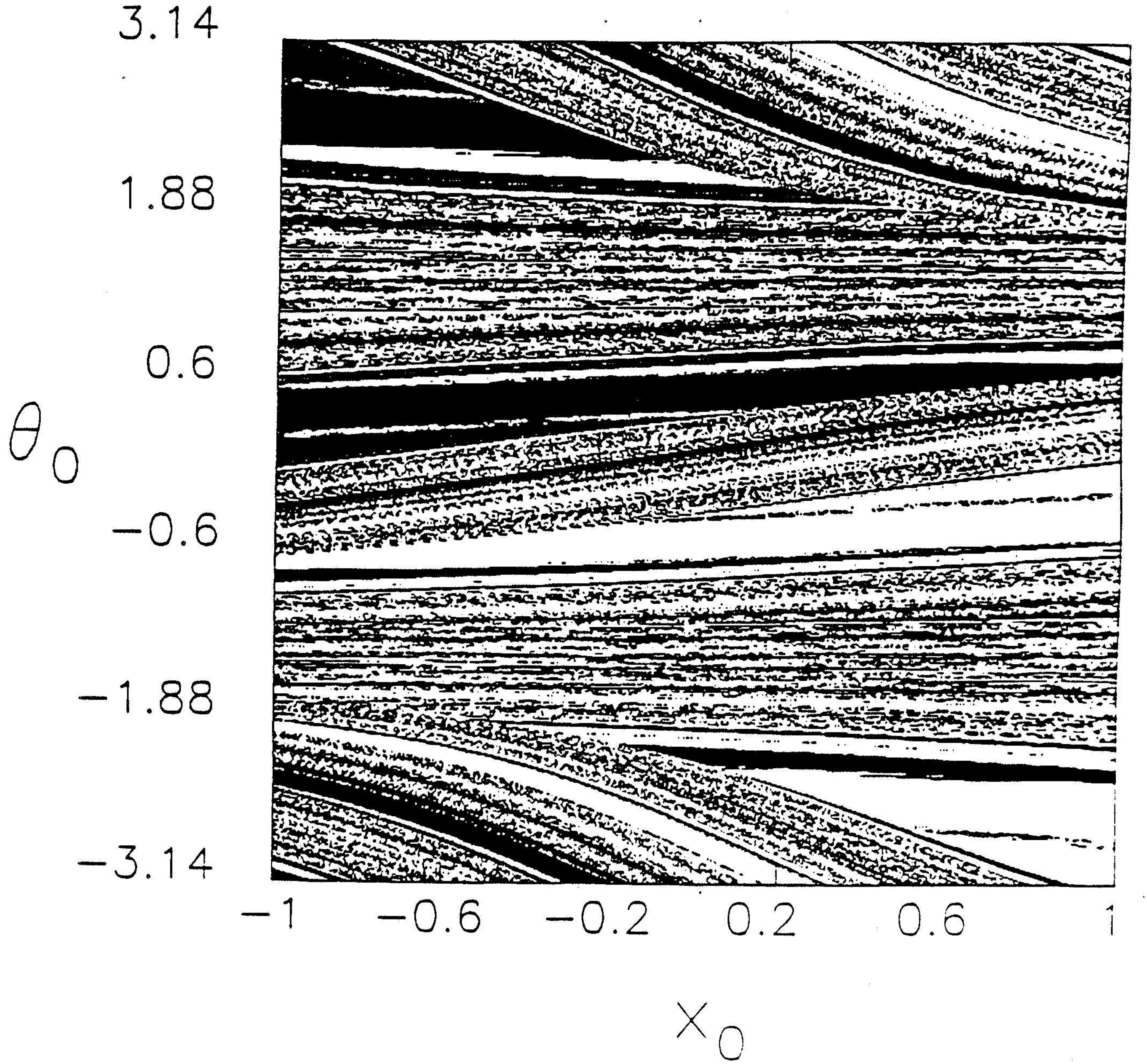
Fig. 5: Possible dynamical sequences for a ball hitting a wedge-like obstacle. Notice that besides the improbable collision with the point edge of the obstacle, slightly different initial conditions lead to vastly different outcomes.

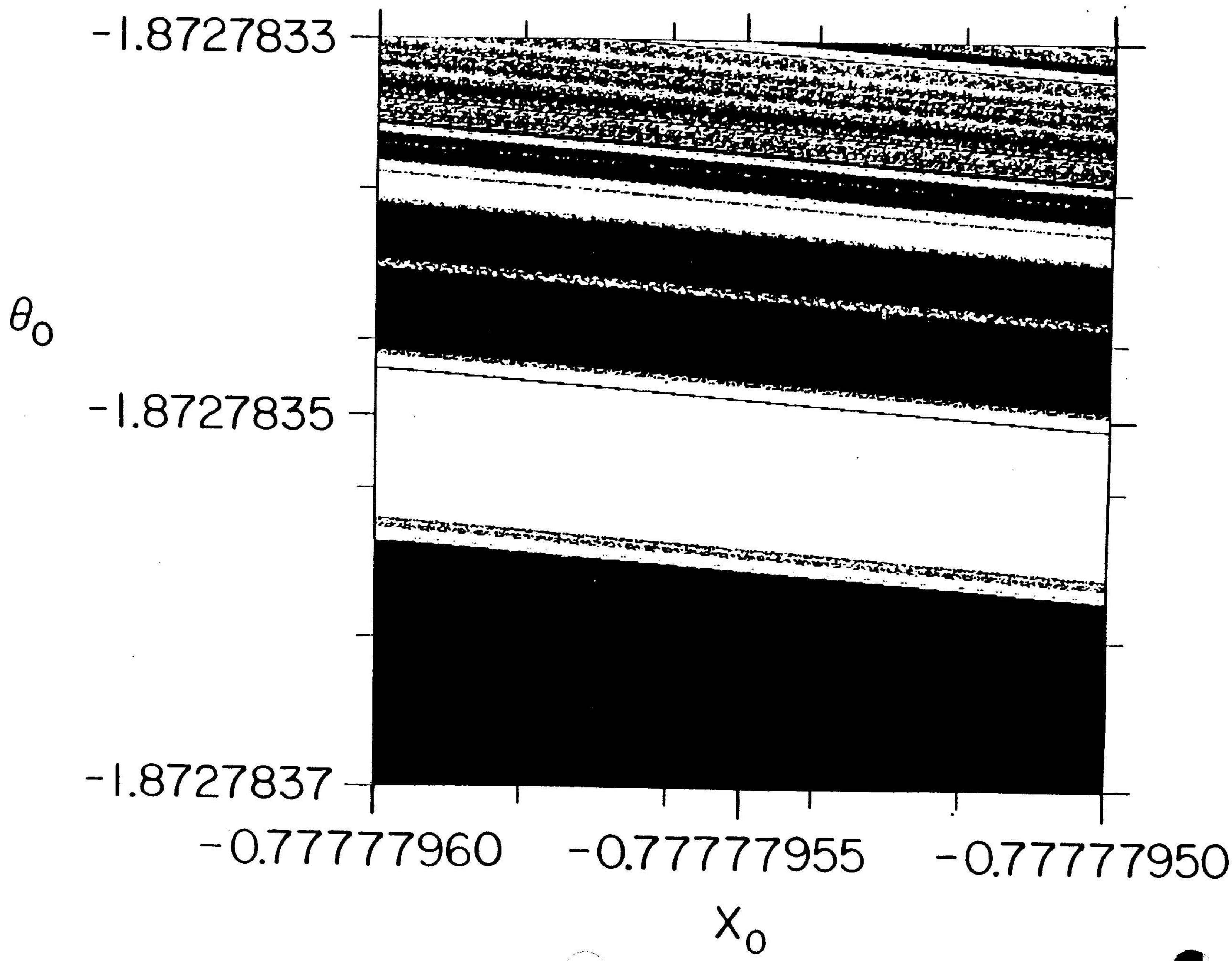
Fig. 6: Sensitivity to initial conditions in the dynamics of collisions between a ball and an obstacle with a curved surface.

Fig. 7: Sequence of iterates for a ball bouncing off perflectly reflecting walls.



L
L
$\vec{v}$
P
r
a
A
b
B
a


3.14
1.88
0.6
θ_0
−0.6
−1.88
−3.14
−1
−0.6
−0.2
0.2
0.6
1
x_0


-1.8727833
-1.8727835
-1.8727837
θ_0
-0.77777960
-0.77777955
-0.77777950
X_0

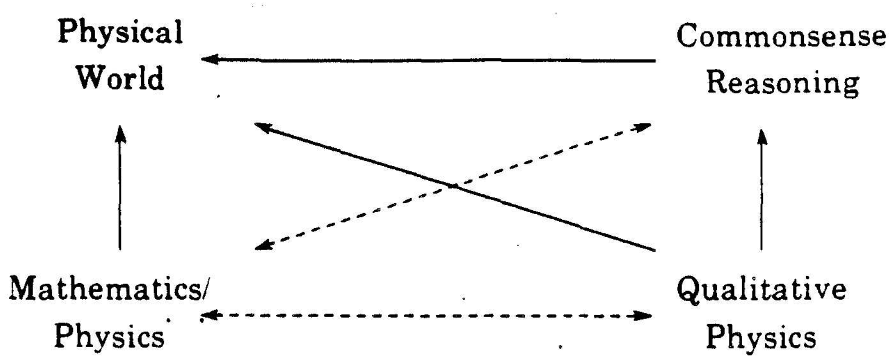


Figure (b)

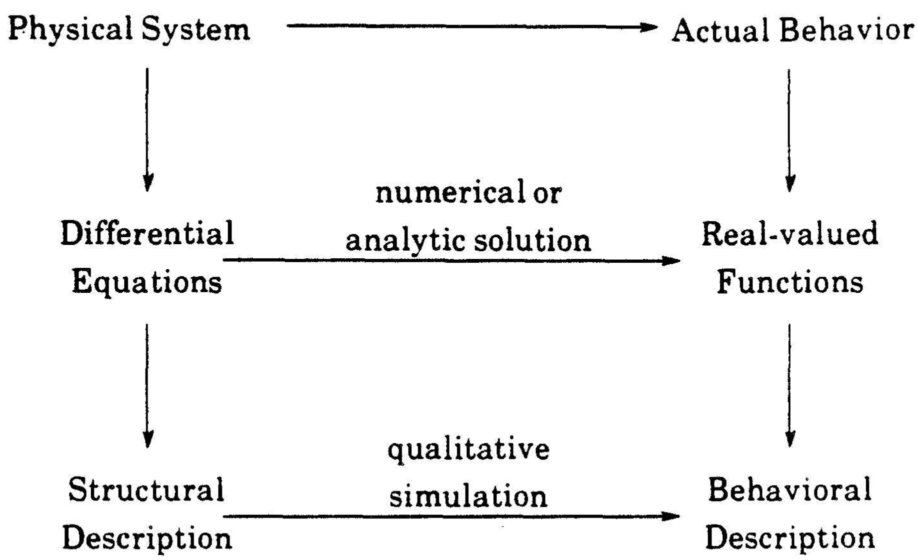


Figure (a)

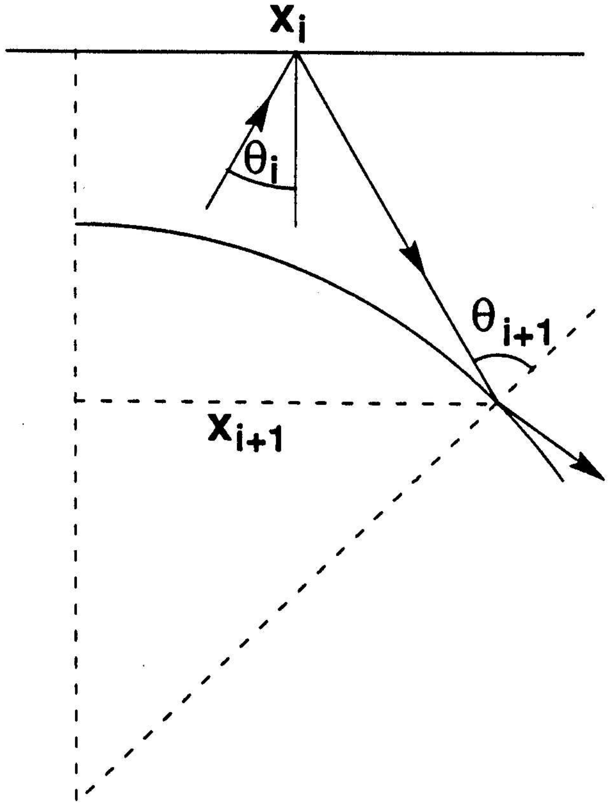


Figure 4

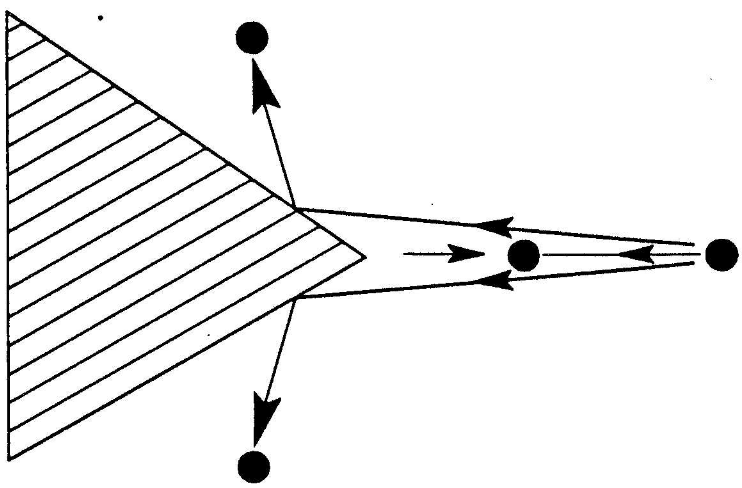


Figure 5

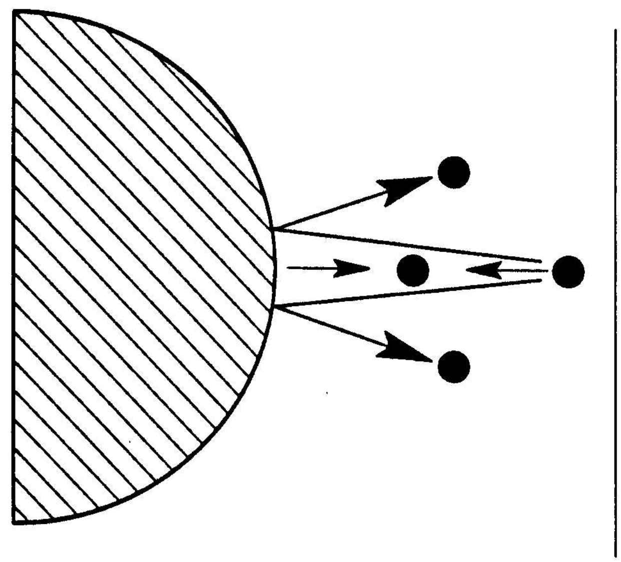


Figure 6

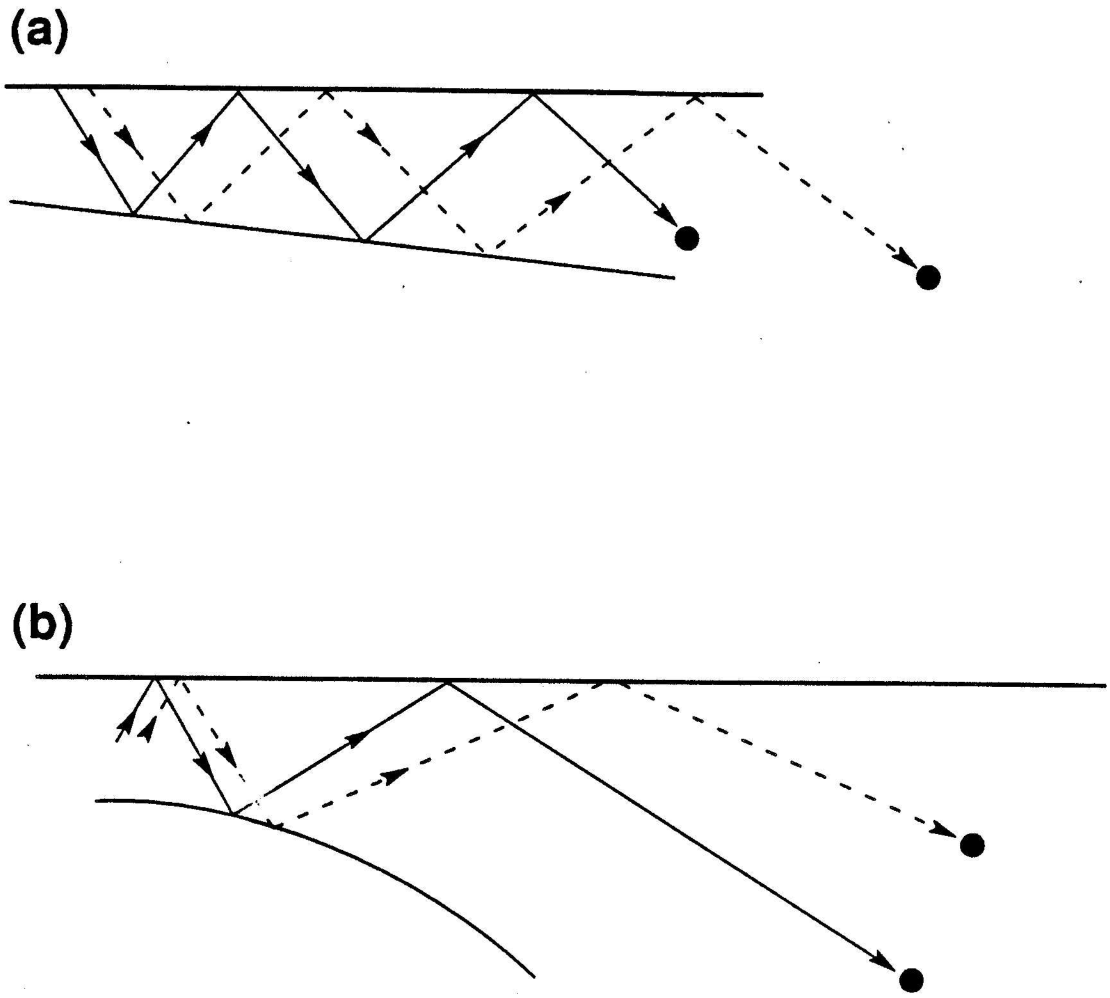


Figure 7